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S. Bertini, G. Corleo and G. Pappalardo: REACTION MECHANISM  
IN THE  $^{27}\text{Al}(d,p)^{28}\text{Al}$  NUCLEAR REACTION. -

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ABSTRACT. -

The experimental data on the  $^{27}\text{Al}(d, p)^{28}\text{Al}$  reaction have been analyzed in the deuteron energy range from 2 to 6 MeV. The method of the cross section fluctuations analysis has been applied to test the statistical nature of the scattering amplitude, and to deduce some nuclear properties as the "coherence energy" and the nuclear radius. By performing suitable averages over the energy it has been shown that under particular approximations the contribution to the cross section of the statistical and the direct mechanism can be estimated.

INTRODUCTION. -

In the last years particular attention has been devoted to the detailed analysis of nuclear reactions which proceed through the mechanism described by the statistical model. Particular emphasis has been given to that important energy region where the levels of the intermediate nucleus overlap (continuum region). Average cross sections as well as fluctuations around average values have been theoretically predicted<sup>(1, 2)</sup> and experimentally well verified<sup>(3, 4)</sup> (x).

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(x) - Ref. (4) gives an extensive survey on this subject.

2.

It seems interesting to investigate the case in which statistical and non statistical mechanisms are present in a nuclear reaction.

As it is known, in this case the scattering amplitude can be written as (see for example T. Ericson<sup>(1)</sup>):

$$(1) \quad S_{\alpha\alpha'}(E) = S_{\alpha\alpha'}^{Di} + S_{\alpha\alpha'}^{Fl}(E)$$

where  $S_{\alpha\alpha'}^{Fl}(E)$  is a random function of  $E$  with zero average and  $S_{\alpha\alpha'}^{Di}$  is a slowly varying function of  $E$ . It follows that, when an average is performed over a suitable energy interval:

$$\langle S_{\alpha\alpha'}^{Fl}(E) \rangle = 0 \quad \text{and} \quad S_{\alpha\alpha'}^{Di} = \langle S_{\alpha\alpha'}(E) \rangle$$

The average cross section can be expressed as:

$$(2) \quad \langle \sigma_{\alpha\alpha'} \rangle = \sigma_{\alpha\alpha'}^{Di} + \langle \sigma_{\alpha\alpha'}^{Fl} \rangle$$

The term due to interference between  $S_{\alpha\alpha'}^{Di}$  and  $S_{\alpha\alpha'}^{Fl}(E)$  goes to zero because of the mentioned property of  $S_{\alpha\alpha'}^{Fl}(E)$ .

The cross section measured with good energy resolution will show fluctuations due to the presence of the  $S_{\alpha\alpha'}^{Fl}(E)$  term in the scattering amplitude (1).

An accurate study of the reaction mechanism must involve a detailed analysis of the cross section fluctuations to test the random nature of  $S_{\alpha\alpha'}^{Fl}(E)$ . In particular, energy correlation functions as well as angular cross-correlations, extracted from the experimental data, must be compared with theoretical predictions<sup>(1, 2)</sup>. Finally, the validity of (2) must be checked. This point, namely that the average cross section can be expressed as the incoherent sum of  $\sigma_{\alpha\alpha'}^{Di}$ , and  $\langle \sigma_{\alpha\alpha'}^{Fl} \rangle$  raises the problem of a consistent calculation of both the above quantities.

Elastic scattering and d, p reactions<sup>(5)</sup> at low energy (say 2 - 10 MeV) seem to be a stimulating field of research in this sense. The works of E. Gadioli<sup>(6)</sup>, M. Corti<sup>(8)</sup> and A. Gallmann<sup>(9)</sup> showed that the statistical and the stripping mechanisms are present in (d, p) reactions. In order to give further contribution in this field we analyzed the experimental data on the  $^{27}\text{Al}(d, p)^{28}\text{Al}$  reaction, recently obtained by the Saclay group in the energy interval from 2 to 6 MeV<sup>(7)</sup>. Measurements of this reaction have been performed also by<sup>(6, 8)</sup> between 1.5 and 2.3 MeV.

I. - EXPERIMENTAL DATA ON THE  $^{27}\text{Al}(d,p)^{28}\text{Al}$  REACTION. -

The ground-state Q values for this reaction is  $Q = 5.199$  MeV (see ref. (18)). Table I gives the excitation energy and the spin of the

TABLE I

Excitation energy and spin of the  $^{28}\text{Al}$  low lying levels.

$^{27}\text{Al}(d,p)^{28}\text{Al}$			
Group	Level	Exc. (MeV)	Spin
$P_{0+1}$	g. s.	0	3+
	1	0.031	2+
$P_{2+3}$	2	0.970	(0+)
	3	1.02	(3+)

$^{28}\text{Al}$  residual nucleus levels. According to the shell model the  $1d_{5/2}$   $2s_{1/2}$  configuration should give rise to the ground state doublet ( $p_{0+1}$  proton group). The spins of this doublet are well established. There is some uncertainty in the spin assignment of the first excited doublet ( $p_{2+3}$  proton group). In the  $^{27}\text{Al}(d,p)^{28}\text{Al}$  Saclay experiment<sup>(7)</sup> the energy of the incoming deuterons was varied from 2 to 3.8 MeV in steps of 50 keV, and from 4 to 6 MeV in steps of 1 MeV. The angular distributions were measured at these energies in steps of  $17^\circ$  from  $7^\circ$  to  $170^\circ$ . The overall energy resolution was of about 20 keV at 4 MeV deuterons.

The proton groups leading to the g. s. and to the first exc. state of  $^{28}\text{Al}$  were not resolved because of the small energy separation (see Table I); similarly the protons leading to the second and to the third excited level were not resolved. Fig. 1 shows a tridimensional plot of the angular distributions relative to the  $p_{0+1}$  proton group.

## II. - ANALYSIS OF THE EXPERIMENTAL DATA. -

In this section a general description of the method of analysis will be given. In particular the formulas used in our case as well as their limitation will be briefly discussed.

## II.1 - Cross section fluctuations.

The angular distributions of Fig. 1 show a fluctuating behaviour in shape and intensity when compared at different deuteron energies. This behaviour is in agreement with the statistical assumptions made in sect. I, concerning the fluctuating term of the scatter-

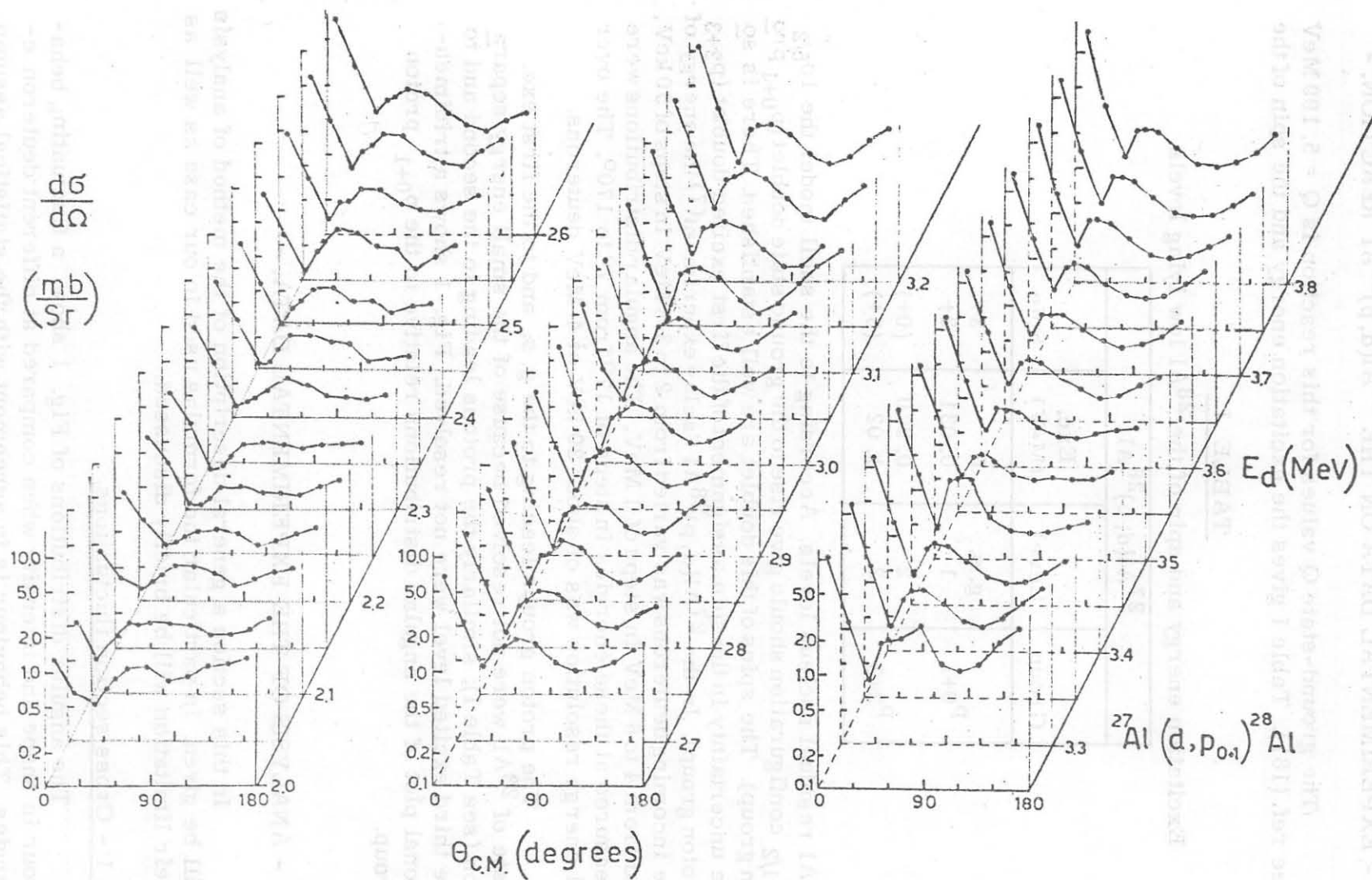


FIG. 1 - Angular distributions of the  $p_{0+1}$  proton group.

ring amplitude. These cross section fluctuations have been the object of several theoretical works. We summarize briefly the main predictions of the theory:

a) The values of each excitation function must be correlated within an energy interval of the order of  $\Gamma$  (coherence energy). The shape and the value of the "correlation function" extracted from the experimental data is theoretically predicted<sup>(1, 2)</sup>.

We used, for our analysis, an autocorrelation function defined by:

$$(3) \quad R(\xi) = \left( \frac{\overline{\sigma(E)}}{\langle \sigma(E) \rangle} - 1 \right) \left( \frac{\overline{\sigma(E+\xi)}}{\langle \sigma(E+\xi) \rangle} - 1 \right)$$

where  $\xi$  is an energy interval and  $\langle \sigma(E) \rangle$  is the average cross section. We will discuss later the proper choice of  $\langle \sigma(E) \rangle$ .

The theoretical value of  $R(\xi)$ , for the case of integrated cross section, is given by<sup>(1)</sup>:

$$(4) \quad R_{st}(\xi) = \frac{\Gamma^2}{\Gamma^2 + \xi^2} \frac{\sum_{ss'} \sum_{Jl'l'} \frac{T^2}{\alpha_{ls}} \frac{T^2}{\alpha_{l's'}}}{\left[ \sum_{ss'} \sum_{Jl'l'} \frac{T}{\alpha_{ls}} \frac{T}{\alpha_{l's'}} \right]^2}$$

where  $T_{\alpha_{ls}}$  are the usual penetrabilities, and  $\Gamma$  is the "coherence energy".

A more elaborated expression has been obtained for the case of a differential cross section:

$$(5) \quad R_{st}(0) = \frac{\sum_{c_1 c'_1 c_2 c'_2} W_{c_1 c'_1 c_2 c'_2}^2(\theta) \frac{T_{c_1} T_{c'_1} T_{c_2} T_{c'_2}}{\sum_{c''_1} T_{c''_1} \sum_{c''_2} T_{c''_2}}}{\left| \sum_{cc'} W_{cc'cc'}(\theta) \frac{T_c T_{c'}}{\sum_{c''} T_{c''}} \right|^2}$$

here  $c$  stands for a channel index  $c \equiv (\alpha_{ls})$ .

The expressions (4) and (5) are valid for the case of a "purely statistical reaction" and under the same conditions of validity of eq. (14) (see sect. II. 3). The further assumption is made that  $\Gamma$  is independent of the spin of the intermediate nucleus levels.

It has been shown<sup>(2)</sup> that if a direct process contributes to the reaction, expressions (4) and (5) are modified according to:

$$(6) \quad R_{st+Di}(\xi) = R_{st}(\xi) \left[ 1 - y_D^2 \right]$$



6.

where

$$(7) \quad y_D = \frac{\sigma_{Di}}{\sigma_{Di} + \langle \sigma_{fl} \rangle}$$

Eq. (6) is valid in the approximation that the term  $y_D$  is independent of the reaction channel.

b) A correlation is expected also between differential excitation functions taken at neighbouring angles  $\theta$  and  $\theta'$ . The value of the cross correlation coefficient is theoretically predicted<sup>(2)</sup>.

We used an angular cross-correlation coefficient defined by

$$(8) \quad C(\theta, \theta') = \left( \frac{\sigma_{\theta'}(E)}{\langle \sigma_{\theta'}(E) \rangle} - 1 \right) \left( \frac{\sigma_{\theta}(E)}{\langle \sigma_{\theta}(E) \rangle} - 1 \right) \frac{1}{\sqrt{C(\theta, \theta)C(\theta', \theta')}}}$$

giving the experimental cross correlation between two differential excitation functions taken at angles  $\theta$  and  $\theta'$ . The factor

$$[C(\theta, \theta)C(\theta', \theta')]^{-1/2}$$

is introduced to normalize to the amplitude of the fluctuations.

According to the "surface emission" model proposed by Brink and Stephen<sup>(2)</sup> the coefficient  $C(\theta, \theta')$ , depends on the difference  $\Delta\theta = |\theta - \theta'|$  irrespectively of the  $\theta$  and  $\theta'$  values and it is related to the nuclear radius  $R$  through the expression

$$(9) \quad C(\Delta\theta) = [j_0(kR\Delta\theta)]^2$$

where  $j_0$  is the spherical Bessel function of zero order and  $k$  is the wave number of the emitted particle.

## II. 2. - Analysis of the fluctuations on the $^{27}\text{Al}(d, p)^{28}\text{Al}$ reaction.

In our case the average cross section has been obtained by using the varying mean method described in ref. (15, 16, 17). The value  $\Delta_{av} = 850$  keV of the averaging interval has been found to be a suitable one for all our cases.

Fig. 2 shows a typical plot of  $R(\theta)$  vs. the averaging interval, obtained from the  $p_{0+1}$  and  $p_{2+3}$  integrated excitation functions.

To obtain the coherence energy  $\Gamma$ , we constructed  $R(\xi)$  from the experimental excitation functions integrated from  $30^\circ$  to  $150^\circ$ . The data of Gadioli<sup>(6)</sup> have been used in this analysis to extend the energy range of data. In Fig. 3 the quantity  $R(\xi)$  is shown as a function of  $\xi$ , for  $p_{0+1}$  and  $p_{2+3}$  proton groups. The obtained values

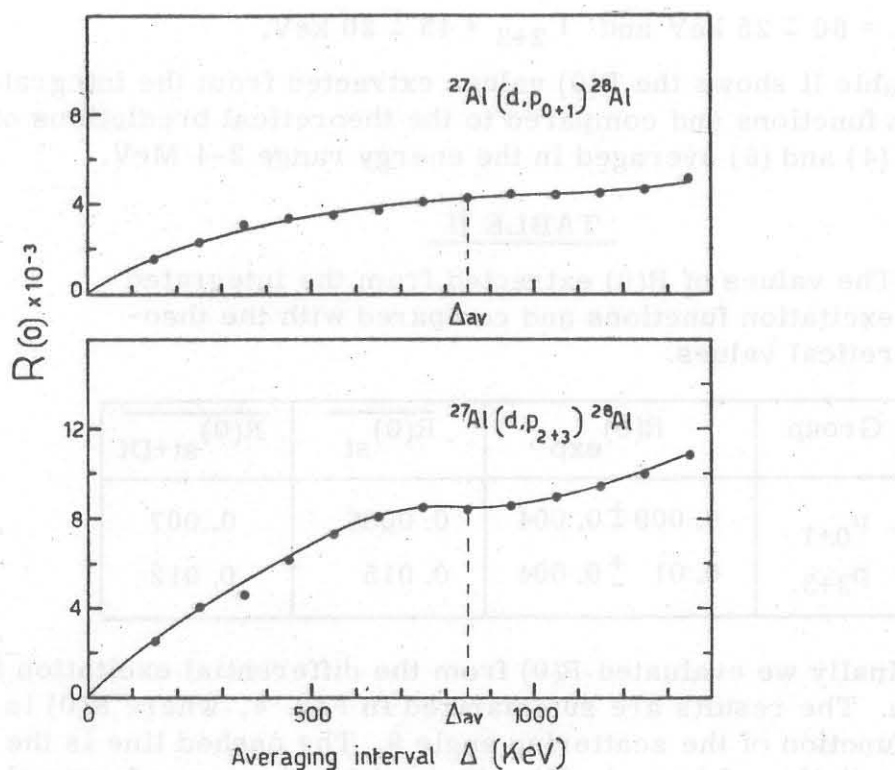


FIG. 2 - The quantity  $R_{\Delta}(0)$  plotted in function of the averaging interval  $\Delta$ . The data refer to the  $p_{0+1}$  and  $p_{2+3}$  integrated excitation functions.

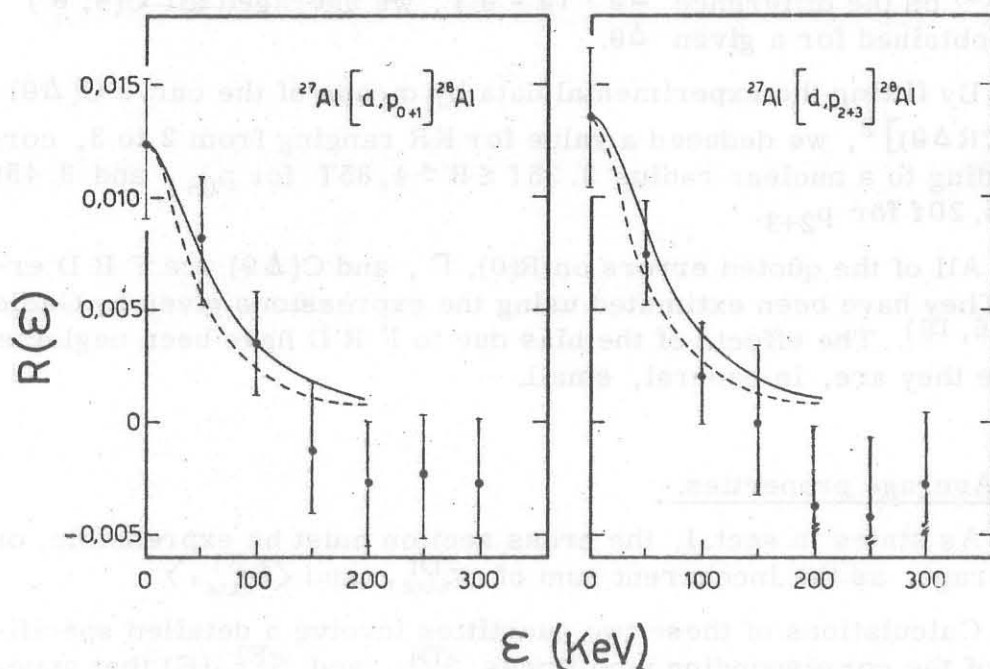


FIG. 3 - Energy auto-correlation functions taken from the  $p_{0+1}$  and  $p_{2+3}$  excitation functions integrated from  $30^{\circ}$  to  $150^{\circ}$ .



8.

are  $\Gamma_{0+1} = 60 \pm 25$  keV and  $\Gamma_{2+3} = 45 \pm 20$  keV.

Table II shows the  $R(0)$  values extracted from the integrated excitation functions and compared to the theoretical predictions of formulas (4) and (6) averaged in the energy range 2-4 MeV.

TABLE II

The values of  $R(0)$  extracted from the integrated excitation functions and compared with the theoretical values.

Group	$R(0)_{\text{exp}}$	$\overline{R(0)}_{\text{st}}$	$\overline{R(0)}_{\text{st+Di}}$
$p_{0+1}$	$0.009 \pm 0.004$	0.0086	0.007
$p_{2+3}$	$0.01 \pm 0.004$	0.015	0.012

Finally we evaluated  $R(0)$  from the differential excitation functions data. The results are summarized in Fig. 4, where  $R(0)$  is plotted as a function of the scattering angle  $\theta$ . The dashed line is the theoretical prediction of formula (5), obtained for the case of a purely statistical reaction. Fig. 5 shows the results of the angular correlation analysis for the  $p_{0+1}$  and  $p_{2+3}$  proton groups respectively. According to the fact that the cross-correlation coefficient  $C(\theta, \theta')$  depends mainly<sup>(2)</sup> on the difference  $\Delta\theta = |\theta - \theta'|$ , we averaged all  $C(\theta, \theta')$  values obtained for a given  $\Delta\theta$ .

By fitting the experimental data by means of the curve  $C(\Delta\theta) = [j_0(KR\Delta\theta)]^2$ , we deduced a value for  $KR$  ranging from 2 to 3, corresponding to a nuclear radius  $3.25 \text{ f} \leq R \leq 4.85 \text{ f}$  for  $p_{0+1}$  and  $3.45 \text{ f} \leq R \leq 5.20 \text{ f}$  for  $p_{2+3}$ .

All of the quoted errors on  $R(0)$ ,  $\Gamma$ , and  $C(\Delta\theta)$  are F R D errors. They have been estimated using the expressions given by Gadioli et al.<sup>(16, 19)</sup>. The effects of the bias due to F R D have been neglected because they are, in general, small.

### II. 3. - Average properties.

As states in sect. I, the cross section must be expressible, on the average, as the incoherent sum of  $\sigma_{\alpha\alpha'}^{Di}$  and  $\langle \sigma_{\alpha\alpha'}^{F1} \rangle$ .

Calculations of these two quantities involve a detailed specification of the corresponding amplitudes  $S_{\alpha\alpha'}^{Di}$  and  $S_{\alpha\alpha'}^{F1}(E)$  that appear in the relation (1).

Let us discuss first the calculation of  $\langle \sigma_{\alpha\alpha'}^{F1} \rangle$ .

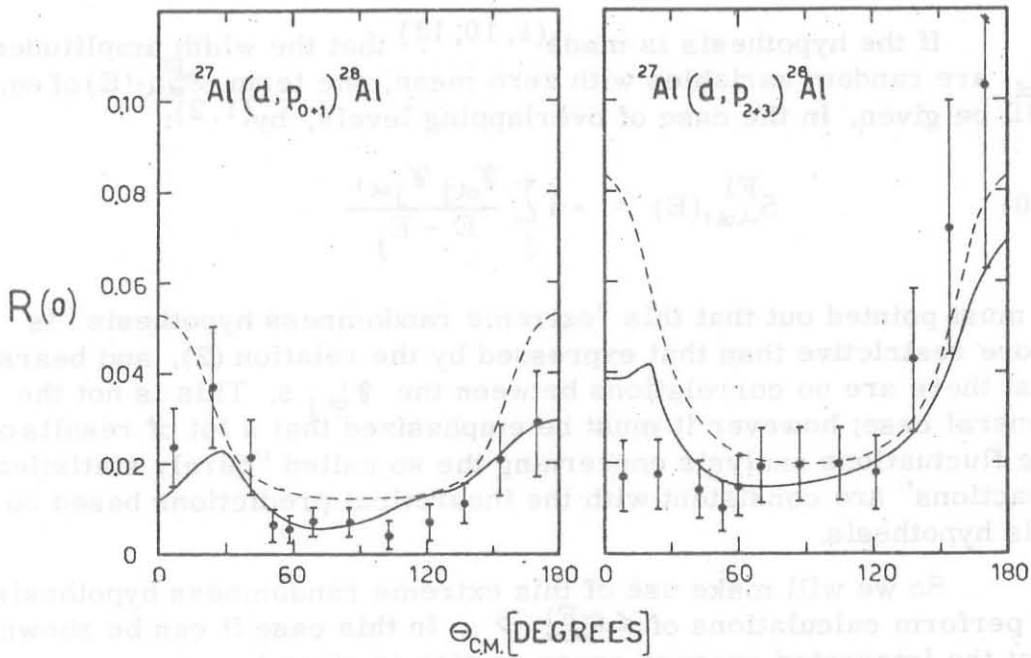


FIG. 4 - The relative mean square deviation  $R(\theta)$  as a function of the centre of mass angle  $\theta$ . Dashed and full lines are the predictions of formulas (5) and (6) respectively.

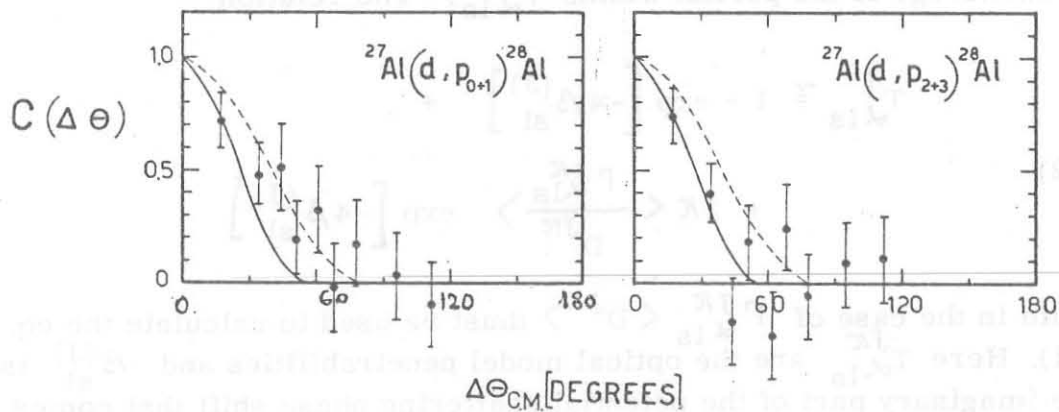


FIG. 5 - Angular correlation function compared to the predictions of formula (9); with  $KR = 2$  (dotted line) and  $KR = 3$  (full line).

10.

If the hypothesis is made<sup>(1, 10, 13)</sup> that the width amplitudes  $\gamma_{\alpha j}$  are random variables with zero mean, the term  $S_{\alpha\alpha'}^{F1}(E)$  of eq. (1) will be given, in the case of overlapping levels, by<sup>(1, 2)</sup>:

$$(10) \quad S_{\alpha\alpha'}^{F1}(E) = -i \sum_j \frac{\gamma_{\alpha j} \gamma_{j\alpha'}}{E - E_j}$$

It must be pointed out that this "extreme randomness hypothesis" is more restrictive than that expressed by the relation (2), and bears that there are no correlations between the  $\gamma_{\alpha j}$  s. This is not the general case; however it must be emphasized that a lot of results of the fluctuations analysis concerning the so called "purely statistical reactions" are consistent with the theoretical predictions based on this hypothesis.

So we will make use of this extreme randomness hypothesis to perform calculations of  $\langle \sigma_{\alpha\alpha'}^{F1} \rangle$ . In this case it can be shown<sup>(1, 10)</sup> that the integrated average cross section is given by:

$$(11) \quad \langle \sigma_{\alpha\alpha'}^{F1} \rangle = 2\pi^2 \lambda^2 \sum \frac{2J+1}{(2I+1)(2i+1)} \frac{\langle \Gamma_{\alpha 1s}^{J\pi} \rangle \langle \Gamma_{\alpha' 1's'}^{J\pi} \rangle}{D^{J\pi} \langle \Gamma^{J\pi} \rangle}$$

where  $D^{J\pi}$  is the average spacing of the intermediate nucleus levels having spin  $J$  and parity  $\pi$ .  $\Gamma_{\alpha 1s}^{J\pi}$  are the partial widths, and  $\Gamma^{J\pi}$  is the total width of the levels. Unfortunately the direct calculation of eq. (11) is in general not possible, because of the very scanty knowledge of the partial widths  $\Gamma_{\alpha 1s}^{J\pi}$ . The relation<sup>(11)</sup>

$$(12) \quad T_{\alpha 1s}^J \approx 1 - \exp \left[ -4\beta_{sl}^{(J)} \right] + 2\pi \left\langle \frac{\Gamma_{\alpha 1s}^{J\pi}}{D^{J\pi}} \right\rangle \exp \left[ -4\beta_{sl}^{(J)} \right]$$

valid in the case of  $\Gamma_{\alpha 1s}^{J\pi} \langle D^{J\pi} \rangle$  must be used to calculate the eq. (11). Here  $T_{\alpha 1s}^{J\pi}$  are the optical model penetrabilities and  $\beta_{sl}^{(J)}$  is the imaginary part of the potential scattering phase shift that comes out from the "direct absorption" through the channel  $\alpha 1s$ . In the hypothesis that the direct contribution in the whole absorption process is small, the  $\beta_{sl}^{(J)}$  becomes negligible and the eq. (14) can be written in the usual form

$$(13) \quad T_{\alpha 1s}^{(J)} \approx 2\pi \left\langle \frac{\Gamma_{\alpha 1s}^{J\pi}}{D^{J\pi}} \right\rangle$$

Eq. (11) becomes<sup>(10)</sup>:

$$(14) \quad \langle \sigma_{\alpha\alpha'}^{F1} \rangle = \pi \lambda^2 \sum \frac{2J+1}{(2I+1)(2i+1)} \frac{T_{\alpha 1s} T_{\alpha' 1's'}}{\sum T_{\alpha'' 1'' s''}}$$

The differential cross section will be given by

$$(15) \quad \left\langle \frac{d\sigma_{\alpha\alpha'}^{F1}}{d\Omega} \right\rangle = \frac{\lambda^2}{4} \sum \frac{(-)^{s-s'}}{(2I+1)(2i+1)} \frac{T_{\alpha 1s} T_{\alpha' 1's'}}{\sum T_{\alpha'' 1'' s''}} \times \\ \times \bar{Z}(1J1J; sL) \bar{Z}(1'J1'J; s'L) P_L(\cos \theta).$$

As in refs. (6, 8, 12) we substituted the sum in the denominators of (14) and (15) by an integral over the level densities of the residual nuclei. That is

$$(16) \quad \sum T_{\alpha'' 1'' s''} = \\ = \sum_{\nu} \sum_{l_{\nu}=0}^{\infty} \sum_{s_{\nu}} \sum_{J_{\nu}} \int_0^{E_{\nu}^{\max}} T_{1\nu}(E_{\nu}) \delta(E_{\nu}^{\pm}, J_{\nu}) dE_{\nu} = g(J)$$

where  $\nu$  specifies a mode of decay of the intermediate nucleus,  $l_{\nu}$  is the orbital angular momentum,  $S_{\nu}$  is the channel spin,  $J_{\nu}$  is the spin of the residual nucleus levels, and  $\rho(E_{\nu}^{\pm}, J_{\nu})$  is the density of residual nucleus levels with spin  $J_{\nu}$  at the excitation energy  $E_{\nu}^{\pm}$ . For the discussion of the involved parameters see sect. II. 4.

In order to evaluate the D. I. contribution to the cross section we performed DWBA calculations assuming a stripping mechanism. However it must be taken in mind that only in a first approximation<sup>(27)</sup> the DWBA amplitude coincides with the average amplitude  $\langle S_{\alpha\alpha'}(E) \rangle$ .

#### II. 4. - Analysis of the average properties in $^{27}\text{Al}(d, p)^{28}\text{Al}$ reaction.

The expressions (14) and (15) of sect. II. 3 have been used to calculate  $\langle \sigma_{\alpha\alpha'}^{F1} \rangle$ , using the optical model penetrabilities given in refs. (20-23). To compute the integral (16) in the denominators of (14) and (15) we used the level density formula given by Lang and Le Couter<sup>(24)</sup>. The values of the involved parameters are the same

as used in ref. (6) for the calculations concerning the  $^{27}\text{Al}(d, \alpha)^{25}\text{Mg}$  reaction: in fact the intermediate nucleus  $^{29}\text{Si}$  is the same for both  $^{27}\text{Al}(d, \alpha)^{25}\text{Mg}$  and  $^{27}\text{Al}(d, p)^{28}\text{Al}$  reactions. In this way we computed  $\langle \sigma_{\alpha, \alpha}^{F1} \rangle$  for the  $^{27}\text{Al}(d, p)^{28}\text{Al}$  reaction in its absolute value without any adjustable parameter. The results of the calculations concerning the integrated excitation functions as well as the angular distributions for both  $p_{0+1}$  and  $p_{2+3}$  proton groups are reported in figs. 6-9 (dotted lines).

The DWBA calculations have been performed for both  $p_{0+1}$  and  $p_{2+3}$  proton groups at different deuteron energies from 2 to 6 MeV. The parameters involved in these calculations are summarized in Table III. The deuteron and proton parameters are those suggested by Melkanoff<sup>(25)</sup> and by Perey<sup>(26)</sup> respectively.

TABLE III

Optical model parameters used in DWBA calculation of the  $^{27}\text{Al}(d, p)^{28}\text{Al}$  reaction.

Particle	Real well				Imaginary well		
	V(MeV)	$r_0$ (fm)	a(fm)	$r_c$	W(MeV)	$r_0$ (fm)	a(fm)
deuteron	60	1.5	0.55	1.5	16	1.5	0.55
proton	52	1.25	0.65	1.25	6.35	1.25	0.47

We normalized the DWBA data to the difference between the experimental average cross section curve and the theoretical value of  $\langle \sigma_{\alpha, \alpha}^{F1} \rangle$ . In the case of the  $p_{0+1}$  group for which only the  $l_n=0$  term contributes significantly, the normalization has been performed over the integrated excitation function. For the  $p_{2+3}$  group  $l_n=0$  and  $l_n=2$  contribute. Then the normalization has been made using the angular distribution at 4 MeV. Figs. 6 and 7 show the results we obtained for the case of integrated cross section. The full line represents the sum of the  $\langle \sigma_{\alpha, \alpha}^{F1} \rangle$  and  $\sigma^{\text{DWBA}}$ . Figs. 8 and 9 shows the experimental average angular distributions compared with the theoretical calculations for different energies.

From the data of the Figs. 6, 7, 8, 9, the ratio

$$Y_D = \frac{\sigma_{Di}}{\sigma_{Di} + \langle \sigma_{\alpha, \alpha}^{F1} \rangle}$$

has been estimated at various angles and energies. This value has been introduced in the formula (6) in order to calculate the damping on the fluctuations amplitude due to the presence of a direct effect. The results are summarized in Table II and in the Fig. 4.

As it can be seen there is a general improvement on the agree

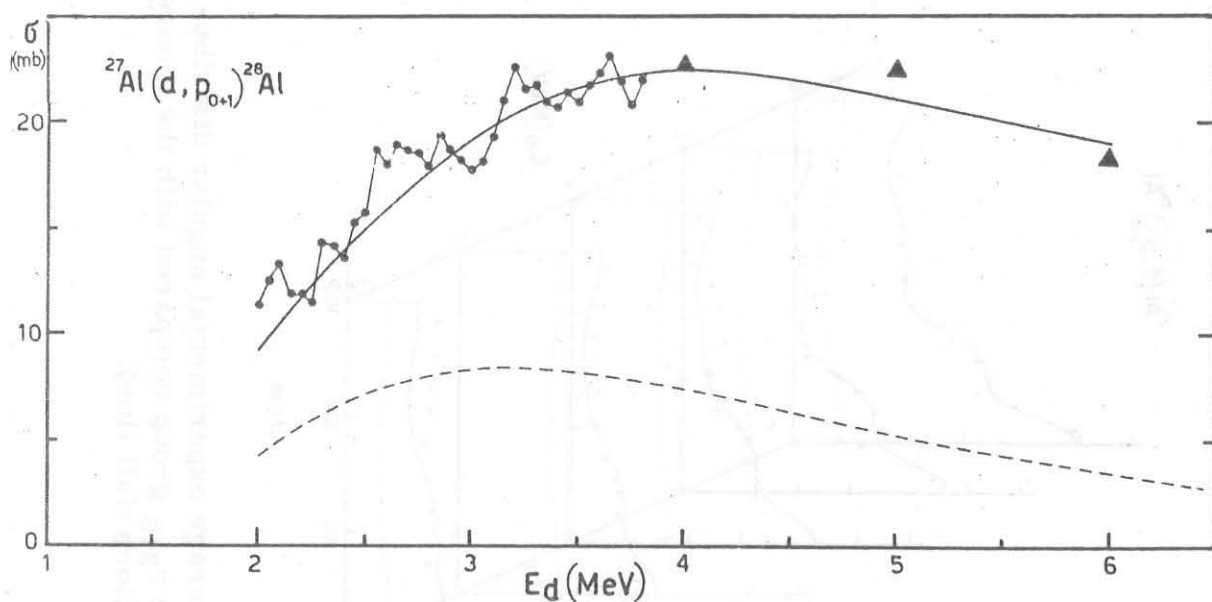


FIG. 6 - Average integrated excitation function of the  $p_{0+1}$  proton group compared with the theoretical predictions (full line). Dotted line is the  $\langle \sigma_{\alpha\alpha'}^{F1} \rangle$  calculation.

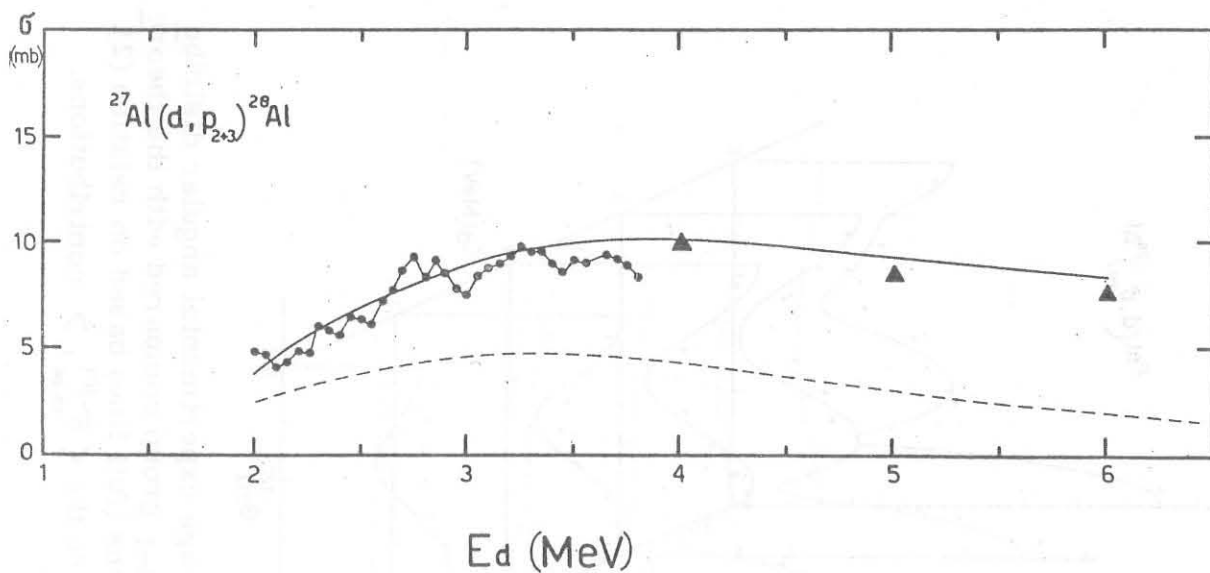


FIG. 7 - Average integrated excitation function of the  $p_{2+3}$  proton group. Full line is the theoretical prediction based on formula (2). Dotted line is the  $\langle \sigma_{\alpha\alpha'}^{F1} \rangle$  contribution.

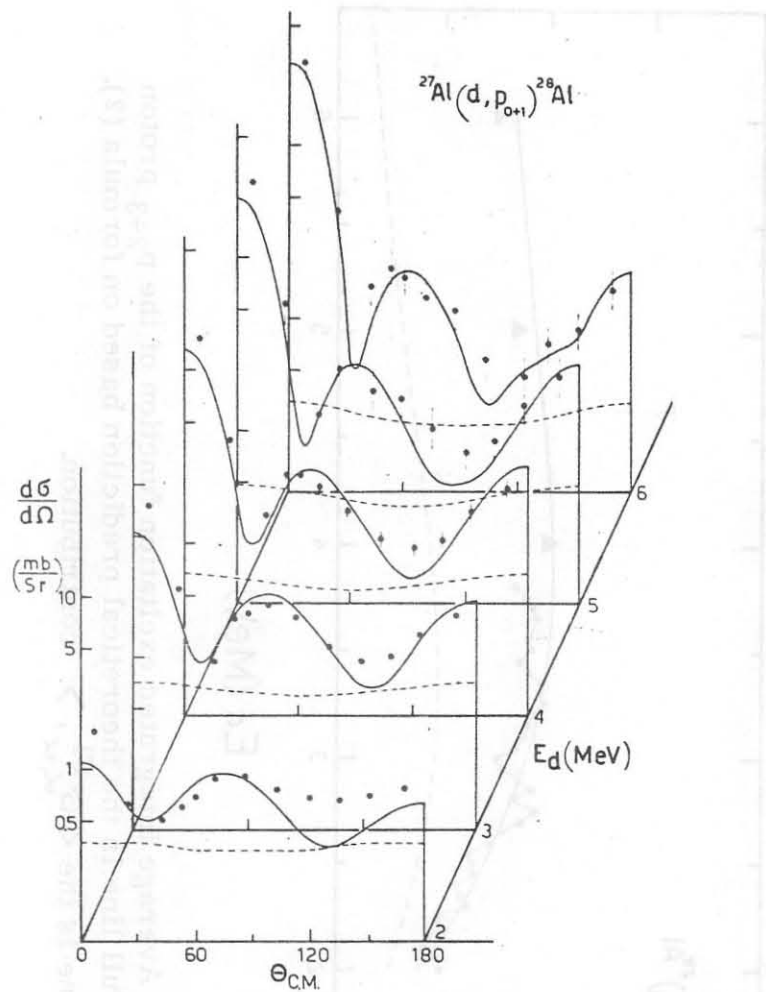


FIG. 8 - Average experimental angular distributions of the  $p_{0+1}$  group compared with the theoretical predictions (full line) based on relation (2). Dotted lines are the  $\langle \sigma_{\alpha_1}^{F1} \rangle$  contributions.

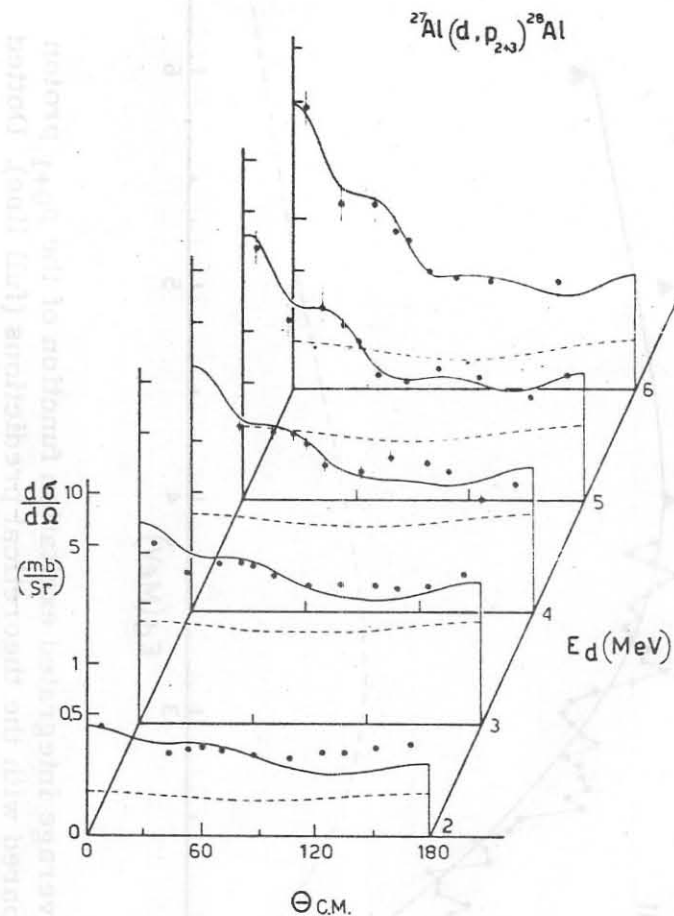


FIG. 9 - Average experimental angular distributions for the  $p_{2+3}$  group compared with the theoretical predictions (full line).



ment between experimental data and theory when the direct effect is taken into account.

### III. - DISCUSSION AND CONCLUSIONS. -

The results of the analysis of the cross section fluctuations are quite consistent with the predictions of the statistical model when the D. I. contribution is taken into account. This general agreement can be considered as a test of the hypothesis made in sect. I concerning the random nature of the scattering amplitude. As a natural consequence of such randomness, the direct process and the statistical one must be separable on the average. This separation has been tested in sect. II. The results summarized in figs. 6-9 show an over all agreement between the theoretical predictions based on the incoherent sum of  $\sigma_{\alpha\alpha}^{Di} + \langle \sigma_{\alpha\alpha} \rangle$ , and the energy averaged experimental data.

As far as the  $\langle \sigma^{F1} \rangle$  calculations are concerned we recall that we have no free parameters at all. In fact the parameters are fixed by the  $^{27}\text{Al}(d, \alpha)^{25}\text{Mg}^{(6)}$  reaction for which the statistical mechanism dominates.

Concerning the DWBA calculations we cannot dispense with the usual normalization factor (spectroscopic factor). The analysis of the  $^{27}\text{Al}(d, p)^{28}\text{Al}$  reaction shows that this factor is constant in the energy range 2-6 MeV, as it is expected.

All this supports the fact that the mechanism separation can be attempted in a more than qualitative way. However it must be remembered that besides the "extreme randomness" hypothesis other assumptions have been made in the course of this work. They can be summarized as follows :

- a) It has been assumed that the "direct absorption" is small in general. This assumption has to be verified. A more careful analysis should be performed to test it.
- b) It has been assumed that the DWBA amplitude gives a good approximation to the direct amplitude. But, as it is known, caution must be used to establish under which physical conditions such approximation is valid<sup>(27)</sup>.
- c) It must be noticed finally that in spite of the fact that  $^{27}\text{Al}$  is deformed nucleus the DWBA calculations performed in the approximation of the spherical shell model give a rather good account in all our cases.

## ACKNOWLEDGEMENTS. -

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