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G. Cattapan and V. Vanzani : OFF-SHELL EFFECTS IN NUCLEON-EXCHANGE REACTIONS.

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SUMMARY. -

Nucleon-nucleus rearrangement reactions proceeding via a nucleon-exchange mechanism are investigated in the framework of a three-body model. Starting from the Alt, Grassberger and Sandhas representation for the transition operators, we derive, by means of a suitable operator multiplier technique, an uncoupled integral equation showing in the inhomogeneous term the role of the nucleon-nucleon off-energy-shell t-matrix. The knock-out triangular diagram amplitude involving the off-energy-shell t-matrix is compared with the corresponding amplitude in the Born approximation for the t-matrix. Our results show that the knock-out amplitude is rather sensitive to the off-shell behaviour of the two-nucleon interactions.

1. - INTRODUCTION. -

Since the Faddeev work on the three-body problem(1) a great deal of theoretical interest has been devoted to the off-shell aspects of the nuclear interactions. Indeed, the off-shell behaviour of the two-body scattering amplitudes represents the input information for solving problems that involve composite particles.

Nuclear rearrangement reactions are possible tools to explore in detail the role of these off-shell effects. For an exact description, the rearrangement scattering problem should be formulated in a N-body context ($N \geq 3$)(2, 3). Generally, for $N > 3$ this treatment appears to be too complicate for practical purposes. For this reason, one usually limits oneself to processes which can be approximately treated in a three-body context. Among them we choose, as testing ground for investigating off-shell effects, the nucleon-nucleus rearrangement reactions proceeding via a nucleon-exchange mechanism.

Our choice is motivated by three main features relative to the theoretical description of such a rearrangement problem. First, to construct the inhomogeneous term of the uncoupled integral equation with compact kernel for the rearrangement scattering operator, one needs only the nucleon-nucleon off-shell t-matrix, which has been extensively studied in the last years and, therefore, is fairly well known. Second, the transition amplitude contains nucleon-nucleus form factors which are simpler, and therefore better known than the nucleus-nucleus ones involved in transfer reactions between two nuclei. Third, from a study of the Feynman diagrams associated with the possible reaction mechanisms(4), it follows that the off-shell behaviour would affect predominantly the cross sections in the forward angle region, where they are larger and therefore better known than the backward region values. On the contrary, in stripping and pick-up reactions similar off-shell effects predominate at backward angles.

We have been stimulated to study the above off-shell effects also by some suggestions coming from earlier studies on the effective interaction in nucleon-nucleus scattering problem(5, 6). There one recognizes that the effective interaction, responsible for the transition, has the nature of a transition operator. In the spirit of the usual distorted-wave Born approximation (DWBA) only the Born term of the transition operator is retained(7). To get a first insight into the role of the t-matrix it was assumed that the off-shell t-matrix had the same form as the free on-shell transition matrix(8). However, it has been proved that this on-shell approximation is not generally adequate, e.g. for break-up reactions below 150-200 MeV(9, 10, 11).

The importance of the nucleon-nucleon off-energy-shell effects in the nucleon-exchange reactions has been already outlined, from a qualitative point of view, within the context of the static limit model for the three-body problem(12). To give a quantitative evaluation for them, we shall compare calculations involving the two-nucleon off-shell t-matrix with calculations involving only its Born approximation.

In order to clarify the role of the nucleon-nucleon t-matrix in the framework of a three-body approach, we shall outline an alternative procedure to the usual Faddeev-Lovelace method(1, 13-15). By means of a suitable operator multiplier technique(16, 17), we obtain one uncoupled integral equation with a compact kernel. The inhomogeneous term, which corresponds, in the Feynman diagram language(18), to the sum of a heavy-particle stripping polar graph and a knock-out triangular graph, will be rewritten in a compact form involving the Green function for the two-nucleon subsystem.

In a three-body context one can construct a generalized distorted-wave model which has the above inhomogeneous term as starting point and is completely equivalent to the Feynman diagram summation method (FDSM)[4, 19-22]. Unfortunately, the simultaneous use of the off-shell t-matrix and of the distorted-waves requires a prohibitive amount of computing time, because of the high-dimensional nature of the integrals involved and of the required transformations in the momentum-space between the arguments of the t-matrix and those of the distorted-waves.

Therefore, we shall resort to the approximate treatment proposed in a recent work on the off-shell effects in break-up reactions[17, x]. According to this paper, we limit ourselves to evaluate the two-nucleon t-matrix operator between asymptotic channel states in the forward scattering angle region and for not too low energies. In these cases one expects that the first terms of the Watson-Faddeev multiple scattering series[23] are responsible for the leading behaviour of the transition amplitude[24].

According to earlier calculations concerning different nuclear reactions, our results show that the knock-out amplitude for nucleon--exchange reactions is rather sensitive to the off-energy-shell behaviour of the two-nucleon t-matrix in the energetic range usually explored in experiments. By comparing the off-energy-shell t-matrix calculations with the Born ones, we find a relevant difference in the absolute magnitude of the cross sections, while the shapes of the angular distributions are almost the same in both cases.

Section 2 deals with a general three-body formulation of nucleon--exchange reactions in terms of symmetric transition operators. Section 3 is devoted to the knock-out triangular diagram amplitude involving the neutron-proton off-energy-shell t-matrix. A convenient regularization procedure is carried out in order to evaluate numerically some singular integrals. In Section 4 we shall be concerned with the comparison between the off-energy-shell t-matrix calculations and the corresponding Born ones.

(x) - Notice that the treatment of nucleon-exchange reactions involves full off-shell t-matrix elements, while break-up calculations involve only half-off-shell elements.

2. - GENERAL THREE-BODY APPROACH IN TERMS OF SYMMETRIC TRANSITION OPERATORS. -

2. 1. - Formalism.

Nucleon-nucleus rearrangement processes A(a, b)B proceeding via an exchange between the incident nucleon a and a nucleon b of the target can be written schematically as

(2. 1) $$a + (b + c) \rightarrow b + (a + c) ,$$

where c is the core both of the target nucleus $A = b + c$ and of the residual nucleus $B = a + c$ $(a, b = p, n;\ a \neq b)$.

We describe the reaction (2. 1) by means of a three-body model. The nuclear cluster c is treated as an inert core.

We start from the nonstandard symmetric form for the transition operators ($\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$)

(2. 2a) $$U_{\beta\alpha}(z) = \bar{\delta}_{\beta\alpha} G_{\alpha}^{-1}(z) + V_{\beta} + V_{\beta} G(z) V_{\alpha} ,$$

(2. 2b) $$U_{\beta\alpha}(z) = \bar{\delta}_{\beta\alpha} G_{\beta}^{-1}(z) + V_{\alpha} + V_{\beta} G(z) V_{\alpha} ,$$

proposed by Alt, Grassberger and Sandhas(14). The indices α and β denote the unbound particle (α, β = a, b, c) in the initial and final channel, respectively, or the asymptotic state with all particles free (α, $\beta = 0$). The operator V_{α} represents the sum of the interactions not contained in the channel α, that is $V_{\alpha} = V - v_{\alpha}$, where V is the sum of all the interactions and v_{α} the interaction between β and γ ($\alpha \neq \beta \neq \gamma$; $v_0 = 0$). The resolvent operators are defined as usual

(2. 3) $$G_{\alpha}(z) = (z - H_{\alpha})^{-1} , \qquad G(z) = (z - H)^{-1} ,$$

where H_{α} and H are the channel and the total Hamiltonian, respectively. The two-body scattering operators t_{α}, acting in the three-particle space and satisfying the well-known equations

(2. 4) $$t_{\alpha}(z) = v_{\alpha} + v_{\alpha} G_0(z) t_{\alpha}(z) = v_{\alpha} + t_{\alpha}(z) G_0(z) v_{\alpha} ,$$

are connected with the above channel resolvents by the relations

(2. 5) $$G_{\alpha}(z) = G_0(z) + G_0(z) t_{\alpha}(z) G_0(z) .$$

The transition operators (2. 2) coincide on-the-energy-shell with the conventional asymmetrical ones

(2.6a) $\qquad U^{-}_{\beta\alpha}(z) = V_\alpha + V_\beta\, G(z) V_\alpha$,

(2.6b) $\qquad U^{+}_{\beta\alpha}(z) = V_\beta + V_\beta\, G(z) V_\alpha$,

and therefore give, in the asymptotic channel state representation, the physical transition amplitudes.

We shall introduce in Sect. 2.2 a suitable operator multiplier technique, by starting from the Lippmann-Schwinger (LS) equations for the symmetric transition operators (2.2)

(2.7a) $\qquad U_{\beta\alpha}(z) = \bar{\delta}_{\beta\alpha} G^{-1}_{\alpha}(z) + \delta_{\beta\alpha} V_\alpha + V_\beta\, G_\beta(z) U_{\beta\alpha}(z)$,

(2.7b) $\qquad U_{\beta\alpha}(z) = \bar{\delta}_{\beta\alpha} G^{-1}_{\beta}(z) + \delta_{\beta\alpha} V_\beta + U_{\beta\alpha}(z) G_\alpha(z) V_\alpha$.

Equations (2.7) have the same kernel as the LS equations for the conventional operators (2.6)

(2.8a) $\qquad U^{-}_{\beta\alpha}(z) = V_\alpha + V_\beta\, G_\beta(z) U^{-}_{\beta\alpha}(z)$,

(2.8b) $\qquad U^{+}_{\beta\alpha}(z) = V_\beta + U^{+}_{\beta\alpha}(z) G_\alpha(z) V_\alpha$,

but differ from them in the inhomogeneous term. It is well known that, owing to the noncompactness of their kernel, the LS equations are not amenable to a solution by usual calculational schemes.

Conventional approaches to nuclear rearrangement processes start, usually, from a Born approximation of the eqs. (2.8) or of some more sophisticated equations (e. g. see eqs. (34, 35) of ref. (25)) having, like the eqs. (2.8) a noncompact kernel. For instance, in the usual DWBA one assumes that the interaction responsible for the transition is given by the non-optical part of the inhomogeneous term of the eq. (2.8a) or (2.8b), i. e. by $V_\alpha - W_\alpha$ in the prior representation or by $V_\beta - W_\beta$ in the post representation (W_α and W_β are the optical potentials in the initial and in the final channel respectively)[26]. For the exchange or knock-out processes (2.1) on a heavy target nucleus one approximates W_α by v_b and W_β by v_a, so that the interaction causing the transition is assumed to be the potential v_c between a and b[7, 12].

A rigorous mathematical basis for deriving correct approximations to the exact transition amplitudes is provided by the Faddeev--Lovelace equations for the operators (2.2)[14]. By iterating them once, one obtains integral equations characterized by a compact kernel and exhibiting in their inhomogeneous term the two-body t-operators.

2.2. - An Operator-Multiplier Technique.

Integral equations with inhomogeneous terms involving two-body scattering operators can be directly derived from the LS equations in the general context of the operator multiplier techniques proposed by Blankenbecler and Sugar[16]. This alternative procedure to the Faddeev-Lovelace one, appears more suggestive because it leads to uncoupled integral equations having at once a compact kernel without need of iteration. The multiplier we use for rearrangement processes ($\alpha \neq \beta$) is slightly different from that proposed in ref. (17). Furthermore, it is applied to the symmetric transition operators and not to the conventional ones, as in ref. (17).

Let us multiply on the left hand side the eq. (2.7a) by the operator ($\alpha \neq \beta \neq \gamma$, $\beta \neq 0$)

$$(2.9a) \qquad M_{\gamma\alpha} = (1 - v_\gamma G_o)^{-1}(1 - v_\alpha G_o)^{-1},$$

which does not introduce spurious bound-state solutions[16, 17]. Using the relations

$$(2.10) \qquad (1 + t_\alpha G_o)(1 - v_\alpha G_o) = 1, \qquad G_\alpha v_\alpha = G_o t_\alpha,$$

the new inhomogeneous term reads

$$(2.11) \qquad M_{\gamma\alpha} G_\alpha^{-1} = G_o^{-1} G_\gamma G_o^{-1} = G_o^{-1} + t_\gamma$$

and coincides with the sum of the two simplest terms appearing in the iterated Faddeev-Lovelace equations for rearrangement processes. After some straightforward manipulations on the kernel of the eq. (2.7a) we obtain the following uncoupled integral equation with compact kernel

$$(2.12) \qquad U_{\beta\alpha} = G_o^{-1} + t_\gamma + (1+t_\gamma G_o)(1+t_\alpha G_o)(v_\alpha G_o v_\gamma + V_\beta G_o t_\beta) G_o U_{\beta\alpha}.$$

To eliminate the explicit appearance of the potentials in the kernel, the properties (2.10) should be used in a way similar to what is done in ref. (17). Multiplying on the right hand side the eq. (2.7b) by the operator

$$(2.9b) \qquad L_{\beta\gamma} = (1 - G_o v_\beta)^{-1} (1 - G_o v_\gamma)^{-1},$$

one obtains an equation equivalent to (2.12) with the same inhomogeneous term(x).

(x) - The inhomogeneous term (2.11) of the eq. (2.12) can also be directly obtained in the framework of the Yakubovskiĭ formalism applied to the three-body case[3].

2.3. - Polar and Triangular Diagram Mechanisms.

In the channel state representation $|\phi_\alpha\rangle$ the first term of eq. (2.12), which contributes only to rearrangement collisions, coincides on-the-energy-shell with the amplitude for the polar diagram describing the transfer of the particle γ (Fig. 1a with $\gamma = c$). As far as the process (2.1) is concerned, one has

$$A_p = \langle \phi_b | G_o^{-1} | \phi_a \rangle = \langle \phi_b | v_b G_o v_a | \phi_a \rangle = \langle \phi_b | v_b | \phi_a \rangle = \tag{2.13}$$

$$= \langle \phi_b | v_a | \phi_a \rangle .$$

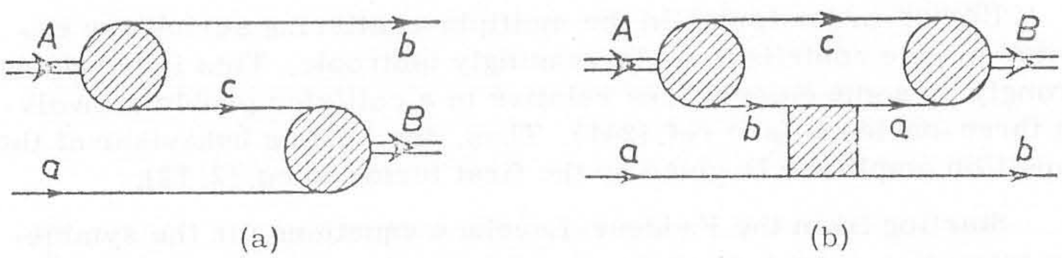

FIG. 1 - Polar (a) and triangular (b) diagrams for the nucleon-exchange reaction (2.1).

These equalities follow from the homogeneous integral equation for two-body bound states. The second term in (2.12) gives in the channel state representation the amplitude for the triangular diagram describing a knock-out mechanism with α - β off-shell interactions (Fig. 1b with $\alpha = a$, $\beta = b$ and $\gamma = c$). One has

$$A_T = \langle \phi_b | t_c | \phi_a \rangle = \langle \phi_b | v_b G_o t_c G_o v_a | \phi_a \rangle . \tag{2.14}$$

Summing the amplitudes (2.13) and (2.14) and taking into account the relation (2.5) one obtains the following compact form for the inhomogeneous term in (2.12)

$$A_p + A_T = \langle \phi_b | v_b G_c v_a | \phi_a \rangle . \tag{2.15}$$

Notice the explicit appearance of the resolvent operator G_c for the a-b subsystem.

Obviously, the terms G_o^{-1} and t_c are the zero-order term and the first-order one, respectively, of the multiple-rearrangement scattering series(23, 24) which can be derived from the Faddeev-Lovelace equations for symmetric transition operators. They correspond to the simplest graphs for single-exchange reactions which can be obtained in the framework of the nonrelativistic Feynman-diagram approach to direct nuclear reactions(18).

From the well-known expressions of the distance of the Feynman diagram singularities from the physical region boundaries, it follows that the triangular singularity dominates at forward angles, while the polar one contributes mainly in the backward direction. Furthermore, for nucleon-exchange on a heavy target nucleus, the triangular singularity is much nearer to the forward physical region boundary than the polar one to the backward boundary.

Notice that the polar and triangular graphs have the same three--ray vertex functions (i.e. the nucleon-nucleus form factors), the latter containing, furthermore, a four-ray vertex function (i.e. the two--nucleon off-shell t-matrix), which will be treated in Section 3.2.

Higher-order terms in the multiple-scattering series are expected to give contributions increasingly isotropic. This is borne out strongly by some calculations relative to a collision problem involving three-nucleons (see ref. (24)). Thus, the leading behaviour of the transition amplitude is given by the first terms in eq. (2.12).

Starting from the Faddeev-Lovelace equations for the symmetric transition operator, neglecting the coupling terms between elastic and rearrangement channels in the equations for the operators U_{aa} and U_{bb}[22], and taking into account only the channel bound state term $|\psi_\alpha\rangle g_\alpha \langle\psi_\alpha|$ in the spectral representation for G_α (α = a, b), one obtains, on-the-energy-shell, the generalized distorted-wave approximation (GDWA)[4]

$$\langle\phi_b|U_{ba}^{GDWA}|\phi_a\rangle =$$

$$= \langle\vec{p}_b{}' m_b'|(1+u_{bb}g_b)|\langle\psi_b|v_b G_c v_a|\psi_a\rangle(1+g_a u_{aa})|\vec{p}_a m_a\rangle, \tag{2.16}$$

where $\vec{p}_a$ ($\vec{p}_b{}'$) is the initial (final) channel momentum, m_α the z-component of the spin of the particle α, $|\psi_\alpha\rangle$ the α-channel bound state and $|\phi_\alpha\rangle = |\vec{p}_\alpha m_\alpha\rangle|\psi_\alpha\rangle$. The quantities $u_{\alpha\alpha} = \langle\psi_\alpha|U_{\alpha\alpha}|\psi_\alpha\rangle$ are the optical scattering operators acting on the plane-wave states $|\vec{p}_\alpha m_\alpha\rangle$. The wave-operators $1 + g_\alpha u_{\alpha\alpha}$ operating on $|\vec{p}_\alpha m_\alpha\rangle$ give the effective two-body distorted-wave states[19].

The operator $v_b G_c v_a$ sandwiched between the internal bound states in eq. (2.16) plays the role of a generalized transition potential. It corresponds to the inhomogeneous term of the eq. (2.12) (see also eq. (2.11)). If the three-particle intermediate state effects are neglected[(x)] and the a-b off-shell t-operator is replaced by its Born

(x) - This means to assume that the equality $G_o v_\alpha|\psi_\alpha\rangle = |\psi_\alpha\rangle$ holds also off-the-energy-shell.

approximation, the eq. (2.16) assumes the usual DWBA form for exchange reactions. For strong interactions it is unlikely that v_c is always a good approximation to t_c. If the triplet deuteron bound state and the singlet virtual state are important in the intermediate state, the Born term is manifestly inadequate. Furthermore, the normalization of the DWBA depends on v_c and it is, generally, different from the normalization based on t_c.

The GDWA (2.16) involves a nine-dimensional integral[22]. For the reasons explained in Sect. 1 and according to the ref. (11) we shall limit ourselves to evaluate the knock-out triangular diagram amplitude (2.14) with the off-energy-shell t_c-matrix contributions.

3. - THE KNOCK-OUT TRIANGULAR DIAGRAM AMPLITUDE. -

3.1. - General Form of the Knock-out Amplitude.

Let us now give an explicit evaluation of the knock-out triangular diagram amplitude (2.14) involving the off-shell t_c-matrix. Three-body states in momentum-space depend on the momentum pair $\vec{p}_\alpha$ and $\vec{k}_\alpha$ (α = a, b, c), where $\vec{p}_\alpha$ is the momentum of particle α in the total center of mass system, and $\vec{k}_\alpha$ is the relative momentum between the particles β and γ(x). In this representation three-particle states with all particles free will be denoted by $|\vec{k}_\alpha \vec{p}_\alpha m_a m_b m_c\rangle$. Omitting spin state specification, one has

$$(3.1) \qquad |\vec{k}_a \vec{p}_a\rangle = |\vec{k}_b \vec{p}_b\rangle = |\vec{k}_c \vec{p}_c\rangle$$

since they describe the same state. Different apices will be used to denote different three-body states.

Let us now insert in eq. (2.14) intermediate momentum integrations and magnetic sums over the complete sets of two-body states $|\vec{k}'_b m'_a m'_c\rangle$ and $|\vec{k}_a m_b m_c\rangle$, and take into account eq. (3.1) and the following relations between momenta specifying the same three-body state

$$(3.2) \qquad \vec{k}_a = -\frac{M_c}{M_b+M_c}\vec{p}_a - \vec{p}_c\,, \qquad \vec{k}_c = \frac{M_a}{M_a+M_b}\vec{p}_c + \vec{p}_a\,;$$

$$(3.3) \qquad \vec{k}'_b = \frac{M_c}{M_a+M_c}\vec{p}\,'_b + \vec{p}\,'_c\,, \qquad \vec{k}'_c = -\frac{M_b}{M_a+M_b}\vec{p}\,'_c - \vec{p}\,'_b\,,$$

(x) - With the symbols $\vec{k}_\alpha$, $\vec{p}_\alpha$ we denote physical momenta and not the normalized ones, as in ref. (13). See the kinematic considerations developed in ref. (22).

where M_α is the mass of the particle α. Changing the integration variables $\vec{k}'_b$ and $\vec{k}_a$ in $\vec{p}'_c$ and $\vec{p}_c$, respectively, and expressing the t_c-matrix elements in the three-body space in terms of the $\hat{t}_c$-matrix elements in the a-b two-body subspace, namely

$$\langle \vec{k}'_c \vec{p}'_c m'_a m'_b m'_c | t_c(z) | \vec{k}_c \vec{p}_c m_a m_b m_c \rangle =$$

(3.4)

$$= \langle \vec{k}'_c m'_a m'_b | \hat{t}_c (z - \frac{\hbar^2 p_c^2}{2\nu_c}) | \vec{k}_c m_a m_b \rangle \, \delta(p'_c - p_c)\, \delta_{m'_c m_c} ,$$

one obtains

$$(3.5) \quad A_T = \sum_{m_b m_c m'_a} \int \frac{f_b^*(\vec{q}'_b m'_a m_c) \langle \vec{q}'_c m'_a m'_b | \hat{t}_c (S_c + i\gamma) | \vec{q}_c m_a m_b \rangle f_a(\vec{q}_a m_b m_c)}{(\varepsilon_b + \frac{\hbar^2 q_b'^2}{2\mu_b})(\varepsilon_a + \frac{\hbar^2 q_a^2}{2\mu_a})} d\vec{p}_c$$

with

$$(3.6) \qquad f_\alpha(\vec{q}_\alpha m_\beta m_\gamma) = \langle \vec{q}_\alpha m_\beta m_\gamma | v_\alpha | \psi_\alpha \rangle , \qquad (\alpha \neq \beta \neq \gamma).$$

The momenta $\vec{k}_a$, $\vec{k}_c$, $\vec{k}'_b$ and $\vec{k}'_c$ evaluated for $\vec{p}_c = \vec{p}'_c$ have been denoted by $\vec{q}_a$, $\vec{q}_c$, $\vec{q}'_b$ and $\vec{q}'_c$ respectively. In eqs. (3.4) and (3.5) μ_α and ν_α are the reduced masses for the β-γ subsystem and for the system consisting of α and β+γ, respectively. The energy shifting S_c, coming from the prescription (3.4), is given by

$$(3.7) \qquad S_c = E - \varepsilon_a - \frac{\hbar^2 p_c^2}{2\nu_c} = \frac{\hbar^2 p_b'^2}{2\nu_b} - \varepsilon_b - \frac{\hbar^2 p_c^2}{2\nu_c} ,$$

where ε_α is the binding energy of the bound state in the channel α and $E = \hbar^2 p_a^2 / 2\nu_a$ is the entrance channel energy in the total center of mass system.

The nucleon-nucleus form factors $f_\alpha(\vec{q}_\alpha m_\beta m_\gamma)$ defined by eq. (3.6) may be written, by means of usual angular momentum expansions(7, 27), in the form

$$(3.8) \quad f_\alpha(\vec{q}_\alpha m_\beta m_\gamma) = -(\frac{2}{\pi})^{\frac{1}{2}} \frac{\hbar^2}{2\mu_\alpha} \sum_{\substack{j_\alpha l_\alpha \\ m_{j_\alpha} m_{l_\alpha}}} N_{l_\alpha \beta j_\alpha l_\alpha} \langle j_\alpha s_\gamma m_{j_\alpha} m_\gamma | s_N m_N \rangle \cdot \langle l_\alpha s_\rho m_{l_\alpha} m_\beta | j_\alpha m_{j_\alpha} \rangle Y_{l_\alpha}^{m_{l_\alpha}}(\hat{q}_\alpha) F_{l_\alpha}(q_\alpha) ,$$

with

$$F_{l_\alpha}(q_\alpha) = i^{-l_\alpha} N_{l_\alpha}^{-1} (q_\alpha^2 + \chi_\alpha^2) \int j_{l_\alpha}(q_\alpha r) u_{l_\alpha}(r) r^2 dr . \tag{3.9}$$

In eqs. (3. 8) and (3. 9) the symbol N stands for the nucleus $(\beta+\gamma)$, while the meaning of s_α and of the bound-state quantum numbers j_α, l_α, m_{j_α}, m_{l_α} is quite transparent ; $u_{l_\alpha}(r)$ is the radial part of the bound-state wave function ψ_α and N_{l_α} its asymptotic normalization constant ; χ_α^2 is expressed in terms of the binding energy ε_α by $\hbar^2 \chi_\alpha^2 = 2\mu_\alpha \varepsilon_\alpha$. The constants $\beta_{j_\alpha l_\alpha}$ and N_{l_α} are related to the spectroscopic factors and single-particle reduced widths, respectively, as conventionally used in direct nuclear reaction theories.

For the sake of completeness, let us give also the momentum-space representation of the amplitude (2. 15). Adding to the amplitude (3. 5) the polar contribution (2. 13) one obtains after some kinematic variable transformations[22] the following expression

$$A_p + A_T = - \sum_{m_b m_c m'_a} \int \frac{f_b^*(\vec{q}'_b m'_a m'_c)\, \psi_c^{(+)}(\vec{q}'_c \vec{q}_c m'_a m'_b m_a m_b ; S_c + i\gamma) f_a(\vec{q}_a m_b m_c)}{\varepsilon_b + \dfrac{\hbar^2 q_b'^2}{2\mu_b}} d\vec{p}_c \tag{3.10}$$

where

$$\psi_c^{(+)}(\vec{q}'_c \vec{q}_c ; m'_a m'_b m_a m_b ; z) = \delta(\vec{q}'_c - \vec{q}_c)\, \delta_{m'_a m_a} \delta_{m'_b m_b} + \frac{\langle \vec{q}'_c m'_a m'_b | \hat{t}_c(z) | \vec{q}_c m_a m_b \rangle}{z - \dfrac{\hbar^2 q_c^2}{2\mu_c}} \tag{3.11}$$

is the two-body scattering state $(1 + \hat{t}_c \hat{G}_o) | \vec{q}_c m_a m_b \rangle$ for the a-b subsystem in the momentum-space representation. The polar amplitude does not involve two-body off-shell interactions and does not influence the behaviour of the transition amplitude in the forward angle region, as already mentioned in Sect. 2. For these reasons it will be ignored in our specific calculations.

3.2. - The Neutron-Proton off-Energy-Shell t-Matrix Elements.

To evaluate the amplitude (3.5) one needs the expression of the off-shell t-matrix elements. We start from the widely used assumption that the two-nucleon transition operator is dominated by the deuteron bound state and the singlet virtual state(13, 24, 28-30). This assumption leads to the use of a nonlocal separable two-nucleon interaction

(3.12) $$v_c(\vec{k}',\vec{k}) = -\frac{\hbar^2}{2\mu_c}\sum_{n=0,1} f_n(\vec{k}')\lambda_n f_n(\vec{k})P_n ,$$

where P_n are projection operators onto the deuteron state (n=0) and the singlet state (n=1). In this approximation for the potential, the two-body LS equation for the t-matrix elements can be easily solved to give

(3.13) $$t_c(\vec{k}';\vec{k};z) = -\frac{\hbar^2}{2\mu_c}\sum_{n=0,1} f_n(\vec{k}')\tau_n(z)f_n(\vec{k})P_n ,$$

where

(3.14) $$\tau_n^{-1}(z) = \lambda_n^{-1}\left\{1+\frac{\hbar^2\lambda_n}{2\mu_c}\int\frac{\left|f_n(\vec{q})\right|^2}{z-\frac{\hbar^2q^2}{2\mu_c}}\,d\vec{q}\right\}$$

If the nucleon-nucleon form factors $f_n(\vec{k})$ are chosen, as usual(24, 29, 30), to be of the Yamaguchi form(31)

(3.15) $$f_n(\vec{k}) = 1/(k^2+\beta_n^2) ,$$

one has

(3.16) $$\tau_n(\sigma+i\gamma) = \lambda_n\frac{\left[\beta_n - i(\sigma+i\gamma)^{1/2}\right]^2}{\left[\beta_n - i(\sigma+i\gamma)^{1/2}\right]^2 - \zeta_n^2}$$

with $\sigma = 2\mu_c S_c/\hbar^2$, $\zeta_n^2 = \lambda_n\pi^2/\beta_n$ and the prescription

(3.17) $$-i(\sigma+i\gamma)^{1/2} = \begin{cases} -i\left|\sigma^{1/2}\right| + \gamma & \text{for } \sigma > 0 \\ \left|(-\sigma)^{1/2}\right| - i\gamma & \text{for } \sigma < 0 \end{cases}$$

The parameters λ_n and β_n are chosen with the prescription of fitting the deuteron binding energy, the triplet and singlet scattering lengths and the singlet effective range. Using the experimental data of ref. (32) for the triplet parameters and of ref. (29) for the singlet ones, one obtains for the np system(x)

(3.18a) $\beta_o = 1.440\ \mathrm{fm}^{-1}$, $\lambda_o = 0.407\ \mathrm{fm}^{-3}$;

(3.18b) $\beta_1 = 1.153\ \mathrm{fm}^{-1}$, $\lambda_1 = 0.145\ \mathrm{fm}^{-3}$.

Since the quantities (3.13) act into total spin-space for the np system, the t-matrix element appearing in (3.5) reads as follows

$$\langle \vec{q}\,'_c m'_a m'_b | \hat{t}_c (S_c + i\gamma) | \vec{q}_c m_a m_b \rangle =$$

(3.19)

$$= - \frac{\hbar^2}{2\mu_c} \sum_{n=0,1} c_n(m'_a m'_b m_a m_b) f_n(\vec{q}\,'_c)\, \tau_n(\sigma + i\gamma) f_n(\vec{q}_c) \, .$$

The constants c_n coming from vector-coupling procedures have the following form

(3.20a) $$c_o(m'_a m'_b m_a m_b) = \frac{1}{2}(1+\delta_{m'_a m'_b}\, \delta_{m_a m_b})\, \delta_{m'_a+m'_b,\, m_a+m_b} \, ,$$

(3.20b) $$c_1(m'_a m'_b m_a m_b) = -\frac{1}{2}(-1)^{m'_a+m_a}\, \delta_{m'_a,\, -m'_b}\, \delta_{m_b,\, -m_a} \, .$$

(x) - Since our three-body model involves a definite pair of nucleons (the incident proton and the emitted neutron, or viceversa) and the inert core c, we do not take an average over the np and nn singlet data to determine the singlet parameters. The average values would be slightly different from the values (3.18b). Furthermore, notice that exchange terms arising from antisymmetric properties with respect to all the nucleons (included the core ones), cannot be taken into account in our three-body context. However, their effect on the shape of the angular distribution is expected to be small (see refs. (10) and (33)).

3.3. - The t-Matrix Amplitudes and the Born Ones.

By inserting eqs. (3.8) and (3.19) in eq. (3.5), one gets an explicit expression for the knock-out triangular diagram amplitude. If the function $\tau_n(\sigma)$ is replaced by the constant λ_n, one has from (3.5) the triangular amplitude in the Born approximation for the two-body t_c-matrix (see eqs. (3.13) and (3.12)). In this approximation the off-energy-shell effects due to $\tau_n(\sigma + i\gamma)$ disappear, because there is no dependence on the two-body energy $S_c = \hbar^2\sigma/2\mu_c$ for the a-b subsystem. Putting

(3.21) $$\alpha_{n(T)} = \tau_n , \qquad \alpha_{n(B)} = \lambda_n ,$$

one can write

(3.22) $$A_X = -\frac{\hbar^2}{\pi\mu_c} \sum_{\substack{j_a l_a \\ j_b l_b}} \sum_{\substack{m_{j_a} m_{l_a} m_c m_b \\ m_{j_b} m_{l_b} m'_a}} N_{l_a} N^*_{l_b} \beta_{j_a l_a} \beta_{j_b l_b} \cdot C^{\,m_{j_a} m_{l_a} m_c m_b \; m_{j_b} m_{l_b} m'_a}_{\,j_a l_a \; j_b l_b} \sum_{n=0,1} c_n(m'_a m'_b m_a m_b) K^{m_{l_a} m_{l_b}}_{n; l_a l_b}(X) ,$$

where

(3.23) $$C^{\,m_{j_a} m_{l_a} m_c m_b \; m_{j_b} m_{l_b} m'_a}_{\,j_a l_a \; j_b l_b} = \langle j_a s_c m_{j_a} m_c | s_A m_A \rangle \langle l_a s_b m_{l_a} m_b | j_a m_{j_a} \rangle \cdot \langle j_b s_c m_{j_b} m_c | s_B m_B \rangle \langle l_b s_a m_{l_b} m'_a | j_b m_{j_b} \rangle ,$$

(3.24) $$K^{m_{l_a} m_{l_b}}_{n; l_a l_b}(X) = \int \alpha_{n(X)}(\sigma + i\gamma) h^{m_{l_a} m_{l_b}}_{n; l_a l_b}(\vec{p})\, d\vec{p} ,$$

and

$$(3.25) \quad h_{n;l_a l_b}^{m_{l_a} m_{l_b}}(\vec{p}) = \frac{Y_{l}^{m^{*}_{l_b}}(\hat{q}'_b) F_{l_b}(q'_b) Y_{l_a}^{m_{l_a}}(\hat{q}_a) F_{l_a}(q_a)}{(q'^2_b + \chi^2_b)(q'^2_c + \beta^2_n)(q^2_c + \beta^2_n)(q^2_a + \chi^2_a)}$$

with $X = T, B$ and $\vec{p} = \vec{p}_c$.

In the following the integrals (3.24) with $X = T$, involving off--energy-shell t-matrix contributions will be called simply t-matrix ampltudes, while the integrals (3.24) with $X = B$, referring to the Born approximation for the t-matrix will be called Born amplitudes. We shall write

$$(3.26) \qquad K_{n;l_a l_b}^{m_{l_a} m_{l_b}}(T) = T_{n;l_a l_b}^{m_{l_a} m_{l_b}} , \qquad K_{n;l_a l_b}^{m_{l_a} m_{l_b}}(B) = B_{n;l_a l_b}^{m_{l_a} m_{l_b}} .$$

3.4. - Regularization of the singular triplet integrals.

In order to make the above formulae suitable for numerical calculations, we shall now perform a convenient regularization of the integrals $T_{0;l_a l_b}^{m_{l_a} m_{l_b}}$. The integrand in (3.24) for $n = 0$, $X = T$ and $E > \varepsilon_a$ has a singular point at $p = p_o + i\gamma$ with

$$(3.27) \qquad p_o = + \left[\frac{2\,\nu_c}{\hbar^2} (E - \varepsilon_a + \varepsilon_o) \right]^{\frac{1}{2}} ,$$

corresponding to the deuteron pole $z = \varepsilon_o + i\gamma$ for the t_c-matrix elements (3.13), (3.14)(x). In equation (3.27)

$$\varepsilon_o = \hbar^2 (\zeta_o - \beta_o)^2 / 2\mu_c$$

is the binding energy of the deuteron. Obviously, the singlet pole does not lie on the integration path, being $\zeta_1 - \beta_1 < 0$.

The integrals $T_{0;l_a l_b}^{m_{l_a} m_{l_b}}$ could be calculated for several continuously decreasing values of the small positive parameter γ and the results should be compared among themselves in order to reach a definite numerical accuracy. However this procedure is unpractical in numerical computation, as already discussed in refs. (21, 34). It is

(x) - Since we exclude too low incident energies (see Sect. 1), we shall perform calculations for $E > \varepsilon_a$.

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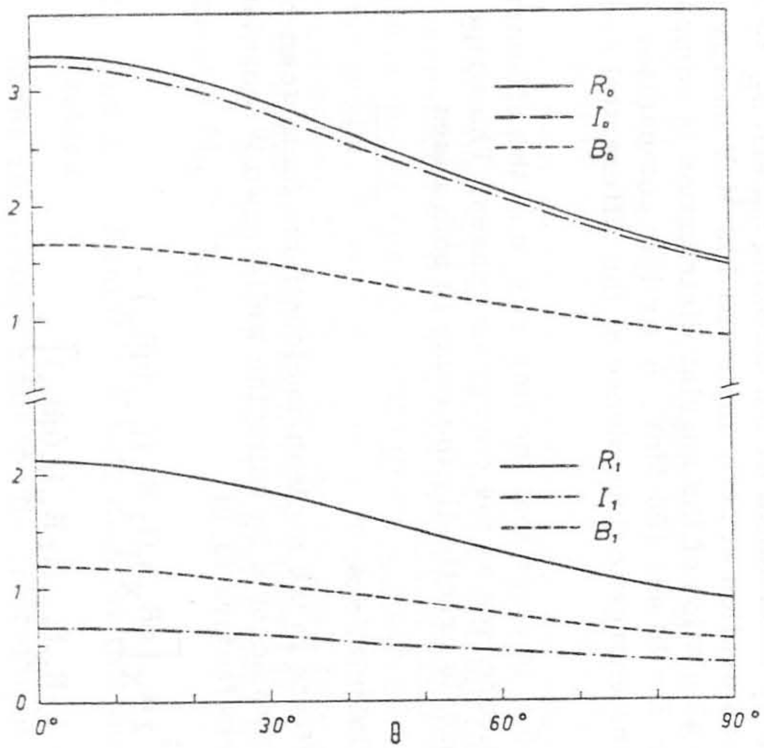

FIG. 2 - Angular dependence of the real and imaginary parts R_n, I_n of the t-matrix amplitudes and of the Born ones B_n at 20 MeV. The upper part of the figure gives the triplet amplitudes (n=0) and the lower one gives the singlet amplitudes (n=1).

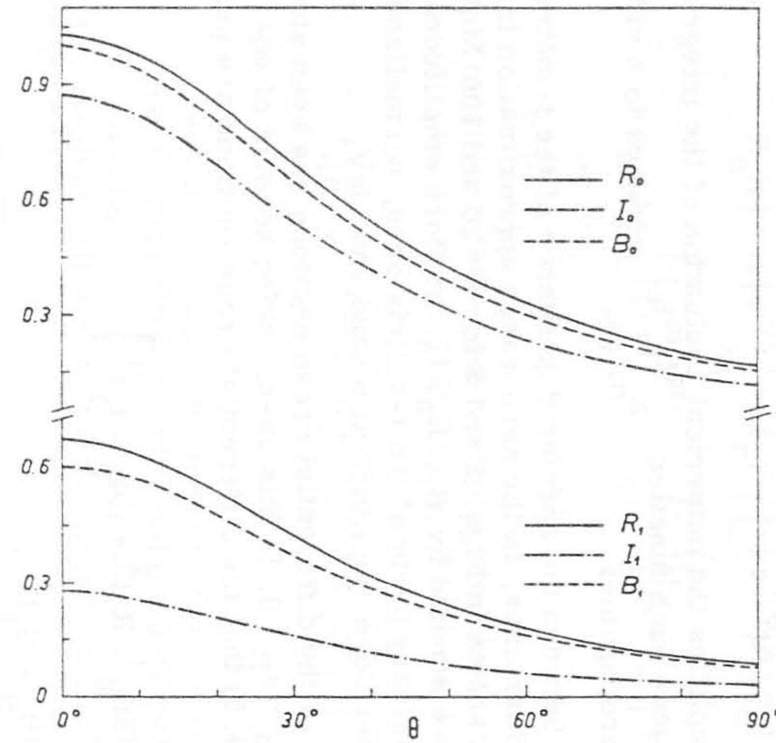

FIG. 3 - Angular dependence of the amplitudes R_n, I_n and B_n at 100 MeV.

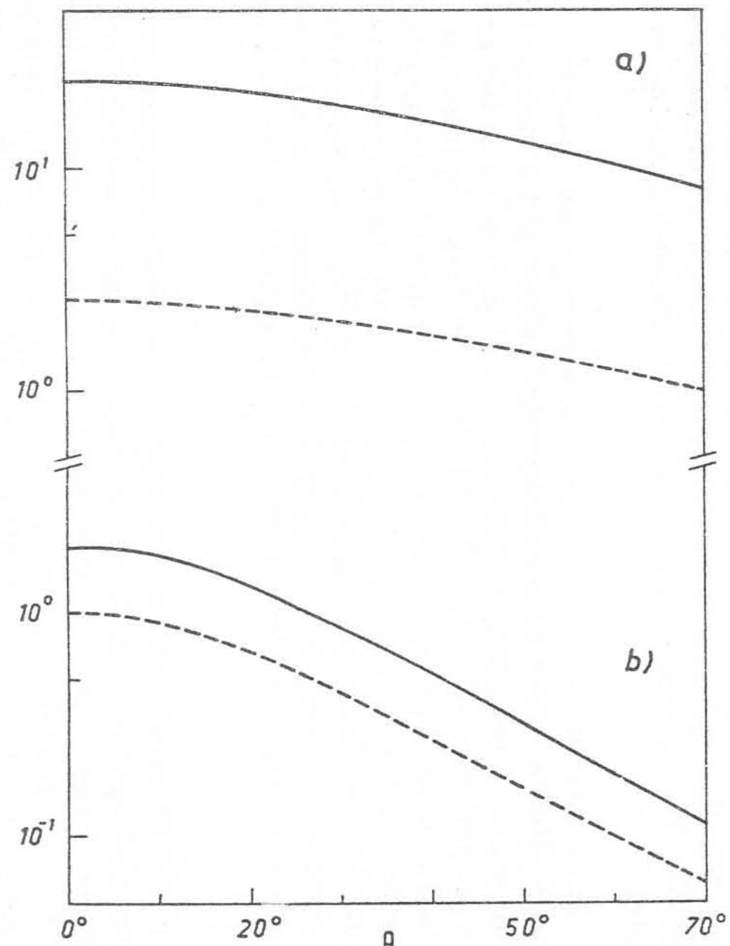

FIG. 4 - Comparison between the angular dependence of S_T (full curve) and S_B (dashed curve) for a) E=20 MeV and b) E=100 MeV.

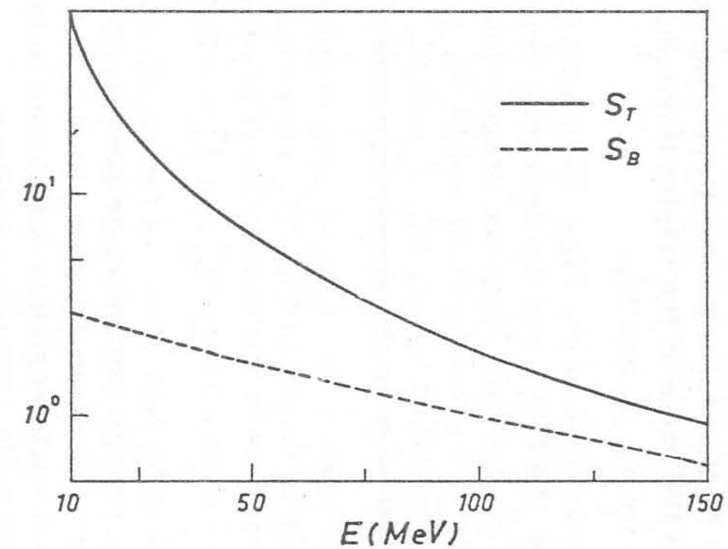

FIG. 5 - Comparison between the energetic dependence of S_T and S_B at $\theta = 0^o$.

evaluated. In Figs. 6 and 7 we plot the t-matrix amplitudes and the Born ones for $l_a=l_b=0$, $R_A \simeq R_B = 4$ fm, in the forward scattering angle region, for E=20 and 100 MeV, respectively. Here also we have normalized the amplitudes to the value of the triplet Born one at 0^o and 100 MeV, denoting them by R_n', I_n' and B_n'.

By analogy with the zero-range case, Figs. 8-10 give the angular dependence, for E=20 and 100 MeV, and the energetic dependence, for $\theta = 0^o$, of the quantities S_T' and S_B'. They are proportional to the differential cross sections and are given by (4. 8) and (4. 9), respectively, with ρ, R_n, I_n and B_n replaced by ρ', R_n', I_n' and B_n'. The normalization factor ρ' is defined as in the zero-range case.

Similar calculations carried out for $l_a=l_b=1$ give results close to the above ones. An example is shown in Fig. 11, where the amplitude B_n'' (i. e. $B^{00}_{n;11}$ normalized as in the above cases) is compared with the corresponding t-matrix amplitudes R_n'' and I_n'' for E=20 MeV.

Calculations performed at 50 MeV give, as far as off-energy-shell effects are concerned, results intermediate between the 20 and 100 MeV ones. The behaviour of the transition amplitudes and of the cross sections in the backward angle region is not plotted in the figures, for the reasons explained in the preceding Sections.

The finite-range results confirm the importance of the off-energy-shell t-matrix contributions in the range of the incident energy usually explored in experiments. In fact, a certain discrepancy between the t-matrix results and the Born ones remains even at 100 MeV.

In comparison with the zero-range results we observe that finite-range effects cause an oscillatory behaviour and a stronger decrease with angle of the differential cross section. Owing to the presence of the propagators (3. 15) in (3. 13), our t-matrix amplitude is more decreasing with angle than the triangular amplitude constructed with a constant four-ray vertex function. Such a strong decrease is needed in order to reproduce the experimental data(x). The occurence of the deep minima in Figs. 8 and 9 is due to the absence, in our formalism, both of mechanisms different from the triangular one as well as of

(x) - For a detailed comparison with experiments one must remove the above simplifying mass restrictions and introduce the channel distorsions. Thus, rather cumbersome and high-dimensional integrals have to be evaluated. In this paper, we are mainly interested in the analysis of the off-energy-shell effects and, to perform this program, we resort to a calculational model which preserves the gross features of the actual transition amplitudes.

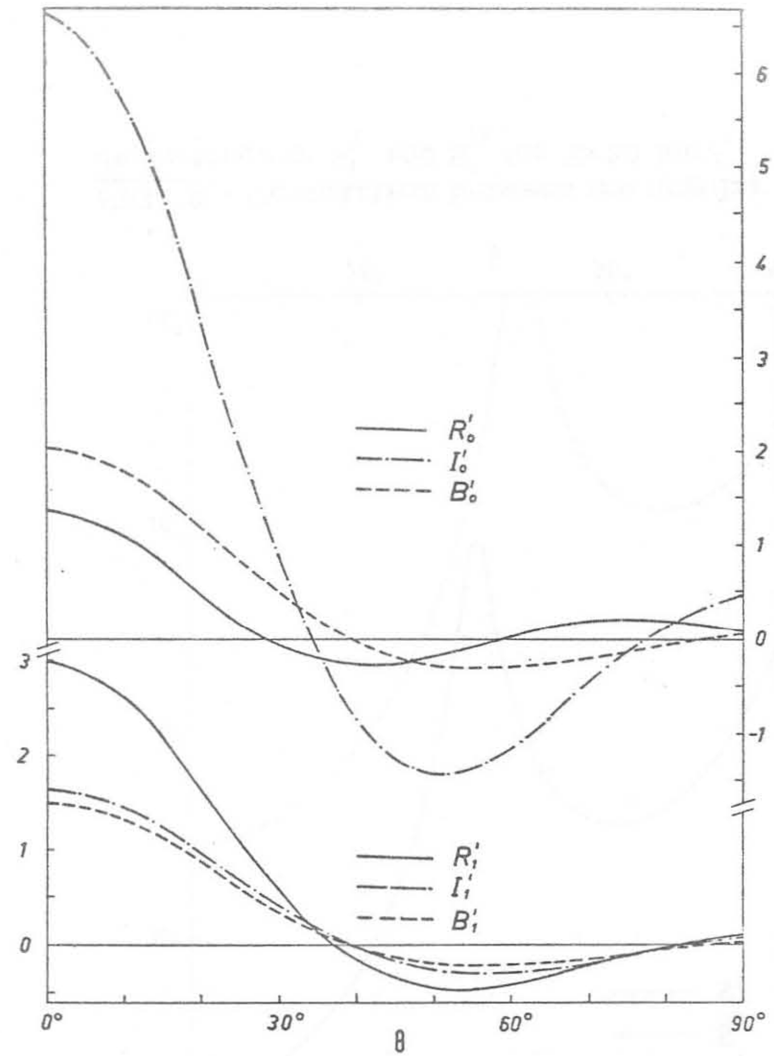

FIG. 6 - Angular dependence of the real and imaginary parts R_n', I_n' of the t-matrix amplitudes and of the Born ones B_n' at 20 MeV. The upper part of the figure gives the triplet amplitudes (vertical right scale) and the lower one gives the singlet amplitudes (vertical left scale).

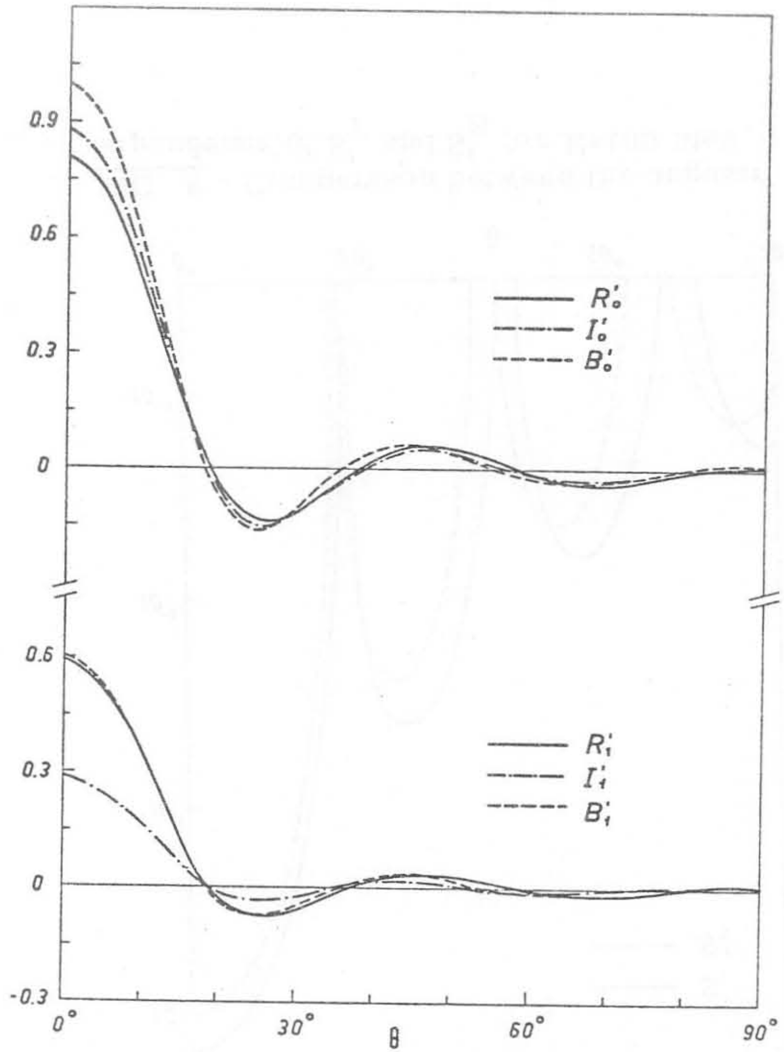

FIG. 7 - Angular dependence of the amplitudes R_n', I_n' and B_n' at 100 MeV.

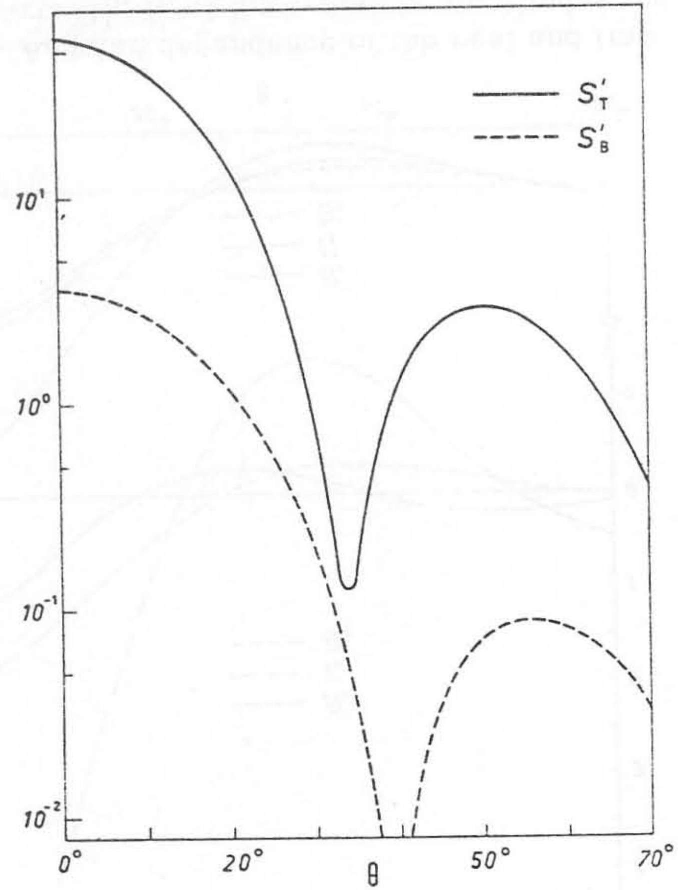

FIG. 8 - Comparison between the angular dependence of S'_T and S'_B for E=20 MeV.

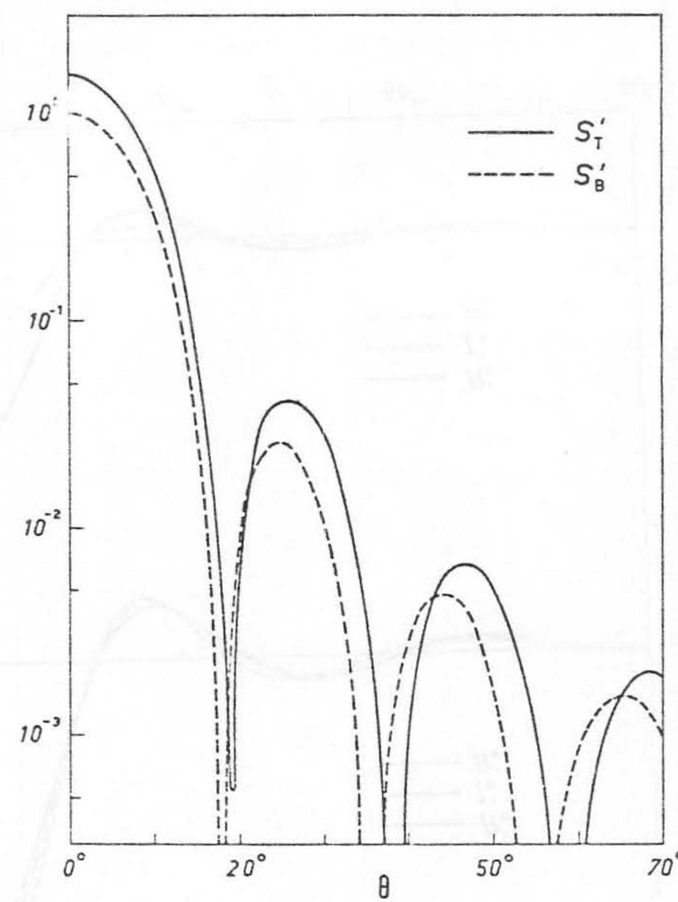

FIG. 9 - Comparison between the angular dependence of S'_T and S'_B for E=100 MeV.

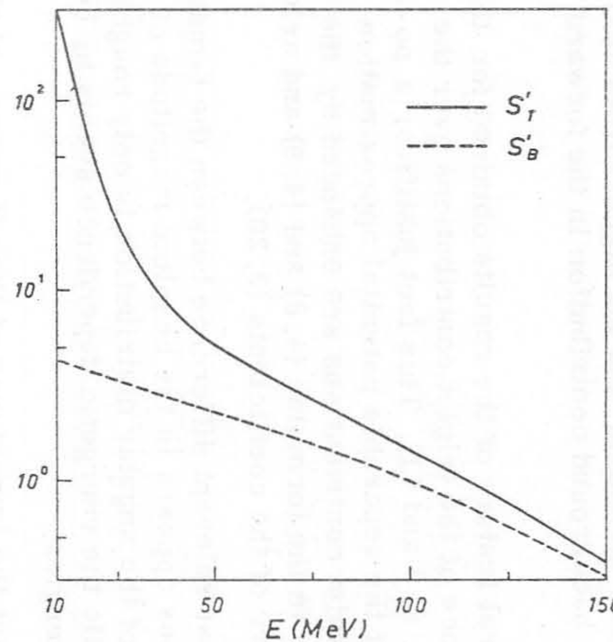

FIG. 10 - Comparison between the energetic dependence of S'_T and S'_B at $\theta = 0^o$.

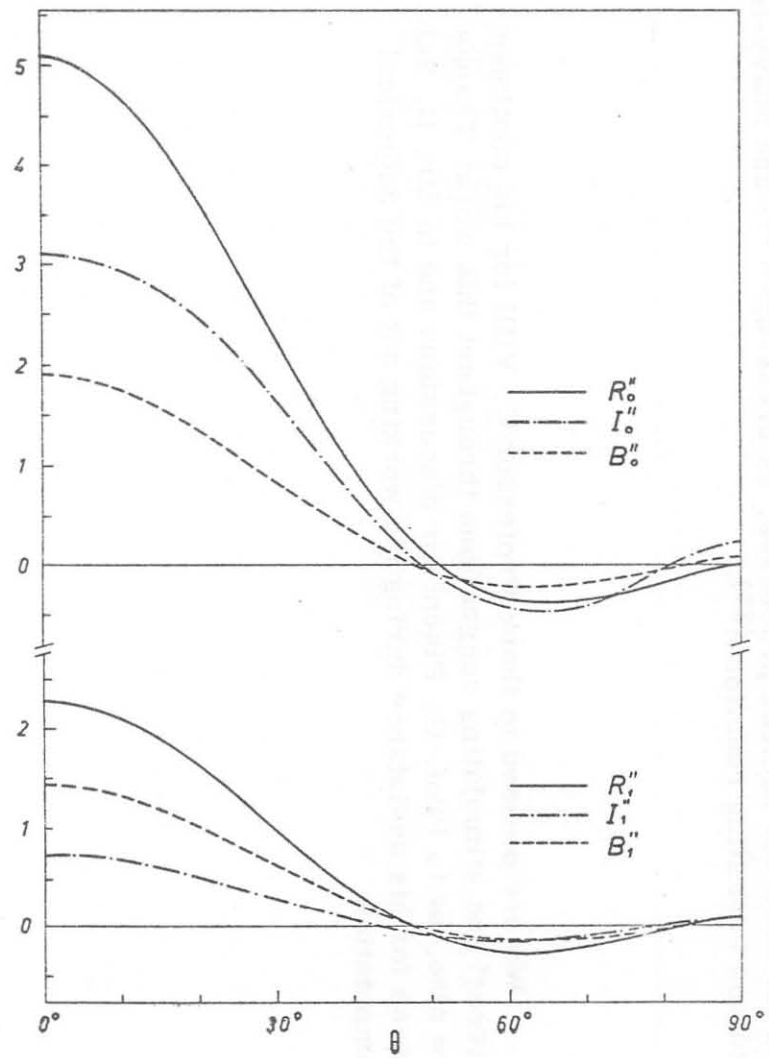

FIG. 11 - Angular dependence of the amplitudes R''_n; I''_n and B''_n at 20 MeV.

channel distorsions (the latter ones correspond to a phenomenological simulation of the higher-order terms in the multiple-scattering series). The polar graph amplitude, which is, in the heavy-core approximation, a slowly increasing function of the angle, and the higher-order terms provide a roughly isotropic background contribution in the forward angle region.

We notice, as a general feature of the results obtained for the amplitudes, the predominance of the triplet contributions over the singlet ones (see Figs. 2, 3, 6, 7 and 11). This fact justifies, a posteriori, the correctness of the separable potential approximation (3.12). Furthermore the triplet contributions are enhanced by the weight factors which appear in the formulae (4.8) and (4.9) and arise from the particular structure of the coefficients (3.20).

In conclusion, the most relevant difference between the t-matrix predictions and the Born ones appears in the absolute magnitude of the cross sections. The shape of the angular distribution is only roughly the same in both cases, while the energetic dependence seems to be rather different at lower energies.

The above results about the importance of the off-energy-shell effects in nucleon-exchange reactions agree with similar results obtained for different nuclear processes, as break-up[9, 11] and heavy-ion neutron tunnelling reactions[27].

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