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V. R. Manfredi and A. Molinari : AN ANALOGY BETWEEN THE DISPERSION CURVE IN LIQUID He II AND THE NUCLEAR ROTATIONAL BANDS. -

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V.R. Manfredi and A. Molinari[(x)]: AN ANALOGY BETWEEN THE DISPERSION CURVE IN LIQUID He II AND THE NUCLEAR ROTATIONAL BANDS. -

As it is well known the connection between energy and angular momentum for a rigid rotor is given by the formula

(1) $$E_I = \frac{\hbar^2 I(I+1)}{2J}$$

where J is the moment of inertia. When applied to nuclear rotational bands formula (1) turns out to be inadequate to reproduce the experimental data, therefore the following expansion has been suggested[(1)]

(2) $$E_I = AI(I+1) + BI^2(I+1)^2 + CI^3(I+1)^3 + \dots\dots$$

Alternatively, instead of (2), one can write

(3) $$E_I = \frac{\hbar^2 I(I+1)}{2J(I)}$$

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where the moment of inertia is a function of the angular momentum (or of the rotational frequency). A number of attempts has been made in this direction[2].

In this note we propose an approach, based on an analogy with liquid He II, to establish the relationship between the moment of inertia J and the rotational frequency ω, which we define in the usual operational way[3]

(4) $$\frac{\hbar^2}{2J} = \frac{\Delta E}{\Delta I(I+1)} = \frac{E(I_1)-E(I_1-2)}{4I_1-2}$$

and

(5) $$\hbar\omega = \frac{\Delta E}{\Delta\sqrt{I(I+1)}}$$

According to the ideas of Mottelson and Valatin[4], inspired by the BCS theory, a transition occurs in a deformed nucleus from the superconducting phase (where the pairing is fully effective) to the independent particle one (rigid rotor) as the rotational frequency is increased. Theoretical calculations by R. A. Sorensen[5], in the frame--work of the two level model, show that this transition follows different paths in the J-$(\hbar\omega)^2$ plane according to different values of the strength of the pairing force G. In particular, for values of G large enough, one reaches a situation in which the moment of inertia J is not even a single valued function of $(\hbar\omega)^2$.

Recent experimental investigations[6] confirm the possible existence of this singular behaviour in the rotational band of ^{162}Er (see Fig. 1).

A similar situation occurs in liquid He II: indeed in this case only the superfluid phase is present at zero temperature and then a gradual transition takes place till the λ-point, where only the normal phase remains. As the temperature raises, the excitations in liquid He II switch gradually from collective ("phonons") to single particle type (passing through "rotons")(x). In order to make the analogy more transparent we consider $(\hbar\omega)^2$ as a function of the moment of inertia J, reversing the relationship of Fig. 1. In this way we get a single valued function whose resemblance with the liquid

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(x) - In liquid He II the situation is complicated by the existence of an upper (the free particle) and a lower branch.

He II excitation curve is indeed rather striking.

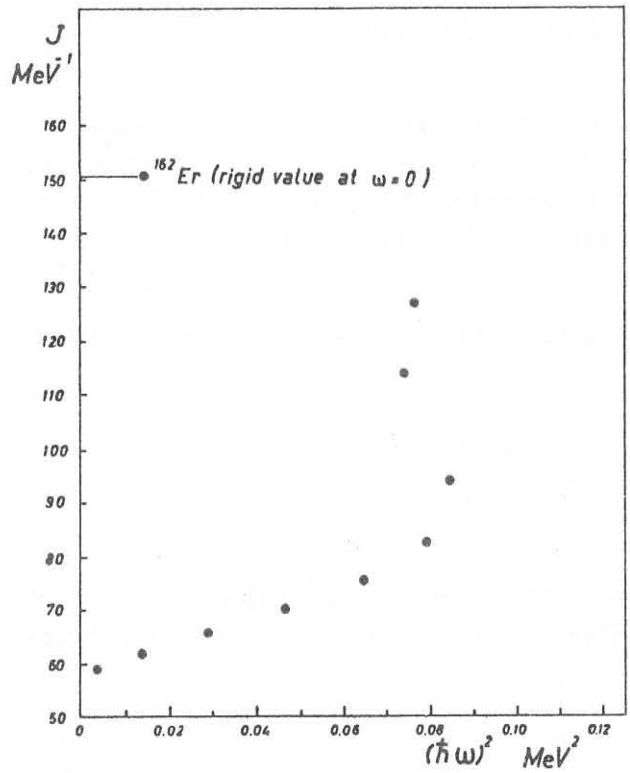

FIG. 1 - Experimental moments of inertia for rotating ^{162}Er. Taken from reference (6).

The actual analytic expression of

$$(\hbar\,\omega)^2 = f(J)$$

must, of course, be provided by a microscopic theory. We point out however that, following the analysis carried out in reference (7), the roton minimum of the liquid He II dispersione curve is "pinched" by two square root branch points in the complex plane of the momentum variable.

We suggest that a similar situation might occur for f(J). Therefore the following expansion should be used

$$(6) \qquad (\hbar\,\omega)^4 = \tilde{A}\,\mathcal{J}^2 + \tilde{B}\,\mathcal{J}^3 + \tilde{C}\,\mathcal{J}^4 + \tilde{D}\,\mathcal{J}^5 + \ldots\ldots$$

where a translation of the J axis has been performed

$$\left[\mathcal{J} = \frac{2}{\hbar^2}\,(J - J_{Min}),\ J_{Min} \simeq 57\,\mathrm{MeV}^{-1} \text{ in the } {}^{162}\mathrm{Er} \text{ case}\right].$$

From (6) we get

(7) $$(\hbar\omega)^2 = A\mathcal{J}\sqrt{1+B\mathcal{J}+C\mathcal{J}^2+D\mathcal{J}^3} = f(\mathcal{J})$$

We have found that four parameters are sufficient to get a fitting of the data. For their determination a numerical analysis of the experiments (with their errors) must be carried out. However, following Landau[8], we like to suggest a simple (but approximate) alternative method.

We want $f(\mathcal{J})$ to be well approximate near the minimum ($\mathcal{J}=\mathcal{J}_o$) by a parabola

(8) $$f(\mathcal{J}) = \nu + \frac{(\mathcal{J}-\mathcal{J}_o)^2}{\mu}$$

and to have the right slope (which is connected with the strength of the pairing force G) for $\mathcal{J}=0$. The following relationship then hold

(9) $$\left(\frac{df}{d\mathcal{J}}\right)_{\mathcal{J}=0} = \varphi(G), \quad \left(\frac{df}{d\mathcal{J}}\right)_{\mathcal{J}=\mathcal{J}_o} = 0, \quad f(\mathcal{J}_o) = \nu, \quad \left(\frac{d^2f}{d\mathcal{J}^2}\right)_{\mathcal{J}=\mathcal{J}_o} = \frac{2}{\mu}$$

By solving the set of equations (9) one gets

(10)
$$A = \varphi(G), \qquad B = \frac{1}{\mathcal{J}_o}\left\{\frac{2\nu}{\varphi^2(G)}\left(\frac{1}{\mu}+\frac{5\nu}{\mathcal{J}_o^2}\right)-3\right\},$$

$$C = \frac{1}{\mathcal{J}_o^2}\left\{3-\frac{\nu}{\varphi^2(G)}\left(\frac{4}{\mu}+\frac{15\nu}{\mathcal{J}_o^2}\right)\right\}, \qquad D = \frac{1}{\mathcal{J}_o^3}\left\{\frac{2\nu}{\varphi^2(G)}\left(\frac{1}{\mu}+\frac{3\nu}{\mathcal{J}_o^2}\right)-1\right\},$$

From an analysis of Fig. 1 the approximate numerical values

(11)
$$\mathcal{J}_o \simeq 63\ \mathrm{MeV}^{-1} \qquad \nu \simeq 0.072\ \mathrm{MeV}^2 \qquad \mu \simeq 1.2\times10^4\ \mathrm{MeV}^{-4}$$

$$\varphi(G) = 3.99\times10^{-3}\ \mathrm{MeV}^3$$

are obtained in the ^{162}Er case.

As it appears from Fig. 2 a good interpolation of the experimental data is achieved with (7). Moreover the roots of the polynomial under square root in (7) are

$$\mathcal{J}_1 = (68+17i)\,\mathrm{MeV}^{-1}, \quad \mathcal{J}_2 = (68-17i)\,\mathrm{MeV}^{-1}, \quad \mathcal{J}_3 = -225\,\mathrm{MeV}^{-1}$$

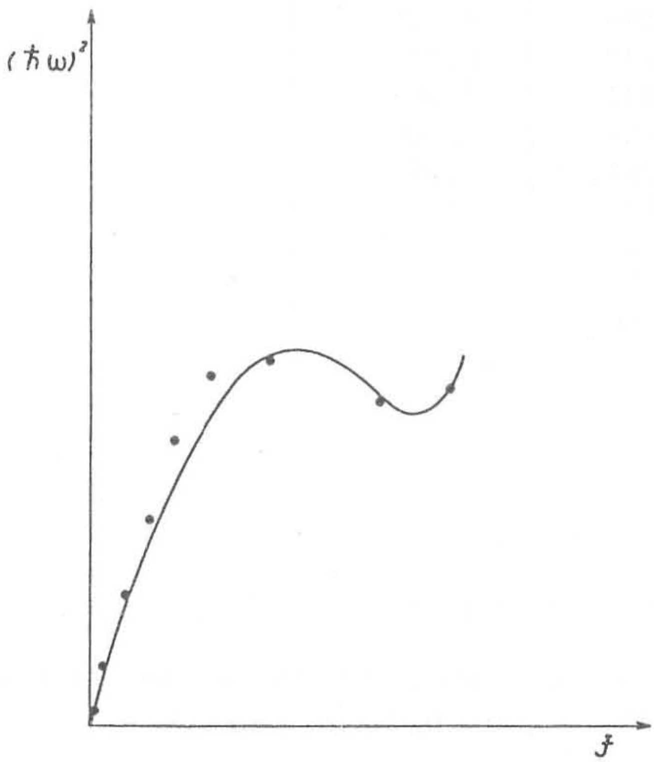

FIG. 2 - Fit of the experimental data with formula (7).

So, taking into account the approximate nature of our fitting procedure and the scarsity of the experimental data at our disposal, we have shown "a posteriori" the analytic properties of $(\hbar\omega)^2$; as a function of the moment of inertia, we have postulated on the basis of an analogy with liquid He II.

Finally we wish to calculate the energy position of the levels of the ground state rotational band in ^{162}Er. Using the relations between angular frequency and angular momentum (4) and (5) we get

$$\left(\frac{2J}{\hbar^2}\right)^2 A\mathcal{J}\sqrt{1+B\mathcal{J}+C\mathcal{J}^2+D\mathcal{J}^3} = \left(\frac{4I-2}{\Delta\sqrt{I(I+1)}}\right)^2 \qquad (12)$$

with the coefficients given by (10) and (11).

Equation (12) fixes in a unique way, for a given I, the corresponding moment of inertia J.

Then the energies of the ground state rotational band of ^{162}Er are given in Table I and compared with the known experimental values[9].

We wish to thank Prof. P. Kinle for making us available Fig. 1 and Prof. R.A. Ricci for his kind and encouraging interest.

TABLE I

I	E(MeV)	
	Theor.	Exp.
2	0.104	0.101
4	0.336	0.327
6	0.675	0.662
8	1.101	1.090
10	1.594	1.595
12	2.137	
14	2.707	
16	3.253	
18	3.806	

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