ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Pisa

INFN/BE-78/7
13 Dicembre 1978

S. Rosati and S. Fantoni: A STRAIGTHFORWARD DERIVATION OF THE FHNC EQUATIONS.

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1. - INTRODUCTION.

The study of the many particle Fermi systems has received a renewed interest in the frame of a variational theory based on Jastrow type[1] trial functions. The accuracy and the details of the quantities calculated so far are comparable with the corresponding ones in the case of Bose systems and all the computational techniques developed for the last case look very suitable to be extended to Fermi systems.

It is well known that the distribution functions find important applications in relation to the many particle systems; the n-body distribution function $\varrho^{(n)}(\vec{r}_1, \ldots, \vec{r}_n)$ is defined as the probability density of finding a set of n particles with coordinates $\vec{r}_1, \ldots \vec{r}_n$ irrespective of the configuration of the remaining particles. By definition, it holds the relation

$$g^{(n)}(\vec{r}_1, \ldots, \vec{r}_n) = \frac{1}{\varrho^n}\,\varrho^{(n)}(\vec{r}_1, \ldots, \vec{r}_n) = \frac{N!}{(N-n)!\,\varrho^n}\,\frac{\int d\vec{r}_{n+1} \ldots d\vec{r}_N \,|\psi(1, \ldots, N)|^2}{\int d\vec{r}_1 \ldots d\vec{r}_N \,|\psi(1, \ldots, N)|^2}, \qquad (1)$$

where ϱ denotes the density and $\psi(1, \ldots N)$ is the wave function of the considered N-particle system. The function $g^{(2)}(1, 2)$ is the most widely used, but even the function $g^{(3)}(1, 2, 3)$ is of relevant interest in many problems so that various approximations have been developed in order to evaluate these two functions.

In the case of the ground state of a Bose system described by a Jastrow function one can readily use the methods developed for the classical systems of many molecules[2]. Some of these methods have been generalized to the case of Fermi systems by Fantoni and Rosati[3] who have derived a set of integral equations (FHNC equations) for calculating the function $g^{(2)}(1, 2)$. The derivation requires a careful analysis of the terms which appear in the cluster expansion and a rather involved procedure for summing up all these terms in an iterative way. Due to the wide use which has been recently made of the FHNC equations, another easy derivation of these equations could be useful which is actually the aim of the present note.

2. - BOSE AND CLASSICAL SYSTEMS.

In this section we shall briefly recall some results related to the distribution functions for the following two cases:

i) The system is an infinite Bose system in its ground state and it is described by a Jastrow function

$$\psi(1, \ldots, N) = \prod_{i<j=1}^{N} f(r_{ij}), \tag{2}$$

where the correlation factor f(r) goes rapidly to 1 as r increases.

ii) The system is an one-component classical system the molecules of which are treated as mass points. The relation (1) can still be used after the substitution

$$\left| \psi(1, \ldots, N) \right|^2 \longrightarrow \exp\left(- \frac{1}{kT} \sum_{i \neq j=1}^{N} u(i, j)\right), \tag{3}$$

where T is the temperature and u(i, j) represents the potential energy of the molecules i and j.

Using the relation (2) eq. (1) can be written as

$$g^{(n)}(\vec{r}_1, \ldots, \vec{r}_n) = \frac{N!}{(N-n)!\,\varrho^n} \frac{\int d\vec{r}_{n+1} \cdots d\vec{r}_N \prod_{i<j=1}^{N} f^2(r_{ij})}{\int d\vec{r}_1 \cdots d\vec{r}_N \prod_{i<j=1}^{N} f^2(r_{ij})} . \tag{4}$$

For a classical system one has to identify $f^2(r_{ij})$ with $\exp(-u(i, j)/kT)$. A commonly used procedure for calculating the distribution functions for infinite systems consist on substituting all the $f^2(r_{ij})$ factors which appear in (4) by

$$f^2(r_{ij}) = 1 + h(r_{ij}) \quad , \tag{5}$$

then expanding in terms of the function h and finally taking the limit for $N \longrightarrow \infty$ at fixed density ϱ. The terms of the cluster expansion can be easily specified and classified in a graphical description by n-body diagrams. A general n-body diagram is characterized by the following elements:

a) The coordinates of a particle are represented by a small circle ("point"); a solid circle, denoted as "internal point", involves the integration over the corresponding coordinates and a density factor ϱ; an open circle indicates the coordinates of a particle in the set 1, ..., n and is called "external point".

b) The correlation $h(r_{ij})$ between the particles i and j is represented by a solid line connecting the particles i and j.

c) Two points may be connected by one correlation line at most and each point must be the extremity of at least one solid line.

It can be proved[4] that the cluster expansion of $g^{(n)}$ is irreducible, which means that the allowed diagrams with n external points are irreducible. (+)

A number of important properties of the N-particle system are directly related to the two-body function $g^{(2)}(\vec{r}_1, \vec{r}_2)$ which reduces to a function $g(r_{12})$ (the so called radial distribution function) in the case of an infinite system. The exact radial distribution function satisfies an integral

equation known as the convolution equation, which can be obtained by various procedures. For instance if one uses the methods of the cluster expansion all the diagrams of the expansion have to be constructed and calculated by an iterative procedure. The generical k-th step is characterized by the two following operations [5]:

- on the basis of the nodal i-j subdiagrams obtained in the preceding step the most general corrispondent set $\alpha^{[k]}$ of i-j subdiagrams which do not contain nodal points is derived;
- the set of all the nodal i-j subdiagrams which have $\alpha^{[k]}$ as chain element is obtained and calculated by means of the relation

$$G^{[k]}(r_{ij}) = \varrho\int\alpha^{[k]}(r_{il})\,\alpha^{[k]}(r_{lj})\,d\vec{r}_1 + \varrho^2\int\alpha^{[k]}(r_{il})\,\alpha^{[k]}(r_{lm})\,\alpha^{[k]}(r_{mj})\,d\vec{r}_1\,d\vec{r}_m + \ldots. =$$

$$= \varrho\int\alpha^{[k]}(r_{il})\left\{\alpha^{[k]}(r_{lj}) + G^{[k]}(r_{lj})\right\}d\vec{r}_1 = (\alpha^{[k]}(r_{il})\,|\,\alpha^{[k]}(r_{lj}) + G^{[k]}(r_{lj})) \quad , \tag{6}$$

where $(a(r_{il}) \,|\, b(r_{lj}))$ denotes the convolution integral

$$(a(r_{il}) \,|\, b(r_{lj})) = \varrho\int a(r_{il})\,b(r_{lj})\,d\vec{r}_1 \quad . \tag{7}$$

In terms of $G^{[k]}(r_{ij})$ one has

$$\alpha^{[k]}(r_{ij}) = f^2(r_{ij})\exp\left\{G^{[k]}(r_{ij}) + \varepsilon(r_{ij})\right\} - G^{[k]}(r_{ij}) - 1 \quad , \tag{8}$$

where the function $\varepsilon(r_{ij})$ represents the contribution due to all the elementary i-j subdiagrams. In the limit $k \longrightarrow \infty$ of the iterative procedure one obtains

$$\begin{aligned} &\alpha(r_{12}) = f^2(r_{12})\exp\left\{G(r_{12}) + \varepsilon(r_{12})\right\} - G(r_{12}) - 1, \\ &G(r_{12}) = (\alpha(r_{13})\,|\,\alpha(r_{32}) + G(r_{32})) \quad . \end{aligned} \tag{9}$$

Then the radial distribution function given by

$$g(r_{12}) = 1 + \alpha(r_{12}) + G(r_{12}) = f^2(r_{12})\exp\left\{G(r_{12}) + \varepsilon(r_{12})\right\} \quad , \tag{10}$$

satisfies the following exact convolution equation [6]

$$\ln \frac{g(r_{12})}{f^2(r_{12})} = \varepsilon(r_{12}) + (g(r_{13}) - 1 - \ln \frac{g(r_{13})}{f^2(r_{13})} + \varepsilon(r_{13}) \Big| g(r_{32}) - 1) \quad . \tag{11}$$

In the approximation $\varepsilon(r) = 0$, the preceding integral equation reduces to the well known hypernetted chain (HNC) equation. The relation (6) in the limit $k \longrightarrow \infty$ can be written in the form

$$(\alpha(r_{13}) \Big| G(r_{32})) = G(r_{12}) - (\alpha(r_{13}) \Big| \alpha(r_{32})) \quad , \tag{12}$$

which means that the convolution of α and G reproduces G except $(\alpha | \alpha)$, the simplest of the chain terms which constitute G having α as chain element. This property can be easily generalized to the case of fermion systems and used to obtain the FHNC equations as discussed in the next section.

3. - FERMI SYSTEMS. THE FHNC EQUATIONS.

Let us consider the case of a Fermi system described by the wave function

$$\psi(1, \ldots, N) = \prod_{i<j=1}^{N} f(r_{ij}) \Phi \quad , \tag{13}$$

where Φ is the Slater determinant made out of the N single particle functions of the Fermi sea. The n-body distribution function can be calculated by using eq. (1) and the cluster expansion in terms of $h(r) = f^2(r) - 1$. The various contributions at the different orders of the cluster expansion can be associated [7] to n-body diagrams with n external points and an arbitrary number of correlation lines which represent the correlation factors $h(r_{ij})$, $-\frac{1}{s} 1(k_F r_{ij})$ and $-\frac{1}{s} 1^2(k_F r_{ij})$; s is the degree of degeneracy of the single particle states, $k_F = (6\pi^2 \varrho/s)^{1/3}$ is the Fermi momentum and $1(x) = x^{-3}(\sin x - x\cos x)$.

The involved diagrams have n external points and an arbitrary number of internal points; two generical points i and j can be correlated by a dinamical correlation $h(r_{ij})$ and by one of the statistical correlations $-\frac{1}{s} 1(k_F r_{ij})$ or $-\frac{1}{s} 1^2(k_F r_{ij})$. Each n-body diagram satisfies to the following conditions:

(i) Two points can be connected by one correlation either dynamical or statistical, the latter multiplied or not by a dynamical one.

(ii) Each internal point is reached by a least one dynamical correlation.

(iii) The statistical correlations $-\frac{1}{s} 1(k_F r)$ occur only in closed loops and there are no common points between two loops; each loop contributes with a factor - 2s.

(iv) Each statistical factor $-\frac{1}{s} 1^2(k_F r)$ has no common points with another statistical correlation.

It can be proved [7] that only the irreducible diagrams with n external points are involved by the cluster expansion; they can be calculated in a similar way to the methods used for classical systems. In this section the discussion will be limited to the radial function $g(r_{12})$, i. e. the limit of $g(1, 2)$ for an infinite number of particles.

Due to the presence of different types of correlations functions, namely the dynamical or the

statistical correlations, one can distinguish the three following situations [o] for an external point of an i-j subdiagram:

a) Case s: the point is affected by only dynamical correlations.

b) Case h: the point is either an extremity of one statistical correlation - $\frac{1}{s}1^2$ or a common extremity of two different statistical correlations - $\frac{1}{s}1$.

c) Case d: the point is an extremity of one statistical correlation - $\frac{1}{s}1$.

For a generic i-j subdiagram the following possibilities have to be considered:

$$(s,s)\ ,\qquad (h,h)\ ,\qquad (s,h)\ \text{or}\ (h,s)\ ,\qquad (d,d)\quad . \tag{14}$$

As an example, (s, s) denotes an i-j subdiagram the points i and j of which are both affected by dynamical correlations only; the other classes of subdiagrams specified in the last equation have corresponding properties in relation to the points i and j.

Let us now indicate as $G_{ss}(r_{12})$ the sum of all the 1-2 nodal diagrams of the type (s, s) and as $G_{hh}(r_{12})$, $G_{sh}(r_{12}) = G_{hs}(r_{12})$ and $G_{dd}(r_{12})$ the sums of all the nodal diagrams specified by the corresponding subscripts. Moreover, let us assume that $\alpha(r_{12})$, $\beta(r_{12})$, $\gamma(r_{12})$ and $\delta(r_{12}) - \frac{1}{s}1(k_F r_{12})$ represent the sums of all the not nodal 1-2 diagrams of the type (s; s), (h, h) (s, h) and (d, d), respectively. The reason for subtracting the term $-\frac{1}{s}1$ in the definition of δ is related to the fact that two consecutive $-\frac{1}{s}1$ correlations cannot be present in a chain connection of δ elements, because each internal point must be extremity of at least one dynamical correlation. The functions G can be constructed by means of chain connections of the elements α, β, γ and δ; due to the previously specified digrammatic rules, only the following connections are possible:

- α with α, β or γ ;
- β with α or γ (namely $\beta_{hh}\ \gamma_{sh}$);
- γ with α, β or γ ;
- δ with δ or $-\frac{1}{s}1$.

G_{ss} can be analogously connected to α, β or γ, G_{hh} to α or γ, G_{sh} to α, β or γ and, finally, G_{dd} to δ or $-\frac{1}{s}1(k_F r)$. It is clear that the convolution integral $(\alpha(r_{13}) \mid G_{ss}(r_{32}))$ coincides with a part of $G_{ss}(r_{12})$ but a similar property holds for the integrals $(\gamma_{sh}(r_{13}) \mid G_{ss}(r_{32}))$ and $(\alpha(r_{13}) \mid G_{hs}(r_{32}))$ as well. The sum of these three convolution integrals reproduces $G_{ss}(r_{12})$ a part the simplest chain terms having α, β and γ as elements. The relation which generalizes the equation (12) to the case of Fermi system is

$$G_{ss}(r_{12}) = (\alpha(r_{13}) + \gamma(r_{13}) \mid G_{ss}(r_{32}) + \alpha(r_{32})) + (\alpha(r_{13}) \mid G_{hs}(r_{32}) + \gamma_{hs}(r_{32}))\quad . \tag{15}$$

In a similar way one gets

$$G_{hh}(r_{12}) = (\gamma_{hs}(r_{13}) \mid G_{hh}(r_{32}) + \beta(r_{32})) + (\gamma_{hs}(r_{13}) + \beta(r_{13}) \mid G_{sh}(r_{32}) + \gamma_{sh}(r_{32})), \tag{16}$$

$$G_{sh}(r_{12}) = (\alpha(r_{13}) + \gamma_{sh}(r_{13}) \mid G_{sh}(r_{32}) + \gamma_{sh}(r_{32})) + (\alpha(r_{13}) \mid G_{hh}(r_{32}) + \beta(r_{32}))\quad . \tag{17}$$

If the function $G_{dd}(r_{12})$ is seen as the function which corresponds to the set of all the 1-2 nodal subdiagrams constructed with δ and $-\frac{1}{s}1$, it must be noticed that there can be present more than one $-\frac{1}{s}1$ factor in a generical chain. However, the value of the integral corresponding to the chain does not vary if the order of the chain elements is changed in such way as it is possible to consider two $-\frac{1}{s}1$ factors as consecutive ad to use the relation

$$(-\frac{1}{s}1(k_F r_{il}) \Big| -\frac{1}{s}1(k_F r_{lj})) = \frac{1}{s}1(k_F r_{ij}) \quad . \tag{18}$$

As a consequence, if one considers all together the chains which have a fixed number of δ elements but different numbers of $-\frac{1}{s}1$ elements, only the chains composed by δ elements only and chains with the δ's plus one $-\frac{1}{s}1$ element are left out because of algebraic cancellations. The convolution of G_{dd} and δ reproduces all the terms of G_{dd} which have or have not one statistical element $-\frac{1}{s}1$, a part the two simplest terms arising from the convolution of δ with δ or $-\frac{1}{s}1$. In conclusion one can write:

$$G_{dd}(r_{12}) = (\delta(r_{13}) \Big| G_{dd}(r_{32}) + \delta(r_{32}) - \frac{1}{s}1(k_F r_{32})) \quad . \tag{19}$$

In terms of the functions G the not nodal elements α, β, γ and δ are given by the relations

$$\alpha(r_{ij}) = F(r_{ij}) - G_{ss}(r_{ij}) - 1 \quad ,$$

$$\beta(r_{ij}) = F(r_{ij})\Big\{G_{hh}(r_{ij}) + \varepsilon_{hh}(r_{ij}) + \Big[G_{sh}(r_{ij}) + \varepsilon_{sh}(r_{ij})\Big]^2 - s\Big[G_{dd}(r_{ij}) + \varepsilon_{dd}(r_{ij})\Big]^2 + 2\,1(k_F r_{ij})\Big[G_{dd}(r_{ij}) + \varepsilon_{dd}(r_{ij})\Big] - \frac{1}{s}1^2(k_F r_{ij})\Big\} - G_{hh}(r_{ij}) \quad , \tag{20}$$

$$\gamma(r_{ij}) = F(r_{ij})\Big[G_{sh}(r_{ij}) + \varepsilon_{sh}(r_{ij})\Big] - G_{sh}(r_{ij}) \quad ,$$

$$\delta(r_{ij}) = F(r_{ij})\Big[G_{dd}(r_{ij}) + \varepsilon_{dd}(r_{ij}) - \frac{1}{s}1(k_F r_{ij})\Big] + \frac{1}{s}1(k_F r_{ij}) - G_{dd}(r_{ij}) \quad ,$$

where

$$F(r_{ij}) = f^2(r_{ij}) \exp\{G_{ss}(r_{ij}) + \varepsilon_{ss}(r_{ij})\} \quad , \tag{21}$$

and ε_{mn} indicates the contribution due to all the elementary i-j subdiagrams of the type (mn).

The integral equations (15), (16), (17) and (19), together with the definitions (20) for α, β, γ and δ, constitute the exact convolution equations derived by Fantoni and Rosati for Fermi systems described by state independent Jastrow correlated functions. It is easy to write these equations in the form given in ref. (3). Firstly eq. (16) is written as

$$(D(\vec{r}_{13}) - \gamma(r_{13}) \,\big|\, G_{hh}(r_{32})) = (\gamma(r_{13}) + \beta(r_{13} \,\big|\, G_{sh}(r_{32})) + (\gamma(r_{13}) \,\big|\, \gamma(r_{32}) + 2\beta(r_{32})) \,, \tag{22}$$

where $D(\vec{r}_{13})$ is the three-dimensional delta function; the subscripts of γ have been omitted since $\gamma_{sh} = \gamma_{hs}$. If one calculates the convolution of $D(\vec{r}_{ij}) - \gamma(r_{ij})$ with both the members of eq. (17) and then eliminates the resulting term $(D(r_{ij}) - \gamma(r_{ij}) \,\big|\, G_{hh}(\vec{r}_{ij}))$ by using eq. (22), the equation for $G_{sh}(r_{12})$ can be readily written as in ref. (3). The equations for the other G's are obtained in a quite similar manner and the FHNC equations can be written as

$$\begin{aligned}
G_{ss}(r_{12}) &= (\alpha(r_{13}) + G_{ss}(r_{13}) \,\big|\, P(r_{32})) \,, \\
G_{sh}(r_{12}) &= (\gamma(r_{13}) + G_{sh}(r_{13}) \,\big|\, P(r_{32})) + (\alpha(r_{13}) \,\big|\, \beta(r_{32})) - (\gamma(r_{13}) \,\big|\, \gamma(r_{32})) \,, \\
G_{hh}(r_{12}) &= (\beta(r_{13}) + G_{hh}(r_{13}) \,\big|\, P(r_{32})) - (\alpha(r_{13}) \,\big|\, \beta(r_{32})) + (\gamma(r_{13}) \,\big|\, \gamma(r_{32})) \,, \\
G_{dd}(r_{12}) &= (\delta(r_{13}) - \frac{1}{s} 1(k_F r_{13}) + G_{dd}(r_{13}) \,\big|\, \delta(r_{32})) \,,
\end{aligned} \tag{23}$$

where

$$P(\vec{r}_{ij}) = \alpha(r_{ij}) + 2\gamma(r_{ij}) + (\alpha(r_{il}) \,\big|\, \beta(r_{lj})) - (\gamma(r_{il}) \,\big|\, \gamma(r_{lj})) \,. \tag{24}$$

It must be noticed that the relations (23) and (24) can be easily obtained from eqns. (15), (16), (17) and (19) by expressing the various functions in terms of their Fourier transforms. However, the previously outlined method is more suitable for writing in different but equivalent forms the FHNC equations which can be derived in more complicate situations (finite nuclei, state dependent correlation factors).

4. - OTHER APPLICATIONS OF THE CONVOLUTION TECHNIQUE.

A quantity which has recently received some attention in the study of the ground state of quantum fluids [8, 9] is the one particle density matrix defined as

$$n(\vec{r}_1, \vec{r}'_1) = N \frac{\int d\vec{r}_2 \ldots d\vec{r}_N \, \psi^+(1, 2 \ldots, N) \psi(1', 2, \ldots, N)}{\int d\vec{r}_1 \ldots d\vec{r}_N \left| \psi(1, \ldots, N) \right|^2} \,. \tag{25}$$

When the wave function $\psi(1, \ldots, N)$ is taken to be of the Jastrow form as given in eq. (13), the P. S. cluster expansion of the r. h. s. of eq. (25) leads to an expression for $n(\vec{r}_1, \vec{r}'_1)$ which results

factorizable into the product of a strength factor n and a function $N(r_{1'})$. If the strength factor is written in the form $n = \exp(Q)$, both Q and $N(r_{1'})$, are given as sums of proper irreducible cluster integrals. In order to calculate Q and $N(r_{1'})$, (the full expressions for these two quantities can be found in ref. 9), the functions G_{mn}, α, β, γ and δ given in section 3 are necessary; moreover, one also needs to evaluate other i-j subdiagrams which obey the diagrammatic rules previously specified, but with the exception that the external point i is either affected by only dynamical correlations of the type $\xi(r) = f(r) - 1$ (represented by wavy lines: case w), or is an extremity of a dashed line superimposed or not to a wavy line (case D). The convolution technique outlined in the preceding section can readily be applied to derive the expression of the functions $G_{ws}(r_{ij})$, $G_{wh}(r_{ij})$ and $G_{Dd}(r_{ij})$ corresponding to the i-j nodal subdiagrams of the types (ws), (wh) and (Dd), respectively. As an example, it is now clear that the sum of the three convolution integrals $(\alpha_{ws}(r_{ik}) \mid G_{ss}(r_{kj}))$, $(\alpha_{ws}(r_{ik}) \mid G_{hs}(r_{kj}))$ and $(\gamma_{wh}(r_{ik}) \mid G_{ss}(r_{kj}))$ gives $G_{ws}(r_{ij})$ a part the two-side chain terms having α_{ws}, γ_{wh} and α, β, γ as elements, i. e.

$$G_{ws}(r_{ij}) = (\alpha_{ws}(r_{ik}) + \gamma_{wh}(r_{ik}) \mid \alpha(r_{kj}) + G_{ss}(r_{kj})) + (\alpha_{ws}(r_{ik}) \mid \gamma_{hs}(r_{kj}) + G_{hs}(r_{kj})) . \tag{26}$$

In a similar way one obtains

$$G_{wh}(r_{ij}) = (\alpha_{ws}(r_{ik}) + \gamma_{wh}(r_{ik}) \mid \gamma_{sh}(r_{kj}) + G_{sh}(r_{kj})) + (\alpha_{ws}(r_{ik}) \mid \beta(r_{kj}) + G_{hh}(r_{kj})) , \tag{27}$$

$$G_{Dd}(r_{ij}) = (\delta_{Dd}(r_{ik}) \mid \delta(r_{kj}) - \frac{1}{s}1(k_F r_{kj}) + G_{dd}(r_{kj})) . \tag{28}$$

The not nodal elements α_{ws}, γ_{wh} and δ_{Dd} are given by the relations

$$\begin{aligned}
\alpha_{ws}(r_{ij}) &= F_{\xi}(r_{ij}) - G_{ws}(r_{ij}) - 1 \\
\gamma_{wh}(r_{ij}) &= F_{\xi}(r_{ij})(G_{wh}(r_{ij}) + \varepsilon_{wh}(r_{ij})) - G_{wh}(r_{ij}) \\
\delta_{Dd}(r_{ij}) &= F_{\xi}(r_{ij})(G_{Dd}(r_{ij}) + \varepsilon_{Dd}(r_{ij}) - \frac{1}{s}1(k_F r_{ij})) - G_{Dd}(r_{ij}) + \frac{1}{s}1(k_F r_{ij}) ,
\end{aligned} \tag{29}$$

where

$$F_{\xi}(r_{ij}) = f(r_{ij}) \exp\left\{G_{ws}(r_{ij}) + \varepsilon_{ws}(r_{ij})\right\} . \tag{30}$$

It is straightforward to see that these equations are equivalent to those given in ref. (9).
It is wortwhile to notice that even in the case of a Jastrow calculation of the Λ_o binding energy in the nuclear matter [10] one is faced with a set of i-j subdiagrams which obey the same rules specified in the present section: the dynamical correlation $\xi(r)$ is then given by $f^2_{\Lambda N}(r)$, where $f_{\Lambda N}(r)$ is the Λ_o-N correlation factor, and the factor $f(r_{ij})$ in eq. (30) must be replaced by $f^2_{\Lambda N}(r_{ij})$.

5. - CONCLUSIONS.

The purpose of the present note is to illustrate the ingredients which are significative to derive the FHNC equations; the followed procedure is very simple and does not require a careful and lengthy summation of diagrams. The convolution technique has been applied in detail to the case of Fermi systems described by state independent Jastrow functions but it is also useful for calculating other interesting quantities such as, for example, the one-body density matrix. Moreover an analogous procedure can be easily developed to treat more complicate problems such as in the case of finite nuclei, or of the infinite nuclear matter described by state dependent Jastrow correlation functions. These arguments will be the object of a following paper.

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(2) - See for example A. Münster, Statistical Thermodynamics (Academic Press, 1969), Vol. 1.

(3) - S. Fantoni and S. Rosati, Nuovo Cimento 25A, 593 (1975).

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(+) - A diagram is irreducible if it is not constituted by two parts which have only one common point and one of the parts contains all the external points.

(5) - See ref. (2), chapter X.

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(o) - The notations s, h and d are based on the diagrammatic representation as adopted in ref. (7) where the dynamical correlation h(r) is represented by a solid line, whilst a helical line and a dashed line are associated to the statistical correlations $-\frac{1}{s}l^2(k_F r)$ and $-\frac{1}{s}l(k_F r)$, respectively.

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