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A. Dellafiore : ON THE AVERAGE MULTIPLICITY OF NEUTRONS EMITTED AFTER MUON CAPTURE IN HEAVY NUCLEI. -

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ABSTRACT. -

It is shown that previously obtained excitation energies account for the observed average number of neutrons emitted after muon capture in heavy nuclei.

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In a previous work[1] it has been shown that the general trend of the total muon capture rates can be explained by a suitable variation along the nuclear table of the average momentum $\overline{\nu}$ of the emitted neutrino. In heavy nuclei the following phaenomenological law was obtained

$$\overline{\nu} \simeq 88 - 0.3\,(Z-20) \qquad [\mathrm{MeV}] \tag{1}$$

This is in contrast with the usual assumption[2,3,4,5] of a constant average neutrino momentum $\langle \nu \rangle \simeq 85$ MeV made in the framework of Primakoff's formula for the total capture rate.

The value of $\overline{\nu}$ agrees with $\langle \nu \rangle$ in intermediate nuclei, but as Z increases $\overline{\nu}$ gradually decreases and in Pb Eq. (1) gives

$$\overline{\nu} \simeq \langle \nu \rangle - 15 \text{ MeV}.$$

These two hypotheses lead to quite different average excitation energies of daughter nuclei in heavy elements. As a matter of fact the momentum of the neutrino is connected to the excitation energy E of the daughter nucleus through the relation $\nu = E_m - E$, where $E_m = m_\mu - \varepsilon_a - \Delta M$, m_μ is the mass of the muon, ε_a its binding energy in the K-orbit of the mesic atom and $\Delta M = M(N+1, Z-1) - M(N, Z)$ is the difference between the masses of the daughter and capturing nucleus.

It is possible to check these two assumptions by comparison with experimental data on the deexcitation of the daughter nuclei. This last process occurs mainly through the emission of neutrons(6). As a neutron to come off the nucleus needs to overcome a threshold energy of the order of 5 ÷ 10 MeV, the average number of the neutrons emitted gives a good description of the mean excitation energy of the nucleus after muon capture.

Experimental data on the multiplicity of the neutrons emitted are available for some nuclei in the range of our interest(7) and are shown in Fig. 1 and Table I. It can be seen that, notwithstanding wide deviations from a smooth line, there is a clear tendency to increase with the atomic number.

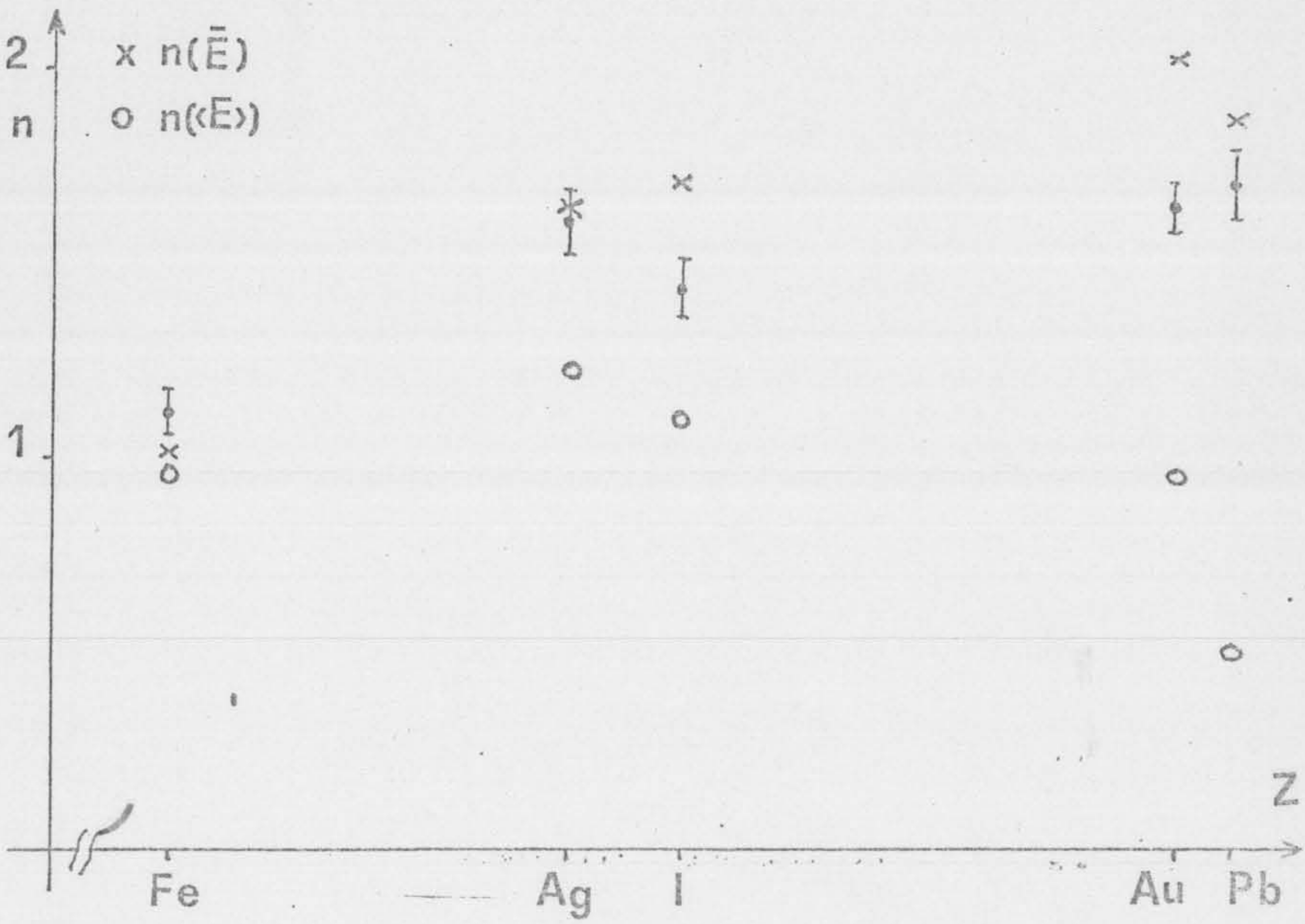

FIG. 1

TABLE I

Element		Q	$\bar{E}$	$n(\bar{E})$	$\langle E \rangle$	$n(\langle E \rangle)$	n exp.
capturing nucleus	daughter nucleus	[MeV]	[MeV]		[MeV]		
Fe	Mn	8.7	15	1.03	14	0.97	1.12 ± 0.04
Ag	Pd	7.6	20	1.66	15	1.25	1.61 ± 0.06
I	Te	7.8	21a	1.78	15	1.11	1.44 ± 0.06
Au	Pt	7	23a	2.21	11	0.97	1.66 ± 0.05
Pb	Tl	5.7	19	1.88	5	≈ 0.5	1.72 ± 0.07

a) For these nuclei $\bar{E}$ has not been calculated in Ref. (1) and values used here are obtained by extrapolation.

Theoretical calculations based on the degenerate Fermi gas model[8] and on the Brueckner picture[9] give multiplicities lower than the observed ones.

We want to show here that the higher excitation energy found in Ref. (1) accounts for the observed values of the neutron multiplicity in heavy nuclei.

In the high mass region it is found that about $80 \div 90\%$ of the emitted neutrons follow an evaporation spectrum[7] so that statistical considerations can be applied to this problem. As a first approach we assume that states excited to an energy greater than the threshold for neutron emission Q, can loose their energy only by evaporating neutrons, and consequently γ rays are emitted only when the excitation energy is lower than Q. In this model the mean number of neutrons evaporated from a nucleus with excitation energy $E > Q$, has been calculated by Le Couteur[10] and is given by

$$r(E) = \frac{1}{2} + \int_Q^E \frac{1}{Q + 2\,\tau\,(E' - Q)}\, dE' \qquad (2)$$

where $\tau(E)$ is the nuclear temperature. Due to the fact that in this model $r(E) = 0$ for $E < Q$ and $r(E) = 1$ for $Q < E < 2Q$, the normalization $r(Q) = 1/2$ has been taken.

Of course we expect that this model gives multiplicities higher than the observed ones. As a matter of fact we have neglected the follo wing effects that all contribute to reduce r(E):

a) about 15% of the emitted neutrons are "direct" ones[7], that is, they escape from the nucleus without sharing their energy with the other nucleons; of course in these processes the multiplicity is one;
b) charged particles emission, even if inhibited by the Coulomb barrier, occurs and is indeed observed in the extent of about 3%;
c) deexcitation through γ emission can take place even at energies higher than Q.

So the results given by Eq. (2) should possibly be corrected in order to take into account these effects.

In order to performe the integration in Eq. (2) we assume the following dependence of the nuclear temperature τ on the excitation energy:

$$\tau(E) = \begin{cases} T = \text{constant} & \text{for } 0 < E < E_x \\ \left[\sqrt{(a/(E-U_o))} - 5/4(E-U_o)\right]^{-1} & \text{for } E > E_x \end{cases}$$

This is suggested by a best fit of the data on neutron and proton resonances (see Appendix). The parameters T, a, U_o, E_x, can be found in the literature[11] and are given in Table II for the nuclei we are interested in.

TABLE II

Element	a $[MeV^{-1}]$	E_x $[MeV]$	U_o $[MeV]$	T $[MeV]$
Mn	7.3	5.3	2.8	1.
Pd	15.3	7.3	2.6	0.66
Te	16.2	6.5	2.2	0.63
Pt	19.7	5	1.5	0.53
Tl	7.5	5.8	1.2	1

Then, Eq. (2) gives

$$r(E) = \begin{cases} 0 & \text{for} \quad E < Q \\ \frac{1}{2} + \frac{E_x}{Q+2T} + \sqrt{aU} - \sqrt{aU_x} - A\log\frac{\sqrt{aU}+y_1}{\sqrt{aU_x}+y_1} - B\log\frac{\sqrt{aU}+y_2}{\sqrt{aU_x}+y_2} & \text{for} \quad E > Q \end{cases}$$

where:

$$U = E - Q - U_o, \qquad U_x = E_x - U_o,$$

$$\begin{matrix} A \\ B \end{matrix} = \frac{1}{4}\left\{ \left(\frac{5}{2}+aQ\right) \pm \frac{\left(\frac{15}{2}+aQ\right)}{\sqrt{1+10/aQ}} \right\} \qquad \begin{matrix} y_1 \\ y_2 \end{matrix} = \frac{aQ}{4}\left\{ 1 \pm \sqrt{1+10/aQ} \right\}$$

A typical graph of r(E) is shown in Fig. 2.

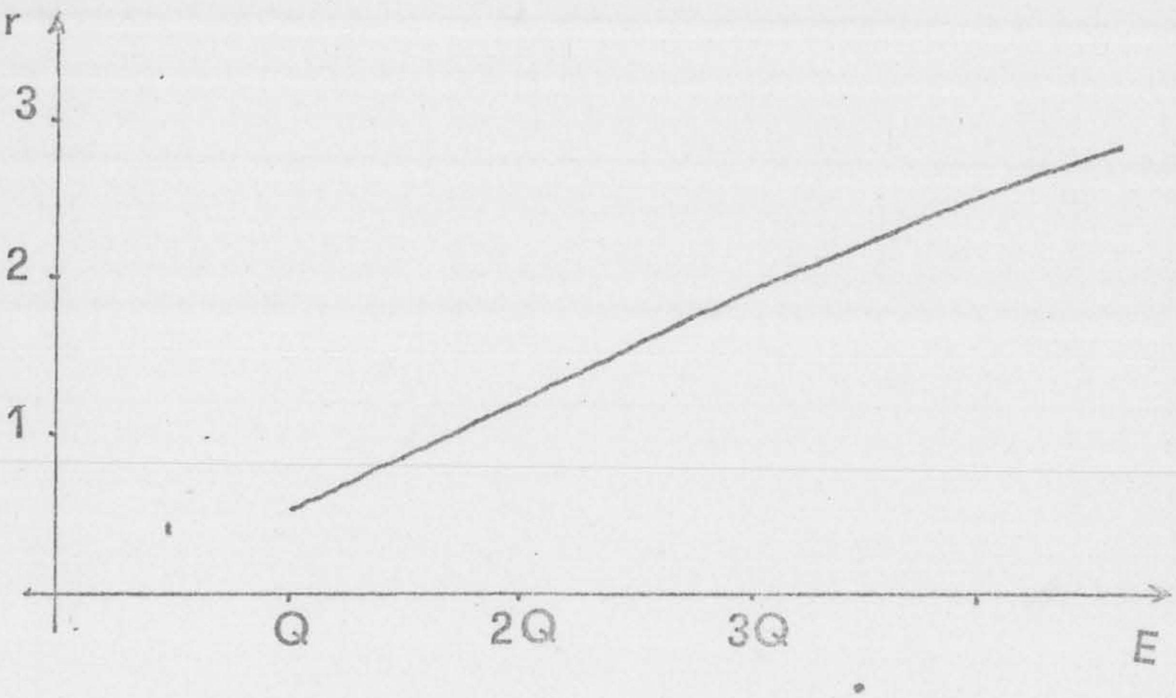

FIG. 2

The mean number of neutrons evaporated after muon capture is obtained by averaging r(E) over the excitation spectrum. If f(E) is the excitation strenght we have

$$n = \int_{o}^{E_m} r(E)\, f(E)\, dE \Big/ \int_{o}^{E_m} f(E)\, dE$$

As we do not know f(E), we shall take $r(\bar{E})$ as the mean number of neutrons given by our model. This value can be corrected for the observed 3% of charged particles emission and for the, let us say, 15% of direct neutron emission, by assuming

$$n(\bar{E}) = \left[r(\bar{E}) \times 0.97 \right] \times 0.85 + 0.15 \tag{4}$$

The results obtained are shown in Table I. In the first two columns we give the average values assumed for the threshold energy Q and the mean excitation energy $\bar{E}$ calculated as in Ref. (1), in the third column the mean number of neutrons given by Eq. (4). In the fourth and fifth column the mean excitation energy $\langle E \rangle$ coming from the assumption $\langle \nu \rangle \simeq 85$ MeV and the consequent mean number of neutrons $n(\langle E \rangle)$ are given.

We do not take into account quantitatively the effect of γ emission so our average multiplicities could possibly be a little higher than the experimental ones.

In order to evaluate the effect of the spread of the excitation strenght around the mean value $\bar{E}$, we have calculated n assuming the following excitation spectrum:

$$f(E) = \begin{cases} \text{constant} & \text{for} \quad 0 < E < C \\ 0 & \text{for} \quad E > C \end{cases}$$

where the energy C is determined by the condition that f(E) reproduces the mean value $\bar{E}$ through the relation

$$\bar{E} = (E_m - \bar{E})^2 \int_{o}^{E_m} E\, f(E)\, dE \Big/ \int_{o}^{E_m} (E_m - E)^2 f(E)\, dE$$

used in Ref. (1).

In all the cases we have found a decrease of less than 2%.

As it can ve seen in Table I and in Fig. 1, n(< E>) is definitely too low in all the heavy nuclei considered which leads to the conclusion that the assumption $\langle \mu \rangle = 85$ MeV is incompatible with the experimental data.

For what concerns $n(\bar{E})$, it reproduces the comprehensive increase of n with the atomic number, furthermore the agreement is quite good for Ag and Pb. The difference between $n(\bar{E})$ and the observed multiplicity in I and Au is perhaps too large to be attributed to γ decay and can be probably due to nuclear structure effects. As a matter of fact, in these two cases $\bar{E}$ has not been calculated in Ref. (1) and has been assumed here by extrapolation so it could be a little too high (a decrease of 1 MeV in $\bar{E}$ gives a decrease of $0.10 \div 0.15$ in $r(\bar{E})$). The slightly too low result obtained in Fe indicates alternatively either that this nucleus is too light for a statistical model beeing applicable here, or that $\bar{E}$ is a little higher. As it has been shown in Ref. (1) exchange forces can increase $\bar{E}$, their effect being more relevant in light nuclei.

The conclusions are that the values of the mean excitation energy obtained in Ref. (1) for heavy nuclei are substantially confirmed by the experimental data on neutron emission, while the assumption of a constant neutrino momentum $\langle \nu \rangle = 85$ MeV leads to excitation energies definitely too low in heavy nuclei.

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APPENDIX. -

The nuclear temperature used here is slightly different than that determined in the study of neutron and proton resonances(11).

If $\varrho(E,J)$ is the density of nuclear levels of given energy E and total angular momentum J, we can have two different definitions:

the "observable" level density $\quad \varrho(E) = \sum_J \varrho(E,J)$

and the "total" level density $\quad W(E) = \sum_J (2J+1)\varrho(E,J)$.

They give two corresponding nuclear temperatures

$$\frac{1}{\tau'} = \frac{d}{dE} \log \varrho(E)$$

and

$$\frac{1}{\tau} = \frac{d}{dE} \log W(E)$$

In Ref. (11) the first one is assumed to be the "nuclear temperature", while we are interested in the second one.

In the Fermi gas model, assumed to be valid at high energies, one obtains

$$\frac{1}{\tau'} = \sqrt{\frac{a}{E-U_o}} - \frac{3}{2(E-U_o)}$$

and

$$\frac{1}{\tau} = \frac{1}{\tau'} + \frac{1}{4(E-U_o)}$$

Here a is a parameter that can be determined from experimental data and U_o takes into account the effect of pairing forces and can be calculated.

For energies lower than about 10 MeV it is found that assuming a constant temperature T fits the experimental data better than the Fermi gas model. If E_x is the tangency point between the two representations, the requirement that the temperature be a continuous function of E gives

$$\frac{1}{T} = \frac{1}{\tau'(E_x-U_o)} + \frac{1}{4(E_x-U_o)} .$$

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