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R. Leonardi : ISOTENSOR CORRELATIONS ON DIPOLE SUM-RULES : A REPLY TO THE PAPER "CORRELATION CORRECTIONS TO SHELL MODEL VALUES OF DIPOLE SUM-RULES" BY A. M. LANE AND A. Z. MEKJIAN.

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It is shown that a recent evaluation of the isotensor correlations on Dipole Sum-Rules is arbitrary.

Recently attention has been given to the possibility of extracting a value of the neutron mean square radius from values of the photonuclear cross-sections for separate isospins(1, 2).

To draw clear-cut conclusions with this theory, it is important to know the order of magnitude of the so-called isotensor term.

Referring to the notations of ref. (1, 2, 3) this would imply the knowledge not only of the cross-sections σ-1 in the channels T and T+1 but also in the T-1. This possibility being at present rather far, (it involves β^- first forbidden Fermi transitions or other similar isovector excitations), the isotensor term has been evaluated with a shell model wave function, thereby introducing a model dependence into the analysis.

However it has been conjectured(4) that this procedure may give the proper order of magnitude of the isotensor term and this would make the previous analysis numerically correct. In a recent paper A. M. Lane et al.(5) pointed out that a more caregul estimate of the isotensor term, taking into account the dynamical correlations, gives results one order of magnitude greater than the shell model estimate and the numerical analysis of ref. (1, 2) must be reviewed.

Any reliable estimate of the isotensor term is welcome and would permit more grounded conclusions on the neutron radii. Unfortunately, we must point out that Lane and Mekjian have not properly taken into account the dynamical correlations in the isotensor term; the isotensor term in their work is not found to be large but, rather, assumed as such. In the following we shall adopt the notation of references (1,2); however, for the reader's convenience, we give also our notations in terms of Lane et al. conventions.

As a first remark we prove that the key formula of Lane et al. is easily obtained from our treatment of the isospin analysis of the giant resonance[1,3] and is exact to any order. (In other words the considerations of the first part of the work of Lane et al. do not play any role in obtaining the formula).

At this purpose we define the physical (q-1) energy-weighted dipole cross-section

$$\sigma_{q-1}(T') \equiv \int E^{q-1}\,\sigma(E,T')\,dE$$

and the corresponding reduced cross-section from[3]

$$\sigma_{q-1}(T') = (2T'+1)\begin{pmatrix} T' & T & 1 \\ T_z & -T_z & 0 \end{pmatrix}^2 \sigma_{q-1,T'} .$$

Finally we define the mean energy in the T' channel

$$E_{T'} \equiv \frac{\langle T'|H|T'\rangle}{\langle T'|T'\rangle} = \frac{\sigma_o(T')}{\sigma_{-1}(T')} = \frac{\sigma_{o,T'}}{\sigma_{-1,T'}}$$

($|T'\rangle$ is the T' dipole excited state and H is the target Hamiltonian). and the isospin splitting

$$\Delta E^+ = E_{T+1} - E_{T'}, \qquad \Delta E^- = E_T - E_{T-1}$$

Utilizing now the decomposition of the reduced cross-sections in terms of their isotensor components[3] we write

$$(1)\quad \sigma_{q-1,T+1} = f_s^q - Tf_v^q - \frac{T(2T-1)}{3}\, f_t^q ; \qquad \sigma_{q-1,T} = f_s^q + f_v^q + \frac{(2T-1)(2T+3)}{3}\, f_t^q$$

$$\sigma_{q-1,T-1} = f_s^q + (T+1) f_v^q - \frac{(2T+3)(T+1)}{3} f_t^q \qquad (1)$$

where in particular for q=0 one has[1, 3]

$$f_s^o \cdot r_s^2 = \frac{1}{6} \langle TT_3 | \sum_{ij} (\vec{r}_i \cdot \vec{r}_j)(\vec{\tau}_i \cdot \vec{\tau}_j) | TT_3 \rangle$$

$$f_v^o \equiv r_v^2 = \frac{1}{2} \langle TT_3 | \sum_i r_i^2 \tau_i^3 | TT_3 \rangle / T_3$$

$$f_t^o \equiv r_t^2 = \frac{1}{4} \langle TT_3 | \sum_{ij} (\vec{r}_i \cdot \vec{r}_j)(3 \tau_i^3 \tau_j^3 - \vec{\tau}_i \cdot \vec{\tau}_j) | TT_3 \rangle / \left[3T_3 - T(T+1) \right]$$

These three quantities called the isoscalar, isovector, isotensor m. s. radii summarize the isotopic properties of the mean square spatial distribution of nucleons. Similarly one has, for q = 1

$$f_s^1 = \frac{1}{4} \, \mathrm{Tr} \langle TT_3 | \sum_{ij} \left[[z_i \tau_i^3, H], z_j \tau_j^3 \right] | TT_3 \rangle$$

$$-T_3 f_v^1 = \frac{3}{8} \langle TT_3 | \sum_{ij} \left\{ \left[z_i \tau_i^+, H \right], z_j \tau_j^- \right\} - \left\{ \left[z_i \tau_i^-, H \right], z_j \tau_j^+ \right\} | TT_3 \rangle$$

$$\frac{2}{3} T(2T-1) f_t^1 = 3 \langle TT_3 | \sum_{ij} \left[[z_i \tau_i^3, H], z_j \tau_j^3 \right] | TT_3 \rangle - f_s^1$$

After a straight-forward calculation one has

$$\Delta E^+ = (T+1) E_T \left\{ \frac{1-\gamma}{a_{-1} + (1-\gamma)} - \frac{1-\gamma'}{a_o + (1-\gamma')} \right\} \times$$
$$\times \left\{ 1 - (T+1) \frac{1-\gamma}{a_{-1} + (1-\gamma)} \right\}^{-1} \equiv (T+1) \Delta^+ \qquad (2)$$

$$\Delta E^- = T E_T \left\{ \frac{1 + (2T+3)/(2T-1)\, \gamma}{a_{-1} + (1-\gamma)} - \frac{1 + (2T+3)/(2T-1)\, \gamma'}{a_o + (1-\gamma')} \right\} \times$$
$$\times \left\{ 1 + T \frac{1 + (2T+3)/(2T-1)\, \gamma}{a_{-1} + (1-\gamma)} \right\}^{-1} \equiv T \Delta^- \qquad (3)$$

where $a_{-1} \equiv \frac{\sigma_{-1}}{c\,r_v^2}$, $a_o \equiv \frac{\sigma_o}{c\,f_v^1}$, σ_{-1} and σ_o are the physical photo-cross sections

$$c = \frac{2\,\pi^2}{3}\,\frac{1}{137}\,, \qquad \gamma = -\frac{(2T-1)r_t^2}{r_v^2}\,, \qquad \gamma' = -\frac{(2T-1)f_t^1}{f_v^1}$$

and simply by cominging to drop out the a_o terms and rearranging one has

$$(4) \qquad \gamma = \frac{(2T-1)\left\{(T+1)\Delta^- + T\Delta^+ - a_{-1}(\Delta^+ - \Delta^-) + \frac{2(2T+1)}{2T-1}\,\gamma'\,\frac{f_v^1}{r_v^2}\right\}}{2(2T+1)E_T - (2T^2+T+1)\Delta^- + T(2T-1)\Delta^+}$$

This formula is identical to the Lane formula[6].

In fact expressing our quantities in terms of the Lane notation one has the following identities;

$$\delta^+ \equiv \Delta^+, \qquad \delta^- \equiv \Delta^-, \qquad \varepsilon \equiv E_T, \qquad \langle D^2 \rangle \equiv \frac{1}{6}\,\frac{\sigma_{-1}}{c}\,,$$

$$M_2 \equiv -\frac{1}{6}\,T(2T-1)\,r_t^2\,, \qquad M_1 \equiv \frac{1}{6}\,T\,r_v^2$$

$$N_2 \equiv -\frac{1}{6}\,T(2T-1)\,f_t^1\,, \qquad \frac{M_2}{M_1} \equiv \gamma$$

The merit of the first two relations is to show us how Δ^+ and Δ^- separately depend on ν and that of the third is to connect γ to $(\Delta^+ + \Delta^-)$ and $(\Delta^+ - \Delta^-)$ on which it critically depends, essentially avoiding other model dependent quantities.

A simple analysis of (2) and (3) indicates that Δ^+/Δ^- for γ (and γ') small is a rather large quantity whereas it decreases to the unity by increasing γ.

From a numerical point of view Δ^+ may be considered as known from experiments ($\simeq 60$ MeV/A) whereas Δ^- is fixed by fixing γ in (4). It is clear that the only possibility to utilize (4) to take into account the effect of the dynamical isotensor correlations on γ is to analyze their effect on Δ^- or, even better, measure Δ^- also. Unfortunately the Lane evaluation of γ is based on the (arbitrary) assumption $\Delta^- \simeq 60$ MeV/A. It is easy to verify that this implies that γ is a priori assumed to be large[6]. Before concluding we analyze the situation from another point of view. The interaction between the excess neutrons and the isovector dipole motion gives rise to a splitting between the excitations with different isospins, and, from general arguments, this interaction is of the form

$$U = U_s(\vec{\tau}\otimes\vec{T})_{scalar} + U_v(\vec{\tau}\otimes\vec{T})_{vector} + U_t(\vec{\tau}\otimes\vec{T})_{tensor}$$

where τ is one and T is the isospin target and

$$(\vec{\tau}\otimes\vec{T})_{scalar} = \text{Identity}, \qquad (\vec{\tau}\otimes\vec{T})_{vector} = -(\vec{\tau}\cdot\vec{T}),$$

$$(\vec{\tau}\otimes\vec{T})_{tensor} = -2(\vec{\tau}\cdot\vec{T})^2 - (\vec{\tau}\cdot\vec{T}) + \frac{4}{3}T(T+1)$$

From the geometry one obtains for the energy in the three channels

$$E_{T+1} = U_s - TU_v - \frac{T(2T-1)}{3}U_t$$

(5)
$$E_T = U_s + U_v + \frac{(2T-1)(2T+3)}{3}U_t$$

$$E_{T-1} = U_s + (T+1)U_v - \frac{(T+1)(2T+3)}{3}U_t$$

and in particular

$$E_{T+1} - E_T \equiv \Delta E^+ = (T+1)\left[-U_v - U_t(2T-1)\right]$$

$$E_T - E_{T-1} \equiv \Delta E^- = T\left[-U_v + U_t(2T+3)\right]$$

For large T nuclei one has $\Delta^+ \simeq \Delta^-$ only if $|2U_tT| \ll |U_v|$.
In the following we analyze the dependence of U_t on our parameter γ.
Inverting the system (5) one easily obtains

(6)
$$U_t = -\frac{1}{2(2T+1)(T+1)}E_{T+1} + \frac{1}{2T(T+1)}E_T - \frac{1}{2T(2T+1)}E_{T-1} \simeq$$

$$\simeq -\frac{1}{4T^2}\left\{-E_{T+1}+2E_T-E_{T-1}\right\} \tag{6}$$

(Similar expressions are obtained for U_s and U_v).

From (1) specialized to the case q=0 one finds that, if γ is small ($\simeq$ shell model value) $\sigma_{-1,T-1} > \sigma_{-1,T} > \sigma_{-1,T+1}$ However increasing γ (i.e. $-r_t^2$) $\sigma_{-1,T}$ decreases, (and consequently E_T increases), whereas $\sigma_{-1,T+1}$ and $\sigma_{-1,T-1}$ increase (E_{T+1} and E_{T-1} both decrease).

It follows from (6) that, on increasing γ, U_t increase. (From a negative value ($\Delta^+ > \Delta^-$) through $0(\Delta^+ \simeq \Delta^-)$ to a positive value ($\Delta^+ < \Delta^-$)). We find in this manner the previous result. We can conclude that, if ΔE^+ is known, the problem of evaluating γ is equivalent to that of evaluating ΔE^- and consequently a serious estimate of γ cannot be based on an arbitrary assumption of ΔE^-.

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