

ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Pisa

INFN/BE-79/4
28 Maggio 1979

E. Barbato: JASTROW CALCULATION OF THE BINDING
ENERGY OF A Λ^0 -PARTICLE IN NUCLEAR MATTER.

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1. - INTRODUCTION.

In the last years several calculations⁽¹⁻⁷⁾ have been performed to evaluate the binding energy B_Λ of a Λ^0 particle in infinite nuclear matter. This parameter plays a fundamental role in the analysis of the Λ^0 -nucleon interaction and can give some information about the features of such an interaction in higher partial waves⁽⁵⁾.

It is commonly accepted that its phenomenological value, as extrapolated from the known Λ^0 -binding energies in finite hypernuclei, is about 30 MeV with an upper limit of 35 MeV⁽⁶⁾. The theoretical estimates, given by various authors, for B_Λ are much larger than the empirical value. Attempts have been made to solve this overbinding problem by considering Λ -N forces which involve tensor components^(8, 9) or by the inclusion of three-body Λ NN forces⁽¹⁰⁾. Yet this problem does not appear to be satisfactorily solved.

Another disturbing feature of the calculations of B_Λ is the large discrepancy between the reaction matrix⁽¹¹⁾ results⁽⁴⁻⁷⁾ and the variational results⁽²⁾. This disagreement is certainly due to the approximations which have been made in the reaction matrix calculations, or in the variational ones, or in both of them. The present paper is mainly devoted to the understanding of this last disagreement, by means of an improvement of the Jastrow⁽¹²⁾ variational calculation.

Mueller and Clark⁽²⁾ have performed a detailed numerical analysis of B_Λ in the framework of the Jastrow approach with a state-independent correlation function. In their paper the quantity B_Λ has been expanded in a cluster series, according to the IY⁽¹³⁾ formalism, and calculated including the two and the three-body cluster contributions; some constraint on both the N- Λ and the N-N correlation factors have been imposed to avoid the Emery⁽¹⁴⁾ difficulty.

In order to ascertain the importance of the many-body cluster contributions, which have been disregarded in ref. (2), a FHNC calculation of B_Λ is performed in the present paper. The quantity B_Λ is expanded by means of the PS cluster expansion⁽¹⁵⁾. The expansion is shown to be linked and irreducible.

Furthermore a set of integral equations are derived; the solutions of these equations permit the calculation of the two-body N- Λ distribution function in the FHNC approximation.

In Section 2 the expression for B_Λ is derived in terms of the N- Λ distribution function.

In Section 3 the N- Λ distribution function is expanded in a cluster series and a set of integral equations is derived by means of the covolution technique.

In Section 4 the N-N and N- Λ potentials and correlation factors, employed in the present calculation, are presented.

Finally Section 5 is devoted to the analysis and the discussion of the results obtained.

2. - DESCRIPTION OF THE METHOD.

The hamiltonian for the system, constituted by A nucleons and one Λ^0 -particle, is taken to be :

$$H^{(A+\Lambda)} = H^{(A)} - \frac{\hbar^2}{2M_\Lambda} \nabla_\Lambda^2 + \sum_{i=1}^A V_{n\Lambda}(i\Lambda), \quad (1)$$

where the hamiltonian $H^{(A)}$ for the nuclear substratum is given by

$$H^{(A)} = - \frac{\hbar^2}{2M_n} \sum_{i=1}^A \nabla_i^2 + \sum_{i < j=1}^A V_{nn}(ij). \quad (2)$$

M_n and M_Λ being the nucleon and Λ^0 masses, respectively.

Let us denote the ground state wave function of the host medium by $\psi^{(A)}$ and that one of the whole system by $\psi^{(A+\Lambda)}$. The binding energy of the Λ^0 particle is defined as :

$$B_\Lambda = \frac{(\psi^{(A)}, H^{(A)} \psi^{(A)})}{(\psi^{(A)}, \psi^{(A)})} - \frac{(\psi^{(A+\Lambda)}, H^{(A+\Lambda)} \psi^{(A+\Lambda)})}{(\psi^{(A+\Lambda)}, \psi^{(A+\Lambda)})}. \quad (3)$$

The wave functions $\psi^{(A)}$ and $\psi^{(A+\Lambda)}$ are taken of the Jastrow type, that is :

$$\psi^{(A)}(1, \dots, A) = \prod_{i < j=2}^A f_{nn}(r_{ij}) \Phi, \quad (4)$$

$$\psi^{(A+\Lambda)}(1, \dots, A, \Lambda) = F(1, \dots, A, \Lambda) \psi^{(A)}(1, \dots, A) = \prod_{i=1}^A f_{n\Lambda}(r_{i\Lambda}) \psi^{(A)}(1, \dots, A), \quad (5)$$

where $f_{nn}(r_{ij})$ and $f_{n\Lambda}(r_{i\Lambda})$ are the state independent N-N and N- Λ correlation factors respectively, and Φ is the ground state wave function of a Fermi gas with density ρ .

The N- Λ potentials are assumed to be central but spin dependent and are written in the form

$$V_{n\Lambda}(i\Lambda) = V^D(i\Lambda) + V^X(i\Lambda) P(i\Lambda), \quad (6)$$

where $P(i\Lambda)$ denotes the space-exchange operator for particles i and Λ^0 and,

$$V^{D,X}(i\Lambda) = V_s^{D,X}(r_{i\Lambda}) P_s + V_t^{D,X}(r_{i\Lambda}) P_t, \quad (7)$$

with P_s and P_t projection operators into the singlet and the triplet states, respectively.

The following expression for B_Λ results :

$$B_\Lambda = -\rho \int d\vec{r} \left\{ \left[\frac{\hbar^2}{4M_\Lambda} \left[\left(\frac{\nabla f_{n\Lambda}(r)}{f_{n\Lambda}(r)} \right)^2 - \frac{\nabla^2 f_{n\Lambda}(r)}{f_{n\Lambda}(r)} \right] + \frac{\hbar^2}{2M_n} \left(\frac{\nabla f_{n\Lambda}(r)}{f_{n\Lambda}(r)} \right)^2 + \bar{V}^D(r) G(r) + \bar{V}^X(r) G^X(r) \right] + \Delta \right\}, \quad (8)$$

where $\bar{V}^{D,X}$ are the N-A spin averaged potentials and are given by

$$\bar{V}^{D,X}(r) = \frac{1}{4} \bar{V}_s^{D,X}(r) + \frac{3}{4} \bar{V}_t^{D,X}(r) \quad (9)$$

and $G(r)$ and $G^X(r)$ are the direct and exchange N-A radial distribution functions and are defined as:

$$G(r_{1A}) = \frac{A^2}{\varrho^2} \frac{\int d\bar{r}_2 \dots d\bar{r}_A (\psi^{(A)})^* F^2 \psi^{(A)}}{\int d\tau^A d\bar{r}_A |\psi^{(A+A)}|^2} \quad (10)$$

$$G^X(r_{1A}) = \frac{A^2}{\varrho^2} \frac{\int d\bar{r}_2 \dots d\bar{r}_A (\psi^{(A)})^* F P(1A) F \psi^{(A)}}{\int d\tau^A d\bar{r}_A |\psi^{(A+A)}|^2} \quad (11)$$

the quantity

$$2\Delta = - \frac{\int d\tau^A d\bar{r}_A F^2 \left[(H^{(A)} \psi^{(A)})^* \psi^{(A)} + (\psi^{(A)})^* (H^{(A)} \psi^{(A)}) \right]}{d\tau^A d\bar{r}_A |\psi^{(A+A)}|^2} + \frac{\int d\tau^A (H^{(A)} \psi^{(A)})^* \psi^{(A)} + (\psi^{(A)})^* H^{(A)} \psi^{(A)}}{\int d\tau^A |\psi^{(A)}|^2} \quad (12)$$

vanishes identically if $\psi^{(A)}$ is an eigenfunction of the hamiltonian $H^{(A)}$. In the present calculation this quantity is assumed to be zero.

3. - CALCULATION OF G AND G^X.

The distribution function G can be expanded in terms of an appropriate series of cluster contributions, which are easily derived by using the PS expansion⁽¹⁵⁾. The details of the derivation are given in the Appendix.

It is useful to associate a diagram to each cluster contribution of the expansion. The diagrammatic representation of ref. (15) will be used, that is, the internal and external indices are represented by solid and open circles; both the correlation factors $h_{nn}(r) = f_{nn}^2(r) - 1$ and $h_{nA}(r) = f_{nA}^2(r) - 1$ are represented by solid lines, while the statistical linkages $-1(k_{fr})/4$ and $-1^2(k_{fr})/4$ are represented by dashed lines and by helical lines, respectively.

The diagrams associated to the cluster contributions satisfy the following rules:

- a solid circle involves a factor ϱ and a summation over the corresponding coordinates;
- each internal point is an extremity of at least one solid line;
- the solid lines can be superimposed on dashed and helical lines;
- the dashed lines are arranged in closed polygons and there are no common points between one polygon and another; each polygon involves a factor -8;
- each helical line has no common points with another statistical line;
- the particle point A is never an extremity of a statistical linkage.

In the Appendix it is proved that the diagrams involved by the cluster expansion are irreducible; as a consequence the following expression for $G(r_{1A})$ holds:

$$G(r_{1A}) = f_{nA}^2(r_{1A}) \left[1 + \sum_m \sum_j \frac{1}{S_j} X_m^j(r_{1A}) \right] \quad (13)$$

where X_m^j indicates the cluster contribution associated to an allowed irreducible diagram with m internal points and the two external points 1 and Λ and S_j is the symmetry number associated with the diagram.

In the evaluation of $G^X(r_{1\Lambda})$ the space-exchange operator $P(1\Lambda)$ is considered to act only on the model wave function Φ in analogy with preceding calculations⁽²⁾. As a consequence the following expression for G^X is used

$$G^X(r_{1\Lambda}) = \frac{A^2}{q^2} \frac{\int d\vec{r}_2 \dots d\vec{r}_A \prod_{i < j=2}^A f_{nn}^2(r_{ij}) F^2 \Phi^* P(1\Lambda) \Phi}{\int d\tau^A dr_A |\psi^{(A+\Lambda)}|^2} \quad (14)$$

The PS expansion of the r. h. s. of eq. (14) leads to a series of irreducible cluster contributions which can be associated to diagrams obeying the diagrammatic rules previously given and the following ones:

- the two external points 1 and Λ are always joined either by a dashed line or by a chain of dashed lines;
- each diagram has a factor -4 .

Some examples of irreducible diagrams which contribute to G and G^X are shown in Figs. 1 and 2 respectively.

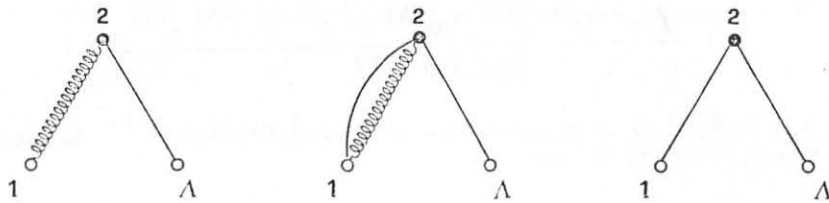


FIG. 1 - Irreducible and topologically distinct diagrams with two external points and one internal point associated to terms contributing to $G(r_{1\Lambda})$.

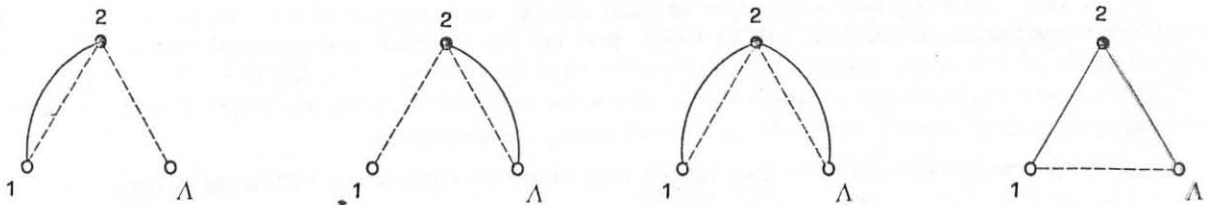


FIG. 2 - Irreducible and topologically distinct diagrams with two external points and one internal point associated to terms contributing to $G^X(r_{1\Lambda})$.

The various cluster terms contributing to the distribution functions G and G^X can be summed by means of a FHNC technique. It is useful to distinguish the following three classes of $N-\Lambda$ diagrams:

- (s Λ) : the point i is affected only by dynamical correlations;
- (h Λ) : the point i is either an extremity of one statistical linkage $-1/4 l^2(k_f r)$ or a common extremity of two different statistical correlations $-1/4 l(k_f r)$.

Let us indicate by $Z_{m\Lambda}(1, \Lambda)$ and $a_{m\Lambda}(1, \Lambda)$ the sums of all the nodal and non-nodal diagrams of the class (m Λ) respectively. Moreover, let us indicate by $g_{mn}(1, 2)$ the whole set of

allowed diagrams of the type mn (ss, sh, hh or dd)⁽¹⁶⁾ having the external points 1 and 2 both corresponding to nucleonic coordinates. The corresponding functions $g_{mn}(r_{12})$ are given by:

$$\begin{aligned}
 g_{ss}(r_{12}) &= F_{nn}(r_{12}) - 1, \\
 g_{sh}(r_{12}) &= F_{nn}(r_{12}) [G_{sh}(r_{12}) + E_{sh}(r_{12})], \\
 g_{hh}(r_{12}) &= F_{nn}(r_{12}) \left\{ -\frac{1}{4} l^2(k_f r_{12}) + E_{hh}(r_{12}) + G_{hh}(r_{12}) + [E_{sh}(r_{12}) + \right. \\
 &\quad \left. + G_{sh}(r_{12})]^2 - 4[E_{dd}(r_{12}) + G_{dd}(r_{12})]^2 + 2 l(k_f r_{12}) [E_{dd}(r_{12}) + G_{dd}(r_{12})] \right\}, \\
 g_{dd}(r_{12}) &= F_{nn}(r_{12}) \left[-\frac{1}{4} l(k_f r_{12}) + G_{dd}(r_{12}) \right],
 \end{aligned} \tag{15}$$

with

$$F_{nn}(r_{12}) = f_{nn}^2(r_{12}) \exp[G_{ss}(r_{12}) + E_{ss}(r_{12})].$$

The functions $G_{mn}(r_{12})$ and $E_{mn}(r_{12})$ correspond to the sums of all the 1-2 nodal and elementary diagrams of the type mn respectively (see ref. (16)).

The set of all the nodal diagrams of the type (sA), $Z_{sA}(1, A)$, is obtained by making the chain connection of $g_{ss}(1, 2)$ with $\alpha_{sA}(2, A)$, of $g_{sh}(1, 2)$ with $\alpha_{sA}(2, A)$ and of $g_{ss}(1, 2)$ with $\alpha_{hA}(2, A)$.

Correspondingly the following integral equation holds for the function $Z_{sA}(r_{1A})$:

$$Z_{sA}(r_{1A}) = (g_{ss}(r_{12}) + g_{sh}(r_{12}) | \alpha_{sA}(r_{2A})) + (g_{ss}(r_{12}) | \alpha_{hA}(r_{2A})), \tag{16}$$

where $(a(r_{1i}) | b(r_{1j}))$ denotes the covolution integral:

$$(a(r_{1i}) | b(r_{1j})) = e \int dr_1 a(r_{1i}) b(r_{1j}). \tag{17}$$

In a similar way one obtains:

$$\begin{aligned}
 Z_{hA}(r_{1A}) &= (g_{hh}(r_{12}) + g_{hs}(r_{12}) | \alpha_{sA}(r_{2A})) + (g_{hs}(r_{12}) | \alpha_{hA}(r_{2A})), \\
 Z_{dA}(r_{1A}) &= (g_{dd}(r_{12}) | \alpha_{dA}(r_{2A})).
 \end{aligned} \tag{18}$$

The functions $\alpha_{mA}(r_{1A})$ which correspond to the non-nodal diagrams $\alpha_{mA}(1, A)$ are given in term of the functions $Z_{mA}(r_{1A})$ by the following relations:

$$\begin{aligned}
 \alpha_{sA}(r_{1A}) &= F_{nA}(r_{1A}) - Z_{sA}(r_{1A}) - 1, \\
 \alpha_{hA}(r_{1A}) &= F_{nA}(r_{1A}) [Z_{hA}(r_{1A}) + E_{hA}(r_{1A})] - Z_{hA}(r_{1A}), \\
 \alpha_{dA}(r_{1A}) &= F_{nA}(r_{1A}) [l(k_f r_{1A}) + Z_{dA}(r_{1A}) + E_{dA}(r_{1A})] - l(k_f r_{1A}) - Z_{dA}(r_{1A}),
 \end{aligned} \tag{19}$$

with:

$$F_{nA}(r_{1A}) = f_{nA}^2(r_{1A}) \exp[Z_{sA}(r_{1A}) + E_{sA}(r_{1A})] , \quad (20)$$

and where the functions $E_{mA}(r_{1A})$ correspond to the contributions of the elementary diagrams of the type (mA) .

In place of the integral equations (16) and (18) together with the relations (19), one can solve the following set of three integral equations, only two of which are coupled:

$$\begin{aligned} \ln \frac{g_{sA}(r_{1A}) + 1}{f_{nA}^2(r_{1A})} &= \left(g_{ss}(r_{12}) + g_{sh}(r_{12}) \right) \left[g_{sA}(r_{2A}) - \ln \frac{g_{sA}(r_{2A}) + 1}{f_{nA}^2(r_{2A})} + E_{sA}(r_{2A}) \right] + \\ &+ \left(g_{ss}(r_{12}) \right) \left[\frac{g_{sA}(r_{2A}) g_{hA}(r_{2A})}{g_{sA}(r_{2A}) + 1} + E_{hA}(r_{2A}) \right] + E_{sA}(r_{1A}) , \\ \frac{g_{hA}(r_{1A})}{g_{sA}(r_{1A}) + 1} &= \left(g_{hh}(r_{12}) + g_{hs}(r_{12}) \right) \left[g_{sA}(r_{2A}) - \ln \frac{g_{sA}(r_{2A}) + 1}{f_{nA}^2(r_{2A})} + E_{sA}(r_{2A}) \right] + \\ &+ \left(g_{hs}(r_{12}) \right) \left[\frac{g_{sA}(r_{2A}) g_{hA}(r_{2A})}{g_{sA}(r_{2A}) + 1} + E_{hA}(r_{2A}) \right] + E_{hA}(r_{1A}) , \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{dA(r_{1A}) - l(k_f r_{1A}) g_{sA}(r_{1A})}{g_{sA}(r_{1A}) + 1} &= \left(g_{dd}(r_{12}) \right) \left[g_{sA}(r_{2A}) \frac{l(k_f r_{2A})}{g_{sA}(r_{2A}) + 1} \right] + \\ &+ \left(g_{dd}(r_{12}) \right) \left[\frac{g_{dA}(r_{2A}) g_{sA}(r_{2A})}{g_{sA}(r_{2A}) + 1} + E_{dA}(r_{2A}) \right] + E_{dA}(r_{1A}) . \end{aligned}$$

The functions $g_{mA}(r_{1A})$ are related to the functions $\alpha_{mA}(r_{1A})$, $Z_{mA}(r_{1A})$ and $E_{mA}(r_{1A})$ through the following equations:

$$\begin{aligned} g_{sA}(r_{1A}) &= \alpha_{sA}(r_{1A}) + Z_{sA}(r_{1A}) = F_{nA}(r_{1A}) - 1 , \\ g_{hA}(r_{1A}) &= \alpha_{hA}(r_{1A}) + Z_{hA}(r_{1A}) = F_{nA}(r_{1A}) [Z_{hA}(r_{1A}) + E_{hA}(r_{1A})] \\ g_{dA}(r_{1A}) &= \alpha_{dA}(r_{1A}) + Z_{dA}(r_{1A}) = F_{nA}(r_{1A}) [Z_{dA}(r_{1A}) + E_{dA}(r_{1A}) + l(k_f r_{1A})] - l(k_f r_{1A}) . \end{aligned} \quad (22)$$

The radial distribution functions G and G^X are given by;

$$G(r_{1A}) = 1 + g_{sA}(r_{1A}) + g_{hA}(r_{1A}) , \quad G^X(r_{1A}) = l(k_f r_{1A}) + g_{dA}(r_{1A}) . \quad (23)$$

4. - POTENTIALS AND CORRELATION FACTORS.

The N-N potentials V_{nn} used in the present calculation are the two hard core central potentials of the Serber type usually denoted as OMY⁽¹⁷⁾ and IY⁽¹⁸⁾ potentials.

The N- Λ potentials, considered here, are the two body effective central potentials H, E and E' which Herndon and Tang⁽¹⁹⁾ obtained by fitting the Λ^0 -binding energy for light hypernuclei and the available Λ^0 -proton scattering data. These potentials are of the general form shown in eq. (6) and (7) and are supplied with a state independent hard core $c_{n\Lambda}$ as well as a space-exchange component given by $V^X = X(X-1)^{-1}V^D$ (X is fixed to the value 0.2).

Calculations have also been performed with another potential, denoted DW, used by Downs and Ware⁽²⁰⁾ in their independent pair model calculation of the Λ^0 -binding energy in nuclear matter. This potential, provided with a state independent hard core, has the same strength in all partial waves.

The radial dependence of the spin averaged potentials $\bar{V}^{D,X}$ (see eq. (9)) is of the form:

$$\bar{V}^{D,X}(r) = \begin{cases} \infty & r \leq c_{n\Lambda} \\ -\bar{V}_0^{D,X} \exp[-\lambda(r-c_{n\Lambda})] & r > c_{n\Lambda} \end{cases} \quad (24)$$

The values of the parameter λ , $c_{n\Lambda}$, \bar{V}_0^D and \bar{V}_0^X are listed in Table I. These potentials have been used to calculate the Λ^0 -binding energy in nuclear matter by means of the Jastrow variational method⁽²⁾ and the reaction matrix method⁽⁶⁾ as well. (For a complete discussion on the merits of these potentials see ref. (19) and (2)).

TABLE I
N- Λ potential parameters (eq. (24)) (b is the intrinsic range).

Potentials	b (fm)	λ (fm ⁻¹)	$c_{n\Lambda}$ (fm)	\bar{V}_0^D (MeV)	\bar{V}_0^X (MeV)
H	2.1	3.935	0.60	548.8	137.2
E	2.0	3.219	0.45	331.6	82.9
E'	2.0	3.219	0.45	319.2	79.8
DW	1.5	3.219	0.40	330.9	0.0

For sake of comparison the correlation factors $f_{nn}(r_{12})$ and $f_{n\Lambda}(r_{1\Lambda})$, used in the present paper, have been chosen to be the "optimal" correlation functions obtained by Mueller and Clark in their variational calculation⁽²⁾. The correlation factor f_{nn} has the form:

$$f_{nn}(r) = \begin{cases} 0 & r \leq c_{nn} \\ \left\{ 1 - \exp[-a(r-c_{nn})] \right\} \left\{ 1 + \beta \exp[-a(r-c_{nn})] \right\} & r > c_{nn} \end{cases} \quad (25)$$

where $c_{nn} = 0.6$ fm is the hard core radius. The values of a and β , for both OMY and IY potentials, are reported in Table II together with the corresponding nuclear matter energies evaluated up to the second and third orders of the cluster expansion. The correlation factor $f_{n\Lambda}$ has the form:

$$f_{n\Lambda}(r) = \begin{cases} 0 & r \leq c_{n\Lambda} \\ \left\{ 1 - \frac{c_{n\Lambda}}{r} \exp[-\gamma(r-c_{n\Lambda})] \right\} \left\{ 1 + \mu \exp[-\nu(r-c_{n\Lambda})] \right\} & r > c_{n\Lambda} \end{cases} \quad (26)$$

where c_{nA} is the hard core of the potential ; the values of the parameters γ , μ and r are given in Table III.

TABLE II

Optimal values of the N-N correlation factor parameters (eq. (25)) and corresponding values for the two and three-body contributions to the expected nuclear matter energy per particle.

Potentials	α (fm ⁻¹)	β	E ⁽²⁾ /A (MeV)	E ⁽³⁾ /A (MeV)
OMY	2.3	1.394	- 12.49	0.02
IY	2.0	1.349	- 6.56	0.51

TABLE III

Results of the present calculation for the A^0 -binding energy in the FHNC approximation, obtained with the correlation factor of eq. (26). B_A (MC) denotes the variational results of Mueller and Clark⁽²⁾. B_A (BR) indicates the reaction matrix results of Bodmer and Rote⁽⁶⁾ at $M_n^x/M_n = 0.638$ and $\Delta_n = 81.4$. The results marked with (a) correspond to an odd-state strength equal to 0,6 of the even-state strength; the results marked with (b) correspond to the same strength in all partial waves.

Potentials	γ (fm ⁻¹)	r (fm ⁻¹)	μ	(a)			(b)		
				B_A (FHNC) - Δ (MeV)	B_A (MC) - Δ (MeV)	B_A (BR) (MeV)	B_A (FHNC) - Δ (MeV)	B_A (MC) - Δ (MeV)	B_A (BR) (MeV)
H/OMY	0.85	1.55	1.248	70.2	65.1	45.6	80.25	74.8	56.8
H/IY	0.90	1.60	1.225	69.8	64.6	----	79.67	74.3	----
E/OMY	0.70	1.40	1.064	66.5	62.8	51.9	74.62	70.0	61.7
E'/OMY	0.70	1.35	0.965	60.5	57.2	47.1	63.1	64.8	56.0
DW/OMY	0.70	1.35	0.871	----	----	----	45.1	42.1	----

A variational calculation of the A^0 -binding energy, evaluated in the FHNC approximation, has been performed with the correlation factor f_{nA} having the form :

$$f_{nA}(r) = \begin{cases} 0 & r \leq c_{nA} \\ 1 - \exp[-\alpha(r - c_{nA})] & r > c_{nA} \end{cases} \quad (27)$$

where α is the unique trial parameter. This type of correlation factor was first used by Downs and Grypeos⁽²¹⁾ and later by Whesthaus and Clark⁽²²⁾ and by Mueller and Clark⁽²⁾. The A^0 -binding energy obtained by these authors using the correlation function given by eq. (27), is about 1 MeV less than the value furnished by the more flexible correlation factor of eq. (26).

5. - ANALYSIS OF RESULTS AND CONCLUDING REMARKS.

Table III shows the results obtained in the FHNC approximation by using the optimal correlation factor f_{nA} of ref. (2) (eq. (26)), for various potential combinations and for fixed values of the

nuclear substratum parameters ($k_f = 1.366 \text{ fm}^{-1}$). The table shows the results found by Mueller and Clark⁽²⁾ with the procedure marked in their paper with (i), together with the corresponding optimal values for the parameters γ , μ and ν . The reaction matrix results of Bodmer and Rote⁽⁶⁾ are also reported.

The columns marked with (a) and (b), in Table III, corresponds to an odd-state strength equal to 0.6 of the even state-strength and to the same strength in all partial waves, respectively. Two observations may be made from the comparison of the FHNC results and those of ref. (2) and (6):

- i) In all the cases considered the many body cluster contributions produce modifications of less than 7%, so that the constraint imposed in ref. (2) on the $N-A$ correlation factor of eq. (26) yields quite good expansion convergence.
- ii) The difference between the theoretical values of B_A calculated by the variational method or by the reaction matrix method is increased by the many body cluster contributions. This difference may be further increased by performing an unconstrained variational calculation.

It should be noted that the quantity Δ (see eq. (8) and (12)) has been disregarded in the present calculation, because the third order estimates obtained by Mueller and Clark⁽²⁾ are negligible and the expansion convergence is quite good. Nevertheless, an evaluation of Δ in the framework of the FHNC approximation would be interesting.

The variational results marked with (a) are affected by another approximation related to the evaluation of the space-exchange radial distribution function $G^X(r_{1A})$ (cfr. eq. (11) and (14)). Nevertheless, observation ii) can also be shown to follow from the results marked with (b) in Table III which are not affected by this approximation.

Analogous conclusion can be drawn from the results obtained by use of the correlation factor (27). Table IV shows the optimal values for the parameter α and the corresponding values of

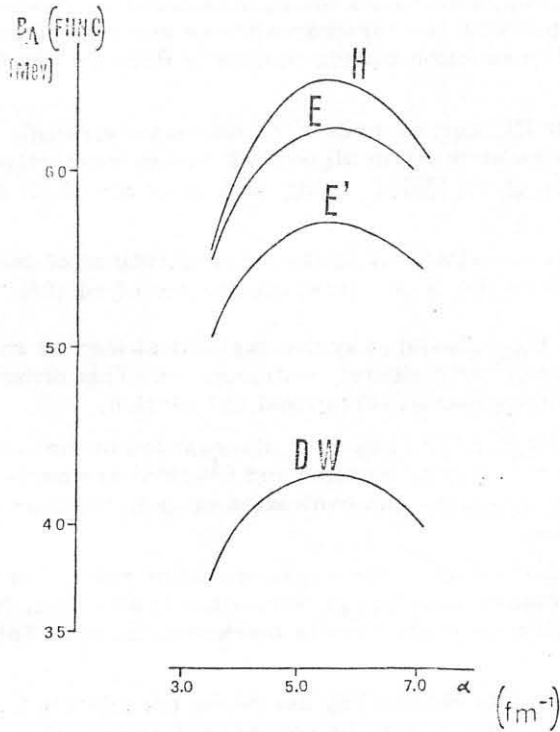
TABLE IV

Optimal values for the $N-A$ correlation factor parameter α (eq. (27)) and corresponding values for the A^0 -binding energy in the FHNC approximation obtained in the present calculation. B_A (MCA) denotes the two and the three-body cluster contributions. The label (a) and (b) have the same meaning as in Table III.

Potentials	α (fm^{-1})	(a)		(b)	
		B_A (FHNC) - Δ (MeV)	B_A (MCA) - Δ (MeV)	B_A (FHNC) - Δ (MeV)	B_A (MCA) - Δ (MeV)
H/OMY	5.4	65.1	63.2	73.7	72.3
H/IY	5.5	65.0	63.3	73.3	72.6
E/OMY	5.5	62.4	61.1	69.4	68.2
E'/OMY	5.3	57.3	55.9	64.0	62.8
DW/OMY	5.3	----	----	42.8	41.5

B_A (FHNC) together with the results for the A^0 -binding evaluated up to the third order (B_A (MCA)). The behaviour of B_A (FHNC) as a function of α is shown in Fig. 3.

The results denoted B_A (MCA) are in good agreement with the corresponding available results obtained by Mueller and Clark^(2, 10). The expansion convergence is very good, but the results, obtained in the FHNC approximation are worst, from the variational point of view, than the



corresponding ones found with the more flexible correlation factor (26).

The results of the present paper confirm the well known inadequacy of two-body effective central potentials in hypernuclear matter calculations. As a consequence potentials containing tensor and (or) three-body components seem to be necessary to solve the A^0 -overbinding problem.

When a realistic Λ -N potential is used together with a purely central N-N potential, it is necessary to adopt a Jastrow wave function with a state dependent N- Λ correlation factor; a corresponding FHNC calculation of B_A could be interesting in order to explore, in a simplified situation, the problems related to the development of the Jastrow variational method with state dependent correlation factors⁽²³⁾.

FIG. 3 - Behaviour of B_A (FHNC) - Δ as a function of the N- Λ correlation factor parameter α (eq. (27)), for the various potential combinations.

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APPENDIX. -

In this Appendix the cluster expansion for $G(r_{1A})$ and $G^X(r_{1A})$ (eq. (10), (11)) are made and are shown to be linked and irreducible.

Let us consider a A - N operator $0 = \sum_{i=1}^A 0(iA)$, where $0(iA)$ has no space-exchange component. In order to derive the PS expansion for the mean value

$$\bar{0} = \frac{\int (\psi^{(A+A)})^* 0 \psi^{(A+A)} d\bar{r}_A d\tau^A}{\int |\psi^{(A+A)}|^2 d\bar{r}_A d\tau^A}$$

the following function must be considered:

$$\begin{aligned} \theta(\alpha, \beta) &= \frac{A \int d\bar{r}_A d\tau^A \Phi^* \check{0}(1A) \prod_{i < j=2}^A f_{nn}^2(r_{ij}, \alpha) \prod_{i=2}^A f_{nA}^2(r_{iA}, \beta) \Phi}{\int d\bar{r}_A d\tau^A \Phi^* \prod_{i < j=2}^A f_{nn}^2(r_{ij}, \alpha) \prod_{i=1}^A f_{nA}^2(r_{iA}, \beta) \Phi} = \\ &= \frac{\sum_{n=0}^{\infty} \sum_{J=0}^n a_{j, n-j} \alpha^j \beta^{n-j}}{\sum_{n=0}^{\infty} \sum_{J=0}^n b_{j, n-j} \alpha^j \beta^{n-j}}, \end{aligned} \tag{A1}$$

where $f_{nn}^2(r, \alpha) = 1 + \alpha h_{nn}(r)$; $f_{nA}^2(r, \beta) = 1 + \beta h_{nA}(r)$ and $\check{0}(1A) = f_{nA}^2(r_{1A}) 0(1A)$. (A2)

The generic coefficient $a_{j, n-j}$ is given by the sum of the expectation value on the Slater determinant Φ of all the possible products of the operator 0 and of n correlation factors, j of the type $h_{nn}(r_{ij})$ and $n-j$ of the type $h_{nA}(r_{iA})$. Grouping all the products having the same topological structure and symmetry number S but m differing particles, one obtains the result:

$$\frac{a_{j, n-j}}{b_{0,0}} = \sum_m \sum_{(\alpha)} \frac{q^m}{S} \frac{A^{-m}}{V} \int d\bar{r}_1 \dots d\bar{r}_m d\bar{r}_A \check{0}(r_{1A}) (h_{nn} \dots h_{nn} h_{nA} \dots h_{nA}) \alpha^j \beta^{n-j} D(\bar{r}_1 \dots \bar{r}_m) \tag{A3}$$

$b_{0,0}$ is the normalization integral; the summation $\sum_{(\alpha)}$ is extended over all the possible combinations of j h_{nn} -factors and $n-j$ h_{nA} -factors involving $\binom{\alpha}{m}$ the indices $1, 2, \dots, m$. The function $D(\bar{r}_1 \dots \bar{r}_m)$ comes from the integration over the variables $\bar{r}_{m+1} \dots \bar{r}_A$ and is fully expressed in eq. (20) of ref. (15). An analogous expression holds for $b_{j, n-j}/b_{0,0}$. The power series expansion of $\theta(\alpha, \beta)$ around $\alpha = \beta = 0$ leads to:

$$\theta(\alpha, \beta) = \sum_k A_k(\alpha, \beta), \tag{A4}$$

where

$$A_k(\alpha, \beta) = 1/k! \left[\frac{d^k}{dt^k} (\alpha t, \beta t) \right]_{t=0}, \tag{A5}$$

the relation

$$\bar{0} = \sum_k A_k(1, 1) = \sum_k A_k, \tag{A6}$$

holds; the coefficients A_k satisfy the recurrence relation⁽¹⁵⁾

$$A_k = \frac{a_k}{b_0} - \frac{b_1}{b_0} A_{k-1} - \frac{b_2}{b_0} A_{k-2} \dots \quad (A7)$$

with $a_\alpha = \sum_{j=1}^{\alpha} a_{j, \alpha-j}$ and $b_\beta = \sum_{j=1}^{\beta} b_{j, \beta-j}$.

The various coefficients A_k can be expressed in term of $a_{k,1}$ and $b_{k,1}$ with the result:

$$A_k = \sum_{l=0}^k \sum_{p=0}^{k-l} \left[\sum_{k_{1,0} + k_{0,1} + 2(k_{0,2} + k_{1,1} + k_{2,0}) + \dots + m(k_{0,m} + k_{1,m-1} + \dots + k_{m,0}) = l} \right. \\ \left. \left(\sum_{j=1}^m \sum_{i=0}^j k_{i, j-i} \right)! \frac{(-b_{0,1}/b_{0,0})^{k_{0,1}}}{k_{0,1}!} \frac{(-b_{1,0}/b_{0,0})^{k_{1,0}}}{k_{1,0}!} \dots \frac{(-b_{0,m}/b_{0,0})^{k_{0,m}}}{k_{0,m}!} \right. \\ \left. \frac{(-b_{1,m-1}/b_{0,0})^{k_{1,m-1}}}{k_{1,m-1}!} \dots \frac{(-b_{m,0}/b_{0,0})^{k_{m,0}}}{k_{m,0}!} \right] \frac{a_{p, k-l-p}}{b_{0,0}} \quad (A8)$$

The following theorem holds:

in eq. (A6) all the integrals coming from the terms of the form $a_{\alpha_0, \beta_0} b_{\alpha_1, \beta_1} \dots b_{\alpha_k, \beta_k}$ ($k > 0$) cancell all the integrals which are unliked or reducible in the point-particle A^0 and are contained in the terms of the type $a_{j, n-j}$.

Let us consider an integral T corresponding to an unliked diagram or to a diagram which is reducible in the point A . This integral can be factored into two parts: one part is linked and irreducible in A , and it is produced by one a_{α_0, β_0} ; the other part is in general unliked and it is contained in various combinations of the form $b_{\alpha_1, \beta_1} b_{\alpha_2, \beta_2} \dots b_{\alpha_n, \beta_n}$. Let the combination $C = a_{\alpha_0, \beta_0} b_{\alpha_1, \beta_1} \dots b_{\alpha_n, \beta_n}$ contain the integral T multiplied by a factor Z ; then also the combinations $C' = a_{\alpha_0 + \alpha_j, \beta_0 + \beta_j} b_{\alpha_1, \beta_1} \dots b_{\alpha_{j-1}, \beta_{j-1}} b_{\alpha_{j+1}, \beta_{j+1}} \dots b_{\alpha_n, \beta_n}$ ($\forall j=1, 2, \dots, n$) contain T with the same factor Z . The coefficient of C in eq. (A8) is:

$$F = (-) \frac{\sum_{i=1}^n k_{\alpha_i, \beta_i} \left(\sum_{i=1}^n k_{\alpha_i, \beta_i} \right)!}{\prod_{i=1}^n k_{\alpha_i, \beta_i}!} \quad (A9)$$

while the coefficient of each C' in eq. (A8) is:

$$F' = (-) \frac{\left(\sum_{i=1}^n k_{\alpha_i, \beta_i} - 1 \right) \left(\sum_{i=1}^n k_{\alpha_i, \beta_i} - 1 \right)!}{\left(\prod_{i=1}^n k_{\alpha_i, \beta_i}! \right) \frac{(k_{\alpha_j, \beta_j} - 1)!}{k_{\alpha_j, \beta_j}!}} \quad (A10)$$

Summing all the F' for all the combination C' one obtains $-F$ and this fact proves that the integral T is not present in the cluster expansion. The theorem then follows from the arbitrariness of the product $b_{\alpha_1, \beta_1} b_{\alpha_2, \beta_2} \dots b_{\alpha_n, \beta_n}$. As a consequence of the theorem, the PS expansion is linked and irreducible in the particle point Λ^0 and one can write:

$$\bar{0} = \sum_{n,k} \left(\frac{a_{n,k}}{b_{0,0}} \right) \text{linked and irred. in } \Lambda. \quad (A11)$$

The integrals contained in the r. h. s. of eq. (A11) corresponding to reducible diagrams cancel as well as in pure nuclear matter⁽¹⁵⁾.

Therefore one can write:

$$\begin{aligned} \bar{0} = \sum_{n,j} \left(\frac{a_{n,j}}{b_{0,0}} \right) \text{irred.} &= \sum_m \frac{\rho^m}{V} \cdot \frac{1}{S} \int d\bar{r}_1 \dots d\bar{r}_m d\bar{r}_\Lambda \left[\tilde{0}(r_{1\Lambda}) h_{nn} \dots h_{nn} h_{n\Lambda} \dots \right. \\ &\left. \dots h_{n\Lambda} A^{-m} D(r_1 \dots r_m) \right] \text{irred.} = \sum_m \frac{\rho^m}{V} \sum_i \frac{1}{S_j} X_m^j, \end{aligned} \quad (A12)$$

where X_m^j is associated to an irreducible diagrams with $m+1$ points ($m \geq 1$) and symmetry number S_j .

The expression for the two-body radial distribution function $G(r_{1\Lambda})$ (eq. (10)) can be derived from eq. (A11) by considering an appropriate δ -function as operator $\bar{0}$.

The evaluation of the mean value of a space-exchange operator $\bar{0}^X = \sum_{i=1}^A \chi(i\Lambda) P(i\Lambda)$ is performed assuming that the space-exchange operator $P(i\Lambda)$ acts only on the model wave function. This leads to the expression:

$$\bar{0}^X = A \frac{\int d\tau^A d\bar{r}_\Lambda \prod_{i < j=2}^A f_{nn}^2(r_{ij}) \prod_{i=2}^A f_{n\Lambda}^2(r_{i\Lambda}) \Phi^* \tilde{\chi}(r_{1\Lambda}) P(1\Lambda) \Phi}{\int d\tau^A d\bar{r}_\Lambda |\Psi^{(A+\Lambda)}|^2}. \quad (A13)$$

The cluster expansion of $\bar{0}^X$ can be easily derived by using the preceding procedure. It must be noted that the function $D(r_1, \dots, r_m)$ which appears in eq. (A3) must be replaced by the function $D'(r_1, \dots, r_m, r_\Lambda)$ given by:

$$D'(r_1, \dots, r_m, r_\Lambda) = \frac{1}{(A-m)!} V^{A-m} \int d\bar{r}_{m+1} \dots d\bar{r}_A \Phi^* P(1\Lambda) \Phi. \quad (A14)$$

It is useful to observe the correspondence between the terms of $D(r_1, \dots, r_m)$ and $D'(r_1, \dots, r_m, r_\Lambda)$: each term T , in D , which does not involve the coordinate r_1 is replaced, in D' , by the product $T' = l(k_{fr_{1\Lambda}})T$: each factor $-l^2(k_{fr_{1j}})/4$ in D , is replaced in D' , by the product $-1/4 l(k_{fr_{1j}})l(k_{fr_{j\Lambda}})$, and each product $(-8)(-1/4l(k_{fr_{1j}})) \dots (-1/4l(k_{fr_{1i}}))$ by the sum of products $(-4)(-1/4l(k_{fr_{1j}})) \dots (-1/4l(k_{fr_{i\Lambda}})) + (-4)(-1/4l(k_{fr_{1i}})) \dots (-1/4l(k_{fr_{j\Lambda}}))$.

It results that:

$$\bar{0}^X = \sum_m \frac{\rho^m}{V} \sum_j \frac{1}{S_j} Y_m^j, \quad (A15)$$

where Y_m^j is associated to an irreducible diagram with $m+1$ points ($m \geq 1$) and symmetry number S_j .

The expression for the two-body radial distribution function $G^X(r_{1\Lambda})$ (eq. (11)) can be derived from eq. (A15) by considering an appropriate δ -function as operator $\bar{0}^X$.