ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Catania

INFN/BE-80/7
23 Luglio 1980

P. G. Fallica, M. Lattuada, F. Riggi, C. Spitaleri and
D. Vinciguerra: THE TREIMAN-YANG CRITERION AND ITS APPLICATION TO THE STUDY OF QUASI-FREE REACTIONS AT LOW ENERGY.

Servizio Documentazione
dei Laboratori Nazionali di Frascati

Istituto Nazionale di Fisica Nucleare
Sezione di Catania

INFN/BE-80/7
23 Luglio 1980

THE TREIMAN-YANG CRITERION AND ITS APPLICATION TO THE STUDY OF QUASI-FREE REACTIONS AT LOW ENERGY[o]

P. G. Fallica[x], M. Lattuada, F. Riggi, C. Spitaleri and D. Vinciguerra

Istituto di Fisica dell'Università di Catania
Istituto di Scienze Matematiche e Fisiche dell'Università di Catania.

ABSTRACT.

The Treiman-Yang criterion has been studied with reference to the study of quasi-free processes at low energies. The conditions to which the experimental set-up must satisfy in order to allow the measurement of a Treiman-Yang distribution are discussed. The application to a specific case, namely the $^9Be(^3He,\alpha\alpha)^4He$ reaction, is also presented.

(o) Work supported in part by Istituto Nazionale di Fisica Nucleare, Catania, and by Centro Siciliano di Fisica Nucleare e di Struttura della Materia, Catania.
(x) Scuola di Specializzazione in Fisica, Università di Catania.

1. - INTRODUCTION.

Nuclear reactions with three bodies in their final state may proceed through a variety of reaction mechanisms ; in general the calculation of the reaction cross-section should include terms from quasi-free processes, expecially when appropriate regions of the phase-space are selected.

In the description of the latter processes the incident particle 0 interacts only with a part (T) of the target nucleus, which has a cluster structure N=S+T, while the cluster S maintains its momentum p_S (see Fig. 1).

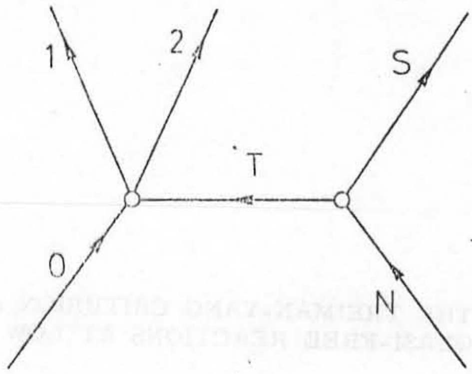


FIG. 1 - Pole diagram for the quasi-free reaction N(0, 12)S.

Many models have been developed to describe such direct reactions ; among these the Feynman graph technique has been widely used(1) but unfortunately it is very difficult in general to select the particular graphs that are dominant for a given process. In particular the role of the pole mechanism has been investigated in detail and attempts have been made to find sensitive criteria able to establish its relative importance. One of these criteria is the study of the distribution with respect to the Treiman-Yang angle ; this criterion, first pointed out by S. B. Treiman and C. N. Yang(2) shows that under given conditions such a distribution should be isotropic, provided the pole mechanism is dominant in the reaction. Despite of the fact that the Treiman-Yang criterion is a necessary but not a sufficient condition for the dominance of the pole graph, due to some triangular graph that may fulfil this criterion(3), it remains strongly indicative of the pole character of the process under consideration.

In this note the Treiman-Yang criterion is considered for the study of quasi-free reactions. In Section 2 the invariance properties that the amplitude of the reaction should have according to this criterion is discussed, while Section 3 is devoted to the calculation of the experimental conditions allowing a Treiman-Yang distribution to be measured. In the last Section the application of the Treiman-Yang criterion to the $^9Be(^3He, \alpha\alpha)^4He$ reaction is presented.

2. - THE TREIMAN-YANG CRITERION.

The amplitude of a reaction of the type $1+2 \rightarrow 3+4$ depends on two invariant variables, that can be chosen for instance between the s, t, u invariants, defined by :

$$s = (q_1 + q_2)^2 = (q_3 + q_4)^2 , \qquad t = (q_1 + q_3)^2 = (q_2 + q_4)^2 , \qquad u = (q_1 + q_4)^2 = (q_2 + q_3)^2 ,$$

where the q_i's are the four-momenta, and $s+t+u = \sum_{i=1}^{4} m_i^2$. In the general case, when more than 4 particles are involved in the process, the number of independent invariants is $3n-10$, n being the number of particles.

For a reaction involving 5 particles, of the type:

$$0 + N \rightarrow 1 + 2 + S$$

we have therefore 5 invariant variables on which the amplitude may depend. In the non-relativistic case the following quantities can be chosen:

$$t = -(\vec{p}_S - \vec{p}_N)^2 + 2(m_S - m_N)(E_S - E_N), \qquad t' = -(\vec{p}_1 - \vec{p}_0)^2 + 2(m_1 - m_0)(E_1 - E_0),$$

$$s' = -(\vec{p}_1 + \vec{p}_2)^2 + 2(m_1 + m_2)(E_1 + E_2), \qquad s = -(\vec{p}_N + \vec{p}_0)^2 + 2(m_N + m_0)(E_N + E_0),$$

$$s'' = -(\vec{p}_S + \vec{p}_2)^2 + 2(m_S + m_2)(E_S + E_2),$$

where p_K, m_K, E_K are the momentum, rest mass and kinetic energy of particle K.

When the amplitude of the reaction is represented by the pole graph in the Feynman series, and the spin of the intermediate particle is zero, it can be expressed as follows[(4)]:

$$M = \text{const}\,\frac{\Gamma(t)\,F(s', t', t)}{t - 2m_T\varepsilon_T}$$

where the term $\Gamma(t)$ is the amplitude of the process $N \rightarrow T + S$, and depends only on the invariant t related to the first vertex in the pole graph, while the second term $F(s', t', t)$ is the amplitude of the reaction $0 + T \rightarrow 1 + 2$, depending on the invariants that can be formed from the momenta of particles 0, 1, 2 in the other vertex (in this case s', t', t).

In the previous equation ε_T is the binding energy of the system T + S in the nucleus N:

$$\varepsilon_T = m_T + m_S - m_N$$

and the system of units in which $\hbar = c = 1$ is used throughout.

In this case it can be seen that the amplitude of the entire process as a whole depends only on three independent variables t, t', s'.

According to the Treiman-Yang criterion the amplitude of the reaction should be invariant when performing a rotation of the $\vec{p}_1 - \vec{p}_2$ plane around the sum of these momenta; if we study the process in the antilaboratory system, i.e. the system in which $\vec{p}_0 = 0$, the direction of $\vec{p}_1 + \vec{p}_2$ is the same of the intermediate particle momentum $\vec{p}_T$.

Under such a rotation the invariants t, t', s' do not change so that the amplitude M depending only on such quantities must remain constant.

If we consider the momenta $\vec{p}\,'_N$, $\vec{p}\,'_S$, $\vec{p}\,'_T$, $\vec{p}\,'_1$, $\vec{p}\,'_2$ (where the primed quantities refer to the antilab system), two planes can be fixed: the first (α) is defined by $\vec{p}\,'_N$, $\vec{p}\,'_S$, $\vec{p}\,'_T$ while the second one (β) by $\vec{p}\,'_T$, $\vec{p}\,'_1$, $\vec{p}\,'_2$.

The Treiman-Yang angle θ_{TY} is denoted as the angle between the planes α and β. In this framework the rotation of $\vec{p}_1'$, $\vec{p}_2'$ around $\vec{p}_T$ can be seen as a rotation of β around the p_T' direction of an angle θ_{TY}.

The Treiman-Yang criterion does not hold in general for the relativistic case when the spin of the intermediate particle is non zero. However, as shown by Shapiro et al. the factorization is also possible for nonrelativistic cases with non zero spin of the intermediate particle[5].

This analysis requires the knowledge of the square of the matrix element modulus of the reaction, averaged over spin states of the initial particles and summed over those of the final particles:

$$\langle |M|^2 \rangle = \frac{I(2\pi)^3}{4m_S m_T} \frac{\left| \frac{m_1+m_2}{m_2} p_2 - p_0 \cos\theta_{02} + p_1 \cos\theta_{12} \right|}{p_2^2 p_1} \frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}$$

where I is the density of the relative flux of particles T and 0:

$$I = \sqrt{\frac{2}{m_{T0}} \left(\frac{S'}{2(m_2+m_1)} - Q' \right)} ,$$

$\vec{p}_0, \vec{p}_1, \vec{p}_2$ are the momenta of particles 0, 1, 2 in the laboratory system and m_S, m_T, m_1, m_2 are the rest masses of the particles involved in the pole reaction (Fig. 1).

3. - EXPERIMENTAL CONDITIONS.

As shown in the previous Section, the application of the Treiman-Yang criterion is related to the comparison of the differential cross-sections measured at different detection conditions corresponding to the same value of the invariants t, t', s' but with differents angles θ_{TY}. In particular a Treiman-Yang distribution is obtained by considering coincidence events corresponding to a given value of the momentum $\vec{p}_T$ of the intermediate particle as a function of the Treiman-Yang angle θ_{TY}.

The Treiman-Yang angle θ_{TY} can be obtained, for a given kinematical situation, by considering that the plane α is defined by $\vec{p}_T' \Lambda \vec{p}_N$; the plane β by $\vec{p}_T' \Lambda \vec{p}_1$. From Fig. 2 it is seen that

$$\cos\theta_{TY} = \frac{(\vec{p}_T' \Lambda \vec{p}_N')\,(\vec{p}_T' \Lambda \vec{p}_1')}{|\vec{p}_T' \Lambda \vec{p}_N'| + |\vec{p}_T' \Lambda \vec{p}_1'|}$$

and by taking into account the identity:

$$(\vec{A} \Lambda \vec{B}) \cdot (\vec{C} \Lambda \vec{D}) = (\vec{A} \cdot \vec{C}) \cdot (\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D}) \cdot (\vec{B} \cdot \vec{C})$$

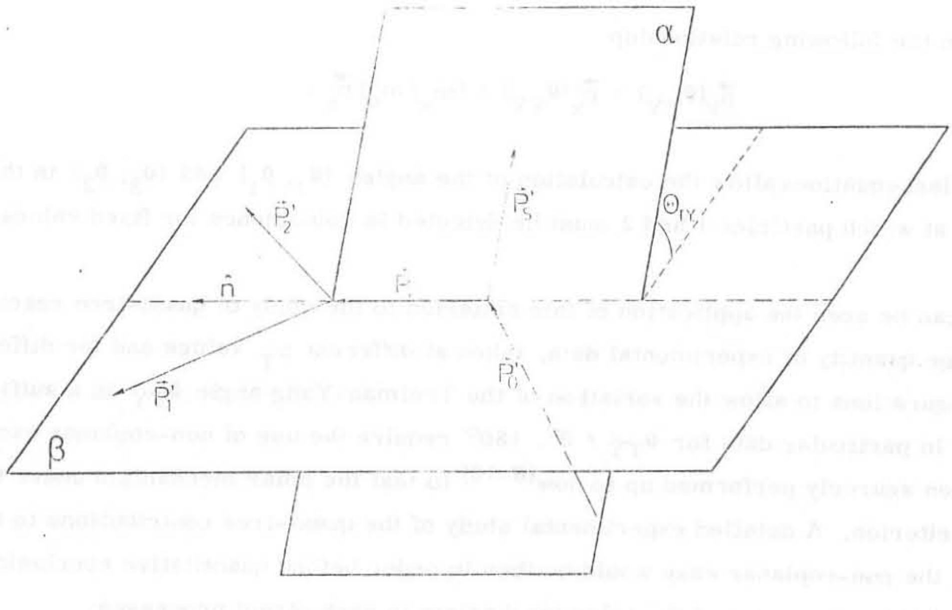


FIG. 2 - The N(0, 12)S reaction as seen in the antilaboratory frame of reference. The angle between α and β planes is the Freiman-Yang(TY) angle θ_{TY}.

it can be shown that

$$\cos\theta_{TY} = \frac{-p_T'^2 p'_{1z} + p'_{Tz}(p'_{Tx}p'_{1x} + p'_{Ty}p'_{1y} + p'_{Tz}p'_{1z})}{\sqrt{p_{Ty}'^2 + p_{Tx}'^2}\,\sqrt{(p'_{Ty}p'_{1z} - p'_{Tz}p'_{1z})^2 + (p'_{Tz}p'_{1x} - p'_{Tx}p'_{1z})^2 + (p'_{Tx}p'_{1y} - p'_{Ty}p'_{1x})^2}}$$

where all the momenta are in the antilaboratory system.

From the experimental point of view the application of the Treiman-Yang criterion requires the knowledge of the detection configurations, once a value for the intermediate particle momentum $\vec{p}'_T$ and the Treiman-Yang angle θ_{TY} have been selected.

To this aim we denote with $\hat{n}$ the unit vector given by

$$\hat{n} = \frac{\vec{p}'_T + \vec{p}'_2}{|\vec{p}'_T + \vec{p}'_2|} .$$

By performing a rotation of the plane β around $\hat{n}$ of an angle θ_{TY} each momentum $\vec{p}'_k$ is transformed into $\vec{p}'_k(\theta_{TY})$ according to

$$\vec{p}'_k(\theta_{TY}) = \vec{p}'_k - \sin\theta_{TY}\vec{p}'_k\,\hat{n} - 2\sin^2\frac{\theta_{TY}}{2}(\vec{p}'_k - \hat{n}(\vec{p}'_k - \hat{n}))$$

and by taking into account that in the transformation from the laboratory to the antilaboratory system we have

$$\vec{p}'_k = \vec{p}_k - (m_k/m_o)\vec{p}_o$$

we can obtain the following relationship

$$\vec{p}_k(\theta_{TY}) = \vec{p}\,'_k(\theta_{TY}) + (m_k/m_o)\vec{p}_o \, .$$

These last equations allow the calculation of the angles (θ_1, ϕ_1) and (θ_2, ϕ_2) in the laboratory system at which particles 1 and 2 must be detected in coincidence for fixed values of $\vec{p}\,'_T$ and θ_{TY}.

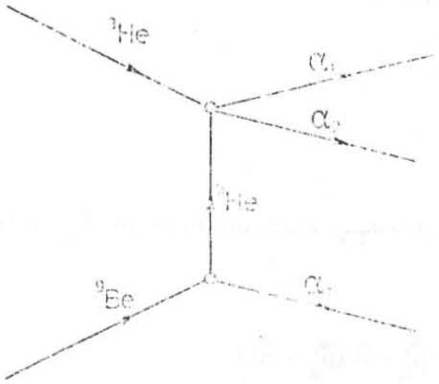
As it can be seen the application of this criterion to the study of quasi-free reactions requires a large quantity of experimental data, taken at different p'_T values and for different detection configurations to allow the variati on of the Treiman-Yang angle θ_{TY} in a sufficiently wi de interval. In particular data for $\theta_{TY} \neq 0^o$, 180^o require the use of non-coplanar experiments, that have been scarcely performed up to now(6-10) to test the polar mechanism under the Treiman-Yang criterion. A detailed experimental study of the quasi-free contributions to three-body reactions in the non-coplanar case would be then in order before quantitative conclusions can be reached about the importance of the polar mechanism in such direct processes.

4. - THE $^9Be(^3He, \alpha\alpha)^4He$ QUASI-FREE REACTION AT LOW ENERGY.

The Treiman-Yang criterion can be applied also to quasi-free reactions that can take place at low incident energies if the process has a sufficiently high Q-value. This is the case for instance of the reaction $^9Be(^3He, \alpha\alpha)^4He$ having Q = 19.009 MeV, that has been widely studied(11-14).

The application of the Treiman-Yang criterion to such reactions at low energies is compli cated by the presence of sequential contributions that make difficult the study of the Treiman-Yang distribution in a sufficiently wide interval.

Fig. 3 shows the schematic diagrams corresponding to the quasi-free process and to the sequential processes proceeding through the formation of the ^8Be nucleus.



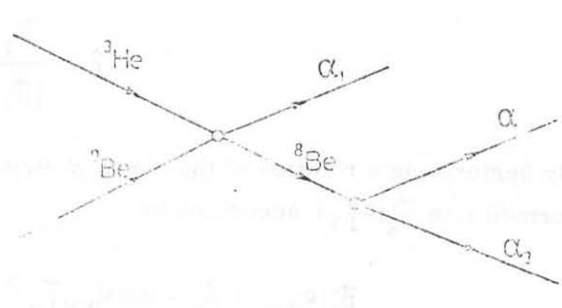


FIG. 3 - Diagrams for the reaction $^9Be(^3He, \alpha\alpha)^4He$. a) Quasi-free process ; b) Sequential process through the ^8Be intermediate nucleus.

The study of this reaction requires then the choice of appropriate kinematical situations in order to separate sequential from quasi-free contributions.

Since the ^4He-^5He motion in ^9Be is mainly in a 3s state, the momentum distribution has its maximum at $p_S = 0$ with a measured FWHM of 118 MeV/c; by taking into account the presence of the sequential peaks, the region of interest is for p_S values between -50 and +50 MeV/c[12].

In the reaction plane all the couples of angles θ_1, θ_2 can be obtained from the relation

$$\theta_1 = \mathrm{arctg} \frac{\left[v + (v^2 - w)^{1/2} \right] \sin \theta_1}{p_o - \left[v + (v^2 - w)^{1/2} \right] \cos \theta_1}$$

where

$$v = m_1 p_o \cos\theta_1 / (m_1 + m_2) ; \qquad w = 2m_1 \left[(m_o - m_2) E_o - m_2 Q \right] / (m_1 + m_2) .$$

Once the starting configuration in the reaction plane has been chosen, a value for the momentum $\vec{p}_S$ must be selected and the angles (θ_1, ϕ_1), (θ_2, ϕ_2) should be calculated corresponding to the Treiman-Yang angle θ_{TY}.

Fig. 4 shows the bahaviour of the minimum $\vec{p}_S$ value as a function of the Treiman-Yang angle.

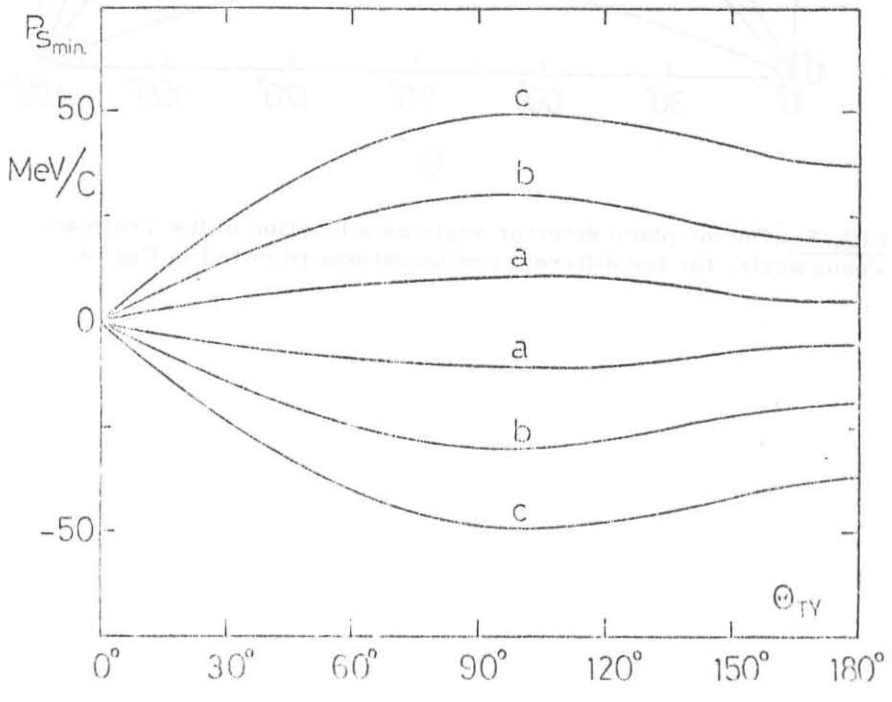


FIG. 4 - Minimum spectator momentum $P_{S_{min}}$ as a function of the Treiman-Yang angle, for different configurations allowing the measurement of a given momentum p_S. a) 10 MeV/c; b) 30 MeV/c; c) 50 MeV/c.

The better choice of the value of $\vec{p}_S$ is made by considering that it should not be too low in order to ensure a value of the azimuthal angle not too close to 0° (in which case due to the finite detection geometry all non-coplanar experiments would be indistinguishable from in-plane experiments, see for instance Fig. 5 ; on the other hand $\vec{p}_S$ should not be too far from zero, in order to measure a cross-section not too low, and to have a clear separation from sequential contributions.

For this reason a value of $|\vec{p}_S| = 30$ MeV/c seems to be the best choice in the case of our experimental conditions.

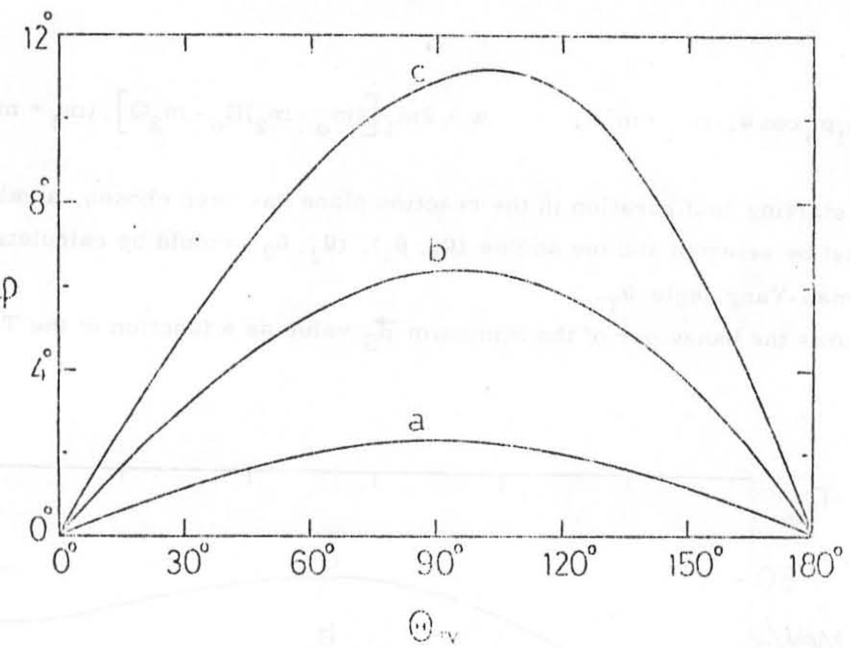


FIG. 5 - Out-of-plane detector angle as a function of the Treiman-Yang angle, for the different configurations reported in Fig. 4.

REFERENCES.

(1) - I. S. Shapiro, Proc. of the Intern. School of Physics 'Enrico Fermi', Course 38, p. 210; Sov. Phys. -Uspekhi 10, 515 (1968), and references therein.
(2) - S. B. Treiman and C. N. Yang, Phys. Letters 8, 140 (1962).
(3) - V. M. Kolybasov, Yad. Fiz. 5, 288 (1967); Sov. J. Nuclear Phys. 5, 202 (1967).
(4) - I. S. Snapiro, JEPT 41, 1616 (1961); Nuclear Phys. 28, 244 (1961).
(5) - I. S. Shapiro, V. M. Kolybasov and G. R. August, Nuclear Phys. 61, 353 (1965).
(6) - A. O. Agangants, Yu. D. Bayukov, V. N. Deza, S. V. Donskov, V. B. Fedorov, N. A. Ivanova, V. D. Kohvansky, V. M. Kolybasov, G. A. Leksin, V. L. Stolin and L. S. Vorobyev, Phys. Letters 27B, 590 (1968).
(7) - R. L. Burman and M. E. Nordberg, Phys. Rev. Letters 21, 229 (1968).
(8) - Yu. D. Bayukov, V. B. Federov, V. D. Khovansky, V. M. Kolybasov, G. A. Leksin, V. L. Stolin and L. S. Vorobyev, Phys. Letters 33B, 416 (1970).
(9) - R. Corfù, J. P. Egger, C. Lunke, C. Nussbaum, J. Rossel, E. Schwarz, J. L. Duran and C. Perin, Phys. Rev. Letters 27, 1661 (1971); Helv. Phys. Acta 43, 443 (1970).
(10) - R. B. Liebert, K. H. Parser and R. L. Burman, Nuclear Phys. A216, 335 (1973).
(11) - J. Kasagi, T. Nakagawa, N. Sekine and T. Tohei, Nuclear Phys. A239, 233 (1975).
(12) - N. Arena, D. Vinciguerra, F. Riggi and C. Spitaleri, Lett. Nuovo Cimento 17, 231 (1976).
(13) - N. Arena, D. Vinciguerra, M. Lattuada, F. Riggi and C. Spitaleri, Nuovo Cimento 45A, 405 (1978).
(14) - S. Barbarino, M. Lattuada, F. Riggi, C. Spitaleri and D. Vinciguerra, Lett. Nuovo Cimento 25, 231 (1979).