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G. Gambarini, I. Iori and N. Molho: PWBA ANALYSIS OF (p,α) REACTIONS WITH INTERFERING MECHANISMS. -

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INTRODUCTION. -

Different arguments supporting the interest of (p, α) reactions have been already given in a previous paper[1] in which (p, α) reactions on light nuclei at 38 MeV incident energy have been studied.

The analysis has been done in PWBA taking into account four possible direct reaction mechanisms[2]. Given the results obtained, the necessity of taking into account the interferential terms has been suggested. Here we will outline how the interference has been evaluated starting from simple models.

In Section I the expressions of the (p, α) differential cross sections for the pick-up (PU), knock-out (KO), and heavy particle pick-up (HPPU) mechanisms will be explicitely developed as an extension of a formalism already known[3,4,5].

Moreover the expression of the differential cross section for the heavy particle knock-out (HPKO) mechanism will be given.

On the ground that the four mechanisms take a part at the same time in the reaction, the differential cross section will be estimated taking into account also the possible interferential terms.

In section II the basical assumptions and approximations will be discussed.

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In Section III computational details of the method used will be given.

In Section IV the results will be reported.

SECTION I. -

We report at first a list of the symbols adopted in the formulas:

M_x x particle mass ,

μ_i, μ_f reduced masses of the system in the initial and final states,

$\vec{k}_p, \vec{k}_\alpha$ momenta of the incident and emitted particles,

μ_{xy} reduced mass of the pair xy (x and y indicate a particle) ,

B_{xy} binding energy of x+y ,

β_{xy} $(2\mu_{xy} B_{xy}/\hbar^2)^{1/2}$,

θ_l reduced width in Teichman Wigner units ,

I_T, I_R, I_C spins of target, residual nucleus and core, with z components m_T, m_R, m_C ,

l_p, l_t, l_α orbital angular momenta, with z components m_p, m_t, m_α,

j_p, j_t total angular momenta with z components μ_p, μ_t ,

ν_p, ν_t z components of intrinsic spins ,

R_p, R_t, R_α cut-off radii ,

O_t, O_α overlap integrals

$\vec{r}_x$ CM coordinates of particle x ,

$\vec{r}_{xy}$ relative coordinates of x and y particles ,

$\vec{r}_i, \vec{r}_f$ coordinates of the light particles with respect to the centers of mass of T and R ,

$\vec{\Omega}_r$ angular coordinates of $\vec{r}$ with respect to the reference system ,

$\vec{\xi}_N$ internal coordinates of nucleus N ,

$\varphi_x^N(\vec{\xi}_x)$ internal wave function of cluster x in the nucleus N.

I. 1. - PU differential cross-section.

The reaction proceeds as

$$T+p = (R+t)+p = R+(p+t) = R+\alpha$$

T consists of a nucleus R bound to a triton considered as a particle of mass 3, charge 1 and spin 1/2 without internal structure.

The effective interaction acts between the proton and the triton.

The differential cross-section is given by :

$$(1) \qquad \frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_\alpha}{k_p} \frac{1}{2(2I_T+1)} \sum_{\nu_p m_T m_R} \left| T^{PU}_{\nu_p m_T m_R} \right|^2$$

where $T^{PU}_{\nu_p m_T m_R}$ is the transition matrix element :

$$T^{PU}_{\nu_p m_T m_R} = \langle \psi_f(\vec{r}_R, \vec{r}_\alpha, \vec{\xi}_\alpha, \vec{\xi}_R) \left| V(\vec{r}_p - \vec{r}_t) \right| \psi_i(\vec{r}_T, \vec{r}_p, \vec{\xi}_T) \rangle$$

ψ_f is the final state wave function ; ψ_i is the initial state wave function; $V(\vec{r}_p - \vec{r}_t)$ is the effective interaction potential.

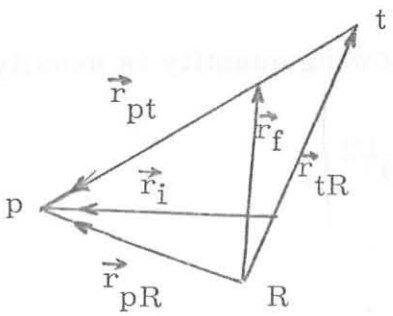
In plane wave approximation one may write :

$$\psi_f = e^{i\vec{k}_\alpha \cdot \vec{r}_f} \varphi_R(\vec{\xi}_R) \phi_\alpha(\vec{r}_p - \vec{r}_t) \varphi^\alpha_t(\vec{\xi}_t) ,$$

$$\psi_i = e^{i\vec{k}_p \cdot \vec{r}_i} \varphi_T(\vec{r}_R, \vec{r}_t, \vec{\xi}_R) .$$

$\phi_\alpha(\vec{r}_p - \vec{r}_t)$ is the internal wave function of the α particle considered as a triton plus a proton.

From the position vector diagram



one gets :

$$\vec{r}_i = (M_R/M_T)\vec{r}_{tR} + \vec{r}_{pt}$$

$$\vec{r}_f = (M_p/M_\alpha)\vec{r}_{pt} + \vec{r}_{tR} .$$

The target nucleus wave function may be written :

$$\varphi_T(\vec{r}_R, \vec{r}_t, \vec{\xi}) = \phi_T(\vec{r}_R - \vec{r}_t)\ \varphi_R^T(\vec{\xi}_R)\ \varphi_t^T(\vec{\xi}_t) .$$

$\phi_T(\vec{r}_R - \vec{r}_t)$ is the wave function of the target nucleus considered as a triton bound to the residual nucleus.

If the interaction between the triton and the residual nucleus is represented with a trirectangular potential well (the Coulomb interaction is neglected), having assumed a jj spin coupling one writes :

$$\varphi_T = \sum_{\nu_p m_T m_R} \left\langle l_t \tfrac{1}{2} m_t \nu_t \middle| j_t \mu_t \right\rangle \left\langle j_t I_R \mu_t m_R \middle| I_T m_T \right\rangle \quad x$$

$$x \left(\frac{3}{R_t^3}\right)^{1/2} \theta_{l_t} \frac{h_{l_t}^{(1)}(i\beta_{tR} r_{tR})}{h_{l_t}^{(1)}(i\beta_{tR} R_t)} Y_{l_t m_t}(\vec{\Omega}_{r_{tR}})\ \varphi_R^T(\vec{\xi}_R)\ \varphi_t^T(\vec{\xi}_t) .$$

Then the transition matrix element may be written

$$T^{PU}_{\nu_p m_T m_R} = \left\langle \tfrac{1}{2} \tfrac{1}{2} \nu_p \nu_t \middle| 0\ 0 \right\rangle \left\langle l_t \tfrac{1}{2} m_t \nu_t \middle| j_t \mu_t \right\rangle \left\langle j_t I_R \mu_t m_R \middle| I_T m_T \right\rangle x$$

$$x \left(\frac{3}{R_t^3}\right)^{1/2} \theta_{l_t} O_t \int \left(e^{-i\vec{k}_\alpha \cdot \vec{r}_f}\ \varphi_\alpha^*(\vec{r}_{pt})\ V(\vec{r}_{pt})\ e^{i\vec{k}_p \cdot \vec{r}_i} \right. \quad x$$

$$\left. x \frac{h_{l_t}^{(1)}(i\beta_{tR} r_{tR})}{h_{l_t}^{(1)}(i\beta_{tR} R_t)} Y_{l_t m_t}(\vec{\Omega}_{r_{tR}}) \right) d\vec{r}_{pt}\ d\vec{r}_{tR} ,$$

where O_t is the degree of overlapping between the internal wave functions of the triton when bound in the α particle and in the target nucleus.

In zero range approximation the following quantity is usually introduced[6] :

$$D_o^2 = 8\pi(\hbar^2/2\mu_{pt})^{3/2} (B_{pt})^{1/2} ,$$

so that

$$T^{PU}_{\nu_p m_T m_R} = \left\langle \frac{1}{2}\ \frac{1}{2}\ \nu_p \nu_t \middle| 0\,0 \right\rangle \left\langle l_t \tfrac{1}{2} m_t \nu_t \middle| j_t \mu_t \right\rangle \quad x \quad \Big\rangle \ x$$

(2)

$$x \left\langle j_t\, I_R \mu_t m_R \middle| I_T m_T \right\rangle D_o \left(\frac{3}{R_t^3}\right)^{1/2} \theta_{l_t} O_t \mathcal{J}_{l_t} \ ; \qquad x$$

when β_{tR} is real, $\mathcal{J}_{l_t}$ is given by

$$\mathcal{J}_{l_t} = \int_{Rt}^{\infty} e^{i\vec{Q}\cdot\vec{r}} \frac{h_{l_t}^{(1)}(i\beta_{tR} r)}{h_{l_t}^{(1)}(i\beta_{tR} R_t)} Y_{l_t m_t}(\vec{\Omega}_r)\, d\vec{r} = \qquad Q),$$

(3)

$$= 4\pi i^{l_t} Y_{l_t m_t}(\vec{\Omega}_Q) \frac{R_t}{Q^2 + \beta_{tR}^2} J_{l_t}(Q, \beta_{tR}, R_t) \qquad \overline{C^R \alpha})$$

with

$$J_l(Q, \beta, R) = QR j_{l-1}(QR) + C_l(\beta R) j_l(QR) ,$$

$$C_l(\beta R) = -i\beta R \frac{h_{l-1}^{(1)}(i\beta R)}{h_l^{(1)}(i\beta R)}$$

$$\vec{Q} = \vec{k}_p (M_R/M_T) - \vec{k}_\alpha ,$$

$j_l(x)$ are spherical Bessel functions of order l and $h_l^{(1)}(x)$ are spherical Hankel functions of the first kind and order l.

I.2. - KO differential cross - section.

The reaction scheme is

$$T + p = (C + \alpha) + p = (C + p) + \alpha = R + \alpha$$

that is the target nucleus consists of a core bound to an α particle considered as a single particle of mass 4, charge 2 and spin 0 without internal structure.

The residual nucleus consists of a proton bound to the same core.

SECTION III. -

Taking into account the four mechanisms, the differential cross-section has the following expression :

$$\frac{d\sigma}{d\Omega} = K \sum_{\nu_p m_T m_R} \left| N_1 (T^{PU} + T^{HPKO}) + N_2 (T^{KO} + T^{HPPU}) \right|^2$$

with

$$K = \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} \frac{1}{2(2I_T + 1)}$$

The quantities T are the matrix elements (2), (8), (11), (12), N_1 and N_2 are coefficients proportional to $\theta_{l_t} O_t$ and $\theta_{l_p} \theta_{l_\alpha} O_\alpha$ respectively. These quantities depend only on the assumed nuclear structure so that $\theta_{l_t} O_t$ is the same for T_{PU} and T_{HPKO} and $\theta_{l_p} \theta_{l_\alpha} O_\alpha$ is the same for T_{KO} and T_{HPPU}.

The four matrix elements are indipendently calculated as functions of the cut-off radii with θ_{l_t}, O_t, θ_{l_p}, θ_{l_α}, O_α equal to unity.

The calculations start with the parameters V_o set roughly equal to values already given.

To make the best-fit analysis one defines the merit function :

$$\chi^2 = \sum_i \frac{1}{p^2(\theta_i)} \left| \sigma^t(\theta_i) - \sigma^s(\theta_i) \right|^2$$

$\sigma^t(\theta_i)$ and $\sigma^s(\theta_i)$ are respectively the calculated and experimental differential cross-sections, $p(\theta_i)$ is a weighting factor which normally is set equal to $\sigma^s(\theta_i)$: in this way the contribution of each term to χ^2 is independent from the absolute value of the cross-section. If one wants to stress the importance of some part of the angular distribution one may change the $p(\theta_i)$.

The calculated cross-section is

$$\sigma^t(\theta_i) = K \sum_{\nu_p m_T m_R} \left| T \right|^2 = K (a_i N_1^2 + b_i N_2^2 + 2\, c_i N_1 N_2)$$

with

$$a_i = \sum_{\nu_p m_T m_R} \left\{ \left| T^{PU} \right|^2 + \left| T^{HPKO} \right|^2 + 2\, \mathrm{Re} (T^{PU}\, T^{HPKO*}) \right\}$$

$$b_i = \sum_{\nu_p m_T m_R} \left\{ \left| T^{KO} \right|^2 + \left| T^{HPPU} \right|^2 + 2\,\mathrm{Re}\left(T^{KO}\, T^{HPPU*}\right) \right\}$$

$$c_i = \sum_{\nu_p m_T m_R} \left\{ \mathrm{Re}\left(T^{PU}\, T^{KO*}\right) + \mathrm{Re}\left(T^{PU}\, T^{HPPU*}\right) + \right.$$

$$\left. + \mathrm{Re}\left(T^{HPKO}\, T^{KO*}\right) + \mathrm{Re}\left(T^{HPKO}\, T^{HPPU*}\right) \right\}$$

Then

$$\chi^2 = K^2 \sum_i \frac{1}{p^2(\theta_i)} \left| a_i N_1^2 + b_i N_2^2 + 2\, c_i N_1 N_2 - T_i^s \right|^2$$

having set :

$$T_i^s = \frac{\sigma^s(\theta_i)}{K}$$

The fitting procedure must minimize χ^2 with respect to the parameters N_1, N_2, and the cut-off radii for each mechanism.

All the possible combinations of the four cut-off radii are taken into account and for each of them the best value of χ^2 is determined.

The first step is to solve the following equation with respect to N_2 giving to N_1 a set of values in a given range

$$\frac{\partial \chi^2}{\partial N_2} = 4K^2 \sum_i \frac{1}{p_i^2} \left\{ b_i N_2^3 + 2 c_i N_1^2 N_2 + a_i b_i N_1^2 N_2 + \right.$$

$$\left. + 3 a_i c_i N_1 N_2^2 + b_i c_i N_1^3 - b_i N_2 T_i^s - c_i N_1 T_i^s \right\} = 0 \ .$$

A set of coupled values of N_1 and N_2 is obtained : one chooses the values giving the smallest χ^2. With this value of N_2 one solves the following equation

$$\frac{\partial \chi^2}{\partial N_1} = 4K^2 \sum_i \frac{1}{p_i^2} \left\{ a_i N_1^3 + 2 c_i N_1 N_2^2 + a_i b_i N_1 N_2^2 + \right.$$

$$\left. + 3 a_i c_i N_1^2 N_2 + a_i c_i N_2^3 - a_i N_1 T_i^s - c_i N_2 T_i^s \right\} = 0 \ .$$

In the same way as before, one determines the new coupled values N_1 and N_2. This procedure is carried on until in two successive iterations, the relative χ^2 differ by an amount less than a fixed ε value (generally we set $\varepsilon = 0.01$).

For each combination of the 4 cut-off radii the χ^2 is determined and among these values the smallest one is choosen.

The V_o values are not really fitting parameters but the code provides the possibility to modify them through some scale factors introduced in the input data.

To save computational time we proceded in the following way. At first we examined the combinations of only two processes, more precisely PU + HPPU. This was done for a large range of radii and we took the results as an indication to limit the values of the radii to be handled.

With a limited set of radii for PU and HPPU and with the complete set of radii for KO we started again the best-fit procedure. At this point we changed, if necessary, the scale factor for the V_o^{KO} value. Then limiting very strictly the number of KO radii too, we introduced the complete set of HPKO cases changing, if necessary, the scale factor for the V_o^{HPKO} value.

It is evident that this procedure may in a sense limit or influence the result, but for these preliminary calculations we did not deem necessary a greater effort.

SECTION IV. -

Some (p, α) reactions on light nuclei at 38 MeV incident energy[1] have been analyzed with the criterion exposed.

The results are shown in Figs. 4, 5, 6 for the nuclei ^{9}Be, ^{11}B, ^{16}O.

An extensive search was not made : the fitting was performed until sufficiently good fits were obtained, to see the possibility of using such procedure.

To point out the importance of the interference term, that has been often disregarded, in Fig. 7 one case is shown as an example. The interference contribution is not at all negligeable and may substantially change the angular distribution.

The results of a fitting procedure are not unique in the sense that more than one group of free parameters (cut-off radii) can give acceptable curves. As the cut-off radii cannot be directly correlated with known properties of nuclei, it is difficult to fix a priori their range of variability. One would need some criterion to choose at

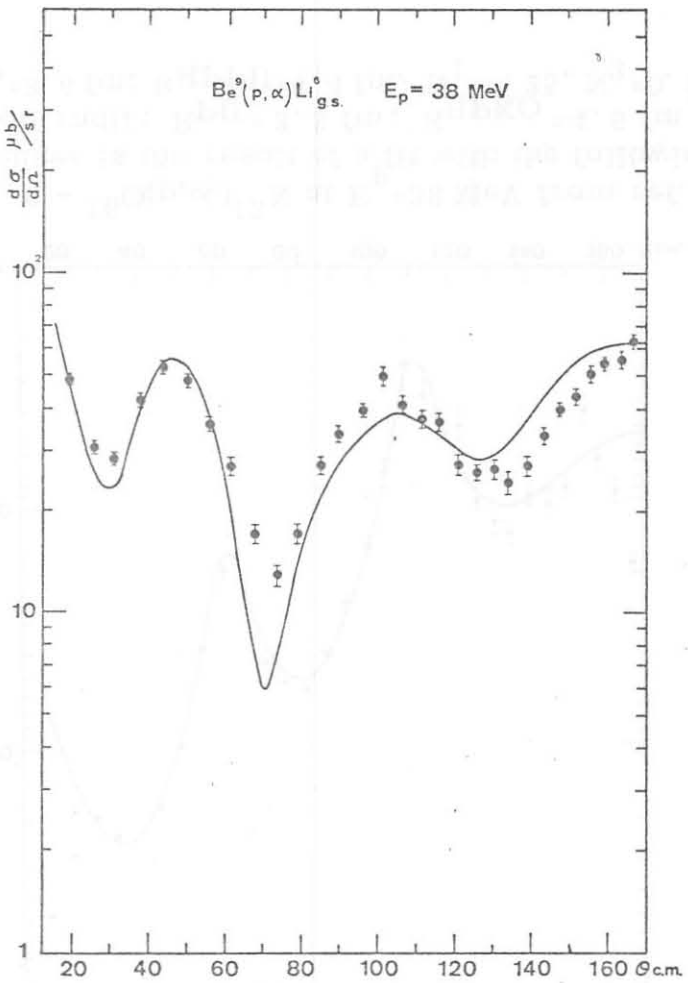


FIG. 4 - $^{9}Be(p,\alpha)^{6}Li$ at E_p=38 MeV from ref. (1); the curve is the result of a fit with the following cut-off radii: R_{PU}=5.4 fm, R_{HPKO}=4.2 fm; R_{KO}=5.4 fm; R_{HPPU}=4.8 fm. N_1 = 0.17, N_2=0.16.

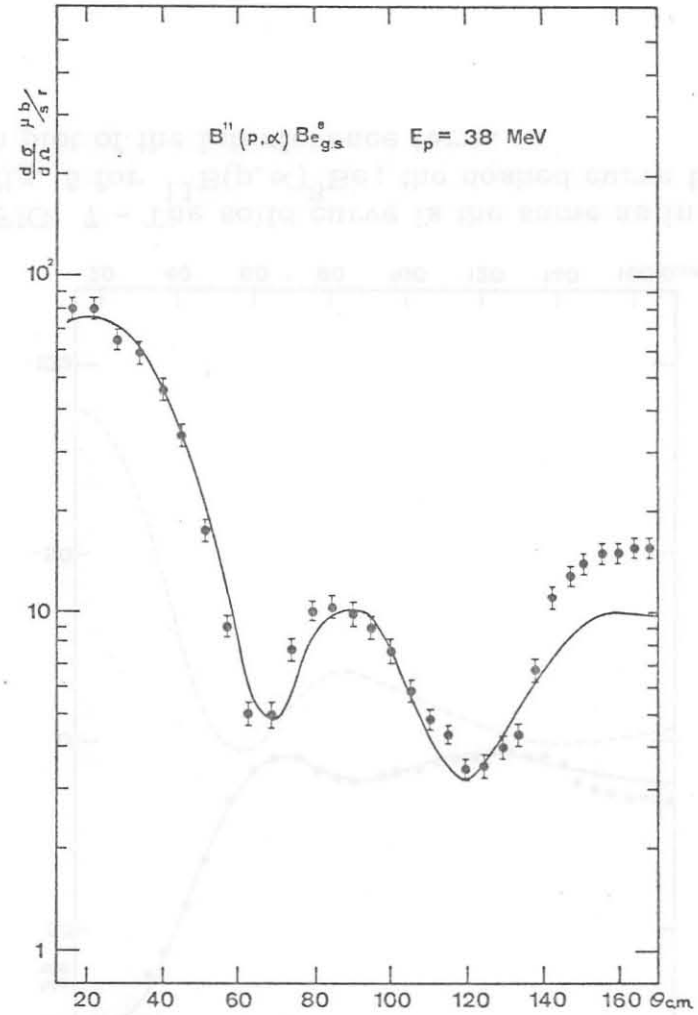


FIG. 5 - $^{11}B(p,\alpha)^{8}Be$ at E_p=38 MeV from ref. (1); the curve is the result of a fit with the following cut-off radii: R_{PU}=3.2 fm; R_{HPKO}=3.4 fm; R_{KO}=4.3 fm; R_{HPPU}=3.4 fm. N_1=0.27, N_2=0.25.

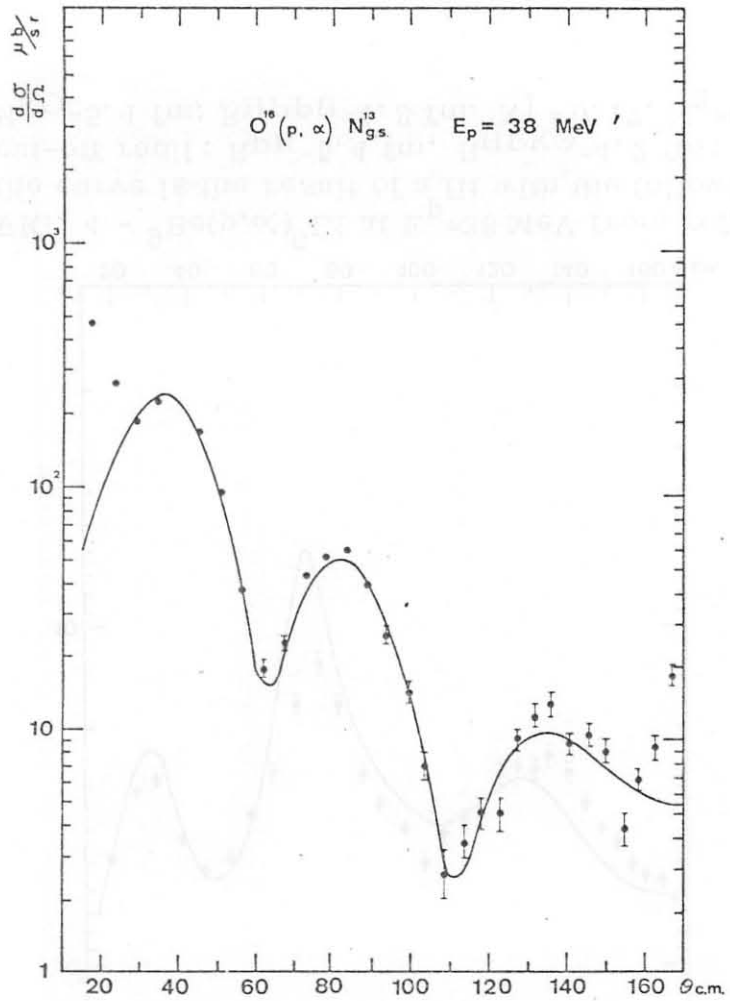


FIG. 6 - $^{16}O(p,\alpha)^{13}N$ at E_p=38 MeV from ref. (1); the curve is the result of a fit with the following cut-off radii: R_{PU} = 3.8 fm; R_{HPKO} = 4.6 fm; R_{KO}=3.6 fm; R_{HPPU}=4.4 fm. N_1=0.25, N_2=0.14.

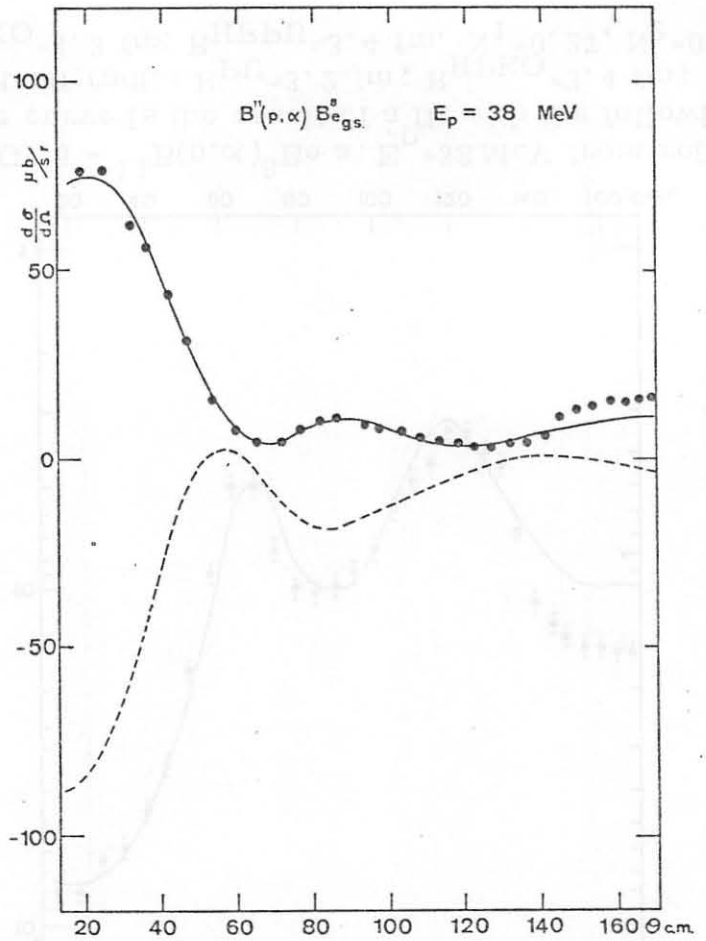


FIG. 7 - The solid curve is the same as in fig. 5 for $^{11}B(p,\alpha)^{8}Be$; the doshed curve is a plot of the interference term.

least roughly these values.

Probably this would be possible from the analysis of experimental results at various energies and for neighbouring nuclei.

For the cases here examined we did not follow any particular criterion to choose the cut-off radii : we let them vary from about 3 to about 7 fermis.

The coefficients N_1 and N_2 obtained are of the same order for ^{9}Be and ^{11}B while for ^{16}O N_2 is smaller than N_1. This seems to be contradictory with what one would expect from the hypothesis on the α cluster structure of this nucleus.

To be sure that such result could not be due to the particular procedure adopted, the fitting was repeated starting with the two mechanisms KO + HPKO, adding then PU and HPPU.

The results practically coincide; this means that they are indipendent from the way followed for the fitting procedure.

The cases here examined are too limited, moreover it is beyond the scope of this work to try to give physical significance to the numbers obtained.

CONCLUSIONS. -

The type of considerations here exposed do not claim to be conclusive, neither do they give definite results.

Starting on the hypothesis that the use of PWBA may be acceptable at least as tentative and that different reaction mechanisms partecipate to this kind of reactions we have tried to see the importance of the interference terms.

The exact evaluation of these terms is strictly bound to the relative precision with which the matrix elements for the different processes are calculated. With the used approximations the interference terms are therefore only indicative, but they seem to have a noticeable importance.

Certainly a greater number of experimental results, toghether with a deepening of the theoretical bases, should allow to extract useful informations on nuclear structure and on nuclear wave functions from the analysis of (p, α) angular distributions.

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