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W. M. Alberico, S. Costa and A. Molinari: AN OUTLOOK OF
NUCLEAR STRUCTURE STUDIES WITH ELECTROMAGNETIC
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1. - INTRODUCTION. -

In this paper we review what we have learned on nuclei with two probes available to explore nuclear structure with electromagnetic interaction: real photons and electrons. We will focus our attention essentially on some of the most recent results and we will have no pretense of being complete.

The electromagnetic interaction is of course well understood and does not perturb too much the system under investigation (the electromagnetic coupling constant being $e^2/hc = 1/137$).

The first experiments in this field go back to the beginning of the fifties and were performed with the bremsstrahlung from electron accelerators⁽¹⁾; betatrons with energies in the range 20 - 70 MeV and synchrotrons in the range up to 350 MeV. The main drawback was the very tiny intensity of the electron current (a fraction of μA) together with the unfavourable feature of the bremsstrahlung energy spectrum, i. e. to be continuous.

Nevertheless outstanding results were obtained: fundamental among them the discovery of the giant dipole resonance, whose energy was determined to be $E_{GDR} \cong 70 - 80 A^{-1/3}$ MeV.

Next the linear accelerators of electrons (notably the ones of Stanford and Saclay) came into play⁽²⁾. These machines, in spite of the modest intensity ($10^{-7} - 10^{-6}$ amp) and the poor energy resolution ($\Delta p/p > 10^{-2}$), made it possible a first determination of the nuclear charge distribution.

The best fit to the experimental data was obtained with the following expression for the charge density:

$$\rho(r) = \frac{\rho_0}{\exp[(r-R)/a] + 1} \left(1 + \omega \frac{r^2}{R^2}\right) \quad (1)$$

Even if none of the parameters entering into (1) is amenable to a direct theoretical evaluation, definite evidence for a finite surface thickness (a) and, less so, for a central depression (ω) was provided.

In spite of these remarkable achievements it is to be noticed that some of the results of the photonuclear reactions, e. g. those of the (γ, p) reaction, can be obtained with high precision in the inverse process (p, γ) which is of course not plagued by the continuous bremsstrahlung spectrum.

Furthermore the advantage of the electromagnetic interaction, namely of not disturbing too much the system, has its counterpart in the smallness of the electromagnetic cross sections of significance to nuclear structure. In fact they are in general $10^{-2} \div 10^{-4}$ times smaller than the strong interaction cross sections.

In our survey we will follow the customary classification (see Fig. 1) of electron scattering experiments :

- a) elastic scattering ;
- b) inelastic scattering from collective levels (including rotational levels in deformed nuclei) ;
- c) "quasi-elastic" peak ;
- d) coincidence experiments of the (e, e'p) type (exclusive reactions),

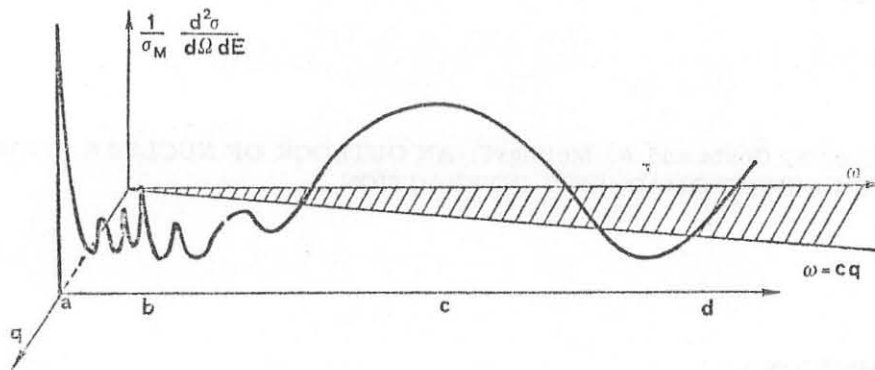


FIG. 1 - A schematic double differential cross section for electron scattering at fixed momentum transfer. One can see the elastic peak (a), the excitation of collective levels (b), the quasi-elastic peak (c), and the beginning of the pion production (d).

Concerning the photons a possible classification on the energy scale is (Fig. 2) :

- a) up to the giant resonances ;
- b) up to the mesonic threshold ;
- c) above the mesonic threshold.

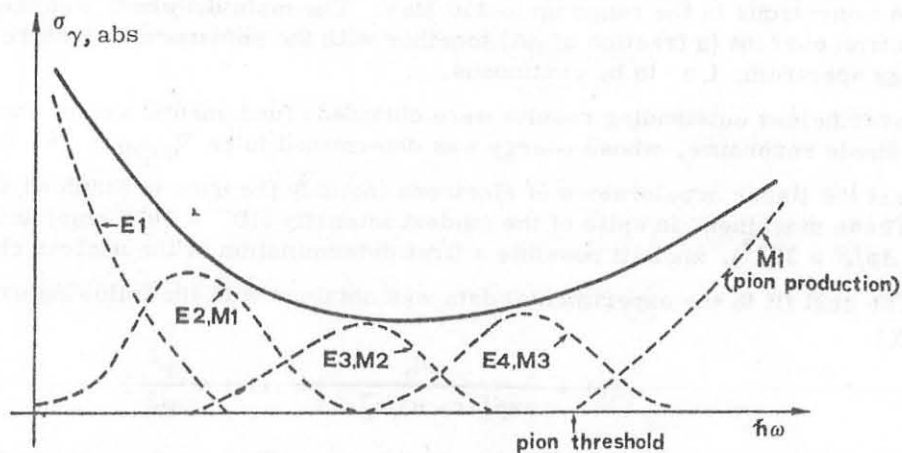


FIG. 2 - Schematic illustration of the γ -photoabsorption cross section. Also shown the hypothetical specific contribution of the various multipoles (taken from ref. (10), p. 280).

2. - ELECTRON SCATTERING. -

The four-momentum $q^\mu = (\vec{q}, \omega/c)$ of the virtual photons carrying the electromagnetic interaction is space-like; therefore the electron scattering experiments are confined in the region $q \geq \omega/c$ in the (ω, q) plane. In other words any momentum greater than ω/c can be transferred for a given ω . On the other hand the photodisintegration experiments lie on the straight line $\omega = cq$ (c speed of light).

It is of importance to realize that the nuclear system responds to a fixed transfer of energy over a large interval of momenta because of its finite size. However also an infinite system can give an inelastic single particle response over an interval of momenta (providing the Fermi momentum k_F has a finite value). These points will be further illustrated in the following.

As a final point let us recall the well-known Lorentz condition:

$$q_\mu A^\mu = q_0 A_0 - \vec{q} \cdot \vec{A} = 0 \quad (2)$$

(A^μ being the electromagnetic four-potential), which assures the gauge invariance of the theory. In the Coulomb gauge ($A_0 = 0$) the transversality of the field of real photons follows immediately. Not so in the case of the field created by electrons where such a gauge is incompatible with the field equation $\square A_0 = -4\pi j_0$ (j_0 is the time component of the electron four-current density). Therefore the field of virtual photons will contain also a longitudinal component, which is the only one effective in the elastic scattering off spin zero nuclei (monopole interaction).

2.1. - Elastic scattering.

A great deal of experimental work has been done recently in the field of elastic scattering of electrons. Perhaps the most exciting result is that the experiments at very large momentum transfer⁽³⁾ ($q \geq 2.5 \text{ fm}^{-1}$) seem to require an oscillating component in the nuclear charge distribution.

From the theoretical viewpoint the shell model predicts indeed the existence of quantum oscillations in the charge density with wavelength⁽⁴⁾:

$$\lambda \geq \frac{\pi}{k_F} = 2.31 \text{ fm} . \quad (3)$$

We recall that in the frame of the shell model the charge density is given by:

$$\rho(r) = \lim_{\vec{r}' \rightarrow \vec{r}} \frac{1}{2\pi i} \oint_{\Gamma} G(\vec{r}, \vec{r}'; E) dE = \sum_{i=1} \left| \varphi_i(\vec{r}) \right|^2 \quad (4)$$

where the one-body Green's function is:

$$G(\vec{r}, \vec{r}'; E) = \sum_{i=1}^{\infty} \frac{\varphi_i(\vec{r}) \varphi_i^*(\vec{r}')}{E - \epsilon_i} , \quad (5)$$

in terms of the single particle energies ϵ_i and wave functions $\varphi_i(\vec{r})$. The integration in (4) is performed along a path Γ in the complex energy plane which encloses the single particle proton eigenvalues (Fig. 3) up to the Fermi surface.

For systems large enough, it can be shown utilizing (4) and (5) that:

- a) only the single particle states of the last shell contribute to the oscillations in the density;
- b) the oscillations are damped, as one proceeds in the internal region, by an inverse power law.

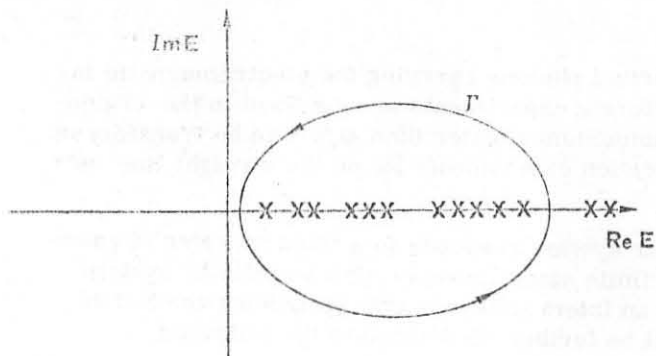


FIG. 3 - Path of integration, in the complex energy plane, of equation (4). The crosses on the real axis are the single particle proton eigenvalues.

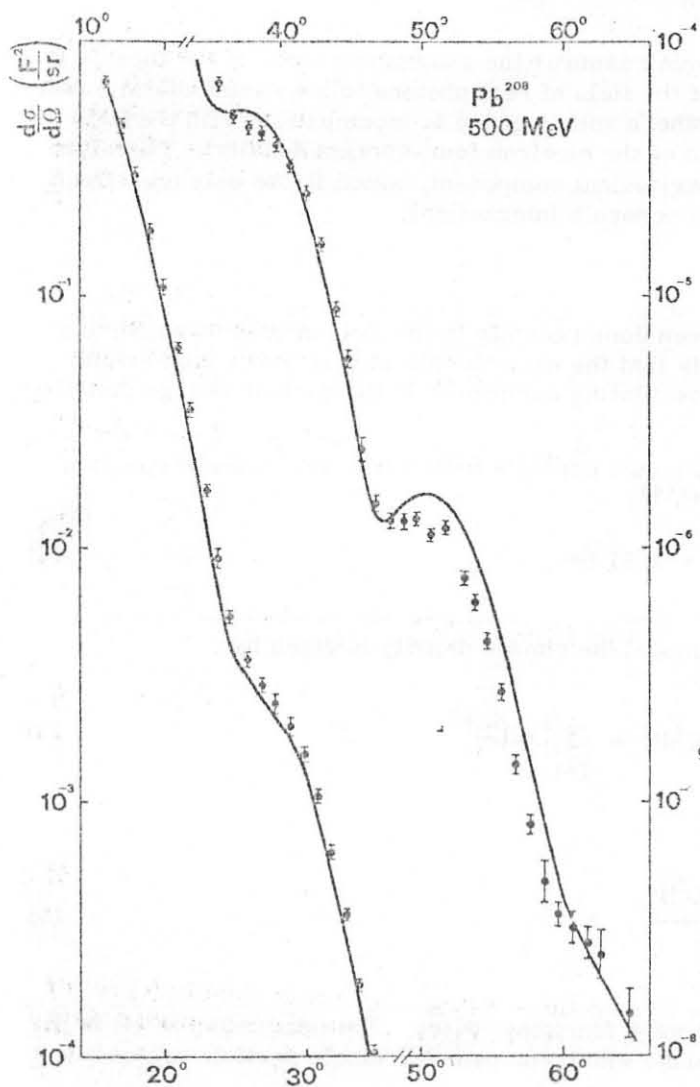


FIG. 4 - Negele's theoretical curve for the scattering of 500 MeV electrons by ^{208}Pb (5).

In the specific case of Pb^{208} the D. D. H. F. (Density Dependent Hartree Fock) theory of Negele(5) is successful in reproducing the experimental data of electron scattering (see Fig.4) as well as those of μ -mesic atoms. The corresponding charge density is characterized by a central maximum due to the $3s_{1/2}$ proton single particle state in the last shell of Pb^{208} (Fig. 5).

Another picture of the density oscillations in finite systems is the one due to Broglia et al.(6). They view the surface of Pb^{208} as a source of density waves propagating inside the system since the loss of translational invariance associated with the surface implies a perturbation of the density (at least in the neighborhood of the impurity).

The occurrence of such static waves in finite nuclei may be thought of in connection with the existence, in the spectrum of

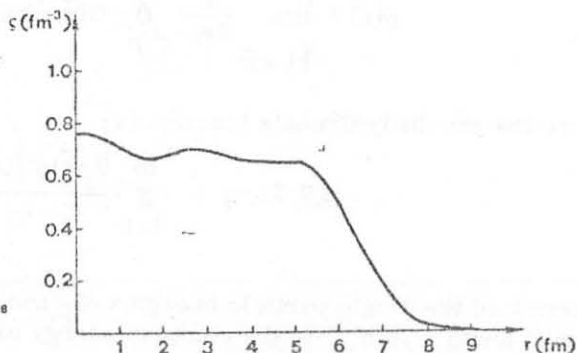


FIG. 5 - Charge density of ^{208}Pb according to the Negele's theory(5).

infinite nuclear matter, of excitations with vanishing energy.

In normal (as opposite to superfluid) infinite nuclear matter one may recognize the existence of two types of such excitations :

- a) single particle-hole states with zero energy, and momentum between 0 and $2k_F$ (corresponding by $\pi/k_F \leq \lambda < \infty$);
- b) collective states (zero sound) with complex momentum in analogy to the "roton" excitations in liquid ^4He .

A detailed analysis carried out in Pb^{208} , considering only excitations of second type, permits a good account of the elastic electron scattering data but for the largest momentum transfers (Fig. 6). The corresponding charge density is displayed in Fig. 7 where it is seen that the static waves associated with the collective excitations are characterized by large wavelength ($\sim 6.5 \text{ fm}$) and exponential damping.

Inclusion of single particle-hole excitations of type (a) is likely to bring agreement between theory and experiment.

To conclude this section we consider the elastic scattering of electrons off an infinite system.

In the crude Born approximation the elastic scattering amplitude is given by:

$$F_{el} = \int e^{i\vec{q} \cdot \vec{r}} \rho(r) d\vec{r} = 4\pi\rho_0 R^3 \frac{j_1(qR)}{qR} \quad (6)$$

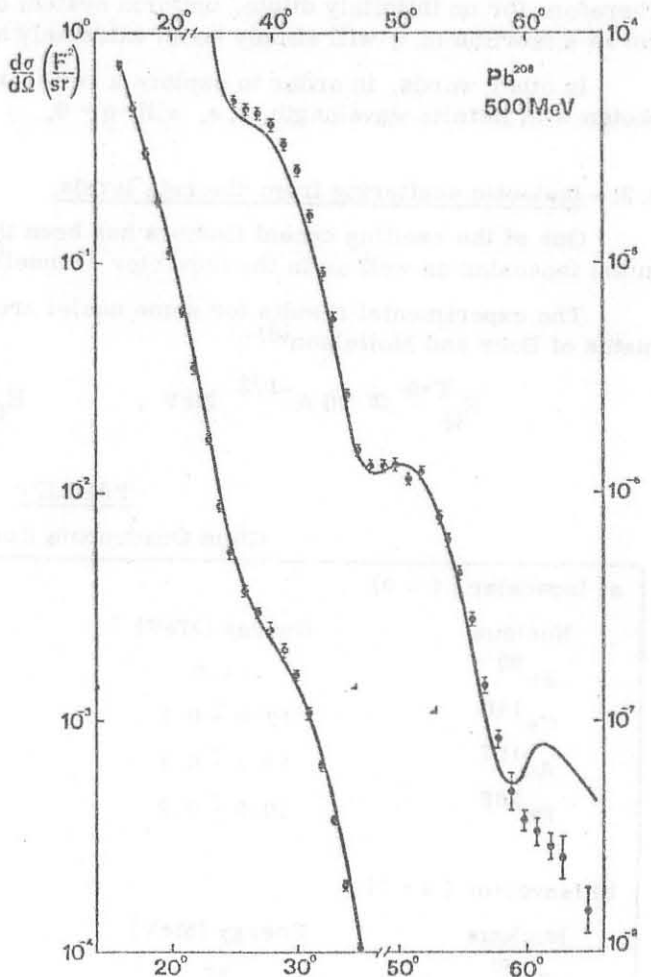
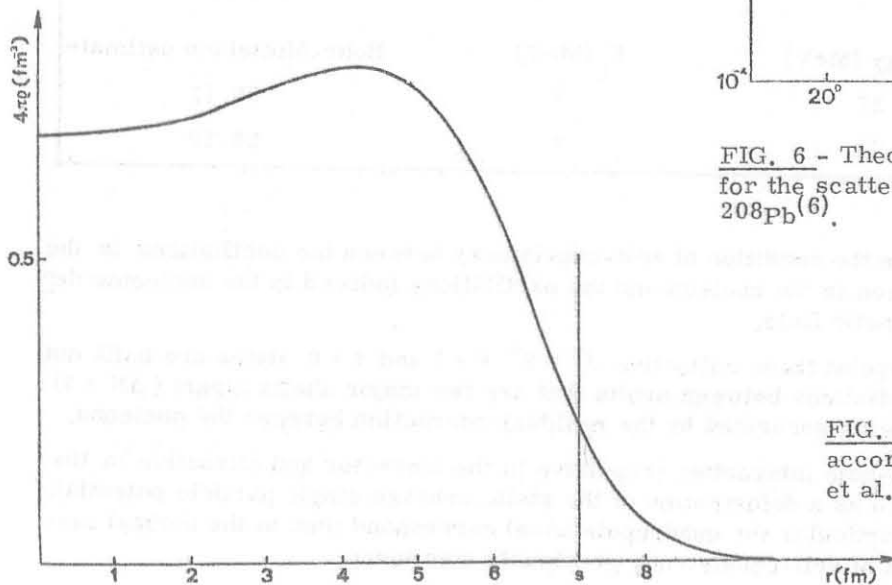


FIG. 6 - Theoretical curve by Broglia et al. for the scattering of 500 MeV electrons by ^{208}Pb (6).

FIG. 7 - Charge density of ^{208}Pb according to the theory of Broglia et al. (6).

for a finite system of spin zero, constant charge density ρ_0 and radius R ($j_1(qR)$ is the spherical Bessel function of order 1). Letting $R \rightarrow \infty$ (and thus $\rho_0 = 3Z/4\pi R^3 \Rightarrow 0$) but at the same time $q \rightarrow 0$ in such a way that $qR \rightarrow 0$, one gets :

$$\lim_{qR \rightarrow 0} F_{el}(q) = 3\pi Z \lim_{qR \rightarrow 0} \frac{\delta(qR)}{(qR)^2} . \quad (7)$$

Therefore for an infinitely dilute, uniform system of Z charges the pattern of the elastic form factor as a function of q will simply be an extremely narrow peak in the origin.

In other words, in order to explore a very large system, an electron should emit a virtual photon with infinite wavelength, i. e. with $q = 0$.

2.2. - Inelastic scattering from discrete levels.

One of the exciting recent findings has been the determination of quadrupole giant resonances in the isoscalar as well as in the isovector channel⁽⁷⁾.

The experimental results for some nuclei are reported in Table I. Also quoted are the estimates of Bohr and Mottelson⁽⁸⁾:

$$E_Q^{T=0} \simeq 60 A^{-1/3} \text{ MeV} , \quad E_Q^{T=1} \simeq 135 A^{-1/3} \text{ MeV} . \quad (8)$$

TABLE I
Giant Quadrupole Resonances

a) Isoscalar ($\tau = 0$)			
Nucleus	Energy (MeV)	Γ_t (MeV)	Bohr-Mottelson estimate
Zr ⁹⁰	14.0	4.8 \pm 0.6	13.39
Ce ¹⁴⁰	12.0 $\bar{+}$ 0.1	2.8 $\bar{+}$ 0.3	11.84
Au ¹⁹⁷	11.2 $\bar{+}$ 0.1	3.3 $\bar{+}$ 0.5	10.31
Pb ²⁰⁸	10.5 $\bar{+}$ 0.2	~ 3	10.13
b) Isovector ($\tau = 1$)			
Nucleus	Energy (MeV)	Γ_t (MeV)	Bohr-Mottelson estimate
Zr ⁹⁰	27	?	30.12
Pb ²⁰⁸	22	?	22.19

The values (8) follow from the condition of self-consistency between the oscillations in the average potential felt by a nucleon in the nucleus and the oscillations induced in the nucleonic density by the external electromagnetic field.

From a microscopic viewpoint these collective $J^\pi = 2^+ \tau = 1$ and $\tau = 0$ states are built out of particle-hole elementary excitations between orbits that are two major shells apart ($\Delta N = 2$): the coherent collective states being generated by the residual interaction between the nucleons.

Alternatively the particle-hole interaction (repulsive in the isovector and attractive in the isoscalar channel) can be viewed as a deformation of the static average single particle potential. The collective excitations (in particular the quadrupole ones) correspond then to the normal modes determined by the condition of self-consistency previously mentioned.

Other types of quadrupole collective states in nuclei are those associated with particle-hole excitations within a major shell ($\Delta N = 0$) i.e. between the orbits in the unfilled shells. In deformed nuclei these excitations provide the basis for the microscopic description of the nuclear rotational degree of freedom.

Bertozzi et al.⁽⁹⁾ at MIT have measured the inelastic form factors from the $I^\pi = 0^+, 2^+, 4^+$ rotational levels of Sm^{152} , Sm^{154} , Er^{166} , Yb^{176} . These experiments are on the forefront of nuclear physics being on the limit of the energy resolution of the electron beams. However no other tool can provide a better control of the nuclear collective model which writes the wavefunctions of the nuclear rotational states as follows :

$$\psi_{MK}^I(q, \Omega) = \sqrt{\frac{2I+1}{8\pi^2}} \frac{1}{\sqrt{2}} \left[\chi_K(q) \mathcal{D}_{M,K}^I(\Omega) + (-1)^{I+K} \chi_{-K}(q) \mathcal{D}_{M,-K}^I(\Omega) \right] \quad (9)$$

where, as it is well known, the $\mathcal{D}_{M,K}^I(\Omega)$; the Wigner functions, describe the "collective" motion, while $\chi(q)$ describes the "intrinsic" one.

Since $\chi(q)$ "should" be the same for all the levels of a rotational band, it is reasonable to expect well defined relationship between the inelastic form factors for the excitations of the different members of the band and the elastic form factor. Departures from these rules would provide most interesting microscopic informations on the breakdown of the adiabatic hypothesis. Of course this is expected to occur for spin values higher than the ones investigated till now and this appears very challenging from the experimental point of view.

As a further point we like to mention that the very low excitation energy of the rotational states makes them the best candidates for an investigation of the so called "dispersive corrections" to the nuclear elastic electron scattering. These long inquired corrections are shown diagrammatically in Fig. 8.

In conclusion we wish to explore the inelastic scattering of electrons off collective nuclear levels in the limit of infinite size as we did for the elastic electron scattering. This is most easily done in the frame of the nuclear hydrodynamical model which views the so called giant resonances as the normal modes of vibrations in a vessel of two interpenetrating fluids (the protons and the neutrons).

These collective modes are created and annihilated by the operators $\hat{q}_{1m}^{+(n)}$ and $\hat{q}_{1m}^{(n)}$ in terms of which the "collective variables" of the giant resonance states (with frequencies $\omega_1^{(n)}$ are so defined :

$$\hat{q}_{1m}^{(n)} = \sqrt{\frac{\hbar}{2B_1^{(n)} \omega_1^{(n)}}} \left[\hat{q}_{1m}^{+(n)} + (-1)^m \hat{q}_{1-m}^{(n)} \right], \quad (10)$$

the constants $B_1^{(n)}$ being :

$$B_1^{(n)} = \frac{m}{K_1^{(n)2}} \frac{Z}{N} \rho_0. \quad (10')$$

The fluctuations of the density :

$$\eta(\vec{r}, t) = \frac{A}{Z} \frac{\rho_p(\vec{r}, t)}{\rho_0} - 1 = - \frac{A}{N} \frac{\rho_N(\vec{r}, t)}{\rho_0} + 1, \quad (11)$$

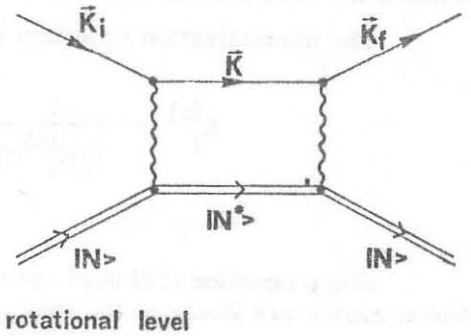


FIG. 8 - Non-static second order contribution to the elastic electron-nucleus scattering.

($\rho_p(\vec{r}, t)$ and $\rho_N(\vec{r}, t)$ being the proton and the neutron time-dependent densities) associated with the nuclear vibrations are then expanded in multipoles as follows :

$$\eta(\vec{r}, t) = \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} (-1)^m q_{l-m}^{(n)} \eta_{lm}^{(n)}(\vec{r}, t) . \quad (11')$$

Each multipole component obeys the equation⁽¹⁰⁾ :

$$u^2 \nabla^2 \eta_{lm}^{(n)}(\vec{r}, t) - \frac{\partial^2 \eta_{lm}^{(n)}(\vec{r}, t)}{\partial t^2} = 0 , \quad (12)$$

whose solution is :

$$\eta_{lm}^{(n)}(\vec{r}, t) = A_1^{(n)} j_l(K_1^{(n)} r) Y_{lm}(\theta, \varphi) e^{i\omega_1^{(n)} t} . \quad (13)$$

The $K_1^{(n)}$ are fixed by the boundary condition :

$$\left. \frac{\partial j_l(K_1^{(n)} r)}{\partial r} \right|_{r=R} = 0 , \quad (13')$$

R being the radius of the system.

The normalization condition gives :

$$A_1^{(n)} = \frac{1}{j_l(K_1^{(n)} R)} \frac{1}{\sqrt{\frac{R^3}{2} \left[1 - \frac{l(l+1)}{K_1^{(n)2} R^2} \right]}} . \quad (13'')$$

Since equation (12) does not contain any mechanism for dispersion the energies of the vibrational modes are linear in the momentum ("phonons") :

$$\hbar \omega_1^{(n)} = u \hbar K_1^{(n)} . \quad (14)$$

The speed of sound u characterizes the velocity of spreading of a density fluctuation in the nucleus and is presently not known. As an orientation one can take :

$$u = c \sqrt{\frac{4b_{\text{symm}} ZN}{mc^2 A}} , \quad (14')$$

b_{symm} being the coefficient of the symmetry energy in the semiempirical mass formula of Bethe-Weizsäcker (~ 50 MeV). With $N=Z=A/2$ one gets $u = (b_{\text{symm}}/mc^2)^{1/2} \approx 0.23 c$. Observe that the estimate (14') for u is close to the value of the Fermi velocity $v_F = \hbar k_F/m = 0.29 c$.

In Born approximation the inelastic form factor for the excitation of a vibration with quantum numbers l and m is :

$$F_1(q) = \sqrt{\frac{3}{4} A} \frac{u \hbar K_1^{(n)}}{b_{\text{symm}}} \frac{1}{j_l(K_1^{(n)} R)} \frac{(2l+1)}{\sqrt{1 - \frac{l(l+1)}{K_1^{(n)2} R^2}}} \frac{1}{R^3} \int_0^R x^2 dx j_l(qx) j_l(K_1^{(n)} x) \quad (15)$$

For a very large system all the multipolarities will lie on a continuous line^(x) in the energy vs. momentum plane (if we neglect the degrees of freedom associated with spin and isospin): therefore in (14) and (15) the index "1" will be redundant and "n" will become a continuous one, namely k itself.

In this case the inelastic form factor becomes:

$$F_1(q) = \sqrt{\frac{3A}{4}} \frac{u\hbar K_1^{(n)}}{b_{\text{symm}}} \frac{1}{j_1(K_1^{(n)}R)} \frac{(2l+1)}{\sqrt{1 - \frac{l(l+1)}{(K_1^{(n)}R)^2}} \frac{1}{(qR)^2} \delta \left[R(q - K_1^{(n)}) \right], \quad (15')$$

which shows again that an infinitely large system responds collectively to an external electromagnetic probe with only one q for a given ω .

Note also that even if a finite system responds to an energy transfer ω with any q ω/c , in the (ω, q) plane the collective response is maximum on the straight line $\omega = uq$ (see eq. (15)).

2.3. - Quasi-elastic scattering.

Let us now deal with the so called quasi-elastic peak, the common interpretation of which is based on a direct interaction of the incident electron with a single nucleon bound in the nuclear structure.

This view is supported by the comparison between the mean interparticle distance in real nuclei (about 1.7 - 1.8 fm) and the wavelength of a virtual photon emitted by a 500 MeV electron scattered at $\theta \approx 60^\circ$ (typical experimental condition). Ignoring surface effects, the situation is typical of infinite nuclear matter.

In Born approximation the corresponding graph is the one illustrated in Fig. 9.

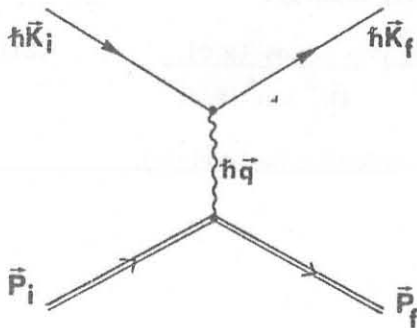


FIG. 9 - Typical graph of the quasi-elastic process (in Born approximation): \vec{p}_i and \vec{p}_f are the initial and final proton momenta, $\hbar\vec{K}_i$ and $\hbar\vec{K}_f$ those of the electron and $\hbar\vec{q}$ the transfer momentum.

From the conservation laws of momentum and energy:

by:

$$\begin{aligned} \vec{K}_i &= \vec{K}_f + \vec{q} \\ \vec{p}_f &= \vec{p}_i + \hbar\vec{q} \\ cK_i &= cK_f + \hbar\omega \\ \hbar\omega + \frac{p_f^2}{2m} - \bar{E}(\vec{p}_i) &= \frac{p_f^2}{2m} - \bar{E}(\vec{p}_f) \end{aligned} \quad (16)$$

with obvious meaning of the symbols, it follows immediately:

$$\hbar\omega = \frac{\hbar^2 q^2}{2m} + \frac{\hbar\vec{q} \cdot \vec{p}_i}{m} + \bar{E}(\vec{p}_i) - \bar{E}(\vec{p}_f) \quad (17)$$

Neglecting the average potential energy $\bar{E}(\vec{p})$ felt by a particle inside and outside the nuclear structure one gets the well known bounds on the excitation energy for a Fermi gas:

(x) - The distinction among the multipolarities of the various vibrations in finite nuclei is intimately connected with the shell structure which of course is washed out in the limit of infinite size.

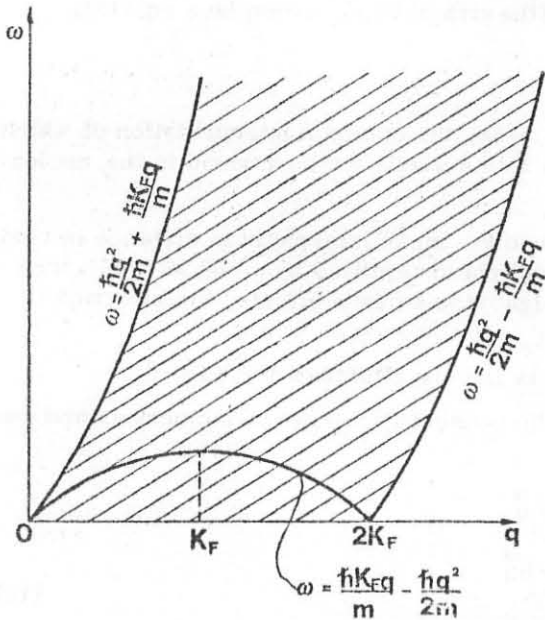
$$\omega = \frac{\hbar q^2}{2m} + \frac{\hbar q k_F}{m} \quad (18)$$

From (18) it is seen that the energy width of the quasi-elastic peak is a direct measure of the Fermi momentum k_F .

As previously mentioned, the system responds to a fixed energy transfer ω over a constant ($= 2k_F$) interval of momenta:

$$-k_F + \sqrt{k_F^2 + \frac{2m\omega}{\hbar}} \leq q \leq k_F + \sqrt{k_F^2 + \frac{2m\omega}{\hbar}} \quad (19)$$

The region in the (ω, q) plane allowed for the electron scattering off a gas of protons is the dashed one in Fig. 10.



This holds as long as we can neglect $\vec{E}(\vec{p})$ and the correlations among the protons, but those imposed by the Pauli principle.

In Born approximation the double differential cross section (with respect to the final energy of the electron, E , and to the scattering angle) is⁽¹¹⁾:

$$\frac{1}{\sigma_{\text{Mott}}} \frac{d^2\sigma}{d\Omega dE} = -\frac{V}{\pi} \text{Im}\Pi(\vec{q}, \omega) \quad (20)$$

It is expressed in terms of the polarization function $\Pi(\vec{q}, \omega)$ ^(x) and the usual definition of the Mott cross section holds, namely:

$$\sigma_{\text{Mott}} = \left(\frac{Ze^2}{\hbar c}\right)^2 \frac{\cos^2(\theta/2)}{4K_i^2 \sin^4(\theta/2)} \quad (21)$$

(V is the volume enclosing the system).

FIG. 10 - The response region in the (ω, q) plane of a free Fermi gas.

(*) - An equivalent, more familiar, expression for the inelastic double differential cross section is:

$$\int_{\text{over resonance}} d(\hbar\omega) \frac{1}{\sigma_{\text{Mott}}} \frac{d^2\sigma}{d\Omega dE} = |F_{\text{no}}(\vec{q})|^2 \quad (21')$$

where

$$F_{\text{no}}(\vec{q}) = e_{\text{no}}(-\vec{q}) = \int e^{i\vec{q}\cdot\vec{x}} \langle \psi_n | \hat{\rho}(\vec{x}) | \psi_0 \rangle d^3x \quad (21'')$$

is the inelastic form factor and the charge operator is so defined:

$$\hat{\rho}(\vec{x}) = \sum_{i=1}^A e_i \delta(\vec{x} - \vec{x}_i) \quad (21''')$$

The polarization function $\Pi^0(\vec{q}, \omega)$ for a perfect Fermi gas (graphically illustrated in Fig. 11) is⁽¹²⁾:

$$\text{Im}\Pi^0(\vec{q}, \omega) = -\frac{mk_F^2}{\hbar^2} \frac{1}{4\pi q} \left[1 - \frac{m^2}{\hbar^2 k_F^2} \left(\frac{\omega}{q} - \frac{\hbar q}{2m} \right)^2 \right], \quad (22a)$$

$$\text{if } q > 2k_F \quad \text{and} \quad \frac{\hbar q^2}{2m} + \frac{\hbar q k_F}{m} \geq \omega \geq \frac{\hbar q^2}{2m} - \frac{\hbar k_F q}{m},$$

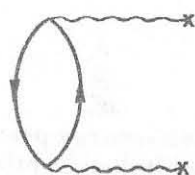
$$\text{Im}\Pi^0(\vec{q}, \omega) = -\frac{mk_F^2}{\hbar^2} \frac{1}{4\pi q} \left[1 - \frac{m^2}{\hbar^2 k_F^2} \left(\frac{\omega}{q} - \frac{\hbar q}{2m} \right)^2 \right], \quad (22b)$$

$$\text{if } q < 2k_F \quad \text{and} \quad \frac{\hbar q^2}{2m} + \frac{\hbar k_F q}{m} \geq \omega \geq \frac{\hbar k_F q}{m} - \frac{\hbar q^2}{2m}$$

and

$$\text{Im}\Pi^0(\vec{q}, \omega) = -\frac{m}{\hbar^3} \frac{2}{4\pi q} \omega^2, \quad (22c)$$

$$\text{if } q < 2k_F \quad \text{and} \quad 0 \leq \omega \leq \frac{\hbar k_F q}{m} - \frac{\hbar q^2}{2m}.$$



$\Pi^0(\vec{q}, \omega)$

FIG. 11 - The zeroth-order polarization function.

In Fig. 12 some of the cross sections are plotted as a function of ω at fixed q .

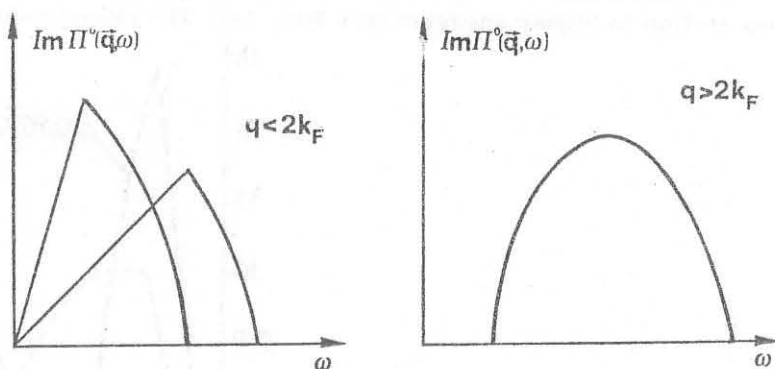


FIG. 12 - Double differential cross section, in Born approximation (up to a factor), of electrons out of a free Fermi gas; two typical situations are illustrated, corresponding to small and large momentum transfers.

To go beyond the free Fermi gas the correlations among nucleons should be taken into account. This is difficult to achieve in general and the best one can do is to use perturbation theory in order to calculate $\Pi(\vec{q}, \omega)$.

For short range correlations induced by an hard-core potential of radius a (≈ 0.4 fm) an attempt has been made by Czyz and Gottfried⁽¹³⁾. The contributions to the polarization function taken into account by these authors are those illustrated in Fig. 13. Note that the dashed lines stand for the G-matrix, which is the solution of the Bethe-Goldstone equation⁽¹⁴⁾:

shift in the maximum of the quasi-elastic peak cross section toward higher energies (see Fig. 15). Note however that the predictions of the Fermi gas model concerning the symmetry of the quasi-elastic peak around $\hbar^2 q^2/2m$, when $q > 2k_F$, seem to be fulfilled (for large ω the mesonic degrees of freedom, of course, come into play)^(x).

An explanation of the larger energies at the maximum of the cross section for the quasi-elastic peak can also be offered in terms of a velocity-dependent average field felt by the particle.

The theory of nuclear matter gives a reliable estimate of $\bar{E}(q)$, namely, in the framework of the reference spectrum method⁽¹⁵⁾:

$$\bar{E}(q) = \begin{cases} \frac{\hbar^2 q^2}{2m^*} - A & \text{if } q \leq 2k_F \\ aq + b & \text{for } 2k_F \leq q \leq 3 \text{ fm}^{-1} \\ \frac{\hbar^2 q^2}{2m} & \text{if } q \geq 3 \text{ fm}^{-1} \end{cases} \quad (24)$$

($m^* = 0.6 m$; $A = 100 \text{ MeV}$).

The response of the system in the (ω, q) plane when an average field is present has been recently investigated by two of us⁽¹⁶⁾. It is found that the velocity dependent field shifts the cross sections toward higher energies, reduces them, but leaves unchanged the sum rule defined below.

Whether short range correlations brought about by a hard-core will still be necessary in order to bring agreement with experiment remains a question to be explored.

We wish now to discuss the so called sum-rule i. e. the integral of the cross section over the excitation energy up to the mesonic threshold.

If the elastic contribution is excluded and the independence on the excitation energy of the σ_{Mott} is taken into account, one gets:

$$S(q) = \frac{1}{Z} \int_{\omega_1}^{\infty} \hbar d\omega \frac{1}{\sigma_{\text{Mott}}} \frac{d^2\sigma}{d\Omega dE} = \frac{1}{Z} \sum_{n=0}^{\infty} \left\{ |F_{n0}(\vec{q})|^2 - Z^2 |F_{00}(\vec{q})|^2 \right\} \quad (25)$$

that is, using closure:

$$S(q) = \frac{1}{Z} \int_{\omega_1}^{\infty} \hbar d\omega \frac{1}{\sigma_{\text{Mott}}} \frac{d^2\sigma}{d\Omega dE} =$$

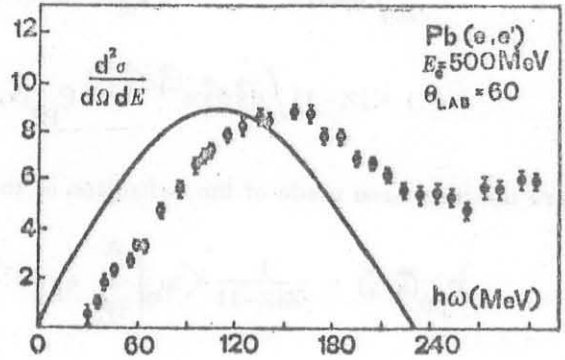


FIG. 15 - Inelastic electron scattering on ^{208}Pb ; the solid curve is the quasi elastic scattering out of a free Fermi gas with $k_F = 1.34 \text{ fm}^{-1}$. The experimental points agree with such a theory but for a shift in the maximum of about 40 MeV (taken from ref. (48)).

(x) - In the Fermi gas model the maximum of the cross section for fixed q and variable ω occurs actually for $\omega = \hbar q^2/2m$ if $q \geq k_F$, but the symmetry of the cross section around $\omega = \hbar q^2/2m$ is preserved only if $q \geq 2k_F$. When $q < k_F$ the locus of the maxima of the cross section is

$$\omega = \frac{\hbar q k_F}{m} - \frac{\hbar q^2}{2m}$$

$$= \frac{1}{Z} \int d\vec{x} d\vec{y} e^{i\vec{q} \cdot (\vec{x} - \vec{y})} \langle \psi_0 | \hat{\rho}^+(\vec{y}) \hat{\rho}(\vec{x}) | \psi_0 \rangle - Z |F_{00}(\vec{q})|^2 \quad (*) \quad (26)$$

$$= 1 + (Z-1) \int d\vec{x} d\vec{y} e^{i\vec{q} \cdot (\vec{x} - \vec{y})} P_{pp}(\vec{x}, \vec{y}) - Z |F_{00}(\vec{q})|^2, \quad (27)$$

where use has been made of the definition of the charge operator $\hat{\rho}(\vec{x})$ (formula (21^m)) and

$$P_{pp}(\vec{x}, \vec{y}) = \frac{1}{Z(Z-1)} \langle \psi_0 | \sum_{i \neq j}^A e_i e_j \delta(\vec{x} - \vec{x}_i) \delta(\vec{y} - \vec{x}_j) | \psi_0 \rangle, \quad (28)$$

is the spatial proton-proton correlation function, normalized to unity; ω_1 is the energy of the first excited state.

The (27) is the famous q-fixed sum rule for Coulomb interaction. For $q=0$ $S(q)$ vanishes and for large q it becomes equal to 1, i.e. the value corresponding to the incoherent scattering of the electron from the Z point protons of the target.

Remember that the elastic cross section, i.e. the coherent scattering of the electron from the Z protons of the target, is proportional to Z^2 .

Care should be taken in using closure: in fact since $\omega < cq$ for small momentum transfer a large fraction of the nuclear spectrum is excluded, so the completeness relation

$$\sum_n | \psi_n \rangle \langle \psi_n | = 1 \quad (29)$$

cannot be used.

On the other hand $\hbar\omega$ cannot exceed $m_\pi c^2$, even if formally the integration is performed up to infinity, otherwise the mesonic degrees of freedom come into play and they are not present in the wavefunctions (29).

In spite of these restrictions Mac Voy and Van Hove⁽¹⁷⁾ in a remarkable paper, whose main results are reported in Fig. 16, made a careful investigation on the role of the short range correlations in the q-fixed sum rule. The conclusion is that the correlations imposed by the Pauli principle are by far the most important reducing the cross section by about 30%.

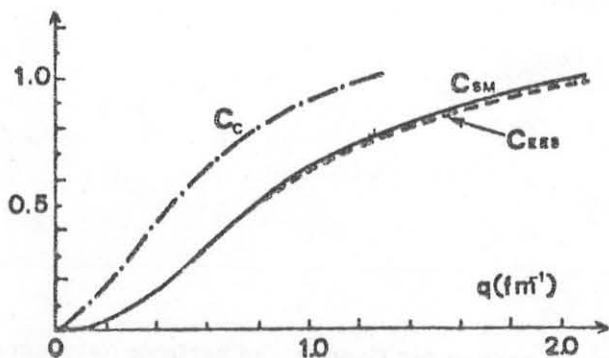


FIG. 16 - Calculation of the constant-q sum rule $S(q)$ in ^{16}O . Only Coulomb interaction is taken into account. C_c corresponds to the classical perfect Fermi gas (no antisymmetrization); C_{SM} is the same but with the Pauli principle; C_{EES} includes the correlations induced by a hard-core of radius 0.4 fm. (Taken from ref. (17)).

(*) - Note that here the elastic form factor is normalized to unity ($F_{00}(0) = 1$).

The dynamical ones, beside of being model dependent, have little effect. If they are brought about by an infinite hard-core of radius ~ 0.4 fm they reduce $S(q)$ of no more than 5%.

A significant theoretical challenge is to understand if the Mac Voy and Van Hove results are compatible with the ones of Czyz and Gottfried. The quoted calculation of Alberico et al.⁽¹⁶⁾ supports, in RPA i. e. with the following polarization function:

$$\Pi_r(\vec{q}, \omega) = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots$$

and a soft core repulsive potential, the Mac Voy and Van Hove results in infinite nuclear matter.

This outcome of course is unfortunate since what we aim at in nuclear physics are "correlations". From this view point perhaps real photons are better suited as we shall see in the following.

It may be worthwhile finally to recall the sum rule for a free Fermi gas :

$$S(q) = \begin{cases} \frac{3}{4} \frac{q}{k_F} \left[1 - \frac{1}{12} \left(\frac{q}{k_F} \right)^2 \right] & \text{for } q < 2k_F \\ 1 & \text{for } q > 2k_F. \end{cases} \quad (30)$$

We wish now to shortly consider the "reaction products" of the quasi-elastic peak: so we shall deal with "exclusive" reactions of the type $(e, e'p)$, a nearly unique tool for establishing the shell structure of nuclei.

The usual theoretical analysis of these processes is made in the plane wave impulse approximation (PWIA) which factorizes the cross section as follows⁽¹⁸⁾:

$$\frac{d^6 \sigma}{d^3 \vec{p}'_e d^3 \vec{p}_f} = g \sigma_{ep}(\vec{p}_e, \vec{p}'_e, \vec{p}_f) S(\vec{p}_i, Q), \quad (31)$$

g being a kinematical coefficient, $\sigma_{ep}(\vec{p}_e, \vec{p}'_e, \vec{p}_f)$ the free electron-proton differential cross section (\vec{p}_e and \vec{p}'_e are the initial and final electron momenta, \vec{p}_f is the final proton momentum) and $S(\vec{p}_i, Q)$ the "spectral function" representing the probability of finding a proton bound in the nucleus with a separation (or removal) energy Q and a momentum \vec{p}_i .

In the frame of the "impulse approximation" there is an interesting relation between the spectral function $S(\vec{p}_i, Q)$ and the total binding energy E_Z of the protons in the target (the so called "Koltun sum rule"⁽¹⁹⁾):

$$E_Z = \frac{1}{2} \int_{-\infty}^{+\infty} dQ \int d\vec{K} \left(\frac{K^2}{2m} - Q \right) S(\vec{K}, Q), \quad (32)$$

which holds irrespective of the structure of the nuclear wavefunction, if only two-body forces are effective. A relation analogous to (32) can be derived for neutrons.

For pure shell model

$$S(\vec{p}_i, Q) = |\psi_{nlj}(\vec{p}_i)|^2 \delta(Q - \epsilon_{nlj}), \quad (33)$$

where $\psi_{nlj}(\vec{p}_i)$ is the Fourier transform of a single particle wave function with eigenvalue ϵ_{nlj} . Therefore the angular distribution of the protons outgoing from a definite missing energy peak permits in principle to distinguish between single particle states with angular momentum $l = 0$ (since $|\psi_{hoj}(0)|^2 \neq 0$) and $l \neq 0$ (being $|\psi_{nlj}(0)|^2 = 0$).

Clearly corrections are necessary to the simple picture of a particle knocked out of a potential in which it is bound in a well defined single particle state. Apart from the multiple scattering of the incoming and outgoing particles which is a large correction to the concept of quasi-free scattering and significantly reduces the cross sections, the extreme single particle model of the nucleus is inadequate for a proper description of the highly excited "hole states" in which the residual nucleus is eventually left and to account for the higher momentum components in the nuclear wavefunction.

Furthermore, while to identify the missing energy E_M with the proton separation energy Q seems quite justified, the identification of the removal energy $Q^{(*)}$ with the single particle energies ϵ_{nlj} requires much care⁽²⁰⁾.

This equivalence is assured by Koopman's theorem which holds under the condition that the orbitals of the single particles do not change during the time of removal.

Such is the situation in a pure Hartree-Fock theory, which gives for the binding energy per particle:

$$\frac{E}{A} = \frac{1}{A} \sum_{i=1}^A \langle i | T + \frac{1}{2} U | i \rangle = \frac{1}{A} \left\{ \frac{1}{2} \sum_i \epsilon_i + \frac{1}{2} \sum_i \langle i | T | i \rangle \right\}. \quad (34)$$

U being the self-consistent one-body potential and T the kinetic energy.

Incidentally for an harmonic oscillator potential it follows⁽²¹⁾ from (34):

$$\frac{E}{A} = \frac{1}{2A} \sum_i \epsilon_i + \frac{a}{\langle r^2 \rangle} \quad (a \text{ is a constant}) \quad (35)$$

where the link among binding energy, single particle energies and size of the system is well exhibited^(o).

In Table II are reported the results for ^{16}O and ^{40}Ca of a plain Hartree-Fock calculation based on the non-local, separable, soft-core Tabakin potential. Also reported are the second order perturbative corrections. The two well known shortcomings of the Hartree-Fock theory are

- (*) - We indicate with Q the separation energy of a nucleon, i. e. the energy required to extract the nucleon from the nucleus, leaving it outside the well with zero kinetic energy. Thinking to the reaction $B(a, ab)C$ where particle "a" knocks out a nucleon "b" from the nucleus B leaving the nucleus C, in general, in an excited state, a peak in the spectrum of the number of the (a + b) coincidences as a function of the missing energy

$$E_M = E_0 - E - T$$

indicates the existence of a bound state of the nucleon B with separation energy: $Q = E_M$. E_0 is the initial energy of the projectile a, while E and T are the final energies of a and b. The excitation energy of the final nucleus C (hole-state) is $E^* = Q - Q_0$, Q_0 being the separation energy when the nucleus C is left in the ground state.

- (o) - Formula (35) is commonly referred to as the Weisskopf rule.

apparent namely:

- a) too little binding energy per nucleon;
- b) too small radius of the system.

TABLE II

Nucleus	16O		40Ca	
First-order energy	-2.41		-3.74	
Second-order potential energy	-4.3		-7.2	
Total B. E. /A	-6.71		-10.94	
Exper. B. E. /A	-7.98		-8.55	
rms radius (th.)	2.38		2.96	
rms radius (ex.)	2.61		3.40	
<u>Removal energies</u>	th.	exp.	th.	exp.
Os	43	40 ± 8	66	50 ± 11
Op _{3/2}	23.1	21.8	--	---
Op _{1/2}	11.6	15.7	28	34 ± 6
Od _{5/2}	--	--	23	22.8
Od _{3/2}	--	--	7.7	15.8

Caption: pure Hartree - Fock calculations with the Tabakin potential (performed by Kerman et al. (22)). Nucleons orbitals (single particle energies) are calculated in first order perturbation theory while the rms radii contain also the second order corrections. Radii are in fm, energies in MeV.

On the contrary the Hartree-Fock eigenvalues ϵ_{nlj} appear to be in reasonable agreement with the experimental removal energies. The second order contribution improves the binding energy per particle, but has little effect on the size of the system: more seriously it destroys the fair agreement of the ϵ_{nlj} with the experiment. From here the need to go beyond Hartree-Fock.

As it is well known, the next step after Hartree-Fock is the so called Brueckner-Hartree-Fock theory⁽²³⁾, where one attempts to take into account the short range correlations brought about by the infinite hard-core repulsive potential replacing v with the G-matrix, solution of the Bethe-Goldstone equation (23).

In Brueckner-Hartree-Fock theory the analogous of (34) for the binding energy per particle is:

$$\frac{E}{A} = \frac{1}{A} \left\{ \sum_i \langle i | T | i \rangle + \frac{1}{2} \sum_{i,j} \langle ij | G(\epsilon_i + \epsilon_j) | ij \rangle \right\}. \quad (36)$$

Without entering into the elaborate mathematical and computational details of this theory, we simply note that a satisfactory situation is not yet reached, at least for the binding energies, which are still too small. This is apparent from Table III.

It is important to realize that in the B. H. F. theory Koopman's theorem is no longer valid since the orbits of the other particles rearrange themselves when a proton is knocked out of the nucleus.

TABLE III
Results of B. H. F. theory for ^{16}O and ^{40}Ca .

Nucleus	Potential used	B. E. /A	rms radius	Removal Energies		
				0s	$0p_{3/2}$	$0p_{1/2}$
^{16}O	HJ ⁽²⁴⁾	2.8	3.06	28.5	13.6	13.6
^{16}O	RH ⁽²⁴⁾	3.4	2.91	32.1	15.5	15.5
^{40}Ca	HJ ⁽²⁴⁾	2.5	3.87	----	----	----
^{40}Ca	RH ⁽²⁴⁾	3.4	3.68	----	----	----
^{16}O	HJ ⁽²⁵⁾	2.37	3.08	35	25.6	24.2
^{16}O	HJ ⁽²⁶⁾	3.8	2.48	44	20.3	15.9
Exp(for ^{16}O)		7.98	2.65	43+5	21.8	15.7
Exp(for ^{40}Ca)		8.55	3.45	----	----	----

HJ = Hamada-Johnston; RH = Reid hard core. Energies are in MeV, radii in fm.

A real step forward in understanding removal energies in finite nuclei is represented by Negele's theory⁽⁵⁾. Using Reid soft-core potential as a starting point for calculating the G-matrix in the local density approximation (LDA), Negele comes up with a local, density-dependent, effective interaction with three additional parameters adjusted to fit the following properties of nuclear matter and finite nuclei:

- a) volume energy;
- b) symmetry energy;
- c) the experimental radius of ^{40}Ca .

This effective interaction is then used for a standard Hartree-Fock calculation (D. D. H. F.). In such a theory the relationship between the binding energy and the single particle energy is given by:

$$\frac{E}{A} = \frac{1}{A} \left\{ \frac{1}{2} \sum_i (\epsilon_i + \langle i | T | i \rangle) - \frac{1}{12} \int \rho(\vec{r}_1) k_F(\vec{r}_1) \rho(\vec{r}_2) v_{D1}(\vec{r}_{12}) d^3 r_1 d^3 r_2 \right\}, \quad (37)$$

where v_{D1} is the long-range ordinary force, which is always positive in order to get saturation. Consequently the last term of (37) is negative so that the nucleus is more strongly bound than the single particle energies indicate. The experimental single particle energies, the binding energy and the size of the system are well accounted for by Negele's theory: this is illustrated in Table IV.

We emphasize that the most critical element in order to get such good results appears to be the density dependence of the interaction.

Note also that formally Koopman's theorem still holds in Negele's theory; in fact the interaction between the remaining nucleons is changed indeed by the removal of a proton because the density is reduced, but this change is already included in Negele's ϵ_i .

In conclusion we hope to have given a feeling of the amount of new ideas and theoretical investigations prompted by the (e, e'p) experiments: the counterpart in nuclear physics of the Franck and Hertz measurements in atomic physics.

TABLE IV
Results of Negele's D.D.H.F. theory.

Nucleus	^{16}O	^{40}Ca			
Theor. binding energy	7.59			7.99	
Exper. binding energy	7.98			8.55	
Proton rms radius (theoretical)	2.71			3.41	
Proton rms radius (experimental)	2.64			3.43	
		0s	0p	0d	1s
Proton removal energies (experimental)	53+11	37+6	16.2	15.7	
$-\epsilon_i$ Negele's eigenvalue	47	30	15.2	11.9	

Energies are in MeV, radii in fm.

3. - PHOTONUCLEAR REACTIONS, -

Real photons are, in principle, a very powerful tool to study nuclear structure. Unfortunately, it is difficult to obtain good quality photon beams, and this fact has in the past strongly biased the experimental work, preventing the full exploitation of this probe.

Therefore, in spite of some thousands papers published in the field⁽²⁷⁾, a lot of work remains to be done on photonuclear reactions. Recent technical improvements have set the instrumental possibilities on a satisfactory, if not yet ideal, level: we therefore look forward to a new generation of exciting photonuclear experiments, whose vanguard came recently to light^(x).

We believe that in the future great effort will be paid to a more systematic study of the decay products of the giant dipole states. For example the degree of polarization of the photoparticles which brings informations on the strength of the spin-orbit coupling inside nuclear structure is worth of analysis.

Also the energy distributions of the photoparticles will be explored more carefully since they are a sensitive test of the nuclear level density $\eta(E)$. To understand the departures of $\eta(E)$ from the one obtained with purely statistical methods means to have a deeper insight of the correlations among nucleons in the nuclear structure.

But in particular we think to the angular distribution of the photonucleons. It has been pointed out in the literature⁽³³⁾ the importance, for this kind of reaction, of the semidirect process, namely the one going through a "doorway" state (see Fig. 17b) with respect to the direct one, illustrated in Fig. 17a, in determining both the absolute magnitude of the photonuclear cross section and the shape of the angular distribution. Since the semidirect process is typically charge-independent, it tends to make similar the angular distribution of protons and neutrons.

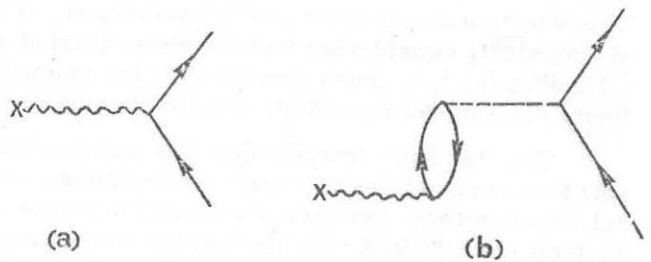


FIG. 17 - A nucleon is emitted by a nucleus by a direct photoabsorption (a) and by the decay of a collective state (b).

(x) - We are thinking particularly at the Saclay⁽²⁸⁾, Livermore⁽²⁹⁾, Glasgow⁽³⁰⁾, M. I. T.⁽³¹⁾, and Mainz⁽³²⁾ impressive results.

Moreover since the collective states of the nuclear system are associated with quadrupole as well as dipole degrees of freedom, the angular distribution of the photoparticles will be asymmetric around 90° .

A recent calculation⁽³⁴⁾ has shown indeed that the differential cross section for the process (γ, p) and (γ, n) peaks forward, at least in some energy range. This question, in our opinion, deserves a more systematic and careful investigation especially on the experimental side, also because it will shed light on the role of multipoles higher than the dipole in the photoprocesses.

To extract the angular distribution coefficients and to perform the quantitative analysis of interfering multipoles, a higher sensitivity can be achieved by using a polarized photon beam: a facility which gives almost completely polarized and quasi-monochromatic photons is under construction at the Frascati Laboratory⁽³⁵⁾. With such a beam, it will be possible to assign unambiguously the multipolarity (E1 or E2) to the coherent scattering and, in E1 approximation, nuclear coherent and incoherent scattering will be easily separated.

By the inverse photonuclear reactions, and with polarized protons, amplitudes and relative phases of the reaction matrix elements can be measured⁽³⁶⁾, improving the understanding of the configurations in a Giant Dipole Resonance (G. D. R.).

As a last example of future work in the low energy region (20 - 30 MeV) we mention the isospin splitting of the G. D. R. Especially in light nuclei, this is an almost open field of research, interesting per se spectroscopically and also for other reasons: according to Leonardi⁽³⁷⁾, for instance, a measurement of the neutron radius can be achieved through the isospin sum rules.

Above the G. D. R. , the incoherent Compton scattering by bound protons becomes significant as the wavelength of the incident photon approaches the internucleonic distance. In the impulse approximation, the scattering amplitude is directly related to the momentum distribution of the nucleons inside the nucleus, and therefore by this method that very important quantity can be measured (as in $(e, e'p)$ experiments, but without worrying about radiative corrections and tails).

Another way to investigate the momentum distribution goes through the (γ, N) experiments. Nuclear photoionization looks in fact particularly suitable to probe high momentum components in the nuclear wavefunction. At intermediate energy (around say 100 MeV) a (γ, p) reaction, for instance, involves initial proton momenta of $1.5 - 2.5 \text{ fm}^{-1}$. In the last years a number of experiments have been carried out on (γ, N) processes at intermediate energy. The error bars are now in some cases sufficiently short to probe, at least in first approximation, the theory.

A pure shell model description has proved definitely incapable of explaining the experimental cross sections: the theoretical predictions are at least a factor of ten too small and, in addition, the observed forward asymmetry of the (γ, n) angular distribution cannot be reproduced. This shortcoming is most easily understood, as it has been pointed out in the literature a number of times⁽³⁸⁾, considering that the momentum of a nucleon emitted in a photoprocess is about $\sqrt{2} mc^2 \hbar \omega / c$, i. e. much greater than the momentum $\hbar \omega / c$ the photon brings in, and the conventional shell model cannot provide single particle wavefunctions with large momentum.

The "ad hoc" introduction (for instance with the Jastrow's method) of short range nucleon-nucleon correlations to repair the deficiency of high momentum components in the pure shell model wavefunction, has been successful to fit the experiments in the lower energy range⁽³⁹⁾, but Jastrow correlations with reasonable short range behaviour seem to be inadequate at energies as high as 100 MeV⁽⁴⁰⁾.

In a more recent model calculation some photonuclear processes have been analyzed taking into account shell model transitions, nucleon-nucleon correlations in the initial and final states and contributions arising from the direct coupling of the electromagnetic field to the mesons exchanged between a n-p pair⁽⁴¹⁾. A diagrammatical description of the exchange contribution is given in Fig. 18.

The agreement obtained with existing experiments on ^4He , ^{12}C and ^{16}O nuclei is satisfactorily good and shows the importance of the exchange contributions (i. e. of the absorption of the photon by a correlated n-p pair) at photon energies above 80 MeV.

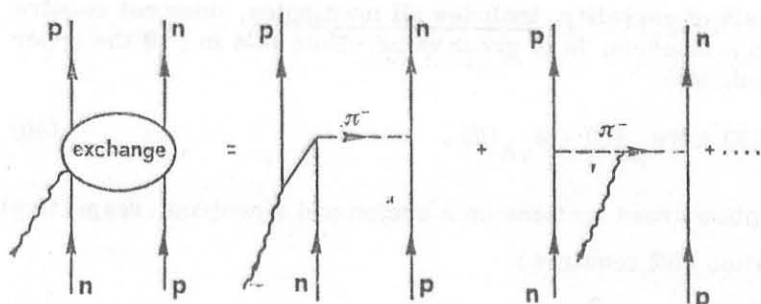


FIG. 18 - A correlated p-n pair absorbs a photon through the exchange current carried by a π^- meson.

This brings us back to the assumptions of the old quasi-deuteron model by Levinger⁽⁴²⁾, and clarifies with the microscopic description of the physical process what was a phenomenological approach to the photoeffect at intermediate energy.

As a final point of this survey we like to touch upon two topics which are still very much open, namely:

- a) how much of multipoles higher than the dipole the nuclear system can absorb;
- b) what happens to the photoeffect above the mesonic threshold.

Of valuable guidance in this kind of problems are the photoabsorption sum rules, the oldest and more celebrated among them being the Levinger and Bethe⁽⁴³⁾ electric dipole one:

$$\int \sigma_{\gamma A}(E)dE = 2\pi^2 \alpha^2 \hbar c \langle \psi_0 | [D, [H, D]] | \psi_0 \rangle = \tag{38}$$

$$= \frac{2\pi^2 \alpha^2 (\hbar c)^2}{mc^2} \frac{NZ}{A} (1+k) = 60 \frac{NZ}{A} (1+k) \text{ mb} \times \text{MeV},$$

α being the electromagnetic coupling constant and D the dipole operator (the other symbols are obvious). In the integral the upper limit is not specified, but logically should be set by the meson mass m_π (~ 140 MeV).

It may be worthwhile to recall that the derivation of (38) requires:

- a) $kR \ll 1$;
- b) the Siegert theorem to hold.

The k term, which physically represents the contribution of the meson exchange currents (or mathematically the contribution of the nuclear forces non double-commuting with the dipole operator) was generally estimated to be 0.4⁽⁴⁴⁾. Furthermore it was commonly accepted that the nuclear dipole absorption was concentrated below, say, 30 MeV.

The recent total photoabsorption measurements on nuclei up to ^{40}Ca , of the Mainz group⁽³²⁾, for energies up to the mesonic threshold yielded for k values between 0.4 and 1.1, depending on the nucleus. In the frame of the Bethe Levinger sum rule attempts have been made in order to explain such a large value of k .

It turns out that tensor correlations are of paramount importance⁽⁴⁵⁾, while short range correlations as those induced by the hard-core are not. This calculations are difficult to perform and are necessarily affected by large uncertainties. Therefore we think it more useful to investigate before hand if the experimental data are to be compared at all with the electric dipole sum rule.

To this purpose the celebrated Gell-Mann, Goldberger and Thirring dispersion sum rule⁽⁴⁶⁾

$$\int_0^{m_\pi} \sigma_{\gamma A}(E)dE = \frac{2\pi^2 \alpha^2}{mc^2} (\hbar c)^2 \frac{NZ}{A} + \int_{m_\pi}^{\infty} \Sigma(E)dE, \tag{39}$$

which is derived under the only hypothesis of causality, includes all multipoles, does not require $kR \ll 1$ and also avoid the use of Siegert's theorem, is of great value. Note that in (39) the upper limit of integration is explicitly indicated, and

$$\Sigma(E) = Z\sigma_{\gamma p}(E) + N\sigma_{\gamma n}(E) - \sigma_{\gamma A}(E), \quad (40)$$

where $\sigma_{\gamma p}$ and $\sigma_{\gamma n}$ are the photoabsorption cross sections on a proton and a neutron, respectively.

Unfortunately, to hold good relation (39) requires:

$$\lim_{E \rightarrow \infty} [F_{\gamma A}(E) - AF_{\gamma N}(E)] = 0, \quad ()$$

where $F_{\gamma A}(E)$ and $F_{\gamma N}(E)$ are the forward scattering amplitudes of a photon out of the nucleus and of the nucleon respectively.

Some time ago this was thought to be true and the integral in the r. h. s. of (39) was estimated, on the basis of the existing high energy data, to be a correction of about 40%. Such a result, by comparison with the Bethe Lvinger sum rule, was interpreted as a strong indication that the nucleus, like the atom, essentially absorbs electric dipole radiation only.

However the "vector meson dominance" i.e. the appearance of the hadronic components of the photon at energies larger than about 5 GeV has demonstrated that assumption (41) is wrong. The photonucleus scattering between 5 and 20 GeV can essentially be viewed as a vector meson (in particular rho meson) scattering following a photon-vector meson transition.

Weise⁽⁴⁷⁾ argues then that since the basic feature of the hadron nucleus scattering at high energies is a "shadowing effect" (the hadron being essentially scattered by the surface) one can conjecture

$$\sigma_{\gamma A}(E) = A_{\text{eff}}(E)\sigma_{\gamma N}(E). \quad (42)$$

Existing data suggest:

$$A_{\text{eff}}(E) \simeq A^{0.9}, \quad (43)$$

in the range $5 \text{ GeV} \leq E \leq 20 \text{ GeV}$, which clearly shows why (41) does not hold.

Without trying to describe the several methods currently attempted to remedy the deficiencies of the Gell-Mann, Goldberger and Thirring sum rule (we refer to the nice paper of Weise⁽⁴⁷⁾ for this purpose), we like to stress how this theme of research emphasizes the intimate connection between low-energy and high-energy phenomena. The integral relationship (39) between these two domains still keeps surprises in the development of both nuclear and particle physics.

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