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G. Cutrona, G. Lanzanò, F. Porto and S. Sambataro: OPTICAL MODEL ANALYSIS OF ELASTIC SCATTERING OF ${ }^{3} \mathrm{He}$ PARTICLES FROM ${ }^{12} \mathrm{C}$ AT $9.5,10.5,11.5 \mathrm{MeV}$.

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## ABSTRACT. -

An optical model analysis has been tried to fit the elastic scattering differential cross sections of ${ }^{3} \mathrm{He}$ from ${ }^{12} \mathrm{C}$ at energies of $9.5,10.5,11.5 \mathrm{MeV}$. Good fits have been obtained. An investigation has been done to see the dependence of the optical parameters on energy.

## RIASSUNTO. -

Le sezioni d'urto differenziali della diffusione elastica di ${ }^{3} \mathrm{He}$ da ${ }^{12} \mathrm{C}$ a 9. 5, 10, 5, 11. 5 MeV sono state analizzate mediante il modello ottico. I risultati ottenuti sono soddisfacenti. E' stata fatta un'indagine per vedere la variazione dei parametri di modello ottico con l'energia.

## 1. - INTRODUCTION. -

The purpose of the present paper is to obtain the set of the best optical parameters for elastic scattering of ${ }^{3} \mathrm{He}$ from ${ }^{12} \mathrm{C}$ at energies of $9.5,10.5$ and 11.5 MeV .

Various sets of optical model parameters of ${ }^{12} \mathrm{C}$ for ${ }^{3} \mathrm{He}$ particle indeed have been obtained by many authors ${ }^{(1-7)}$ over a wide range of energies, but we think our research is needed because all the parameters did not always give good fits and differed from each other dependently on the initial set of parameters, the angles at which the measurements were made and the incident energy of ${ }^{3} \mathrm{He}$ particles ${ }^{(T)}$.

Moreover, at these energies, the differential elastic cross sections of ${ }^{3} \mathrm{He}$ by ${ }^{12} \mathrm{C}$ show backward peaks. Similar structures have also been found in the study of scattering of tritons by other light nuclei ${ }^{(8)}$ and it may be interesting to che ck, in our case, if they are due to resonances in compound nucleus or exchange reaction or indeed if they may be explained in terms of the appropriate optical potentials.

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## 2. - EXPERIMENTAL DATA AND POTENTIAL FORMS USED IN THE ANALYSIS. -

The experimental data at $9.5,10.5$ and 11.5 MeV used in this analysis are those of W . Bohne et al. ${ }^{(6)}$. As we noted in the introduction, these angular distributions show peaks at backward angles. The parameters used by these authors to reproduce with DWBA calculation the experimental angular distributions of the ${ }^{12} \mathrm{C}\left({ }^{3} \mathrm{He}, \alpha\right)^{11} \mathrm{C}$ reaction, do not account very well for elastic scattering especially at backward angles. We think it is useful to analyse again these experimental data and to see whether it is possible to fit also the backward structures with reasonable optical parameters. The optical model potential used in the present analysis has the form:

$$
\begin{equation*}
V(r)=-\left\{V f\left(x_{R}\right)+i W f\left(x_{I}\right)+i 4 W_{1} \frac{d}{d x_{D}} f\left(x_{D}\right)+V_{S 0} \frac{(\sigma \cdot L)}{r}\left(\frac{\nvdash}{m_{p}^{c}}\right)^{2} \frac{d}{d r} f\left(x_{S 0}\right)\right\}+V_{c}(r) \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{f}\left(\mathrm{x}_{\mathrm{i}}\right)=\left[1+\exp \left(\mathrm{x}_{\mathrm{i}}\right)\right]^{-1} \tag{2}
\end{equation*}
$$

with

$$
x_{i}=\left(r-r_{i} A^{1 / 3}\right) / a_{i}
$$

is the usual Saxon-Woods form factor and $\mathrm{V}_{\mathrm{c}}(\mathrm{r})$ is the Coulomb potential due to an uniformly charged sphere of radius $R_{c}=r_{c} A^{1 / 3} f m$.

The computations were carried out by the CDC 7600 computer at Casalecchio Computation Center, using the least - squares fitting program Mercy, in which the maximum number of simultaneously parameters in search is eleven.

The best fit to the experimental data was found by minimizing the quantity $\chi^{2}$ defined by:

$$
\begin{equation*}
\chi^{2}=\frac{1}{N-m} \sum_{i=1}^{N}\left[\frac{\sigma_{t h}\left(\theta_{i}\right)-\sigma_{\exp }\left(\theta_{i}\right)}{\Delta \sigma_{\exp }\left(\theta_{i}\right)}\right]^{2} \tag{3}
\end{equation*}
$$

where $\sigma_{\text {th }}\left(\theta_{\mathrm{i}}\right)$ and $\sigma_{\exp }\left(\theta_{\mathrm{i}}\right)$ are the calculated and experimental cross sections, respectively, at angle $\theta_{i}$, and $\Delta \sigma_{\exp }\left(\theta_{i}\right)$ is an experimental error of $\sigma_{\exp }$, which we fixed for all points at $10 \%$. $N$ is the number of the experimental points and $m$ is the number of the parameters in search.

In the anlaysis, two combinations of parameters were tried, i. e., the combinations obtained from setting

$$
\begin{array}{ll}
\text { a) } W_{1}=V_{S 0}=0 & ; \tag{4}
\end{array} \quad \text { b) } \mathrm{W}=\mathrm{V}_{\mathrm{S} 0}=0
$$

in the formula (1).

## 3. - ANALYSIS. -

Initially, a detailed analysis was done in the 11.5 MeV experimental angular distribution. Due to the presence of bumps at backward angles, we thought useful to carry out our analysis in two steps. Firstly we considered the experimental points ranging from $15^{\circ}$ to $90^{\circ}$ approximately, and afterwards we extended it to the entire angular distribution.

Moreover, because of the sensitivity of the fit to the input parameters, we started the search by using, in the first step, the optical model parameter sets reported by H. T. Fortune et al. (2), W. Bohne et al. ${ }^{(6)}$, J. Y. Park ${ }^{(3)}$ and T. Fujisawa et al. ${ }^{(7)}$, as displayed in Table I. In sets $1,2,3$ the authors used volume absorption potential (combination 4a), in the set 4 the surface absorption potential (combination 4b) was used.

TABLE I - Starting parameters used in the present search.

| Set <br> No. | Authors | $V(\mathrm{MeV})$ | $r_{\mathrm{v}}(\mathrm{fm})$ | $\mathrm{a}_{\mathrm{v}}(\mathrm{fm})$ | $\mathrm{W}(\mathrm{MeV})$ | $\mathrm{W}_{1}(\mathrm{MeV})$ | $\mathrm{r}_{\mathrm{w}}(\mathrm{fm})$ | $\mathrm{a}_{\mathrm{w}}(\mathrm{fm})$ | $\mathrm{r}_{\mathrm{c}}(\mathrm{fm})$ |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Fortune et al. | 165.1 | 1.31 | 0.79 | 9.96 | - | 2.14 | 0.8 | 1.31 |
| 2 | Bohne et al. | 140.0 | 1.2 | 0.58 | 28.0 | - | 1.4 | 0.58 | 1.2 |
| 3 | Park | 147.5 | 1.2 | 0.575 | 17.5 | - | 1.2 | 0.575 | 1.4 |
| 4 | Fujisawa et al. | 142.3 | 1.1 | 0.72 | - | 13.45 | 1.31 | 0.82 | 1.4 |



The variations of $\chi^{2} / \mathrm{N}$ for each set are shown in Fig. 1 against real potential depth $V$, variable from 100 to 200 MeV , with the other parameters fixed to the values listed in Table I.

A five parameters search (V, W or $\mathrm{W}_{1}, \mathrm{a}_{\mathrm{V}}, \mathrm{r}_{\mathrm{w}}$, $a_{w}$ ) has then been done in the minima of $\chi^{2} / \mathrm{N}$ near to the value of 150 MeV , so that the requirement $\mathrm{V}\left({ }^{\mathrm{n}} \mathrm{He}\right)=\mathrm{n} 50$ was satisfied ${ }^{(6)}$. Because of the well known ambiguity ${ }^{(9,10)} \mathrm{Vr}^{\mathrm{n}}$, we have chosen to keep fixed the value of real potential radius $r_{v}$ given by the authors ${ }^{(2,6,3,7)}$.

The search started with Park's parameters (set 3) was not able to fit experimental points with reasonable set of parameters. Better results were obtained in the search started with the remaining sets. These best fit parameters are given in Table II.

TABLE II - Optical model parameters that gave best fits: $r \mathrm{v}$ is fixed, $\overline{\theta_{\text {max }} \text { for }} \chi^{2}$ fit $=100^{\circ}$.

| Set <br> No. | $\mathrm{V}(\mathrm{MeV})$ | $r_{\mathrm{V}}(\mathrm{fm})$ | $a_{\mathrm{V}}(\mathrm{fm})$ | $\mathrm{W}(\mathrm{MeV})$ | $W_{1}(\mathrm{MeV})$ | $r_{\mathrm{w}}(\mathrm{fm})$ | $a_{\mathrm{w}}(\mathrm{fm})$ | $x^{2} / \mathrm{N}-\mathrm{m}$ |
| :--- | :---: | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| 1 a$)$ | 169.5 | 1.31 | 0.658 | 10.1 | - | 1.31 | 1.34 | 3.6 |
| $2 \mathrm{a})$ | 138.4 | 1.2 | 0.558 | 43.2 | - | 0.86 | 0.83 | 8.2 |
| $4 \mathrm{a})$ | 137.8 | 1.1 | 0.71 | - | 14.2 | 1.33 | 0.72 | 6.2 |

In Figs. 2 and 3 the best fits obtained are shown. We can note that the curve relative to set $2 a$ is not in so excellent accord with experimental data as the others.

In Fig. 4, the theoretical curves calculated with set la (full line) and set 4a dashed line) are compared with the entire experimental angular distribution. Though the overall calculated curves are not able to well reproduce the behaviour of the backward angles experimental points, however we observe that especially the curve relative to set la foresees the presence of the backward structures. So we extended our analysis to fit all experimental points, by starting from the best fit parameter sets 1a and 4 a and doing a five parameter search, as previously, with $\mathrm{r}_{\mathrm{v}}$ fixed at the same initial value. In Table III and in Fig. 5 the results of this search are shown. Theugh the $\chi^{2}$-value for the cur ve obtained by means of set 4 a (dashed line), is slightly lower, however the full-line curve obtained by means of set 1 b , is a better visual fit and it is able to reproduce correctly the backward angles structures.


FIG. 2 - Optical model fits with para$\overline{\text { meter }}$ sets la (full-line) and 2a (dashed line), compared with elastic scattering cross section at 11.5 MeV .


FIG. 4 - Theoretical curves calculated with parameter sets 1a (full--line) and 4a (dashed line) obtained fitting only full points.


FIG. 3 - Optical model fit with parameter set 4 a , compared with elastic scattering cross section at 11.5 MeV.


FIG. 5 - Optical model fits with parameter sets 1 b (full-line) and 4 b (dashed line).

$$
\frac{\text { TABLE III }}{\chi^{2} \text { fit }=160^{\circ} \text {. Optical model parameters that gave best fits: } \theta_{\max } \text { for }}
$$

| Set <br> No. | $V(\mathrm{MeV})$ | $r_{v}(\mathrm{fm})$ | $a_{V}(\mathrm{fm})$ | $W(\mathrm{MeV})$ | $W_{1}(\mathrm{MeV})$ | $r_{w}(\mathrm{fm})$ | $a_{w}(\mathrm{fm})$ | $\chi^{2} / \mathrm{N}-\mathrm{m}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 b$)$ | 165.7 | 1.31 | 0.708 | 5.82 | - | 2.12 | 1.14 | 11.0 |
| $4 \mathrm{~b})$ | 151.7 | 1.1 | 0.72 | - | 38.0 | 1.67 | 0.27 | 8.5 |
| 5 | 152.4 | 1.39 | 0.708 | 5.82 | - | 2.12 | 1.14 | 9.7 |

A further improvement of this curve was obtained, see Fig. 6, by putting $r_{V}=r_{C}$ in grid, $V$ in search, and the remaining parame ters as in set 1b. The best parameters so obtained are also re ported in Table III, set number 5. In this way we were also able to get some information about the ambiguity $\mathrm{Vr}_{\mathrm{v}}^{\mathrm{n}}$, drawing the va lue $n \approx 1$.33. The inclusion of a spin-orbit coupling term $V_{S 0}$ did not cause any noticeable improvement in the fit.

In attempts to fit the 9.5 and 10.5 MeV data both with $\theta_{\text {max }}=$ $=100^{\circ}$ and $\theta_{\max }=160^{\circ}$, we chose to a search in which we fixed the geometrical parameters of the sets $1 \mathrm{a}, 4 \mathrm{a}$ and $1 \mathrm{~b}, 4 \mathrm{~b}$, 5 respectively. None of these fits is very good even for large changes in the potential well depths.

FIG. 6 - Optical model fit with parameter set 5 .


Good fits are instead obtained in a five parameter search by using as starting values the parameter sets 1a and 5 as shown in Fig. 7, but the parameters, especially those relative to the ima ginary potential, change from their initial values. These bets fit parameters are listed in Table IV.

FIG. 7 - Optical model fit of angular distributions at 9.5 and 10.5 MeV with $\theta_{\max }=$ $=100^{\circ}$ (part a) and $\theta_{\max }=160^{\circ}$ (part b). The best-fit parameters are reported in Table IV.


TABLE IV - Optical model parameters that gave best fits at 9.5 and 10.5 MeV .

| Bombarding <br> Energy <br> $(\mathrm{MeV})$ | $\mathrm{V}(\mathrm{MeV})$ | $\mathrm{r}_{\mathrm{V}}(\mathrm{fm})$ | $\mathrm{a}_{\mathrm{V}}(\mathrm{fm})$ | $\mathrm{W}(\mathrm{MeV})$ | $r_{\mathrm{W}}(\mathrm{fm})$ | $\mathrm{a}_{\mathrm{W}}(\mathrm{fm})$ | $\chi^{2} / \mathrm{N}-\mathrm{m}$ | $\theta_{\max }^{\chi^{2}} \mathrm{for}_{\mathrm{fit}}$ <br> 10.5 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9.5 | 169.7 | 1.31 | 0.635 | 6.20 | 1.98 | 1.02 | 3.3 | $100^{\circ}$ |
| 10.5 | 181.6 | 1.31 | 0.657 | 8.23 | 1.24 | 1.26 | 6.7 | $100^{\circ}$ |
| 9.5 | 171.4 | 1.39 | 0.795 | 4.82 | 2.47 | 1.54 | 9.4 | $160^{\circ}$ |

## 4. - DISCUSSION AND CONCLUSIONS.-

As pointed out in the previous section, it seems possible to fit the entire experimental angular distribution at 11.5 MeV with parameters very close to those reported by Fortune et al. ${ }^{(2)}$ in their study in the energy range $16-18 \mathrm{MeV}$.

However, when we extended the serach to lower energies, 10.5 and 9.5 MeV , the parameters so obtained have not smooth variations with energy, in fitting either the forward angles experimental points or the entire angular distributions. As we are dealing with data not averaged in energy, it could be expected that this energy variation of the particle model parameters is not a true indica-. tion of an energy dependence but it could be due to the presence of statistical fluctuations.

Under the assumption that such a fluctuations are not present, we agree with the conclusions of other authors ${ }^{(2,7)}$ that the simple spherical optical model potential is inadequate and a proper description of the elastic scattering process must take into account the coupling between the ground state and the first excited state.

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