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P. D'Agostino, V. D'Amico, G. Fazio and F. Mezzanares :

KINEMATIC OF THE $P+T \rightarrow A_i + A_{j-k} \rightarrow A_1 + A_2 + A_3$ TYPE REACTIONS IN THE RCS.

284

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P. D'Agostino[(+)], V. D'Amico[(*)], G. Fazio[(*)] and F. Mezzanares[(*)]: KINEMATIC OF THE $P+T \longrightarrow A_i+A_{j-k} \longrightarrow A_1+A_2+A_3$ TYPE REACTIONS IN THE RCS[(o)].

1. - INTRODUCTION.

In this paper we show a kinematic study of the reactions with three bodies in the final state

$$P + T \longrightarrow A_1 + A_2 + A_3 \tag{1}$$

useful for the determination of the best experimental conditions. First , we apply the classical mechanics[1, 2, 3] in the laboratory system (LS), then we transform the expressions in the system of the relative coordinates (RCS); in the latter system obtained results are to be read.

2. - THE KINEMATIC CURVE IN THE (LS).

If one applies the conservation principles

$$E_p + Q = E_1 + E_2 + E_3$$

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$$

to the (1) one obtains

$$a_1 E_1 + a_2 E_2 + 2c_{12} E_1^{1/2} E_2^{1/2} - 2c_1 E_1^{1/2} - 2c_2 E_2^{1/2} - b = 0 \tag{2}$$

where the quantities concerning the undetected particle 3 are written as a function of the other two (1 and 2), and

$E_p, m_p, \vec{p}, m_t$ are referred to the projectile and to the target respectively
$E_i, m_i, \vec{p}_i$ (i=1, 2) are referred to the i-th emitted particle

$a_i = m_i + m_3$
$c_i = (m_i m_p E_p)^{1/2} \cos \theta_i$
$c_{12} = (m_1 m_2)^{1/2} \cos \theta_{12}$

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$\cos\theta_{12} = \cos\theta_1 \cos\theta_2 + \mathrm{sen}\,\theta_1\,\mathrm{sen}\,\theta_2 \cos(\phi_1 - \phi_2)$

$b = m_3 Q + (m_3 - m_p) E_p$

$M = m_p + m_t \cong m_1 + m_2 + m_3$

Equation (2) represents an ellipse in the $E_1^{1/2}$ $E_2^{1/2}$ plane because

$$a_1 a_2 - c_{12}^2 = m_3 M + m_1 m_2\,\mathrm{sen}^2\theta_{12} > 0$$

The physical part is restricted to the part of the ellipse where both $E_1^{1/2}$ and $E_2^{1/2}$ are positive (they are in fact proportional to the momenta abs. val. $E_i^{1/2} = (2m_i)^{-1/2} p_i$).
The physical solutions of (2) can be obtained from

$$E_j^{1/2} = \frac{a_{ji} \pm (a_{ji}^2 + a_j b_i)^{1/2}}{a_j} \qquad (3)$$

where

$$a_{ji} = c_j - c_{12} E_i^{1/2} \quad ; \quad b_i = b + 2c_i E_i^{1/2} - a_i E_i$$

for each physical value of $E_i^{1/2}$

2.1. - <u>The sequential decay.</u>

Let (1) proceed by sequential decay with formation of an intermediate system

$$\left.\begin{array}{ll} P + T \longrightarrow A_1 + A_{2,3} & \\ \longrightarrow A_2 + A_{1,3} & \\ \longrightarrow A_3 + A_{1,2} & \end{array}\right\} \longrightarrow A_1 + A_2 + A_3 \qquad \begin{array}{r}(4a)\\(4b)\\(4c)\end{array}$$

In case (4a) for a definite internal energy $E_{2,3}$ of the bound state, one can obtain only two allowed values

$$E_{1S}^{1/2} = \frac{c_1 \pm (c_1^2 + M\, b_{2,3})^{1/2}}{M}$$

where $b_{2,3} = m_{2,3} Q_{1,23} + (m_{2,3} - m_p) E_p$

$m_{2,3}$ = mass of the intermediate system

$Q_{1,23}$ = Q of (4a)

For the two allowed values $E_{1S}^{1/2} > 0$ one can draw two straight lines at the most. They are parallel to the E_2 axis and their intersections with (2) give the points all around which the coincidences A_1, A_2 will accumulate.

Obviously, if the level with internal energy E_{2-3} has a finite width Γ , one must consider the intersections of a family of parallel straight lines and the coincidences will accumulate on a strip around the kinematic curve. Similarly for (4b), one can obtain, at the most, four points of intersection related to E_{1-3} = const sequential decay.

E_{1-2} = const hypothesis (4c) leads to the conic

$$(m_{1,2}-m_1)E_1+(m_{1,2}-m_2)E_2-2c_{12}E_1^{1/2}E_2^{1/2}-b_{1,2} = 0 \qquad (5)$$

with

$$b_{1,2} = m_{1,2}Q_{1,2} \quad ; \quad Q_{1,2} = Q \text{ of the } A_{1,2} \longrightarrow A_1 + A_2 \qquad (6)$$

equation (5) is an allipse because

$$(m_{1,2}-m_1)(m_{1,2}-m_2)-m_1m_2\cos^2\theta_{12} = m_{1,2}Q_{1,2}+m_1m_2\text{sen}^2\theta_{1,2} > 0$$

being $Q_{1,2} > 0$ if the decay (6) takes place.

The coincidences of the A_1, A_2 sequential decay will accumulate around the points of the intersection of the ellipse (2) with (5).

2. 2. - The kinematic curve in the E_1, E_2 plane.

Generally, from (2) one obtains in the E_1, E_2 plane a quartic, whose points can be directly obtained by squaring the positive values of (3), taking into account the limits of E_1 for various ellipse positions. The quartic physical part (Fig. 1) obviously coincides with the analogous part of the ellipse.

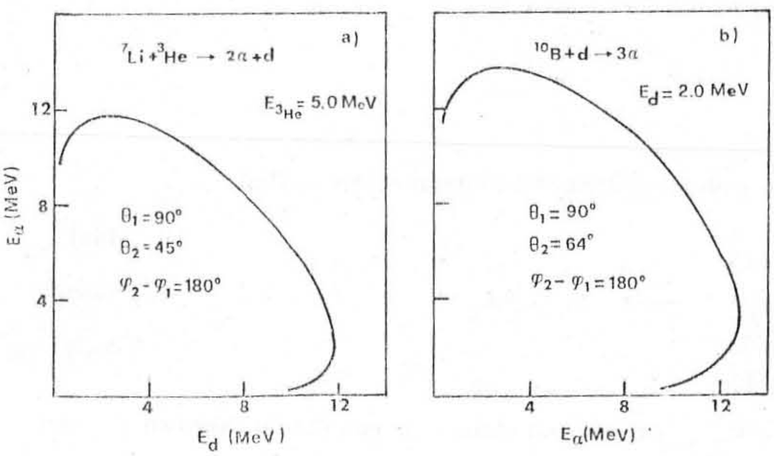

FIG. 1 - Kinematic curves of the reactions of the $P+T \rightarrow A_1+A_2+A_3$ type.

The coincidences in the E_1, E_2 plane, on account of the finiteness of angular and energetic resolution, will accumulate in a strip around the kinematic curve (θ_1, θ_2) and can be attributed to this curve by a special method[1].

Therefore, from a bidimensional spectrum $N(E_1, E_2)$ we go to one N(s) as a function of the curvilinear abscissa

$$s = \int_{E_{i_o}}^{E_i} \left[1 + \left(\frac{dE_j}{dE_i}\right)^2\right]^{1/2} dE_i = \int_{E_{i_o}}^{E_i} \left(1 + \frac{E_j \Delta_j}{E_i \Delta_i}\right)^{1/2} dE_i \qquad (7)$$

with $\Delta_i = a_{ji}^2 + a_j b_i$

Since the integrand of (7) diverges, in the inversion points, one must choose on the kinematic curve, a point C beyond which the integration will be inverted:

$$s = \int_{E_{j_o}}^{E_j} \left[1 + \left(\frac{dE_i}{dE_j}\right)^2\right]^{1/2} dE_j$$

3. - THE RELATIVE SYSTEM.

The usefulness has been pointed out[1, 2] of the study of the reaction (1) in the relative system and the importance of their choice as far as sequential decay to be studied is concerned.

Having defined a reference system, we give the explicit expression of the involved quantity (Fig. 2)

$$p_{i-jk} = \mu_{i-jk} \left[\frac{\vec{p}_i}{m_i} - \frac{1}{m_i + m_k} (\vec{p}_j + \vec{p}_k) \right] = \vec{p}_i - \frac{m_i}{M} \vec{p}$$

being the momentum of the "i" particle in CMS is

$$\begin{cases} p^2_{i-jk} = \frac{2m_i}{M^2} \left[M^2 E_i + m_i m_p E_p - 2Mc_i E_i^{1/2} \right] \\ E_{i-jk} = \frac{p^2_{i-jk}}{2\mu_{i-jk}} \\ \mu_{i-jk} = \frac{m_i(m_j + m_k)}{M} \\ \phi_{i-jk} = \phi_i \\ \mathrm{tang}\ \theta_{i-jk} = \frac{p_i \mathrm{sen} \theta_i}{p_i \cos\theta_i - (m_i/M)p} \end{cases}$$

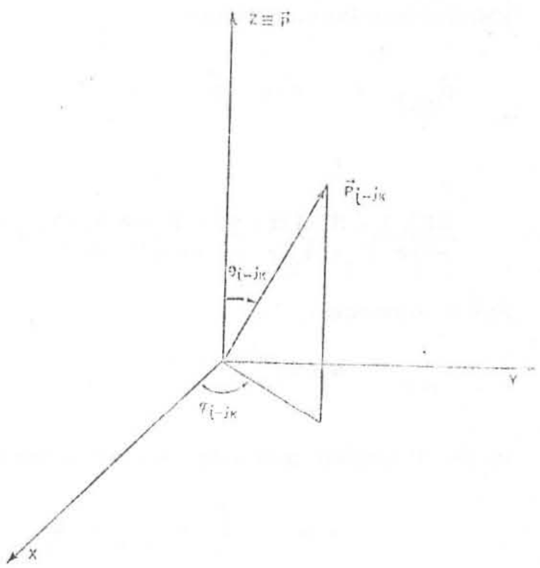

FIG. 2 - p_{i-jk} ; θ_{i-jk} ; ϕ_{i-jk} in the S system.

while

$$\vec{p}_{j-k} = \mu_{j-k} \left[\frac{\vec{p}_j}{m_j} - \frac{\vec{p}_k}{m_k} \right] = \vec{p}_j + \frac{m_j}{m_j + m_k} \left[\vec{p}_i - \vec{p} \right]$$

being the momentum of the "j" particle in the j-k CMS and E_{j-k} the internal energy of the j-k system, we have

$$E_{j-k} = \frac{m_t}{M} E_p + Q - E_{i-jk} \quad ; \quad p_{j-k} = (2\mu_{j-k} E_{j-k})^{1/2} \quad ; \quad \mu_{j-k} = \frac{m_j m_k}{m_j + m_k}$$

$$\mathrm{tang}\ \phi_{j-k} = \frac{p_j \mathrm{sen}\ \theta_j \mathrm{sen}\ \phi_j + d_i p_i \mathrm{sen}\ \theta_i \mathrm{sen}\ \phi_i}{p_j \mathrm{sen}\ \theta_j \cos\ \phi_j + d_i p_i \mathrm{sen}\ \theta_i \mathrm{sen}\ \phi_i}$$

$$\mathrm{tang}\ \theta_{j-k} = \frac{\left[p_j^2 \mathrm{sen}^2 \theta_j + d_i^2 p_i^2 \mathrm{sen}^2\ \theta_i + 2d_i p_i p_j \mathrm{sen}\ \theta_i \mathrm{sen}\ \theta_j \cos(\phi_j - \phi_i) \right]^{1/2}}{p_j + d_i (p_i \cos\ \theta_i - p)}$$

$$d_i = \frac{m_j}{m_j + m_k}$$

The comparison between the sings of tang ϕ_{j-k} and of the numerator (or denominator) gives the sign of sen ϕ_{j-k} (or cos ϕ_{j-k}) and enables us to define ϕ_{j-k} $(0 \div 2\pi)$.

4. - THE RELATIVE ANGLES.

It seems suitable to refer the angular correlations of the products of the j-k complex decay, to a S' system (Fig. 3) rotated with respect to the S system, assuming as a polar axis

$$\vec{z} = \vec{p}_{jk-i} = - \vec{p}_{i-jk}$$

and assuming for the azimuthal angles the reference semiplane containing

$$\vec{p}_{jk-i} \quad \text{and} \quad \vec{p}$$

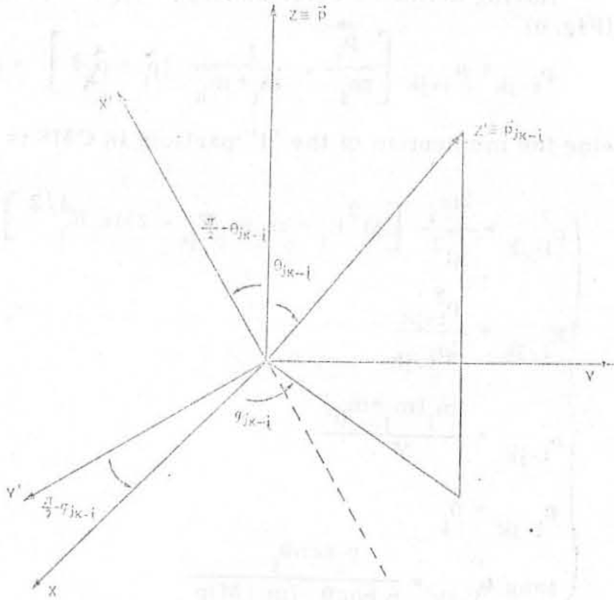

FIG. 3 - RCS for $P + T \longrightarrow A_i + A_{j-k} \longrightarrow A_1 + A_2 + A_3$ reactions.

Since, obviously,

$$\theta_{jk-i} = \pi - \theta_{i-jk} \qquad ; \qquad \phi_{jk-i} = \pi + \phi_{i-jk}$$

is the S' system and with respect to the already defined angles we have

$$\cos\theta_r = - \left[\cos\theta_{j-k} \cos\theta_{i-jk} + \operatorname{sen}\theta_{j-k} \operatorname{sen}\theta_{i-jk} \cos(\phi_{j-k} - \phi_{i-jk}) \right]$$

$$\operatorname{tang}\phi_r = \frac{\operatorname{sen}\theta_{j-k} \operatorname{sen}(\phi_{j-k} - \phi_{i-jk})}{\cos\theta_{j-k} \operatorname{sen}\theta_{i-jk} - \operatorname{sen}\theta_{j-k} \cos\theta_{i-jk} \cos(\phi_{j-k} - \phi_{i-jk})}$$

it is enough to define ϕ_r $(0 \div 2\pi)$ to make the same consideration as for ϕ_{j-k}

5. - THE JACOBIAN OF THE TRANSFORMATION.

The density N(s) obtained as a function of the curvilinear abscissa along the kinematic curve, will be referred to RCS by the suitable Jacobian of transformation. Obviously

$$N(E_i, \Omega_i, \Omega_j)\, dE_i\, d\Omega_i\, d\Omega_j = N(s) \frac{ds}{dE_i}\, dE_i\, d\Omega_i\, d\Omega_j =$$

$$= N(E_{i-jk}, \Omega_{i-jk}, \Omega_{j-k})\, dE_{i-jk}\, d\Omega_{i-jk}\, d\Omega_{j-k}$$

then

$$N(E_{i-jk}, \Omega_{i-jk}, \Omega_{j-k}) = J_{i-jk}\, N(s)$$

with

$$J_{i-jk} = \frac{\partial\,(E_i, \Omega_i, \Omega_j)}{\partial\,(E_{i-jk}, \Omega_{i-jk}, \Omega_{j-k})} \left| \frac{\delta s}{\partial E_i} \right|$$

Since it is possible

$$\frac{\delta\,(\vec{p}_{i-jk}, \vec{p}_{j-k})}{\delta\,(\vec{p}_i, \vec{p}_j)} = 1$$

it will be

$$p^2_{i-jk}\, dp_{i-jk}\, p^2_{j-k}\, dp_{j-k}\, d\Omega_{i-jk}\, d\Omega_{j-k} = p_i^2\, dp_i\, p_j^2\, dp_j\, d\Omega_i\, d\Omega_j$$

and then

$$J_{i-jk} = \frac{\mu_{i-jk}\, p_{i-jk}\, p^2_{j-k}\, dp_{j-k}}{m_i\, p_i^2\, p_j^2\, dp_j} \left| \frac{\partial s(E_i, E_j)}{\partial E_i} \right| =$$

$$= \left(\frac{m_k}{M^3}\right)^{1/2} \frac{(E_{i-jk} E_{j-k})^{1/2}}{E_i E_j} \left(E_i \Delta_i + E_j \Delta_j \right)^{1/2}$$

If "i" and "j" are the detected particles and "i" the first emitted one. In(Fig. 4) we show the Jacobians vs. the curvilinear abscissa for some reactions of the $P + T \longrightarrow A_1 + A_2 + A_3$ type.

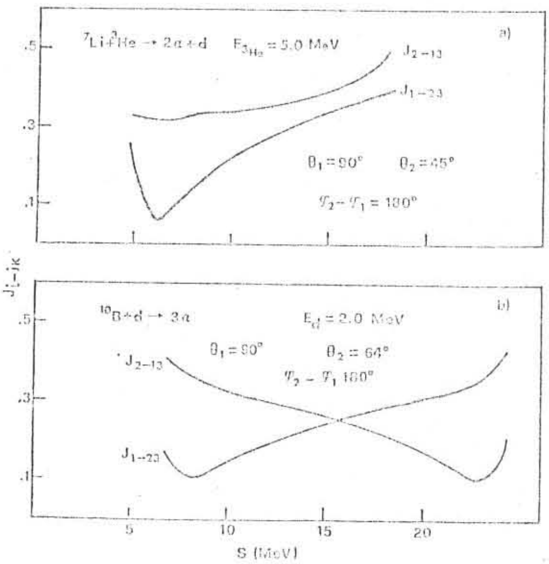

FIG. 4 - Jacobians of the transformation vs. curvilinear abscissa.

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