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S. Notarrigo, F. Porto, A. Rubbino, S. Sambataro and A. Straz zeri: ANGULAR DISTRIBUTIONS OF THE ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, \alpha\right){ }^{25} \mathrm{Mg}$ REACTION AT 10 MeV ANALYZED BY MEANS OF A DIFFRACTIONAL MODEL. -

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S. Notarrigo, F. Porto, A. Rubbino, S. Sambataro and A. Strazzeri: ANGULAR DISTRIBUTIONS OF THE ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, ~ \propto\right){ }^{25} \mathrm{Mg}$ REACTION AT 10 MeV ANALYSED BY MEANS OF A DIFFRACTIONAL MODEL.

## SUMMARY -

The angular distribution of the $\alpha$ particles from the ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, \alpha_{\mathrm{o}}\right){ }^{25} \mathrm{Mg}$ reaction have been measured at an incident ener gy of 9.8 MeV . The data are compared with a diffractional formula to gether with other data from the literature.

## RIASSUNTO -

E' stata misurata la distribuzione angolare delle particelle $\alpha$ emesse nella reazione ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, \alpha_{\mathrm{o}}\right)^{25} \mathrm{Mg}$ ad energia delle particel le incidenti di 9.8 MeV .

I risultati assieme a quelli di altri autori vengono confrontati con una formula di tipo diffrazionale.

## 1. - INTRODUCTION -

D. W.B.A. calculations are universally employed to analyse direct reactions. However, in case of strong absorbtion, the experimental angular distributions suggest the analysis by means of dif-
fractional models $(1,4)$. These models have had a good success; yet the parameters used in fitting the data have hitherto only a phenomenologi cal meaning, and do not show any definite correlation with the physi cal quantities characterizing the nuclear interaction.

A particular diffractional model was used in the analysis of the ${ }^{26}{ }_{\mathrm{Mg}}\left({ }^{3} \mathrm{He}, \chi\right){ }^{25} \mathrm{Mg}$ reaction at $5 \mathrm{MeV}{ }^{(4)}$; the good agreement found between the experimental data and this model led us to extend its application at higher energies. For this reason we measured the angular distribution of ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, \quad \alpha_{\mathrm{o}}\right)^{25} \mathrm{Mg}$ reaction at 9.8 MeV .

## 2. - EXPERIMENTAL ARRANGEMENT AND RESULTS -

The helium -3 particles were supplied by the 5.5 MeV Van de Graaff accelerator of the Institut fllr Kernphysik at Frankfurt/M. The magnesium 26 target was $0.4 \mathrm{mg} / \mathrm{cm}^{2}$ thick and was deposited on a Nickel-foil, $0.45 \mathrm{mg} / \mathrm{cm}^{2}$ thick. The target was at $45^{\circ}$ to the beam direction. The details of the experimental set up have already been given in a previous paper(5). The angular distribution of the $\alpha$ particles leading to the ground state of the residual nucleus in the ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He},{ }_{\mathrm{o}}^{\mathrm{o}}\right.$ ) ${ }^{25}{ }^{2} \mathrm{Mg}$ reaction at 9.8 MeV is shown in fig. 1. In this figure the data are compared with recent data at 10.2 MeV appeared in the mean while in the litera ture ${ }^{(6)}$. They are normalized at the point at $50^{\circ}$. At backward angles the two curves do not agree. We cannot say if this discrepancy is due to experimental systematic errors or to statistical fluctuations in the cross section as evidenced at $5 \mathrm{MeV}^{(5)}$. In fact our target had a thickness cor responding to about 200 KeV while Nurzynski's ${ }^{(6)}$ target was about $50 \overline{\mathrm{~K}} \mathrm{eV}$.


FIG. 1 - Angular distribution for the ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, \propto\right)^{25} \mathrm{Mg}$ reaction leading to the ground state of ${ }^{25} \mathrm{Mg}$. Black points a re our results; circles are the results of ref.(6).

## 3. - HAUSER-FESHBACH CALCULATIONS -

Since we expect some statistical contribution also at 10 MeV , however less pronounced as compared to the 5 MeV data, we calculated the angular distributions of the $\alpha$-particle for the ${ }^{26}{ }_{\mathrm{Mg}}\left({ }^{3} \mathrm{He}, \alpha\right){ }^{25} \mathrm{Mg}$ reaction by means of the formula (7)

$$
\frac{d \sigma_{\alpha} \alpha^{\prime}}{d \omega}=\sum_{J}(-)^{s-s^{\prime}} \bar{Z}(\ell J \ell J, s L) \bar{Z}\left(\ell^{\prime} J \ell ' J, s^{\prime} L\right) x
$$

(1)

$$
x P_{L}(\cos \theta) \frac{T_{\ell} T_{\ell} \ell^{\prime}}{g(J)} \frac{x^{2}}{4} \frac{1}{\left(2 I_{\alpha^{\prime}}+1\right)\left(2 i_{\alpha}+1\right)}
$$

with

$$
\left.g(J)=\sum_{\nu} \sum_{\ell \nu} \sum_{s \nu} \sum_{I \nu} \int_{O_{\ell \nu}}^{E_{\max }} T_{\nu}\right) \rho\left(E_{\nu}^{*}, I_{\nu}\right) d E_{\nu}
$$

where $\nu$ indicates the channels in which the compound nucleus states decay, $T_{\ell}$ are the transmission coefficients and $\rho\left(E_{\nu}^{*}, I_{\nu}\right)$ is the level density in ( $\nu, I)$-channel at energy $E^{*}$. For the level density we used the expression due to Lang and Le Couteur (8); this expression de pends chiefly on the following parameters: the radius of the residual nucleus $R \simeq r_{o} A^{1 / 3} \mathrm{fm}$, the pairing-energy $\triangle$ and the a-parameter related to the spacing of the single nucleon states near the top of the Fermi distribution ${ }^{(9)}$. For these parameters we used the following values: for $\mathrm{a}^{(10)}$.

$$
\begin{aligned}
& \mathrm{a}\left({ }^{28} \mathrm{Si}\right)=4.8 \mathrm{MeV}^{-1} ; \mathrm{a}\left({ }^{28} \mathrm{Al}\right)=5.6 \mathrm{MeV}^{-1} \\
& \mathrm{a}\left({ }^{25} \mathrm{Mg}\right)=4.5 \mathrm{MeV}^{-1}
\end{aligned}
$$

for $\Delta^{(11)}$

$$
\Delta\left({ }^{28} \mathrm{Si}\right)=-4.3 \mathrm{MeV} ; \Delta\left({ }^{25} \mathrm{Mg}\right)=-2.1 \mathrm{MeV}
$$

and $\mathrm{r}_{\mathrm{O}}=1.4 \mathrm{fm}$.
The above parameters were fixed at $5 \mathrm{MeV}^{(5)}$.
The transmission coefficients were calculated with an optical model potential which fitted the elastic scattering $(6,12)$.

The parameters for ${ }^{3} \mathrm{He}$ and $\propto$ particles are reported in table I. The angular distributions calculated by formula (1) are shown in fig. 2 together with the Nurzynski's experimental results.

TABLE I

| Particle | Potential <br> Form | $V_{i}$ <br> $(\mathrm{MeV})$ | $W_{j}$ <br> $(\mathrm{MeV})$ | $r_{o}$ <br> $(\mathrm{fm})$ | $a$ <br> $(\mathrm{fm})$ | $\mathrm{r}_{\mathrm{o}}^{\prime}$ <br> $(\mathrm{fm})$ | $\mathrm{a}^{\prime}$ <br> $(\mathrm{fm})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $3^{3} \mathrm{He}$ | S.W. | 151.8 | 14.6 | 1.08 | 0.80 | 1.78 | 0.60 |
| $\alpha$ | S.W. | 220 | 17 | 1.41 | 0.60 | 1.41 | 0.60 |

## 4. - COMPARISON WITH A DIFFRACTIONAL FORMULA -

Under the hypothesis of strong absorption, which is valid for the $\left({ }^{3} \mathrm{He}, \alpha\right)$ reactions, it seems that the dependence of the transition amplitude from the detailed shape of the effective interaction is not ea sily observable ${ }^{(1-4,13,14)}$.

Thus the stripping radial integrals $I(\ell)$ appearing in the DWBA differential cross-section show a considerable localization in $\ell$-space ${ }^{(14,15)}$.

This fact suggest the following phenomenological parametrization of these integrals $(1,4)$ :

$$
\begin{equation*}
I(\ell) \propto \frac{d}{d l}\left[\left(1-\exp \frac{\pi\left(l_{0}-\ell\right)}{\beta}\right)\right]^{-1} \exp \left(2 i \sigma_{l}\right) \tag{2}
\end{equation*}
$$

With such an assumption in a previous paper a diffractional formula was obtained by the methods of complex angular momenta(16).

The results of the comparison between the prediction of this formula and our experimental data are shown in fig. 3. In fig. 4 the same comparison has been made with the Nurzynsky's data after subtraction of the statistical contribution.

As in the previous paper ${ }^{(4)}$ the points in the angular distributions corresponding to angles $\theta \leq 25^{\circ}$ and $\theta \geq 140^{\circ}$ were neglected because the model breaks down at these extreme angles. We choose, as free parameters in the fitting procedure the same one's as in ref. (4), the normalization constant $h$ which in some way is related to the spectroscopic factor, the parameter $\ell_{0}$ which corresponds to the angular mo-


FIG. 2 - Angular distributions for the ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, \alpha\right){ }^{25} \mathrm{Mg}$ reaction leading to the ground state and the first four excited levels in 25 Mg . The data are those of J. Nurzynski, taken from ref. 6. The solid lines are the angular distributions calculated as described in


FIG. 3 - Angular distribu tion (in arbitrary units) for the ${ }^{26}{ }_{\mathrm{Mg}}\left({ }^{3} \mathrm{He}, \alpha\right){ }^{25} \mathrm{Mg}$ reac tion at 9.8 MeV , leading to the ground state of 25 Mg . The solid line is the bestfitted curve of the diffractional model (see text) with the parameter values given in table II.
mentum where one has the maximum stre ight of the stripping integral according to the parametrization (2), the para meter $\dot{\beta}$ which gives a measure of the $\overline{\mathrm{de}}$ gree of localization in $\ell$-space, the para meter $G_{i}$ which represents the immaginary part of the $\sigma_{\ell}$ phase: finally the pa rameters $\sigma_{r}^{\prime}$ and $\sigma_{i}^{\prime}$ which represent the real and immaginary part respectively of the variation with $\ell$ of the phase $\sigma_{\ell}$ around $\ell_{0}$ (see ref. (4)).

The parameters obtained from the best-fits are given in the table II and III togheter with the corresponding $\sigma_{i}, \sigma_{r}^{\prime}$ and $\sigma_{i}^{\prime}$ calculated with the following approximate formula of the Coulomb phase shift, valid for $|\ell| \gg 1(17)$ :

$$
\sigma_{l} \approx n \log (1+\ell)
$$

where n is the Coulomb parameter.
Since it is expect that the statistical contribution is percentually higher at backward angles we also fitted the Nurzynski's data only up to $\theta=120^{\circ}$.

The results of the comparison are shown in fig. 5 and the parameters obtained from the best-fits are reported in table IV.

TABLE II

| h | $\mathrm{l}_{\mathrm{O}}$ | $\beta$ | $\sigma_{\mathrm{i}}$ | $\sigma_{\mathrm{r}}^{\prime}$ | $\sigma_{\mathrm{i}}^{\prime}$ | $\left.\begin{array}{c}\mathrm{r}_{\mathrm{o}} \\ \mathrm{fm}\end{array}\right)$ | d <br> $(\mathrm{fm})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 960 | 4.88 | 0.77 | -0.41 <br> $(0.31)$ | -0.33 <br> $(0.31)$ | -2.92 <br> $(-0.041)$ | 1.24 | 0.17 |

From the tables II, III and IV it can be seen that the $\sigma_{i}$ parameters are generally smaller than the corresponding Coulomb phase shifts: this could be expected because the empirical parameter $\varsigma_{i}$ takesinto ac count nuclear contributions which tend to reduce it (being of opposite sign).


FIG. 4 - Angular distributions for the ${ }^{26} \mathrm{Mg}\left({ }^{3} \mathrm{He}, \alpha\right){ }^{25} \mathrm{Mg}$ reaction leading to the ground state and the first four excited levels in ${ }^{25} \mathrm{Mg}$. The data are those of J. Nurzynski, taken from ref. 6. The solid lines are the best--fitted curves of the diffractional model (see text) with the parameter values given in table III.


FIG. 5 - As in Fig. 4. Best-fits have been performed up to $\theta=120^{\circ}$; the parameter values are given in table IV.
8.

TABLE III

| groups | h | $\ell_{0}$ | $\beta$ | $\sigma_{\mathrm{i}}$ | $\sigma_{\mathrm{r}}^{\prime}$ | $\sigma_{\mathrm{i}}$ | $\sigma_{\mathrm{o}}^{\prime}$ <br> $(\mathrm{fm})$ | d <br> $(\mathrm{fm})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{\mathrm{o}}$ | 1134 | 4.95 | 2.72 | 0.545 <br> $(0.793)$ | 0.360 <br> $(0.256)$ | -0.078 <br> $(-0.256)$ | 1.23 | 0.59 |
| $\alpha_{1}$ | 142.5 | 4.89 | 1.32 | 0.277 <br> $(0.416)$ | 0.305 <br> $(0.300)$ | 0.051 <br> $(-0.067)$ | 1.23 | 0.29 |
| $\alpha_{2}$ | 61.11 | 3.38 | 1.49 | 0.535 <br> $(0.635)$ | 0.063 <br> $(0.382)$ | 0.074 <br> $(-0.130)$ | 1.01 | 0.31 |
| $\alpha_{3}$ | 11.58 | 3.25 | 0.69 | 0.629 <br> $(0.307)$ | 0.624 <br> $(0.431)$ | 0.0005 <br> $(-0.070)$ | 1.00 | 0.15 |
| $\alpha_{4}$ | 3.09 | 5.41 | 1.27 | -0.270 <br> $(0.374)$ | 1.380 <br> $(0.284)$ | -0.2000 <br> $(-0.056)$ | 1.34 | 0.29 |

TABLE IV

| groups | h | $\ell_{0}$ | $\beta$ | $\sigma_{\mathrm{i}}$ | $\sigma_{\mathrm{r}}^{\prime}$ | $\sigma_{\mathrm{i}}^{\prime}$ | $\boldsymbol{\sigma}_{\mathrm{o}}^{\prime}$ <br> $(\mathrm{fm})$ | d <br> $(\mathrm{fm})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{\mathrm{o}}$ | 1272 | 4.70 | 2.09 | 0.495 <br> $(0.645)$ | 0.356 <br> $(0.285)$ | -0.276 <br> $(0.285)$ | 1.19 | 0.45 |
| $\alpha_{1}$ | 165 | 4.94 | 1.39 | 0.290 <br> $(0.427)$ | 0.309 <br> $(0.296)$ | 0.046 <br> $(-0.069)$ | 1.24 | 0.30 |
| $\alpha_{2}$ | 194 | 3.70 | 2.06 | 0.720 <br> $(0.765)$ | 0.210 <br> $(0.333)$ | 0.099 <br> $(-0.146)$ | 1.06 | 0.45 |
| $\alpha_{3}$ | 18.05 | 3.63 | 0.83 | -0.037 <br> $(0.339)$ | 0.570 <br> $(0.394)$ | -0.059 <br> $(-0.071)$ | 1.06 | 0.18 |
| $\alpha_{4}$ | 3.66 | 5.76 | 1.17 | -0.280 <br> $(0.330)$ | 1.080 <br> $(0.271)$ | -0.320 <br> $(-0.047)$ | 1.40 | 0.26 |

In the same tables the $r_{0}$ values, connected to the interaction radius $R$, and the surface-width $d$ of the interaction region are repor ted as derived according to the semiclassical picture $(4,18)$ :

$$
\begin{aligned}
& \ell_{o}\left(\ell_{o}+1\right)=K R(K R-2 n) \\
& \left(2 \ell_{0}+1\right) \beta=2 \pi K d(K R-n)
\end{aligned}
$$

where

$$
R=\frac{r_{o}}{2}\left(A_{T}^{1 / 3}+A_{R}^{1 / 3}+A_{3}^{1 / 3}+A_{\alpha}^{1 / 3}\right)
$$

and

$$
\mathrm{n}=\frac{1}{2} \frac{\mathrm{e}^{2}}{\mathrm{~h}^{2}}\left[\frac{\mathrm{~m}^{3} \mathrm{He}^{Z_{T}} \mathrm{Z}_{3_{H e}}}{\mathrm{~K}_{3 \mathrm{He}}}+\frac{\mathrm{m}_{\alpha} \mathrm{Z}_{\mathrm{R}} \mathrm{Z}_{\alpha}}{\mathrm{K}_{\alpha}}\right]
$$

Finally in fig. 6 the $r_{o}$-values at 10 MeV are shown (see table IV) versus $r_{o}$-values at 5 MeV (see ref. (4)); the "errors" represent the cor responding $d$-values for the five $\alpha$-groups.


FIG. $6-\left(r_{o}\right.$-values $) \pm$ (d-va lues) at 10 MeV (see table IV) versus ( $\mathrm{r}_{\mathrm{O}}$-values) $\pm$ (dvalues) at 5 MeV (see ref. 4).

The values of $r_{0}$ at different energies are not very different.

In conclusion, the present analysis seems to show that our diffractional model describes adeguately the angular distribution also at energies around 10 MeV , in spi te of its simplicity and of the relatively small set of parameters.

More experimental and theoretical work seems worthwhile in order to understand why that is so, because it is not clear, at the moment, whether such an agreement, has some deep significance or it is due to the compensating effect of the parame ters.

This accomplishement would be extre mely valuable in order to extract, later on, in a very simple manner, as compared to D.W.B.A. calculations, relevant spectroscopic informations from the experimental data.

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