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G. Inglima, E. Rosato, R. Caracciolo and A. De Rosa:

$^{32-33}S(\tau,\alpha)^{31-33}S$ REACTIONS AS A TEST OF DWBA IN DESCRIBING HIGH Q-VALUE (τ,α) REACTIONS.

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ABSTRACT. -

A possible explanation of the very high spectroscopic factors found in the ^{33}S $(\tau,\alpha)^{32}S$ reaction has been searched for in the probable failure of DWBA in describing (τ,α) reactions with very high Q-values.

A complete analysis of the DWBA methods has been made, studying, at the same time, the effect of a radial cut-off and of the finite range corrections on the theoretical angular distributions.

The results, however, are still unable to explain completely the great discrepancy between theory and experiment.

1. - INTRODUCTION. -

During our spectroscopic investigations about nuclei in the s-d shell[1,2], which had been undertaken in order to further test some recent nuclear models[3,4] we studied the reactions $^{32}S(\tau, d)^{33}Cl$ and $^{32-33}S$ $(\tau,\alpha)^{31-32}S$. Thus it was possible to extract the spectroscopic factors from these reactions, by analysing them in terms of DWBA, and certainly really interesting and unexpected results were obtained[1,2]. In fact, while for the stripping reaction $^{32}S(\tau, d)^{31}S$ and for the pick-up reaction $^{32}S(\tau,\alpha)^{31}S$ the S-factors which had been obtained were in a very good agreement with the theoretical ones from the MSDI[3] and ICVM[4] models and with those from other experimental works[5 ÷ 8], the spectroscopic factors extracted from the $^{33}S(\tau,\alpha)^{32}S$ reaction were found to be much larger than those theoretically predicted. (This reaction was studied here for the first time). In particular the pick-up S-factor for the transition to the ground state in ^{32}S results about six times larger than the one corresponding to the ground-state transition in ^{33}Cl for the stripping reaction. On the contrary, in the hypothesis of charge independence of nuclear forces, they should be identical, independently of the nuclear model adopted[3].

Now, the $^{33}S(\tau,\alpha)^{32}S$ reaction has very high Q-values (11.94 MeV for the ground-state transition), so very large angular momentum transfers are involved, which are very different from the semiclassical angular momentum transfers $|\vec{K}_{in} - \vec{K}_{out}|$. R. Due to this angular momentum mismatch DWBA calculations may be no more reliable[9], this being a possible reason for such very large spectroscopic factors. Thus we wanted to test accurately the use of DWBA in the case of the $^{32-33}S(\tau,\alpha)^{31-32}S$ reaction, also trying to see if the introduction of a radial cut-off or of the finite range corrections could reduce the discrepancy between experimental data and theory.

2. - DWBA ANALYSIS. -

The (τ, d) and (τ,α) reactions on $^{32-33}S$, at a bombarding energy of 10.4 MeV, proceed by means of a prevalently direct mechanism. Consequently it is possible to analyse the experimental angular distributions in terms of DWBA. For what concerns the stripping reaction on a ^{32}S target we may then write:

(1) $$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = 4.42\,(2\,J_f + 1)\,C^2S^{\ell,j}\left(\frac{d\sigma}{d\Omega}\right)^{\ell,j}_{DW}$$

where 4.42 is the normalization factor suggested by Bassel, J_f the spin of the final nucleus, C a Clebsch-Gordan coefficient relative to isospin, S the spectroscopic factor and $(d\sigma/d\Omega)^{\ell,j}_{DW}$ the differential cross section calculated by means of DWBA. For the pick-up reaction a similar relation exists:

(2) $$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \frac{23\;C^2S^{\ell,j}}{2\,J + 1}\left(\frac{d\sigma}{d\Omega}\right)^{\ell,j}_{DW}$$

where J is now the transferred angular momentum and 23 the normalization factor for (τ,α) reactions. $(d\sigma/d\Omega)_{DW}$ depends exclusively on the reaction kinematics, while S contains informations about the structure of the target and residual nuclei, as it represents the probability that the final nucleus has the same configuration of the target plus one proton in some orbit (ℓ, j) in the case of the stripping reaction, or minus one neutron in the pick-up case. So it is possible to obtain some information about ℓ-transfers, spin and parities of the final nucleus and about single-particle strenghts of its different excited states.

The theoretical calculations relative to the reactions we have studied were carried out by means of the DWUCK[10] computer code, using the same optical model parameters of ref. [1].

In Fig. 1 and 2 are reported the experimental angular distributions of the reactions studied here together with DWBA curves. As it is apparent from the figures the quality of the fits is satisfactory for almost all the transitions. In table I, II and III are shown the spectroscopic factors which were extracted from these reactions.

The uncertainty on the S-factors has been estimated to be about 20% as it is usual in this kind of experiments.

Further comments about these results are reported in refs. (1, 2).

3. - DISCUSSION. -

3.1. - High Q-value reactions. -

DWBA calculations are generally satisfactory for direct reactions as most of the details seems to be not very important for what concerns the final cross section. In the case of high Q-value reactions, however, one must pay attention in using relationships (1) and (2), because in these conditions there are important contributions also from low partial waves, i. e. just those ones which are not sufficiently determined by the elastic scattering in the entrance and exit channels. In fact for this to happen the semiclassical condition:

$$\ell - \left|\vec{K}_{in} - \vec{K}_{out}\right| . \; R = \Delta L \tag{3}$$

must be satisfied at least in an approximate way. If the Q-value is very high, as it is the case for the (τ, α) reaction on ^{33}S, for most of the transitions to the various levels in ^{32}S, $\Delta L \gg \ell$, so DWBA could loose much of its reliability. The best way to test the validity of theoretical calculations, in this case, is to look at the decomposition in partial waves of the scattering amplitudes or, which is equivalent, at the decomposition of the cross section $\sigma_{\ell, j}$ it terms of factors of the Legendre polynomials:

$$\sigma_{\ell j} = \text{const} \sum_{m=-\ell}^{\ell} \left| \sum_{L_b, L_a} \Gamma^{\ell, m}_{L_b, L_a} \; f^{\ell, j}_{L_b, L_a} \; P^{m}_{L_b}(\theta) \right|^2 \tag{4}$$

Here $\Gamma^{\ell, m}_{L_b, L_a}$ are the weighting factors for the various partial waves and their explicit expression is:

$$\Gamma^{\ell, m}_{L_b, L_a} = i^{L_a - L_b - \ell} (2L_b + 1) \left[\frac{(L_b - m)!}{(L_b + m)!} \right]^{1/2} \times$$

(4)

$$\times \langle L_b \, \ell \, 0 0 \,|\, L_a \, 0 \rangle \langle L_b \, \ell \, m \, {-m} \,|\, L_a \, 0 \rangle$$

TABLE I - Spectroscopic factors from the $^{32}S(\tau, d)^{33}Cl$ reaction

E_x (MeV)	ℓ	j^π	C^2S, experiment 10.4 MeV[a]	15.0 MeV[b]	29.7 MeV[c]	34.5 MeV[d]	C^2S, theory MSDI[e]	ICVM[f]
0.0	2	3/2+	0.54	0.90	0.70	0.63	0.64	0.77
0.810	0	1/2+	0.22	0.29	0.32	0.37	0.27	0.28
2.358	2	3/2+				0.061	0.07	0.04
		5/2+				0.033		
2.686	3	7/2-	0.52	0.73	0.50	0.41		
2.860	1	3/2-	0.72	0.55	0.50	0.58		

a) Present work
b) Ref. (5)
c) Ref. (6)
d) Ref. (6)
e) Ref. (3)
f) Ref. (4)

TABLE II - Spectroscopic factors from the $^{32}S(\tau,\alpha)^{31}S$ reaction

E_x (MeV)	ℓ	J^π	C^2S, experiment 8 MeV[a]	10.4 MeV[b]	15 MeV[c]	33.6 MeV[d]	C^2S, theory MSDI[e]
0.0	0	1/2+	4.4	0.96	0.9	1.04	1.13
1.24	2	3/2+	3.8	0.70	1.1	0.94	0.81
2.23	2	5/2+	7	2.12	2.9	2.77	2.275

a) Ref. (11)
b) Present work
c) Ref. (7)
d) Ref. (8)
e) Ref. (3)

TABLE III - Spectroscopic Factors from the $^{33}S(\tau,\alpha)^{32}S$ Reaction

E_x (MeV)	ℓ	j	J^π	C^2S, experiment 10.4 MeV[a]	C^2S, theory MSDI[b]
0.0	2	3/2	0+	4.0	0.64
2.23	0	1/2	2+	9.6	0.54
	2	3/2		2.7	0
	2	5/2		2.7	0
	4	7/2		0.43	-
3.78	2	3/2	0+	0.75	0.07

a) Present work
b) Ref. (3)

This decomposition allows one to establish which are the relative contributions to the cross section of the different radial integrals and whether they are or not localized in L-space (i. e. how the different partial waves do interfere among themselves).

3.2. - L-Space Localization. -

Figs. 3 and 4 show the behaviour of the elastic scattering reflection coefficients and of the quantity $\Gamma \cdot f$ for different values of L, in the case of the $^{33}S(\tau, \alpha)^{32}S$ and $^{32}S(\tau, \alpha)^{31}S$ reactions. As one can easily see, the localization of the pick-up radial integrals becomes more pronounced as the Q-value of the corresponding transition decreases. On the contrary (see an example in Fig. 5) for the stripping reaction $^{32}S(\tau, d)^{33}Cl$, L-space localization always is very goog. Fig. 3 shows that for the transition to the groundstate in ^{32}S(Q = 11. 94 MeV) the contributions to the reaction cross-section of the various partial waves are spreaded over a wide range of L-values, although the relative contributions of low partial waves are not very large. The same spreading is also present for the radial integrals corresponding to the transitions to the excited levels at 2. 23 MeV (in the assumption $\ell = 2$) and at 3. 78 MeV in ^{32}S, but at a lower extent (expecially for $\Gamma^{20}_{L,L-2} \cdot |f^{\ell=2}_{L,L-2}|$. This behaviour is not surprising, because for these transitions the classical condition (3) is violated, as $\Delta L \approx 4 \gg$. On the contrary, in the case of an $\ell = 4$ transfer (no mismatch) for the transition to the first excited state in ^{32}S, the radial integrals with the largest contributions to the cross-section would be those from a well defined region, approximately centered around $|\eta_L| \approx 0.5$,

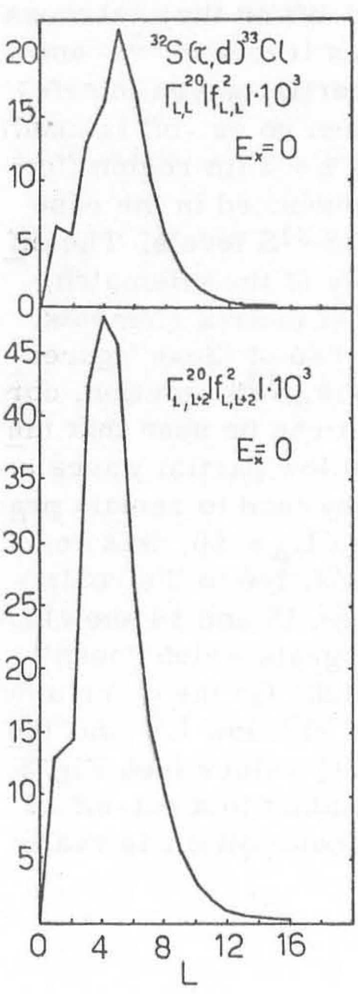

FIG. 5 - Moduli of the radial integrals $\Gamma^{\ell,0}_{Ld, L\tau}$ $f^{\ell}_{Ld, L\tau}$ for the transition to the ground state in ^{33}Cl from the $^{32}S(\tau, d)^{33}Cl$ reaction (in this case the less important terms are those with $m \neq 0$ and $L_\tau = L_d - \ell$).

while a still poorer localization would be in the case of an $\ell = 0$ transfer. All these considerations are also supported by the analysis of the radial integrals for the $^{32}S\ (\tau, \alpha)^{31}S$ reaction (see Fig. 4).

3. 3. - Effect of a Cut-Off on the Radial Integrals. -

The dependence of the theoretical angular distributions on a radial cut-off is shown in Figs. 6, 7, 8 and 9. It is apparent that the effect of a cut-off is stronger for very high Q-values, while in the case of transitions corresponding to lower Q-values, cut-offs of 5-6 fm don't notably change the shape of the angular distributions, although they affect the value of the total cross sections. This suggests that perhaps a radial cut-off of 2. 5-3 fm could have been used in the DWBA analysis of the $^{33}S(\tau,\alpha)^{32}S$ reaction, not with the aim of obtaining better fits, but at least to have spectroscopic factors more consistent with the theoretical values. In the case of the $^{32}S(\tau,\alpha)^{31}S$ reaction the introduction of a radial cut-off would give, on the contrary, spectroscopic factors too lower than the calculated ones.

In Fig. 10 is shown the effect of a radial cut-off on the total cross sections for different transitions leading to the various levels in ^{32}S and ^{31}S. In all these cases there are two regions where partial waves interfere destructively (the total cross section decreases when no cut-off is used) and one of these is just for the contributions from the 2 ÷ 4 fm region (low partial waves). Here too the interference is more pronounced in the case of greater mismatch for the transitions to both ^{32}S and ^{31}S levels. The different effect of a radial cut-off, according to the entity of the mismatch, may be understood following the behaviour of the radial matrix elements, which is shown in Figs. 11, 12, 13 and 14. In the first two of these figures are reported the radial integrals relative to the $^{33}S(\tau,\alpha_0)^{32}S$ reaction, corresponding to $L_\tau = L_\alpha - 2$ (for this transition $\ell = 2$). It can be seen that there are strong oscillations for the integrals relative to low partial waves as soon as the radial cut-off gets over 1. 5 fm, while they tend to remain practically constant for matrix elements corresponding to $L_\alpha \geqslant 10$, this demonstrating that only the former ones are actually sensitive to the contributions from the interior of the nucleus. Besides, Figs. 13 and 14 show that this is a very general characteristic of the radial integrals which doesn't depend in an essential way on the entity of the mismatch. On the other side, the relative importance of the radial matrix elements with low L's and those with high L's is noticeable only in the case of high Q-values (see Fig. 3, in which are also shown the radial integrals corresponding to a cut-off of 3 fm) so in such conditions the use of a cut-off to improve DWBA is really questionable.

3. 4. - Finite Range Corrections. -

The introduction of the finite range correction parameter (as it is clear from Fig. 15 where DWBA calculations are shown for the transitions to the g. s. and to the 3. 78 MeV state in ^{32}S) leads to theoretical angular distributions which are almost completely structure-less and therefore inadequate for the reactions studied in this work. Such a violent effect of this factor on angular distributions is due principally to the assumption of an imaginary diffuseness of only 0. 45 fm. Making use of larger values for $a_i(0.70 \div 0.90$ fm) the angular distributions show some more diffractiveness but are still unable to reproduce the experimental data. Finite range corrections, for the remaining transitions, did not give better results.

4. - CONCLUSIONS. -

It is clear, from this analysis, that the discrepancy in the ground state spectroscopic factors can be only in part explained in terms of a failure of DWBA in treating high Q-value (τ, α) reactions, as in this case it does not seem to be so bad as one could be induced to expect on the basis of the very high Q-value (11.94 MeV). The same is true also for the remaining transitions studied here. The investigation of the radial integrals shows in fact that they are sufficiently localized in L-space, low partial waves being not very important for the cross-sections, so that the reactions we have analysed may be considered of surface type and making use of DWBA should be reasonable. On the other hand the introduction of a cut-off radius of $2 \div 3$ fm seems to be not completely justified, also if it could reduce the difference between stripping and pick-up spectroscopic factors in the case of the ^{33}S $(\tau, \alpha)^{32}S$ reaction, because it would have the opposite effect for the other pick-up reaction.

Cut-off radii larger than 3 fm would be absolutely arbitrary, as they notably modify the theoretical angular distributions, which no more fit the experimental data.

For the same reason finite range corrections should be completely avoided.

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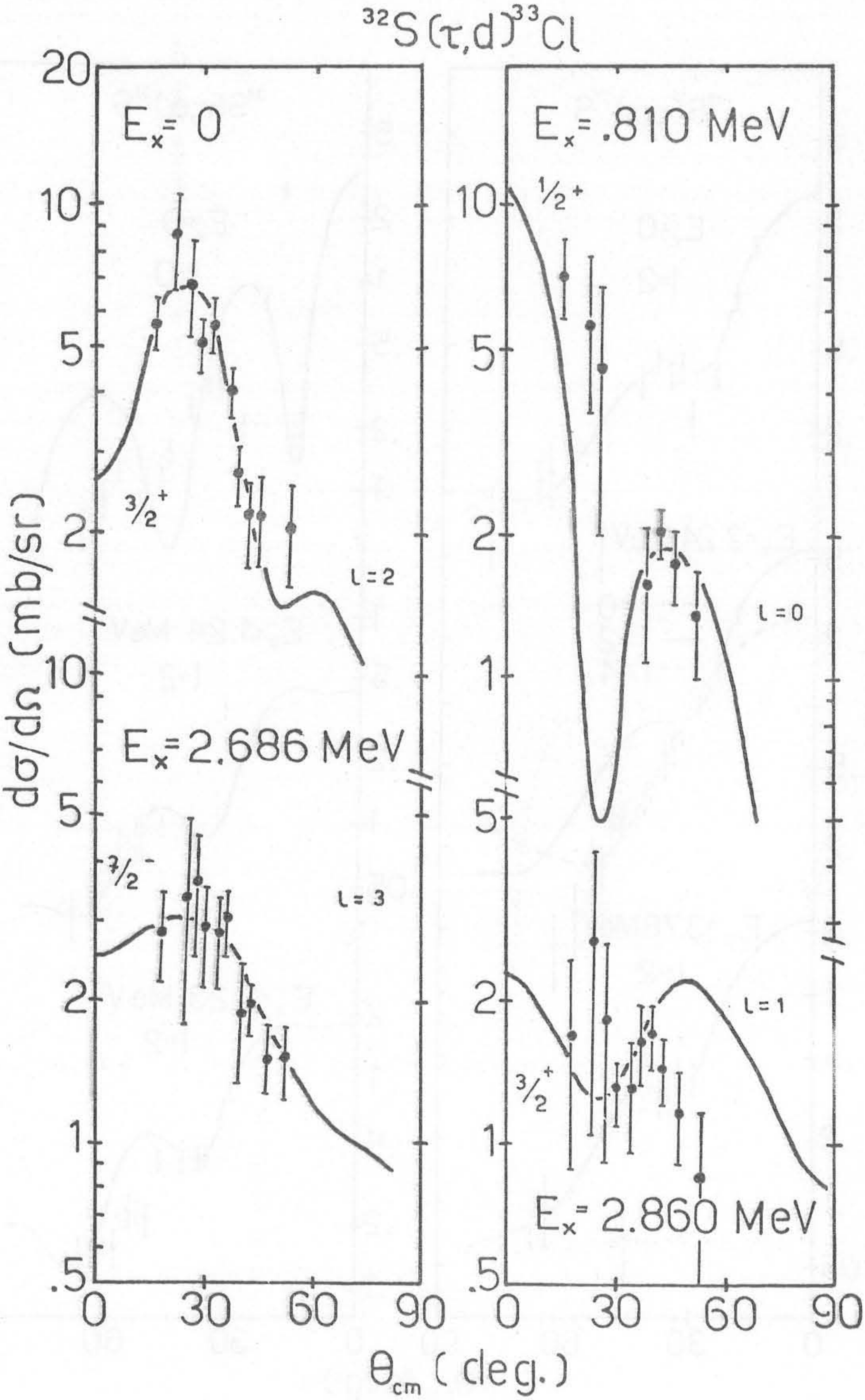

FIG. 1 - Angular distributions of deuteron groups from the $^{32}S(\tau, d)^{33}Cl$ reaction; DWBA curves, excitation energies and ℓ - values are also shown. Errors include background subtraction correction.

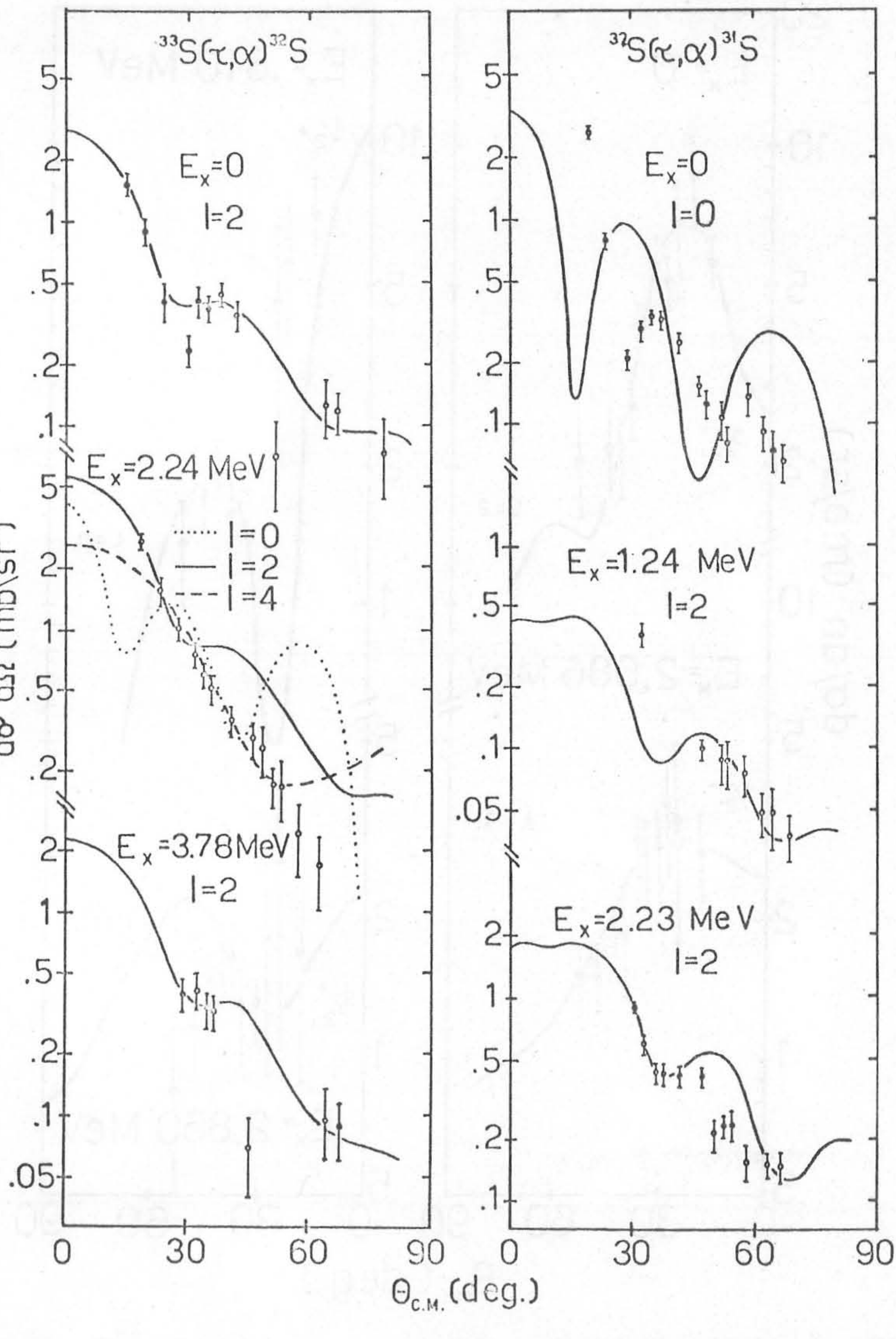

FIG. 2 - Angular distribution of α - particle groups from the $^{33}S(\tau,\alpha)$ ^{32}S and $^{32}S(\tau,\alpha)^{31}S$ reactions. See also caption of Fig. 1.

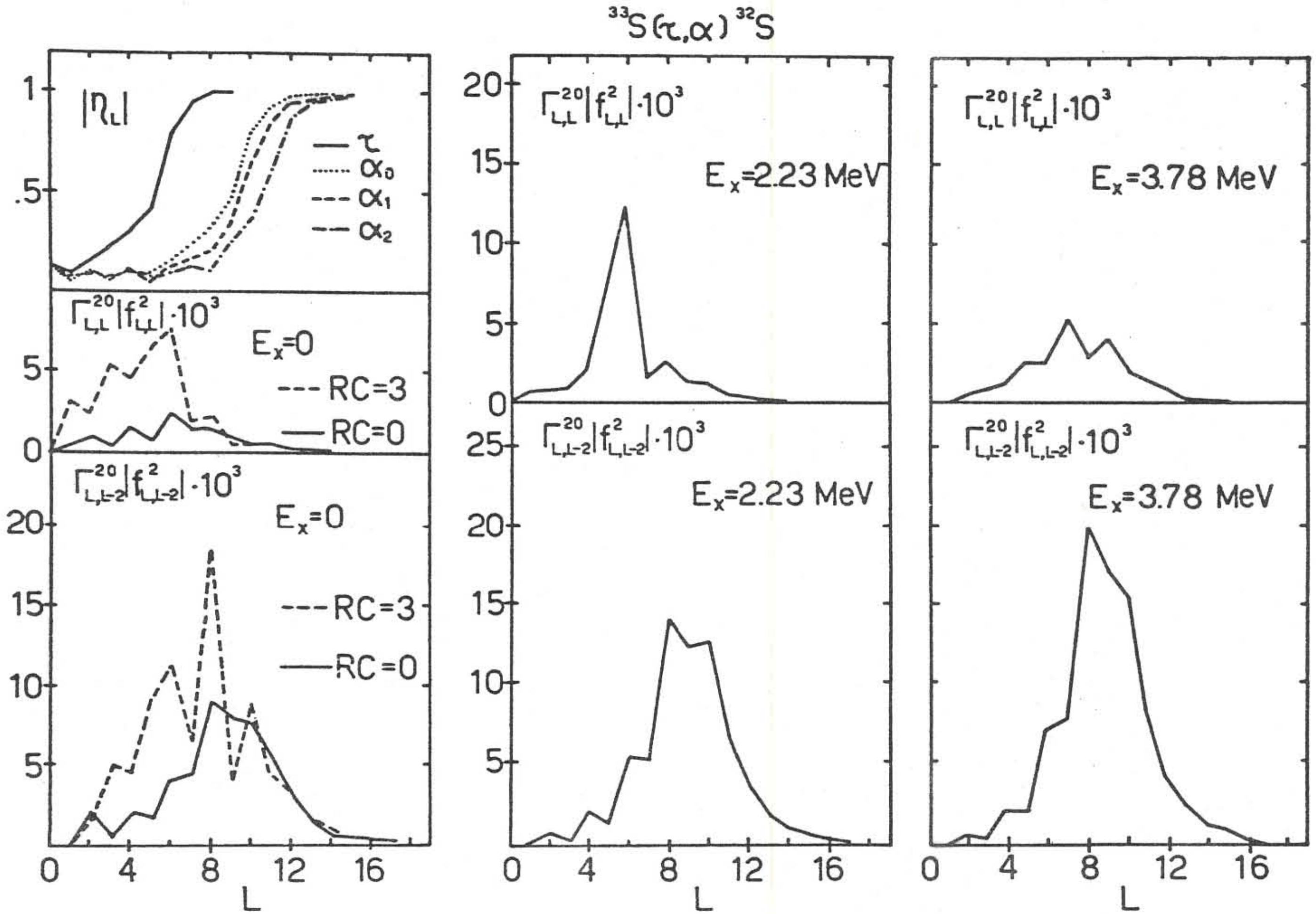

FIG. 3 - Reflection coefficients $|\eta_L|$ for elastic τ and α scattering and moduli of the radial integrals $\Gamma^{\ell,0}_{L\alpha,L\tau}\, f^{\ell}_{L\alpha,L\tau}$ as functions of L for the $^{33}S(\tau,\alpha)^{32}$ reaction. Terms with m $\neq$ 0 and $L_\tau = L_\alpha + \ell$ are less important and not shown in the figure. For the transition to the ground state in ^{32}S the radial integrals corresponding to a cut-off of 3 fm are also shown.

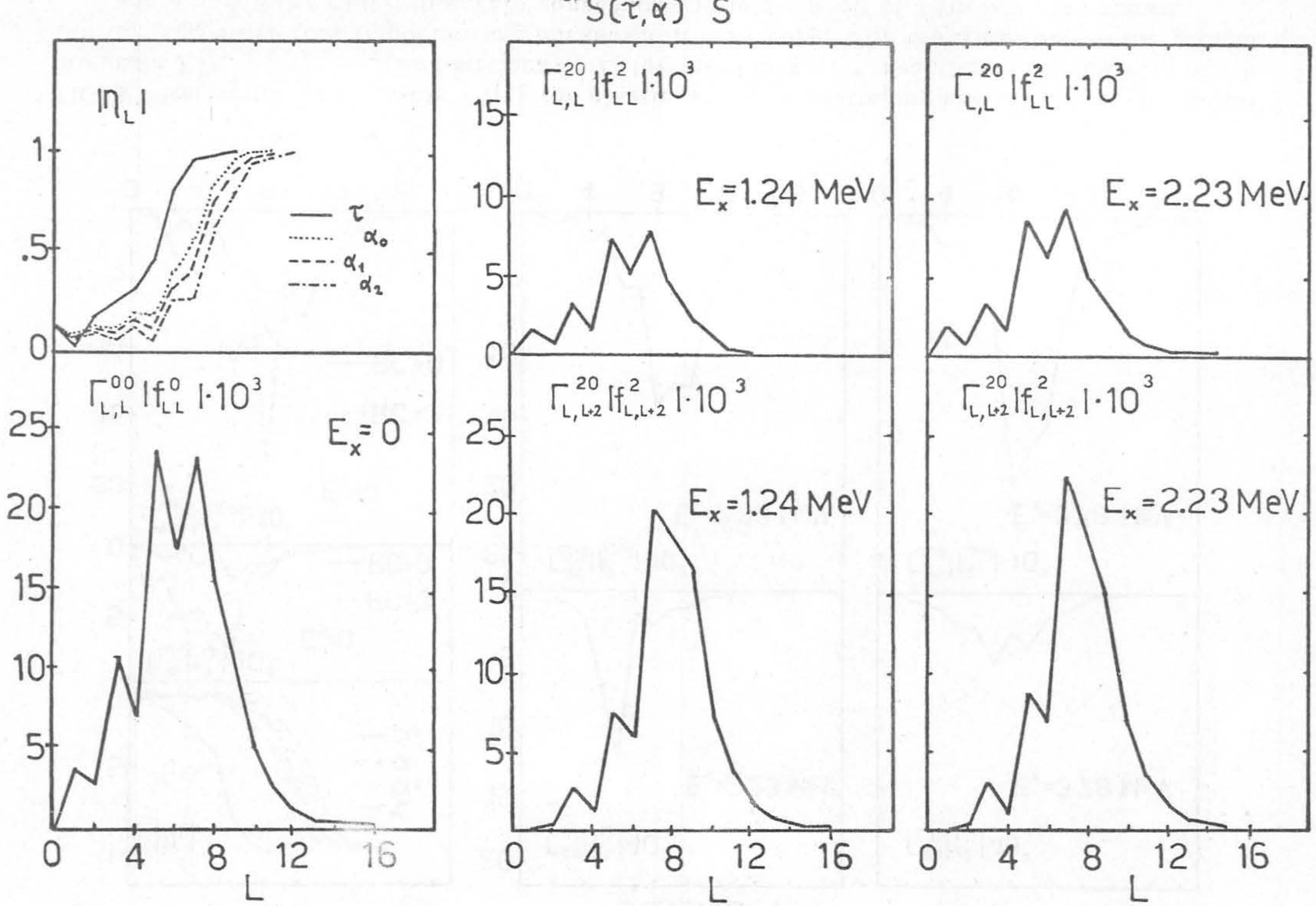

FIG. 4 - Same as Fig. 3 for the $^{32}S(\tau,\alpha)^{31}S$ reaction.

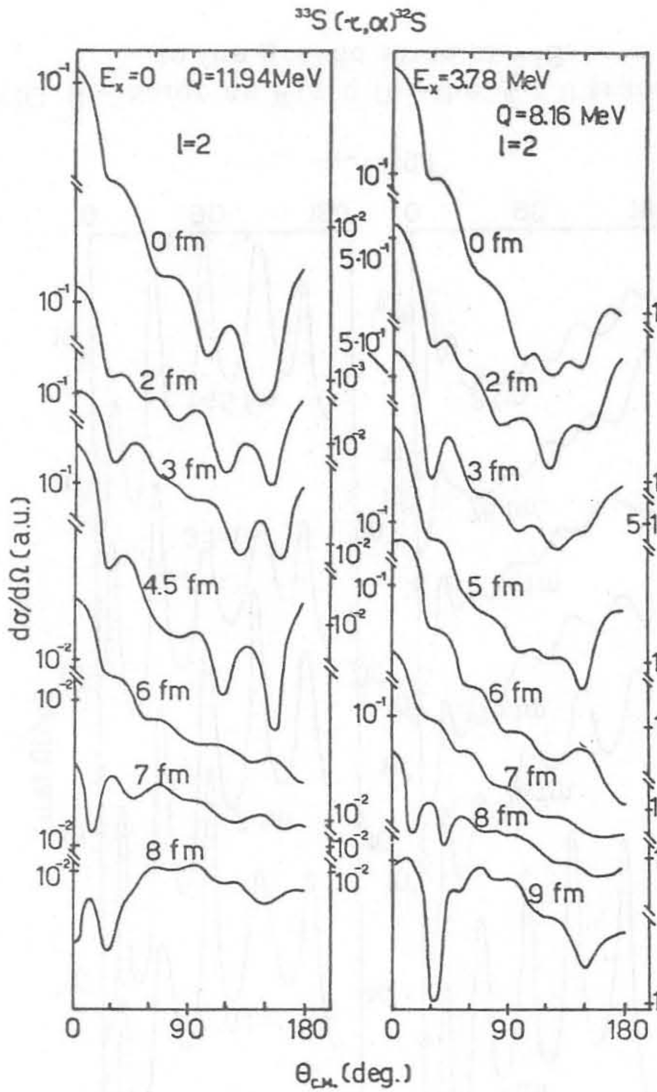

FIG. 6 - Effect of a radial cut-off on DWBA angular distributions for the $\ell = 2$ transitions to the ground state and to the 3.78 MeV level in ^{32}S.

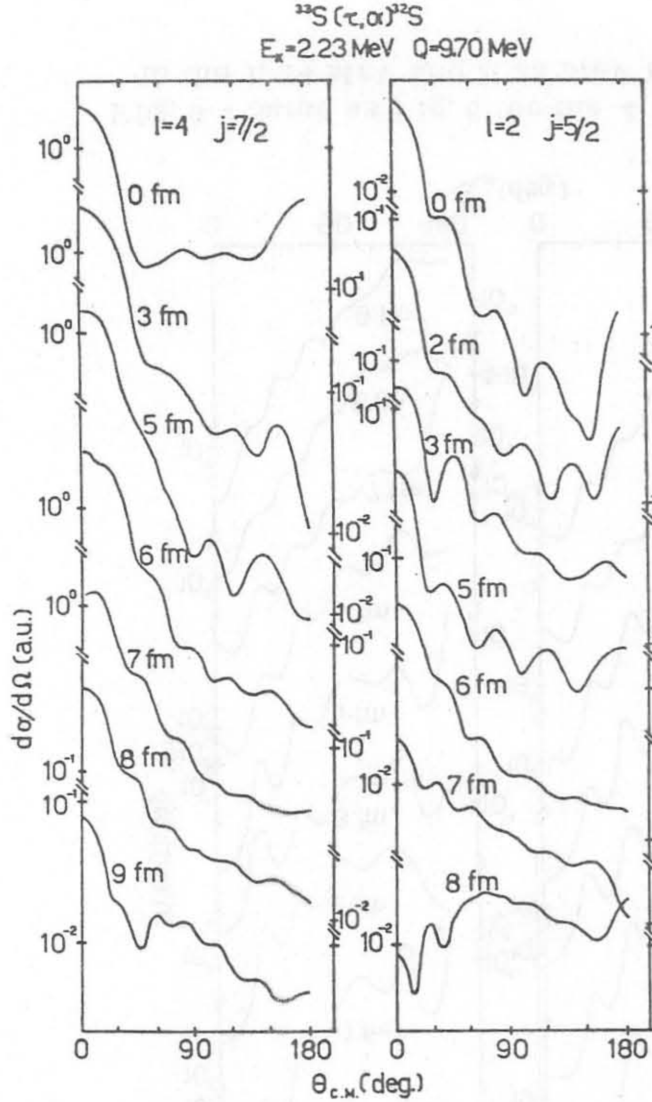

FIG. 7 - Same as Fig. 6 for the transition to the 2.23 MeV state in ^{32}S corresponding to the two cases $\ell = 4 (j=7/2)$ and $\ell = 2 (j=5/2)$.

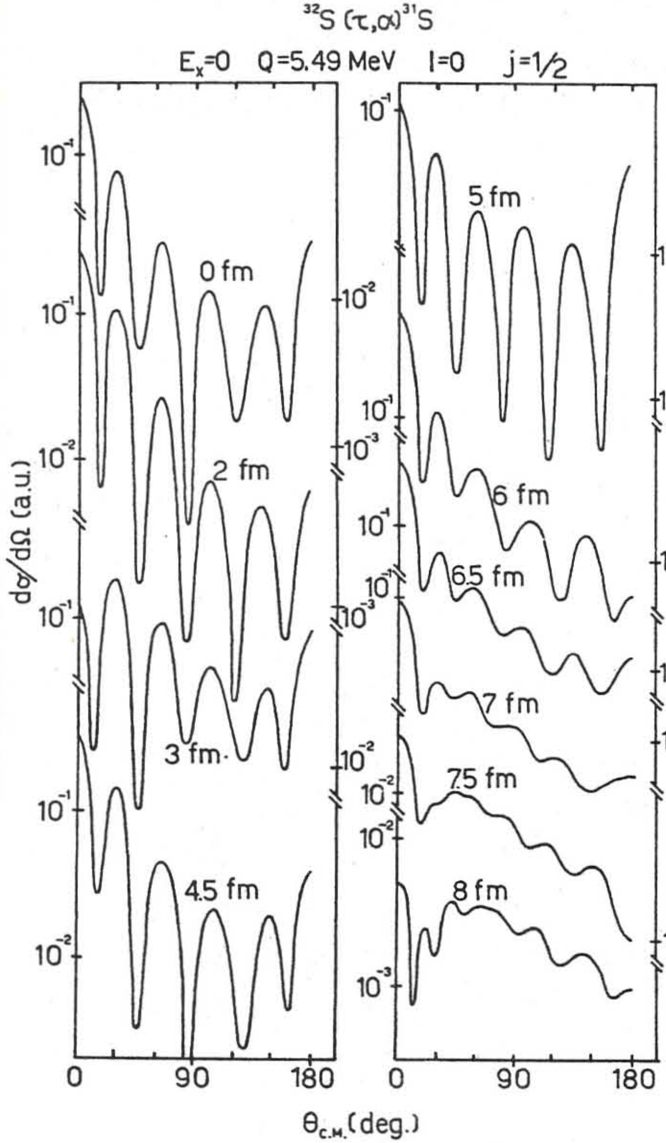

FIG. 8 - Same as Fig. 6 for the ℓ = 0 transition to the ground state of ^{31}S.

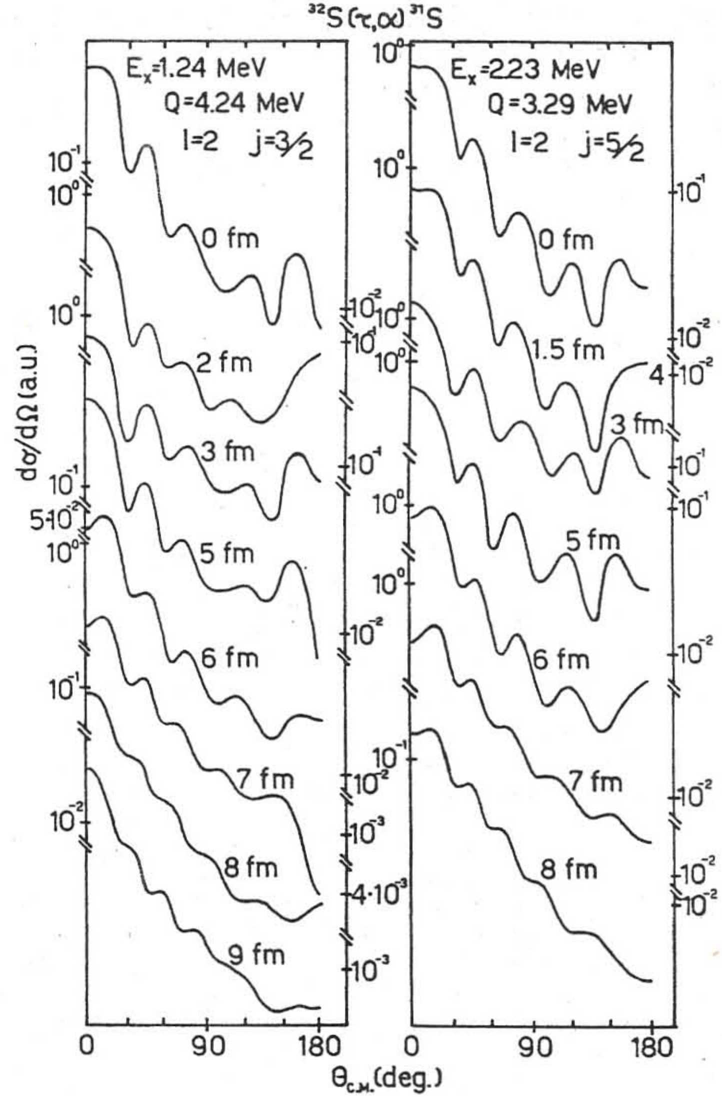

FIG. 9 - Same as Fig. 6 for the ℓ = 2 transitions to the 1.24 MeV and 2.23 MeV levels in ^{31}S.

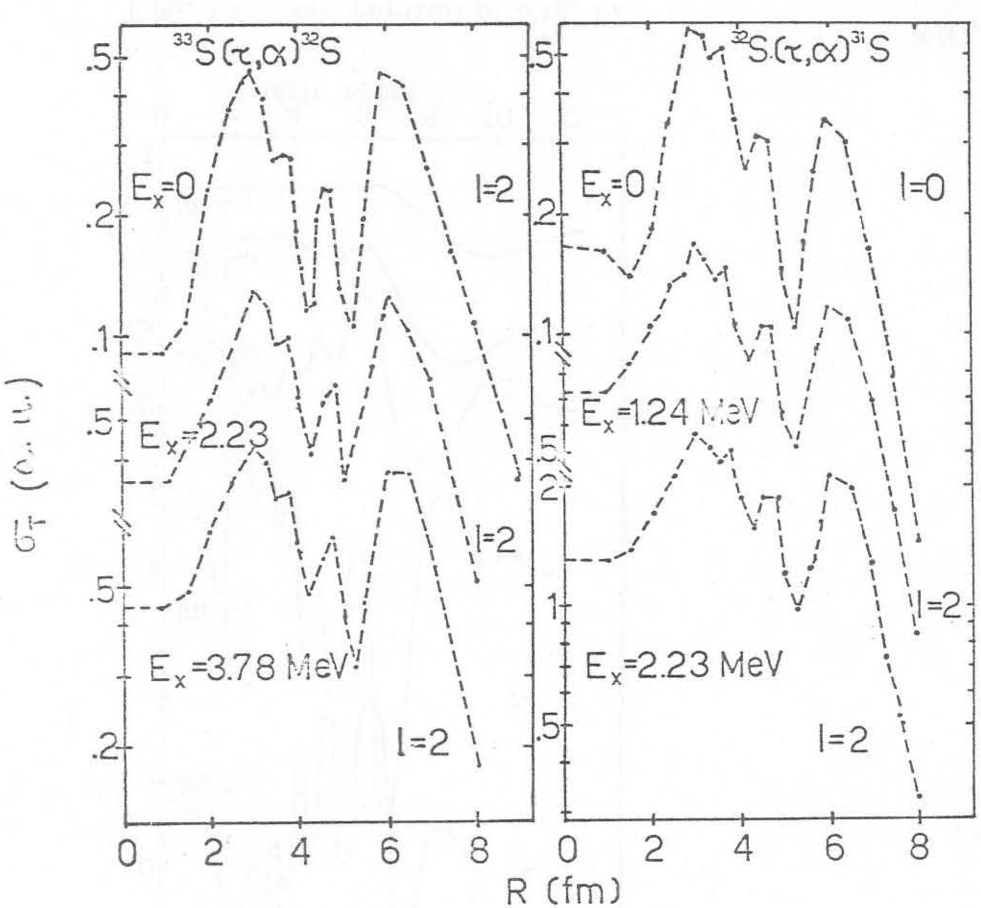

FIG. 10 - Total cross sections as functions of the cut-off radius for the $^{33}S(\tau,\alpha)^{32}S$ and $^{32}S(\tau,\alpha)^{31}S$ reactions. In the case of the transition to the 2.23 MeV state in ^{32}S only the value corresponding to $\ell = 2$, $j = 5/2$ is reported.

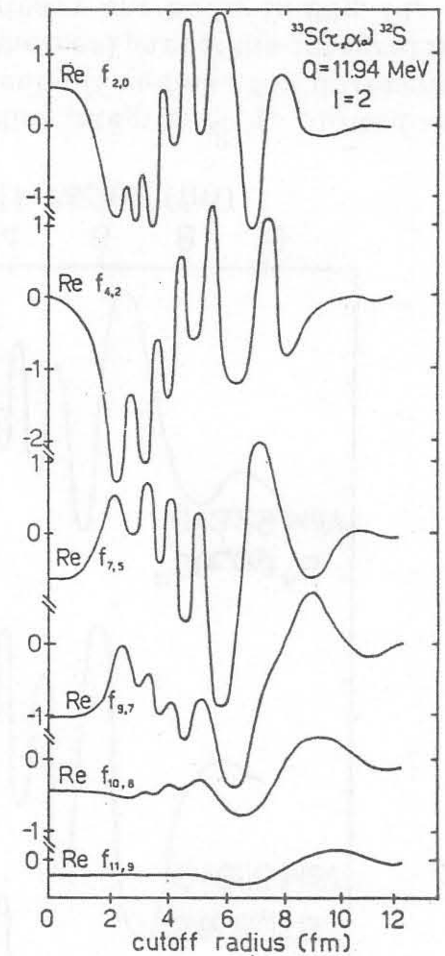

FIG. 11 - Effect of a cut-off on some radial matrix elements for the $\ell = 2$ transition to the ground state of ^{32}S (Q = 11.94 MeV). Only the real part of the integrals is shown, the imaginary one being in Fig. 12.

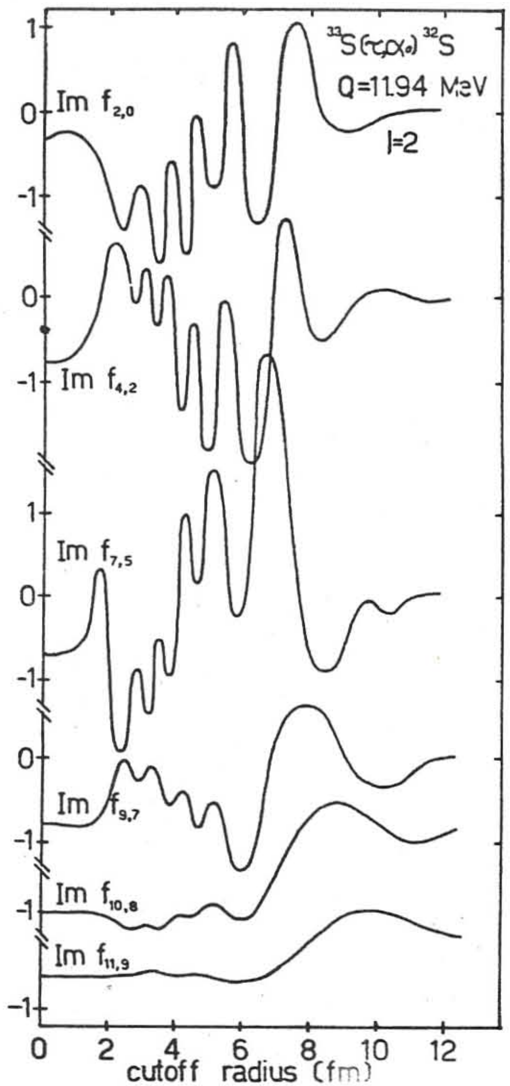

FIG. 12 - See caption of Fig. 11.

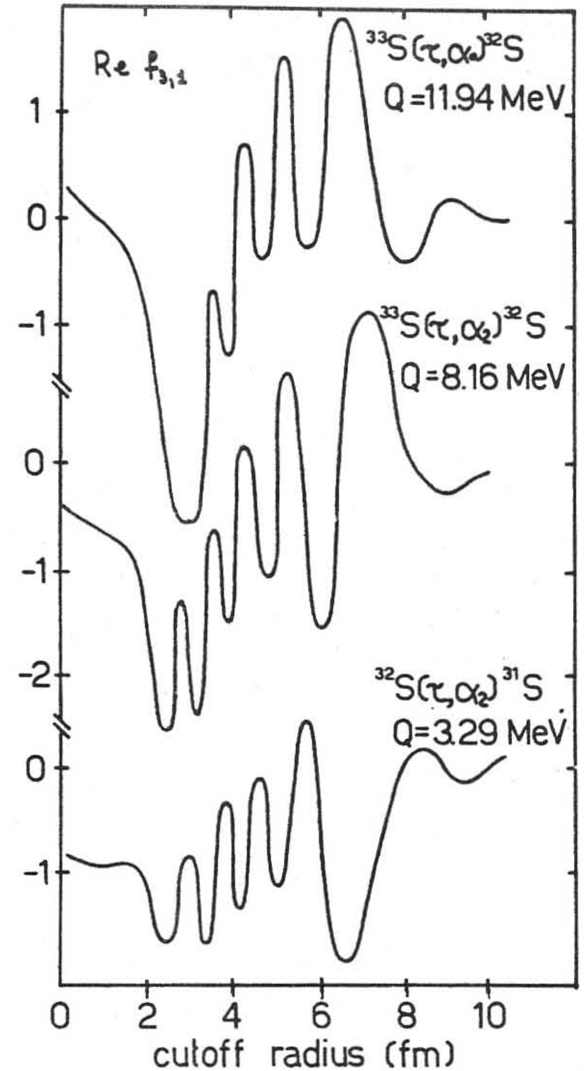

FIG. 13 - The radial integral $f^2_{3,1}$, plotted as a function of the cut-off radius, for different Q-values. Only the real part of the integral is shown, the imaginary one being in Fig. 14.

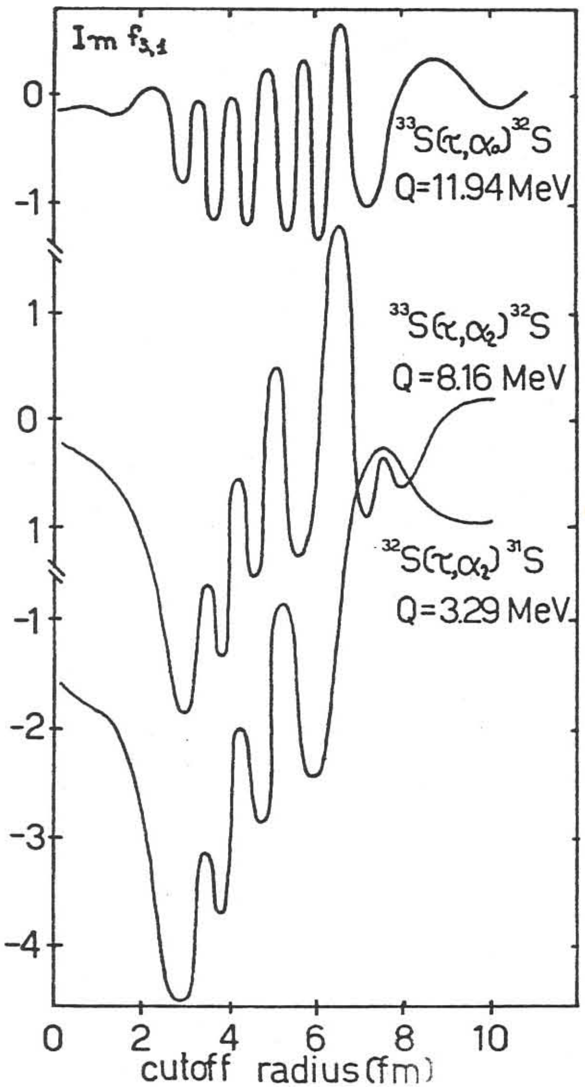

FIG. 14 - See caption of Fig. 13.

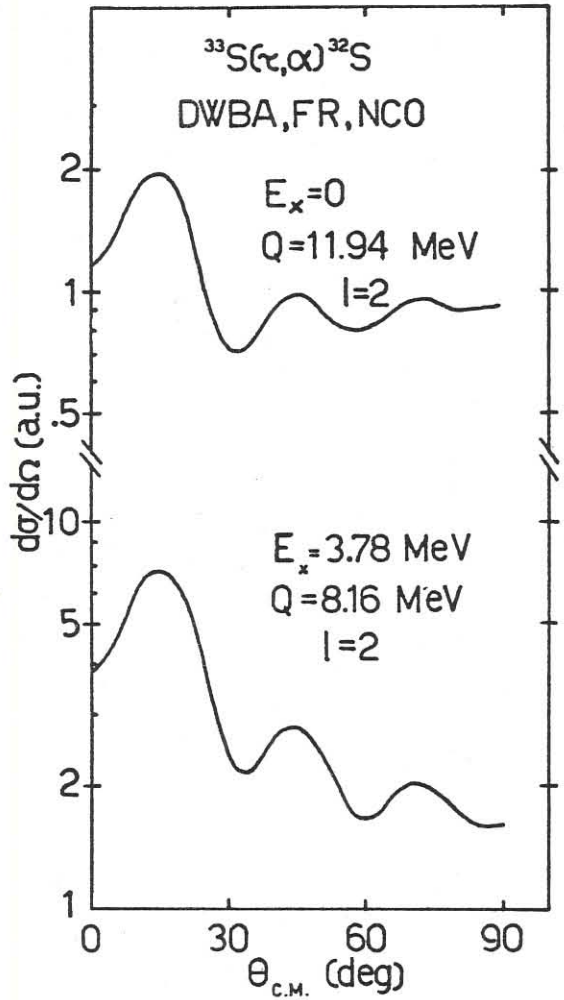

FIG. 15 - Effect of finite range corrections on angular distributions for the transitions to the ground state and to the 3.78 MeV level in ^{32}S. The finite range parameter is R = 2 fm.