ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Milano

INFN/BE-70/9
3 Dicembre 1970

E. Gadioli : INTERACTION OF 20-45 MeV PROTONS WITH HEAVY NUCLEI. -

Reparto Tipografico
dei Laboratori Nazionali di Frascati

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ABSTRACT. -

The paper deals with the separation of non statistical and statistical effects in reactions induced by 20-45 MeV protons on heavy nuclei (160 < A < 210) below the high fissility region.

The theoretical background is introduced and experiments that could give information on Compound Nucleus formation cross section σ_{CN} and Direct or non Compound Nucleus formation cross section σ_{DI} are discussed. Results concerning (p, xn) reactions (x=3, 4) induced in ^{169}Tm, ^{181}Ta, ^{209}Bi and the ^{197}Au(p,α)^{194}Pt reaction are considered.

(x) - Invited paper to the 160th meeting of the American Chemical Society-Division of Nuclear Chemistry and Technology.

1. - INTRODUCTION. -

The interaction of protons of intermediate energy (20-45 MeV) with heavy nuclei will be considered and an attempt will be done to evaluate the indipendent contributions to the reaction mechanism of non statistical and statistical effects.

Though a great effort has been done to study this subject at lower and higher energies, no such systematic study has been performed in this energy region.

For instance it is not possible to find in literature a quantitative estimation of the fraction of reaction cross section leading to statistical effects and the complementary fraction leading to non statistical effects (though local fluctuations of these quantitatives are expected, it seems reasonable to assume that the relative percentages of the two different effects should be quite smooth functions of the mass number of target nuclei).

Heavy nuclei are chosen for such a kind of analysis because in their case it is much more simple to analyse the statistical contribution to the process. Neutron emission from excited heavy nuclei is always predominant with respect to charged particles and γ rays emission (except for very low excitation energies where γ emission can compete favorably with neutron emission(1)) and the result of any analysis are less biased by an eventually erroneous choice of the parameters relevant to the calculation.

The knowledge of the statistical contribution greatly simplifies the analysis of non statistical effects and the study of very improbable events, once the main features of the process are known, greatly helps to fix the value of the parameters entering into the calculations.

One has a similar situation in other regions of the nucleidic chart where neutron emission from excited nuclei is strongly inhibited by the high value of the neutron binding energy and excited nuclei decay predominantly by proton decay.

Studies of such a kind are very important both by a fundamental point of view (the study of the interaction mechanism) and by a technological point of view.

In next sections the general theoretical ideas will be briefly outlined and three kind of experiments, that seem to be most promising for a systematic search in this field, discussed.

2. - STATISTICAL AND NON STATISTICAL EFFECTS. -

Since the classical Fermi's works on neutron induced reactions and the discovery of proton resonances in 1935, nuclear scientists usually divide nuclear reactions into two broad classes: compound nucleus (CN) reactions and direct reactions.

At sufficiently low excitation energy of the intermediate system $E \sim E_{inc} + B_{inc} \sim 8$ MeV, compound nucleus and direct reactions are well separated on a time scale. The CN characterized by states of complex energy $E = \mathrm{Re}\,E - i\,\Gamma/2$, survives a mean time $\tau = \hbar/\Gamma$, that is long when compared to nuclear transit time for the incoming particle, that is the characteristic time for direct interactions $\tau_d \sim \beta^{-1} A^{1/3}\, 10^{-23}$ sec.

The mean CN lifetime is also long when compared to the nuclear relaxation time $\tau_R \sim h/D$ (D is the mean spacing between CN levels); statistical equilibrium is achieved before decay and the CN decay is governed only by the available configurations in phase space of final products.

As the excitation energy increases, the compound nucleus states overlap (Γ becomes greater than D) and statistical equilibrium is hardly achieved before the CN decay; also the separation in time between CN and so called direct effects is much less sharp. Ericson[2] has then shown that more than a time criterion a randomness criterion should be introduced to discriminate between the two different types of reactions. To go a little more into details, the nuclear matrix element, the modulus squared of which gives the cross section for a given process, is decomposed, in Ericson treatment, into an average and a fluctuating term following a Gaussian distribution with zero average:

$$S_{\alpha\beta} = \langle S_{\alpha\beta} \rangle + \hat{S}_{\alpha\beta} \qquad (1)$$

$\langle S_{\alpha\beta} \rangle$ is associated to the direct interaction, $\hat{S}_{\alpha\beta}$ to the statistical interaction.

If one makes a good resolution experiment, in the measured cross section strong interference between direct and statistical effects becomes apparent. The term statistical interaction rather than CN interaction is used in this case due to the difficulty of extrapolating at high energies the simple features of the model that has been introduced to describe low energy phenomena; as in the low energy case, however, the decay of the intermediate system (projectile and target) is regulated, on the average, only by all the available configurations in phase space of final products.

The interference between statistical and non statistical effects vanishes if an experiment with a low resolution beam is performed. In that case the cross section is just the sum of a non statistical and an average statistical term.

In an experiment with a low resolution beam the energy average is done on C N states. Another kind of average can be done if many final reaction channels are not resolved. In both cases the cross section can, with a good approximation, be considered the sum of average statistical and non statistical effects.

In a good resolution experiment, fluctuations in the measured cross sections are to be expected both due to the fluctuating part of the matrix element $\hat{S}_{\alpha\beta}$ and to the interference between $\langle S_{\alpha\beta}\rangle$ and $\hat{S}_{\alpha\beta}$. The amplitude of the fluctuations can be evaluated. In the case of purely statistical reactions ($\langle S_{\alpha\beta}\rangle = 0$) the following relation holds:

$$\left\langle \left(\frac{\sigma_{\alpha\beta}(E) - \langle \sigma_{\alpha\beta}(E)\rangle}{\langle \sigma_{\alpha\beta}(E)\rangle} \right)^2 \right\rangle \sim \frac{1}{N} \qquad (2)$$

N is the number of channels inchoerently contributing to the reaction. If non statistical effects are present, the preceding ratio is further reduced. For this reason, when many channels contribute to a reaction, though in principle is important to speak about fluctuations, in practice, the deviations from the average cross section are of increasingly negligible importance. In experiments that will be discussed in present paper, reactions leading to many unresolved final states are studied; to each of these states many channel contribute inchoerently; for this reason, interference effects in all the considered cases will be neglected.

3. - REACTION CROSS SECTION, C N FORMATION CROSS SECTION AND DIRECT. INTERACTION CROSS SECTION. -

The treatment given by H. Feshbach[3] will be strictly followed. At high projectile energies, the Optical Model (OM) allows one to predict the reaction and the elastic scattering cross sections. The compound elastic cross section is considered to be negligible. By definition OM allows one to calculate the average transition matrix $\overline{S^{J\pi}_{\alpha\alpha}}$; this is done through complex phase shifts:

$$\overline{S^{J\pi}_{\alpha\alpha}} = \overline{S(\alpha 1s|\alpha 1s|J\pi)} = 1 - \exp\left[2i(\xi^J_{1s} + i\,\eta^J_{1s})\right] \qquad (3)$$

If $\overline{S^{J\pi}_{\alpha\alpha}}$ is compared with the average matrix element predicted by general Nuclear Reaction Theory (N R T):

$$(4) \qquad \langle S^{J\pi}_{\alpha\alpha} \rangle = 1 - \exp(2i\,\delta^{J}_{1s}) + \pi \exp(2i\,\delta^{J}_{1s}) \langle \Gamma(\alpha\, 1s \,|\, J\pi)/D^{J\pi} \rangle$$

the potential scattering phase shift $\delta^{J}_{1s} = \alpha^{J}_{1s} + i\beta^{J}_{1s}$, channel widths and C N level spacings $D^{J\pi}$ are connected to O M phase shifts. As a consequence the transmission coefficients

$$(5) \qquad T^{J}_{1s} = 1 - \exp(-4\,\eta^{J}_{1s})$$

can be expressed by means of the potential scattering phase shift and the ratio $\Gamma(\alpha\, 1s \,|\, J\pi)/D^{J\pi}$:

$$(6) \qquad T^{J}_{1s} = 1 - \exp(-4\beta^{J}_{1s})\,(1 - \pi \langle \Gamma(\alpha\, 1s \,|\, J\pi)/D^{J\pi} \rangle)^{2} = 1 - \exp(-4\beta^{J}_{1s}) \times$$
$$\times (1 - 2\pi \langle \Gamma(\alpha\, 1s \,|\, J\pi)/D^{J\pi} \rangle + \pi^{2} \langle \Gamma(\alpha\, 1s \,|\, J\pi)/D^{J\pi} \rangle^{2}).$$

Since the reaction cross section is given by:

$$(7) \qquad \sigma_{R} = \sum_{s} \sum_{J,1} \left[(2J+1)\pi\lambda^{2}/(2i+1)(2I+1) \right] T^{J}_{1s}$$

one obtains

$$(8) \qquad \sigma_{R} = \sum_{s} \sum_{J,1} \frac{(2J+1)\pi\lambda^{2}}{(2i+1)(2I+1)} \left\{ (1 - \exp\left[-4\beta^{J}_{1s}\right]) + \exp\left[-4\beta^{J}_{1s}\right] \times \right.$$
$$\left. \times \left(2\pi \left\langle \frac{\Gamma(\alpha\, 1s \,|\, J\pi)}{D^{J\pi}} \right\rangle - \pi^{2} \left\langle \frac{\Gamma(\alpha\, 1s \,|\, J\pi)}{D^{J\pi}} \right\rangle^{2} \right) \right\}.$$

By definition of potential scattering phase shift

$$(9) \qquad \sigma_{Di} = \sum_{s} \sum_{J,1} \frac{(2J+1)\pi\lambda^{2}}{(2i+1)(2I+1)} \left(1 - \exp(-4\beta^{J}_{1s})\right)$$

is the cross section for inelastic scattering through "direct" interaction and as a consequence

$$\sigma_R = \sigma_{Di} + \sigma_{CN} \tag{10}$$

where

$$\sigma_{CN} = \sum_s \sum_{J,1} \frac{(2J+1)\pi\lambda\!\!\!^{-2}}{(2i+1)(2I+1)} (\exp(-4\beta^J_{1s})(2\pi\langle\Gamma(\alpha 1s|J\pi)/D^{J\pi}\rangle - \tag{11}$$
$$- \pi^2\langle\Gamma(\alpha 1s|J\pi)/D^{J\pi}\rangle^2))$$

is the so called "compound nucleus formation cross section". The O M and N R T provide then a clear cut division between direct and compound processes, but really one does not posses estimations of β^J_{1s} and as a consequence one cannot predict the values of σ_{Di} by theoretical means on general basis. This value can be tentatively obtained by Monte Carlo techniques or estimated by means of refined intermediate structure statistical models or extracted by proper analysis of experimental data.

Experimental evidence concerning reactions induced by intermediate energy protons on heavy nuclei, below the high fissility region, and theoretical calculations of non statistical effects, suggest that relation (10) could be rewritten as

$$\sigma_R = \sigma_{CN} + \sigma^n_{Di} + \sigma^p_{Di} \tag{10a}$$

where σ^n_{Di} and σ^p_{Di} are respectively the cross sections for non statistical emission of one neutron and one proton.

4. - MONTE CARLO CALCULATIONS OF PROMPT NUCLEON EMISSION IN REACTIONS INDUCED BY HIGH ENERGY PROJECTILES. -

The model usually introduced to describe the direct or non statistical interaction of an high energy projectile with a target, is the Serber model.

The fast incident projectile is supposed to make free interactions with the nucleons which constitute the nucleus; after each interaction the struck nucleons can make further interactions with other nucleons of the nucleus. The struck and incident nucleons can reach the nuclear surface; if their energy is bigger than a fixed value (cut-off energy) they are emitted.

It can be possible that the incident energy is shared between so many nucleons that they cannot leave the nucleus. In this case a CN is created and emission is possible when by a statistical phenomenon to a single nucleon sufficient energy is given to leave the nucleus.

The nucleus is described as a degenerate Fermi gas. The nucleon momentum distribution is the usual one for a Fermi gas:

(12) $$\frac{dn}{dp}\,dp = \frac{\Omega p^2\,dp}{2\pi^2\hbar^3}$$

here Ω is the nuclear volume.

The maximum value of the momentum, the Fermi value, is related to the nuclear density by:

(13) $$k_f^2 = 3\,\pi^2\,\rho^{2/3}\ .$$

In early calculations the nuclear density was assumed to have a square distribution with a radius $R = 1.3\ A^{1/3}$ fm[4].

In later calculations trapezoidal and step density distributions were introduced[5], simulating as a first approximation a Fermi distribution

(14) $$\rho\,(r) = \rho_o / (1+\exp\left[(r-c)/a\right])$$

$c = 1.07\,A^{1/3}$ fm, $a = 0.545$ fm.

Non uniform density distributions are introduced to increase the agreement between experiment and calculations for very short nuclear cascades like (p, p), (p, n), (p, pn), (p, 2p).

These cascades are thought to arise when the projectile strikes the nucleus at a large impact parameter in the diffuse edge of the nucleus. In this case due to the low nucleon density, the probability that the struck and (or)the incident nucleons leave the nucleus without further interactions is rather high. Refraction and reflection effects in nucleon path can be taken into account as nucleons move from a density region to a different one, but comparison with experiment seems to indicate that agreement between calculations and experiment is worsened when such effects are considered.

Usually the calculations do not consider the possibility of complex particle emission as a result of the incident nucleon interaction with a cluster of nucleons. In fact, however, experimental results seem to suggest the presence of nucleon clusters at the diffuse nucleus edge.

To take into account the possibility of α's clusters in the low density nuclear region, a correlation index which goes from 0 to 1 and gives the probability of α clustering can be attributed to each constant density step[6].

The model can in principle describe all kinds of nucleon emissions before statistical equilibrium be established. It appears however that the results are very sensible to details of the calculation like nuclear radius, cut-off energy....

If we ask for the range of applicability of the model, the following conditions should at least be fulfilled:

i) the incident nucleon wave lenght should be small when compared to the mean distance between nucleons ($\sim 10^{-13}$ cm).

ii) the collision time for the projectile should be small when compared to the mean collision time between nucleons ($\sim 2 \times 10^{-23}$ sec),

iii) the incident particle energy should be great when compared to the residual interaction energy of various struck nucleons.

These requirements are hardly satisfied for projectile energies lower than 100 MeV.

It must be stressed however that, also at higher energies, at the end of each cascade process the preceding requirements could not be satisfied.

Comparison with experiment, however, seems to show that, in spite of many minor discrepancies, the applicability of the model might be greater than expected. This is probably due to partial compensation of opposite sources of error. A very detailed comparison between the model previsions and the experimental data is however lacking for projectile energies in the interval of interest for us.

5. - INTERMEDIATE STRUCTURE STATISTICAL MODELS. -

A new method to try to predict the energy dependence of the yield of residual nuclei following non statistical nucleon emission has been developed by Griffin for neutron emission[7] and extended by Blann to include charged particle emission[8].

This method follows an old idea due to Weisskopf. The projectile is supposed to interact firstly with one or few nucleons. Following this first interaction a state of moderate complexity is produced; i.e. few excitons (particles and holes) are excited. Most of the states cor responding to the excited compound nucleus are of much greater complexity and are characterized by an average exciton number that in Fermi gas model can be estimated as

$$\bar{n} = 2\, g t \ln 2 \qquad (15)$$

where g is the nucleon state density at Fermi energy and t the thermodinamic temperature[9]. The complex CN states can be reached starting from the moderate complexity initial states through nucleon interactions that, in first approximation, can be considered two body interactions. In this approximation, if, at a given stage of the process, the exciton number is n', at the next stage it can be n' or n'$\pm$2. In fact it can be shown that if n'$\ll \bar{n}$, there is a great probability that the state n'+2 is reached. According to the model the nucleon interactions inside the nucleus proceed toward states of increasing complexity corresponding to higher exciton numbers. At each stage nucleon emission is possible. The relative probability can be estimated and is given by the expression:

$$P(\varepsilon)\,d\varepsilon = \frac{2s+1}{\pi^2 \hbar^3}\, m \varepsilon \sigma \frac{\rho_{n'-1}(U)}{\rho_{n'}(E)} \tau_{n'} d\varepsilon \qquad (16)$$

where m is the reduced mass, σ the inverse cross section, ε the emitted nucleon energy, E the CN energy, $U=E-B_n-\Delta_n-\varepsilon$ the residual nucleus energy, $\tau_{n'}$ the mean life of the n' exciton state, ρ state densities.

The total probability for nucleon emission before statistical equilibrium is reached is assumed to be given by:

$$P_t(\varepsilon)\,d\varepsilon = \sum_{\substack{n=n' \\ \Delta n=2}}^{\bar{n}} P_n(\varepsilon)\,d\varepsilon \qquad (17)$$

Introducing explicitly the expression for the density of a p particle h hole state:

$$\rho_{p,h}(E) = \frac{g(gE)^{p+h-1}}{p!\,h!\,(p+h-1)!}, \qquad (18)$$

and assuming constant lifetimes for all states characterised by different exciton numbers, one obtains:

(19) $$P_t(\mathcal{E})\,d\mathcal{E} = K\,\frac{2s+1}{gE}\,m\,\mathcal{E}\,\sigma\left\{\sum_{n}\left(\frac{U}{E}\right)^{n-2} p(n-1)\right\}d\mathcal{E}$$

where $n = p+h$.

The Griffin-Blann model can allow to predict the energy distribution of nuclei following nucleon emission, before statistical equilibrium is reached, much more easily than Monte Carlo calculations based on the impulse approximation. At low excitation energies it is also basically much more founded by a theoretical point of view. The approximations of the model must not be forgotten however. In deriving expression (17), for instance, the depletion of nuclear states due to nucleon emission is neglected. When precompound emission is not negligible the approximation turns out to be erroneous. This approximation and the assumption of constant lifetimes for the different exciton states suggest that the model should be more nearly correct if $U \ll E$, that is when sum (19) converges very rapidly. In this case non statistical emission is mainly due to the first stages of the process.

If the preceding inequality is fulfilled we also have

(20) $$\ln \frac{P_t(\mathcal{E})}{\mathcal{E}\sigma} \sim (n'-2)\ln U + C.$$

The model has been successfully applied to (p, n) reactions[7, 10, 11], to (α, p) reactions[10, 12], to (α, xn), (p, xn), (p, pxn) reactions[8], to (α, n)[13] and (n, p) reactions[14]. The initial configuration exciton number is found to be $n' = 3$ in the case of proton or neutron induced reactions and $n' = 5$ in the case of α induced reactions.

6. - EXPERIMENTS THAT COULD GIVE INFORMATION CONCERNING THE CROSS SECTIONS CORRESPONDING TO C N FORMATION AND TO DIRECT INTERACTION PROCESSES. -

6.1. - Level density expressions. -

Before discussing suitable experiments, it turns out to be useful to make some comments on level density expressions and related parameters due to their great importance in quantitative analyses of some of the experimental results. It is well known that in heavy element region, at slow neutron resonance energy, the level density value drops abruptly for near doubly magic nuclei.

In a recent paper Gadioli et al.[15] have used a very simple model for describing the level density of doubly magic nuclei.

The nucleus is pictured as a two fermion gas with gaps D at the filling of magic shells. This simple model neglects completely deformation effects that could give rise to shell effects at non magic nucleon numbers[16]. For this reason it is a little dangerous to extend the model to predict level density of nuclei having a mass number quite different from ^{208}Pb. This procedure has been however applied in the analysis of (p, xn) and (p, α) reactions we will discuss. This mainly for the following reasons:

a) when it is applied to nuclei with mass number A$\sim$194-198, as the ones involved in the reaction ^{197}Au(p, α) ^{194}Pt, the model provides very satisfactory results (see section 6.4);

b) more sophisticated models based on the calculation of single particle states in realistic deformed well potentials would require a number of parameters the values of which are not known at present with sufficient accuracy.

In the particular case of (p, xn) excitation functions, the analysis results are relatively insensitive to the choice of level density expressions and parameters, so that the obtained results are little biased by possible shortcomings of the above quoted model.

For the details of the calculations we refer to original papers[17, 18]. In the following, only data strictly relevant to the argument we are interested in will be quoted and discussed.

6.2. - Fission excitation functions, near threshold, of low fissility nuclei. -

The fission excitation functions of low fissility nuclei (170 $\lesssim$ A $\lesssim$ 210), near threshold, increase very rapidly with energy of incoming particle, i.e. several orders of magnitude for a few MeV increase of the incident particle energy. See for instance Fig. 1 concerning reaction ^{209}Bi(p, f). For these nuclei fission is a first chance phenomenon; fission following particle emission from C N can contribute only a negligible amount to the process. Let now consider two projectile-target combinations leading to the same C N. If, in first approximation, angular momentum effects are neglected and the different projectiles are called a and b, at the same C N energy it should be:

(21) $$\left(\frac{\Gamma_f}{\sum_i \Gamma_i}\right)_a = \left(\frac{\Gamma_f}{\sum_i \Gamma_i}\right)_b$$

or taking into account that $\sum_i \Gamma_i \sim \Gamma_n$:

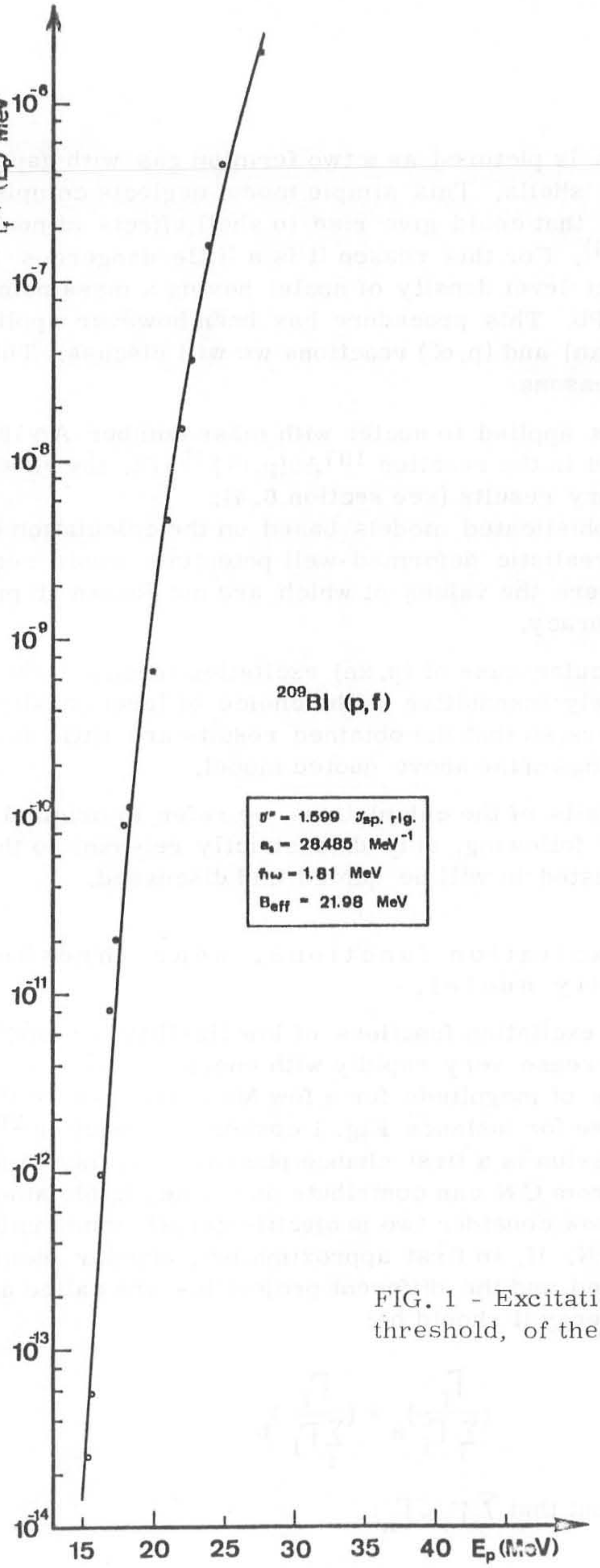

FIG. 1 - Excitation function, near threshold, of the reaction $^{209}Bi(p,f)$.

(22) $$\left(\frac{\Gamma_f}{\Gamma_n}\right)_a \cong \left(\frac{\Gamma_f}{\Gamma_n}\right)_b$$

From (22) it follows

(23) $$\left(\frac{\sigma_f}{\sigma_{CN}}\right)_a \sim \left(\frac{\sigma_f}{\sigma_{CN}}\right)_b$$

a relation that allows one to evaluate the C N formation cross section σ^a_{CN} corresponding to projectile a and target A if one knows (i) σ^b_{CN} corresponding to projectile b and target B, (ii) the corresponding fission excitation functions.

In practice to reach accurate results angular momentum effects cannot be neglected, expecially in the case that a or b are heavy ions.

In this case[19]

(24) $$\sigma^i_f \sim \sigma^i_{CN} \frac{\Gamma_f}{\Gamma_n} (\langle J_i \rangle)$$

where $\langle J_i \rangle$ is the average spin of C N created by absorption of projectile i into the target nucleus. If the J dependence of the ratio Γ_f/Γ_n is known (let us call R_{ab} the ratio $(\Gamma_f/\Gamma_n)(\langle J_a \rangle)/(\Gamma_f/\Gamma_n)(\langle J_b \rangle)$ it is

(25) $$\left(\frac{\sigma_f}{\sigma_{CN}}\right)_a / \left(\frac{\sigma_f}{\sigma_{CN}}\right)_b = R_{ab}$$

If protons, deuterons, α's are taken as possible projectiles, starting from ^{171}Yb up to ^{209}Bi, about 35 stable nuclei could be used as targets in experiments of the kind we discussed. Fission thresholds range from about 28 MeV for the lightest nuclei to about 18 MeV for the heaviest ones. The number of possible target nuclei increases strongly if heavy ions are used as projectiles. Published data suitable for such a kind of analysis are the ones of Khodai-Joopari[20] concerning reactions ^{209}Bi(p, f) and ^{206}Pb(α, f).

This couple of target nuclei is not the best suited to study angular momentum effects due to the high spin of ^{209}Bi(9/2). For this reason the spins of ^{210}Po created by absorption of protons in Bi and α's in Pb are not greatly different. For the same reason, however, this couple of target nuclei minimizes the errors that can be associated to the estimation of angular momentum effects. To match the excitation energies of C N for the two different incoming channels one can proceed as follows. Parameters giving the best fit to the $^{206}Pb(\alpha, f)$ excitation function, when Bohr and Wheeler formula is used[19] are estimated and at the proper α's energies, matching proton energies, the quantity

$$\sigma^{\alpha}_{f_{BF}} = \sigma^{\alpha}_{CN} \frac{\Gamma_f}{\Gamma_n}(\langle J_\alpha \rangle) \tag{26}$$

is calculated. The quantity σ^p_{CN} is subsequently obtained by means of the formula:

$$\sigma^p_{CN} = \sigma^{\alpha}_{CN} \frac{\sigma^p_{f\,exp}}{\sigma^{\alpha}_{f_{BF}}} \frac{1}{R_{p\alpha_{th}}} \tag{27}$$

(exp = experimental, th = theoretical, BF = Best fit).

The C N formation cross section for α particles can be estimated assuming $\sigma^{\alpha}_{CN} \sim \sigma^{\alpha}_R$, where σ^{α}_R is the Reaction cross section for α's as calculated by Huizenga and Igo[21].

The results one obtains are shown for proton energies ranging from about 15 to about 30 MeV in Fig. 2. The σ^p_{CN} values obtained are compared with σ^p_R, the total reaction cross section calculated by means of O M with parameters suitably choosen to fit the experimental reaction cross section data on neighbouring nuclei.

It must be noted that although σ^p_R so calculated, if erroneous, should be an overestimation of the true value[22], the σ^p_{CN} values obtained with formula (27) are greatly bigger than σ^p_R in the energy range from 15 to 23 MeV where second chance fissions are very unlike. The disagreement can hardly be ascribed to the assuption $\sigma^{\alpha}_{CN} \sim \sigma^{\alpha}_R$. It is probably due to a systematic error in the estimation of proton or α energies; i.e. either E_p is underestimated or E_α overestimated by an amount of the order of 200 keV. This assumption is substantiated by the systematic disagreement one obtains in best fit estimations of the effective fission threshold of ^{210}Po when data from the reactions $^{209}Bi(p, f)$ and $^{206}Pb(\alpha, f)$ are considered. If one takes into account that the choosen data are the best suited ones for an analysis of such a kind, the need for new and improuved experimental research in the field is apparent.

Relations (23) and (25) are very important not only for the evalua tion of the CN formation cross section for a given kind of particle once the one for another kind of particle is known, but also because provide consistency checks in systematic analyses covering this nuclear region.

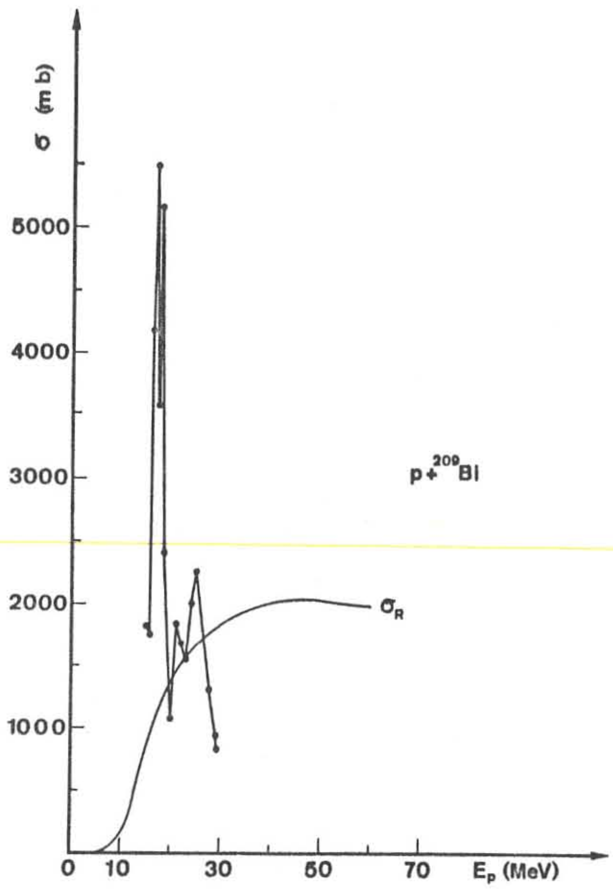

FIG. 2 - Comparison between σ^p_{CN} from Khodai-Joopari's data concerning fission of ^{210}Po and σ_R for the reaction $p+^{209}Bi$ (smooth curve).

6.3. - (p, xn) Excitation Functions. -

The excitation functions of these reactions have a characteristic shape. They increase sharply with energy above reaction threshold, go through a broad maximum and show a further decrease as the energy is increased. The high energy tail decreases much more slowly with the energy than expected on the basis of a CN picture of the process. The tail is attributed to the presence of non statistical effects. According to this hypothesis, one attempts to reproduce the experimental excitation functions by means of a sum of statistical and non statistical contributions:

$$(28) \quad \sigma_{p,xn}(E_i) \sim \sigma_{CN}(E_i)\, P_{xn}(E) + \sigma^n_{Di}(E_i) \frac{\int_0^{U_{max}} P(E,U)\, P_{(x-1)n}(U)\, dU}{\int_0^{U_{max}} P(E,U)\, dU}$$

E_i is the incident particle energy, E the intermediate system excitation energy, $P_{xn}(E)$ and $P_{(x-1)n}(E)$ the probabilities of emitting x and (x-1) neutrons from C N and residual nucleus left after non statistical neutron emission. The energy distribution P(E, U) of residual nuclei after non statistical neutron emission can be calculated according to the precompound model using formula (19).

Normalisation at the tail of excitation functions, where the statistical contribution is negligible, allows one to estimate the value of $\sigma^n_{Di}(E_i)$, the total cross section for precompound neutron emission. A recent analysis of (p, 3n) and (p, 4n) excitation functions on heavy nuclei (^{169}Tm, ^{181}Ta, ^{209}Bi)(17) gave for σ^n_{Di}, in all considered cases, at an incident energy $E_i \sim 45$ MeV a value $\sigma^n_{Di} \sim 500$ mb. At lower energies ($E_i \sim 18$ MeV) it is reported in literature(11) a value σ^n_{Di} given by $\sigma^n_{Di} \sim 0.1\ \sigma_{CN}$ (σ_{CN} is the C N formation cross section).

From this estimation one can deduce as broad limits for σ^n_{Di} the values 60 and 120 mb. The first is obtained by estrapolating at lower energies the σ_{CN} value for $E_i \sim 30$-40 MeV (see later), the second by assuming $\sigma_{CN} \sim \sigma_R$ (σ_R is the reaction cross section). How does σ^n_{Di} vary between the quoted limits? At present no clear information on this point exists. This fact obviously indicates a shortcoming of present formulation of precompound model. In fact, while this model should theoretically predict the absolute fraction of reactions leading to precompound and compound emissions, the neglect or the impossibility of taking correctly into account the depletion of states due to precompound emission does not allow to make detailed predictions on this point. On the other hand, phenomenological evidence collected during the years and concerning many kind of reactions seems to indicate quite smooth variations, with energy, of the yield of non statistical effects. For this reason, as a first approximation, a linear variation for σ^n_{Di} can be assumed. After that, one can calculate the second term of right hand side of relation (28) ($P_{xn}(E)$ and $P_{(x-1)n}(U)$ can be calculated by standard procedures) and, by a best fit to excitation functions in the maximum region, the energy variation of the C N formation cross section. The results one obtains for the reactions ^{169}Tm, ^{181}Ta, ^{209}Bi(p, xn) (x=3, 4) are summarized in Fig. 3. Fig. 4 shows a typical fit to a considered excitation function. Details of the analysis concerning the parameters entering into the calculation and the calculus procedure can be found in ref. (17). It is useful to comment some points of the treatment. It can be shown that the results one obtains depend little from the choice of the level density expressions and parameters. Taking into account that all other parameters, binding energies, pairing energies and so on, can be choosen by indipendent analyses of different experimental data, the analysis of (p, xn) excitation functions seems to be able to fix with reasonable certainty the fraction of precompound and compound effects contributing to the reaction. A point however requires some caution. The precompound contribution to the reaction appears to be great. This result is common in analysis of data that at present are in progress(10) and leads to an internal inconsistency of precompound model that in present formulation, as stressed in sect. 5, implies a preponderant compound contribution to the reaction. One can observe, however, that the precompound contribution to (p, xn) excitation functions is most important at residual nucleus excitation energies U appreciably smaller than E. Summation (19) as a consequence converges rapidly and the precompound model formulae used should allow to reproduce correctly experimental data.

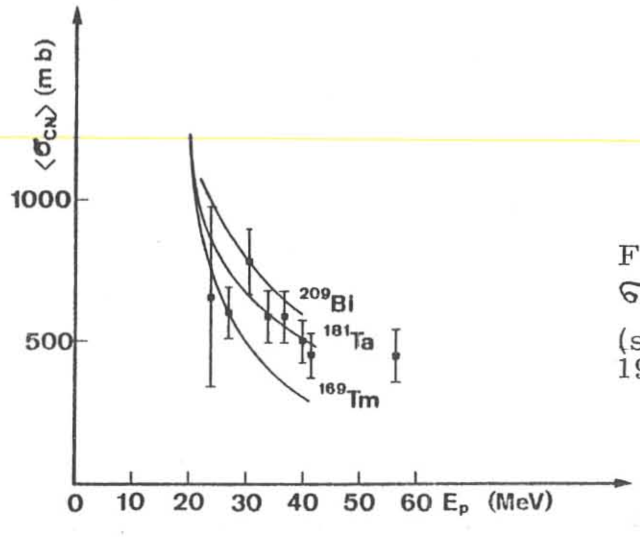

FIG. 3 - Comparison between σ^{p}_{CN} from (p, xn) reactions (smooth curves) and the $^{197}Au(p,\alpha)$ ^{194}Pt reaction (points).

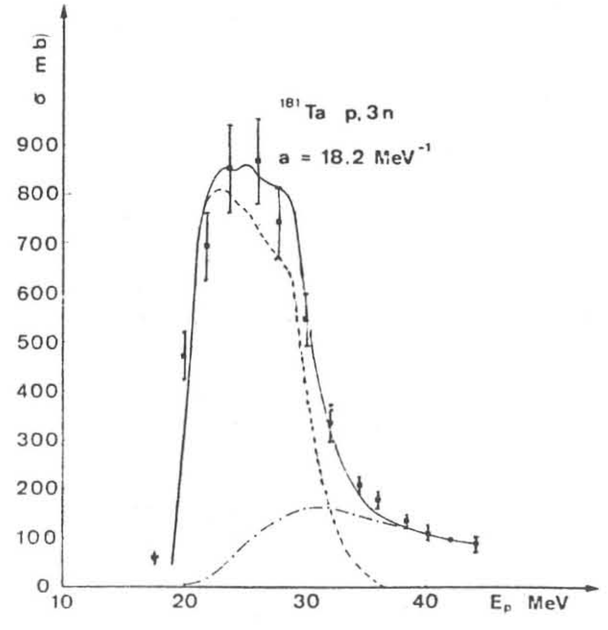

FIG. 4 - Typical fit to a (p, 3n) excitation function. The dashed and dot and dash curves give the statistical and precompound contributions to the excitation function, calculated according to formula (28).

6.4.- Statistical α particle emission in (p,α) reactions.-

Emission of charged particles in a statistical process is a very unlike phenomenon in heavy element region. The detailed study of the process constitutes, as a consequence, a severe test of theory and greatly helps in fixing the parameters entering into the calculations. α particles can be emitted, in a statistical process, both by C N and residual nuclei following either statistical or precompound neutron emission. The total statistical contribution is then given by:

(29) $$\sigma_{stat}(E,\varepsilon) = \sigma_{CN}(E,\varepsilon) + \sigma^{x}(E,\varepsilon)$$

where $\sigma_{CN}(E,\varepsilon)$ gives the yield of α's from C N before and after possible neutron emissions and $\sigma^{x}(E,\varepsilon)$ the yield of α's emitted after precompound neutron emission.

For the two contributions, at a given angle, the following expressions hold:

(30) $$\sigma_{CN}(E,\theta,\varepsilon) = \frac{\sigma^{P}_{CN}}{4\pi}\left\{\frac{\Gamma_{\alpha}(E,\varepsilon)}{\sum_i \Gamma_i(E)} + \frac{\Gamma_n(E)}{\sum_i \Gamma_i(E)}\left[\frac{\Gamma_{\alpha}(E_1,\varepsilon)}{\sum_i \Gamma_i(E_1)} + \frac{\Gamma_n(E_1)}{\sum_i \Gamma_i(E_1)}\left(\frac{\Gamma_{\alpha}(E_2,\varepsilon)}{\sum_i \Gamma_i(E_2)} + \ldots\right)\right]\right\} = \sigma^{P}_{CN}\,\alpha(E,\varepsilon)$$

(31) $$\sigma^{x}(E,\theta,\varepsilon) = \frac{\sigma^{n}_{Di}}{4\pi}\,\frac{\int_0^{U_{max}} P(E,\varepsilon')\,\alpha(U_{max}-\varepsilon',\varepsilon)\,d\varepsilon'}{\int_0^{U_{max}} P(E,\varepsilon')\,d\varepsilon'}$$

where:

(32) $$E = E_{inc} + B_{inc} - \Delta_{CN}$$

(33) $$E_j = E_{inc} + B_{inc} - \sum_{k=1}^{j} B_n(k) - \sum_{k=1}^{j} \bar{T}_n(k) - \Delta_j .$$

E_{inc} and B_{inc} are the kinetic and the binding energies of incoming particle, $B_n(k)$ and $\bar{T}_n(k)$ the binding and kinetic energies of the k-th emitted neutron, Δ_{CN} and Δ_j are the pairing energies of the CN and the j-th residual nucleus. $P(E, \mathcal{E}')$ is given by expression (19) and the Γ's are the usual level widths.

In order to estimate from experimental data the statistical contribution to the process, the theoretical spectrum $\sigma_{stat}(E, \mathcal{E})$ must be normalized to the experimental one. To obtain meaningful results the spectral region where statistical effects predominate must be known. It is useful to introduce the ratio:

$$R(\mathcal{E}) = \frac{n(\mathcal{E})}{\mathcal{E}\sigma_c(\mathcal{E})\rho(E_{max} - \mathcal{E}, 0)} \tag{34}$$

This quantity in presence of both statistical and non statistical effects has the general expression(18):

$$R(\mathcal{E}) = \text{const} + \frac{\rho^x(E_{max} - \mathcal{E}, 0)}{\rho(E_{max} - \mathcal{E}, 0)} \sum_l C_l \tag{35}$$

where

$\rho^x(E_{max} - \mathcal{E}, 0)$, $\rho^x(E_{max} - \mathcal{E}, 0) \ll \rho(E_{max} - \mathcal{E}, 0)$, is the level density of residual nucleus levels available in the non statistical process and C_l are quantities dependent from the transferred angular momentum l, θ, $\mathcal{E}$ and spectroscopic factors.

The ratio ρ^x/ρ is a quantity strongly decreasing with $E_{max} - \mathcal{E}$, that is strongly increasing with $\mathcal{E}$. From (35) it follows that statistical effects predominate at $\mathcal{E}$ values where $R(\mathcal{E})$ is nearly constant. The decomposition of backward angle spectra into statistical and non statistical contributions allows one to estimate the total statistical contribution to the reaction and through formulae (29), (30), (31) the CN formation cross section σ^p_{CN}, once σ^n_{Di} si known. In Fig. 3 the CN formation cross sections estimated at different energies from the reaction $^{197}Au(p,\alpha)\,^{194}Pt$ and the ones estimated from ^{169}Tm, ^{181}Ta, $^{209}Bi(p, xn)$ reactions (x=3, 4) are compared. σ^n_{Di} has been assumed to vary linearly starting with the value $\sim$90 mb at E_p=18 MeV up to $\sim$500 mb for E_p= =40-45 MeV, as in preceding section. In the case of values of σ^p_{CN} estimated from (p,α) reaction, the influence of assumptions concerning precompound model is minimized because $\sigma^x(E, \mathcal{E}) \ll \sigma_{CN}(E, \mathcal{E})$. The influence of level density expressions and parameters is however much stronger than in the case of (p, xn) reactions. For nuclei involved in the quoted (p, α) reaction, the Lang and Le Couteur level density expression[23] and a values slightly reduced with respect to the ones one could obtain from data corresponding to lower energies have been used. The numerical values, estimated according to model of ref. (15) are a = =17.1 (^{198}Hg), 17.2 (^{197}Hg), 17.4(^{197}Au), 17.7(^{194}Pt) MeV^{-1}. For details concerning this point see ref. (18). The agreement between different estimations of σ^{p}_{CN} in Fig. 3 is very satisfactory.

As a comment one can note that a values lower than the ones utilised in the analysis of the $^{197}Au(p,\alpha)$ ^{194}Pt reaction would decrease slightly the corresponding calculated σ^{p}_{CN} values and the agreement would be essentially unchanged (for a $\sim$ 14 MeV^{-1} for all involved nuclei, the decrease in σ^{p}_{CN} is less than 20% at $E_p \sim 40$ MeV). a values lower than the used ones seem however to be hardly justifiable by a theoretical poin of view. Values higher than the used ones would increase strongly σ^{p}_{CN} from $^{197}Au(p,\alpha)$ leading to a consistent disagreement with σ^{p}_{CN} values from (p, xn) data.

6.5.- Final remarks.-

From the analysis of (p, xn) and (p, α) data in heavy element region, the conclusion is reached that also at quite low proton energies the CN formation cross section is much lower than the reaction cross section and both σ^{n}_{Di} and $\sigma^{p}_{Di} \sim \sigma_{R} - \sigma_{CN} - \sigma^{n}_{Di}$, the cross sections for precompound neutron and proton emissions, give an important contribution to the reaction cross section. Complementary experimental information of the greatest interest would concern (p, pxn) reactions.

The results obtained were quite unespected and, if confirmed, could greatly help in understanding the details of the interaction nucleon--nucleus at intermediate energy. In this same field experiments concerning fission excitation functions seem to be most promising.

It is a pleasure to thank Prof. G. Tagliaferri and my colleagues C. Birattari, A.M. Grassi Strini, I. Iori, N. Molho, G. Strini, L. Zetta that with their work have made possible to prepare this survey.

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