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COMPUTER SIMULATION OF QUASI-FREE REACTIONS.

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1. INTRODUCTION.

Quasi-free (QF) scattering and reactions are often used as a tool of investigation of the cluster structure[1-8]. Fig. 1 shows a schema-

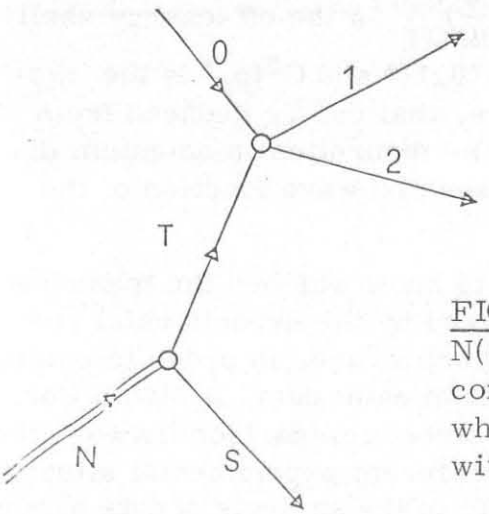

FIG. 1 - Diagram for the quasi-free N(0, 12)S reaction; the particles S is considered spectator of the process, while the incident particle 0 interacts with cluster T.

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tic diagram for the N(0, 12)S reaction. The target nucleus N is assumed decomposed, in the lower vertex, in the clusters S+T, where S is considered spectator of the T(0, 1)2 virtual reaction.

The conservation laws require that:

$$\begin{aligned} E_0 + Q &= E_1 + E_2 + E_S \\ \vec{p}_0 &= \vec{p}_1 + \vec{p}_2 + \vec{p}_S \end{aligned} \qquad (1)$$

Coincidence detection of particles 1 and 2 usually gives a bidimensional E_1-E_2 spectrum, in which the events fall along a kinematical locus defined by (1). The spread of these events around the theoretical curve depends on different resolution effects (finite detector geometry, target thickness, beam size, ...).

When QF data are interpreted in the framework of the plane wave impulse approximation (PWIA), the differential cross-section can be expressed as follows[9]:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = (KF) \cdot \left(\frac{d\sigma}{d\Omega}\right)^{CM}_{OT} \cdot G^2(p_S) \qquad (2)$$

where (KF) is a kinematical factor, $\left(\frac{d\sigma}{d\Omega}\right)^{CM}_{OT}$ is the off-energy shell cross-section of the virtual reaction T(0, 1)2 and $G^2(p_S)$ is the momentum distribution of the two clusters, that can be deduced from experimental data and compared with the theoretical momentum distribution computed by means of the assumed wave function of the target nucleus.

It is very important in many cases to know whether the measured momentum distribution has been distorted by the experimental resolution effects, and how large these distortion are, in order to compare the theoretical model with the experimental data. A Monte Carlo simulation of the experiment allows a direct comparison between theory and experiment, as well as among different experimental situations. Such comparison is very useful not only in the analysis of data already extracted from a particular experiment but also in the evaluation of the necessary conditions to which the experimental set-up must satisfy to get a selected precision.

In this paper we describe a simulation technique including the energy losses of the beam and of the outgoing particles in the target and those effects due to the finite geometry of the two detectors in coincidence.

Basically the following procedure has been used: a random value, according to the chosen $G^2(\vec{p}_S)$, is extracted for the momentum $\vec{p}_S$ of the spectator particle. Then a direction (θ, ϕ) is selected, random distributed within the limits of the detectors. By choosing also the interaction depth along the target and the corresponding value for the reaction energy, one is able to determine completely, from kinematic considerations, the final state of the three body system. If the directions of the two momenta $\vec{p}_1$, $\vec{p}_2$ give rise to a detectable event for the coincidence, the event is stored, otherwise not; the entire process is then repeated until a sufficiently good statistics is achieved.

2. - DETAILS OF THE METHOD.

2. 1. - Starting momentum distribution.

When PWIA is used in the analysis of QF data, the factor $G^2(p_S)$ is simply proportional to the square of the Fourier transform of the intercluster wave function χ_{rel} [10]:

$$G(p_S) \propto \int e^{-i \vec{p}_S \cdot \vec{r}/\hbar} \chi_{rel}(\vec{r})\, d\vec{r} \qquad (3)$$

By expanding the plane wave in terms of the Bessel functions and the Legendre polynomials, and expressing the intercluster wave function χ_{rel} by means of its radial part and the spherical harmonics, it can be shown that, under suitable conditions, the following relation holds:

$$G_\ell(p_S) \propto \int_0^\infty R(r) \cdot j_\ell(p_S r)\, r^2\, dr \qquad (4)$$

where R(r) is the radial part of χ_{rel} and j_ℓ is the Bessel function of ℓ-th order, where ℓ is the assumed value for the relative motion of the two cluster in the target nucleus.

The Bessel functions are univocally determined by the ℓ-value, their expression being

$$j_\ell(p_S r) = (-\frac{r}{p_S})^\ell (\frac{1}{r}\frac{d}{dr})^\ell (\frac{\sin p_S r}{p_S r}) \qquad (5)$$

The momentum distribution is then determined by the choice of the radial part of the intercluster wave function, R(r). Various different

expressions have been considered for light nuclei: for instance:

i) Hänkel function with cut-off

$$R(r) = \begin{cases} 0 & r \leq R_c \\ e^{-Kr}/r & r > R_c \end{cases} \qquad (6)$$

where R_c is the cut-off radius and $K=(2\mu B)^{1/2}/\hbar$ is the wave number related to the binding energy B of the system, whose reduced mass is μ.

ii) Eckart functions of order n.

Their general expression is given by:

$$R(r) = \left[1 - \exp(-br)\right]^{n+1} e^{-Kr}/r \qquad (7)$$

where b is a parameter connected to the FWHM value of the resulting momentum distribution and K has the same meaning as before. The main difference between Eckart function with n=0 and $n \geq 1$, lies in their shape at small intercluster distance, the Eckart function with $n \geq 1$ being similar each other.

The program integrates eq. (4) from r_{min} to r_{max} by means of the Romberg's method[11], with an accuracy that can be selected by the user.

2. 2. - Interaction energy.

To simulate the effect of the (solid) target thickness on the experimental results, the following form has been chosen for the interaction probability as a function of the thickness Δx:

$$f(\Delta x) = \exp(-\eta \cdot \Delta x) \qquad (8)$$

where η is a parameter that can be fixed according to the nature of the process. For each event the interaction energy E_0 is determined by computing the energy loss of the incident particle in traversing a thickness Δx (see sect. 2. 4).

2. 3. - Kinematics.

The final state of the system is determined by the nine quantities $\vec{p}_1, \vec{p}_2, \vec{p}_S$. The conservation laws reduce to five these variables; then if we fix five of the nine variables, we are able to determine completely the state.

Once $\vec{p}_S$ has been extracted according to the theoretical momentum distribution $G^2(\vec{p}_S)$, and a direction (θ_i, ϕ_i) is uniformly chosen within the limits of detector i, the program determines p_i from the second order equation:

$$A\, p_i^2 + B\, p_i + C = 0$$

where

$$\begin{aligned} A &= \frac{1}{2}\,(1/m_1 + 1/m_2) \\ B &= \Big[\, p_S \operatorname{sen}\theta_S \cos\phi_S \operatorname{sen}\theta_i(\cos\phi_i + \operatorname{sen}\phi) \\ &\quad + p_S \cos\theta_S \cos\theta_i - p_o \cos\theta_i \,\Big]\, /m_j \\ C &= \frac{1}{2}\, p_S^2\,(1/m_j + 1/m_S) + \frac{1}{2}\, p_o^2 (1/m_j - 1/m_o) \\ &\quad - Q - p_o p_S \cos\theta_S / m_j \end{aligned} \qquad \begin{matrix} i=1,2 \\ j=2,1 \end{matrix} \qquad (9)$$

Then the vector $\vec{p}_j$ is determined, from the following relations:

$$p_j = \Big[\, 2\, m_j (E_o - E_i - E_S + Q) \,\Big]^{1/2}$$

$$\cos\theta_j = (p_o - p_i \cos\theta_i - p_S \cos\theta_S)/(-p_i \operatorname{sen}\theta_i \cos\phi_i - p_S \operatorname{sen}\theta_S \cos\phi_S)$$

$$\cos\phi_j = p_{jx}/(p_j \operatorname{sen}\theta_j)$$

If the event is acceptable, i. e. if the direction of $\vec{p}_j$ lies within the limits of detector j, the energy losses can be computed and the resulting variables will define the final state.

2. 4. - Energy losses.

In our case the energy losses of the beam as well as of particle 1 and 2 after the reaction are determined by means of Bethe formula (see ref. (12)), with no correction for energy losses by bremsstrahlung, Cerenkov radiation, polarization effects and charge exchange. Under these approximations the stopping power is given by[12]

$$\frac{dE}{d\xi} = \text{const} \cdot \frac{Z_i^2}{\beta^2} \frac{Z}{W} \left[\lg \left(\frac{2mc^2\beta^2}{K_i \cdot Z \cdot (1-\beta^2)} \right) - \beta^2 - C_k/Z \right]$$

where Z_i, Z are the atomic numbers of the incident particle and of the target respectively, W the atomic weight of target, v is the speed of the incident particle ($\beta = v/c$), m the electron mass, K_i the mean ionization potential. The quantity C_k/Z is the correction factor for k electron shell. Although C_k/Z is usually computed from the theoretical results of Walske, a modified formula, more suitable for use on electronic computers, is used in the subroutine computing the energy losses.

2. 5. - Storage of the events.

The quantities $\vec{p}_1, \vec{p}_2, \vec{p}_3$ define completely the final state of the system and constitute a single event. These variables, together with the reaction energy E_o, the relative angle θ_{12} between the two detected particles and the time difference Δt_{12} in the arrival of the particles at the detectors can be stored on unidimensional or bidimensional spectra to give the distributions of the variables of interest.

3. - TEST RUN.

To check the program a test run of 5000 events from the reaction $^9Be(^3He, \alpha\alpha)^4He$ has been studied. Obviously it should be pointed out that all Monte Carlo programs do not produce identical results for the simulated spectra when running on different computers. The comparison with the test run is then possible only if based on the overall result of the calculations, by taking into account that these are affected by the usual statistical uncertainties.

The experimental conditions have been selected in order to reproduce the situation of the measurement reported in ref. (4), namely a 50

$\mu g/cm^2$ target thickness of ^9Be with a 40 $\mu g/cm^2$ ^{12}C backing and a circular geometry with radius 2 mm. for the two detectors placed at 65 mm. from the target.

A theoretical momentum distribution has been computed starting from an Hänkel function with a cut-off radius of 1. 2 fm. Figg. 2-3

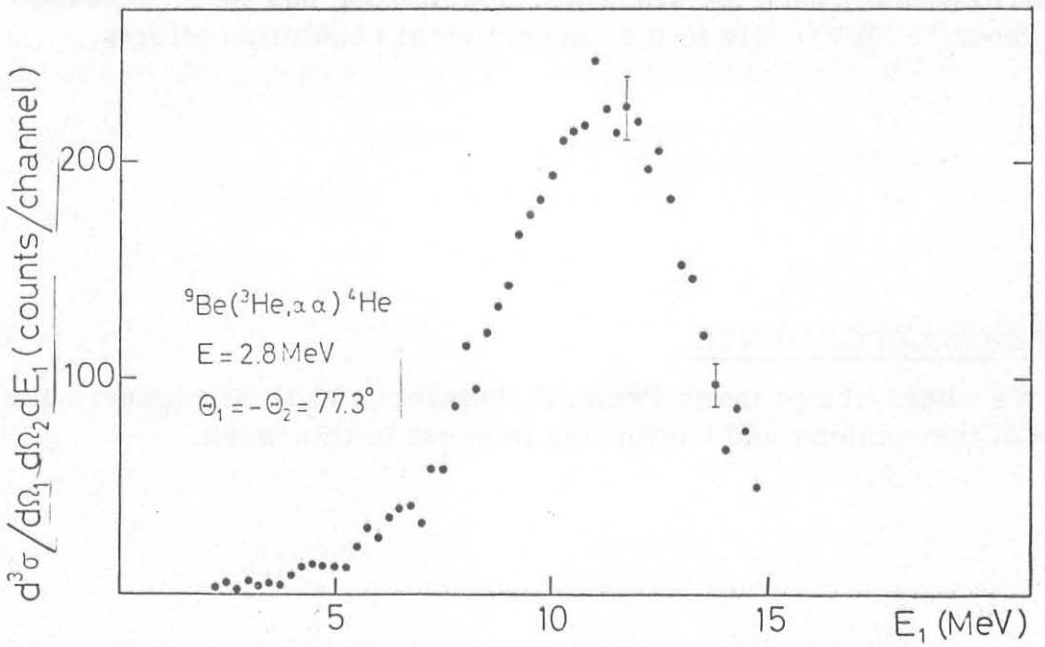

FIG. 2 - Projection of the simulated bidimensional spectrum on the E_1 axis.

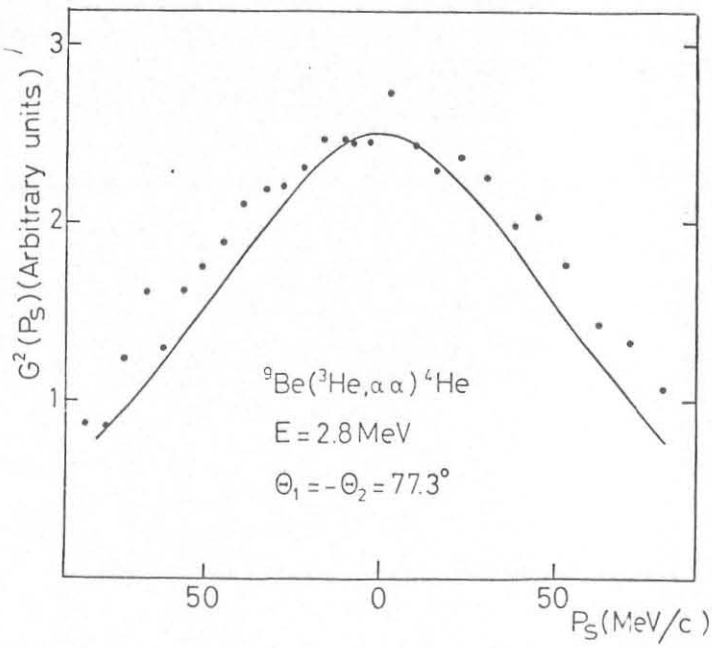

FIG. 3 - Momentum distribution deduced from the simulated data according to eq. (2); the continuous curve is the theoretical starting distribution.

show the obtained results.

In Fig. 2 the projection of the bidimensional spectrum on the E_1 axis is shown, while Fig. 3 shows the momentum distribution $G^2(p_S)$, extracted according to eq. (2). The continuous curve in Fig. 3 is the theoretical distribution, whose FWHM is 120 MeV/c; as it can be seen from the figure the simulated distribution has been increased by about 15 MeV/c due to the experimental resolution effects.

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