G. Alberi, C. Cernigoi, I. Gabrielli and L. Taffara:

A KINEMATICAL ASPECT OF TIE REACTION ${ }^{6} \mathrm{Li}\left(\pi^{-}, 2 n\right){ }^{4} \mathrm{He}$

A KINEMATICAL ASPECT OF THE REACTION
${ }^{6} \mathrm{Li}\left(\pi^{-}, 2 n\right){ }^{4} \mathrm{He}$

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[^0]Recently ( ${ }^{1,2}$ ) some experiments on pion absorption by light nuclei have been performed with the ảim of finding a new method of investigation of nuclear structure ( ${ }^{3}$ ). The basic idea is that, since the absorption can be considered a direct process, the cross-section, as function of the final state variables, can give information on the wave function of the nucleus. More precisely, assuming only a two nucleon interaction, the total momentum distribution of the emitted pair is sensitive to the form of the wave function of the relative motion of the c.m. of the nucleon pair with respect to the c.m. of the residual nucleus. This is immediately shown if one assumes the following form of the wave function of the nucleus:

$$
\begin{equation*}
\psi_{A}\left(\vec{r}_{1}, \ldots, \vec{r}_{A}\right)=\sum_{i} \psi_{2}^{(i)}\left(\vec{r}_{1}-\vec{r}_{2}\right) x^{(i)}(\overrightarrow{\mathrm{R}}) \psi_{\mathrm{A}-2}\left(\vec{r}_{3} \ldots \vec{r}_{A}\right) \tag{1}
\end{equation*}
$$

where

$$
\vec{R}=\frac{\vec{r}_{1}+\vec{r}_{2}}{2}-\frac{\sum_{i=3}^{A} \vec{r}_{i}}{(\mathrm{~A}-2)}
$$

(spin and antisymmetrization are neglected). If the reaction leaves the re sidual (A-2) nucleus unexcited, the differential cross-section in the impulse approximation is given by:
(2) $\left.\quad \mathrm{d} \sigma \sim\left|\Sigma_{i}\langle\overrightarrow{\mathrm{k}}| T\right| \pi \psi_{2}{ }^{(\mathrm{i})}\right\rangle\left.\left\langle\overrightarrow{\mathrm{K}} \mid \chi^{(\mathrm{i})}\right\rangle\right|^{2} \times$

$$
\times \frac{\left(\mathrm{k}^{2}+\mathrm{K}^{2} / 4\right) \mathrm{A}^{2}}{\cos ^{2} \vartheta \sqrt{\cos ^{2} \vartheta-\mathrm{A}^{2}}} \mathrm{~K} \text { d } \mathrm{K} \text { d } \Omega_{1} \text { d } \Omega_{2}
$$

where

$$
\begin{aligned}
A=\frac{K^{2} / 4-k^{2}}{K^{2} / 4+k^{2}}, \cos \vartheta= & \cos \vartheta_{1} \cos \vartheta_{2}+\sin \vartheta_{1} \sin \vartheta_{2} \times \\
& \times \cos \left(\varphi_{1}-\varphi_{2}\right)
\end{aligned}
$$

$\vartheta_{1}, \varphi_{1}, \vartheta_{2}, \varphi_{2}$ are the spherical coordinates of the momenta of the two nucleons; $\overrightarrow{\mathrm{K}}$ and $\overrightarrow{\mathrm{k}}$ are the total and relative momentum respectively
of the emitted nucleons. The phase space factor is calculated in the C.M. reference frame.

The simplest way to look at the form of the wave function of the nucleus is the measurement of the following quantity:

$$
\begin{align*}
&\left.\left.\frac{\mathrm{d} \sigma}{\mathrm{dK} d \Omega_{1} \mathrm{~d} \Omega_{2}}\right|_{\begin{array}{c}
\cos \vartheta_{1}=1 \\
\cos \vartheta_{2}=-1
\end{array}} ^{\sim}\left|\sum_{i}\langle\overrightarrow{\mathrm{k}}| T\right| \pi \psi_{2}^{(i)}\right\rangle\left.\left\langle\overrightarrow{\mathrm{K}} \mid \chi^{(\mathrm{i})}\right\rangle\right|^{2} \times  \tag{3}\\
& \times \mathrm{K} \frac{\left(\mathrm{k}^{2}+\mathrm{K}^{2} / 4\right) \mathrm{A}^{2}}{\sqrt{1-\mathrm{A}^{2}}}
\end{align*}
$$

Since experimentally the two nucleons are detected by means of counters of finite dimensions, it is possible to evaluate the quantity (3) only by assuming a vanishing solid angle of detection. In the experiment of ref. ${ }^{2}$ ) this approximation does not hold; on the other hand it is possible experimentally to measure, with fair accuracy, the relative angle of the emitted neutrons. According to this fact, the following quantity can be measured with a good accuracy:
(4) $\frac{d \sigma}{d K} \sim K\left(k^{2}+K^{2} / 4\right) A^{2} \int \frac{f(\vec{K}, \vec{k})}{\cos ^{2} \vartheta \sqrt{\cos ^{2} \vartheta-A^{2}}} d \Omega_{1} d \Omega_{2}$
where

$$
f(\vec{K}, \vec{k})=\mid \sum_{\dot{i}},\langle\vec{k}| T\left|\pi \psi_{2}^{(i)}><\vec{K}\right| \chi^{(i)}>\left.\right|^{2}
$$

The region of integration is determined by the experimental geometry (two opposite solid angles). For the case of $\mathrm{Li}^{6}$, if we do general assumptions on the wave function $\chi$ and on the $T \mid \pi>$ operatorr, as remarked by Koltun and Reitan $\left({ }^{5}\right)$, the function $f(\vec{K}, \vec{k})$ is angle independent. Therefore, in this case, the integral is easily calculated and the distributions (3) and (4) are connected by the following relation:
(5) $\left.\frac{\mathrm{d} \sigma}{\mathrm{dK} \mathrm{d} \Omega_{1} \mathrm{~d} \Omega_{2}}\right|_{\substack{\cos \vartheta_{1}=1 \\ \cos \vartheta_{2}=-1}} \sim \frac{\mathrm{~d} \sigma}{\mathrm{dK}}\left\{\begin{array}{l}\frac{\mathrm{A}^{2}}{1-\mathrm{A}^{2}}|\mathrm{~A}|>|\cos \alpha| \\ \frac{\mathrm{A}^{2}}{\left.\sqrt{1-\mathrm{A}^{2}}-\sqrt{1-\left(\frac{\mathrm{A}}{\cos \alpha}\right)^{2}}\right] \sqrt{1-\mathrm{A}^{2}}}|\mathrm{~A}|<|\cos \alpha|\end{array}\right.$
$\alpha$ is the minimum angle between the momenta of the two nucleons allowed by the geometry. The above expression gives us the possibility to deduce the quantity $d \sigma /\left.d K \quad d \Omega_{1} d \Omega_{2}\right|_{\cos \vartheta_{1}=1, \cos \vartheta_{2}=-1 \text { from the experimen }, ~} ^{\text {for }}$ tal curve $d \sigma / d K$. In Fig. 1 the experimental data for $d \sigma / d K$ (for pions at rest) are plotted together with the theoretical curves, obtained with a Butler form factor for three values of the parameter $R\left({ }^{6}\right)$. Only points corresponding to the ${ }^{4} \mathrm{He}$, as residual nucleus, are considered.

In Fig. 2 the points calculated by formula (5) from the experimental data of Fig. 1 are reported. For comparison the experimental data of ref. $\left(^{1}\right)$, normalized at the value of $\mathrm{K}=10 \mathrm{MeV} / \mathrm{c}$ on the theoretical curve of ref. $\left({ }^{6}\right)$, are also reported.

The agreement of the theory with the experimental data, corrected by formula (5), is now more satisfactory as it was before ( ${ }^{6}$ ). The rough model of Li and the pole approximation used in ref. ( ${ }^{6}$ ) are therefore able to fit fairly well the experimental data. However, in our opinion, the di stribution $\mathrm{d} \sigma / \mathrm{dK}$ is more reliable for a comparison between theories and experiments, because of better experimental precision and greater sensiti vity to the parameter involved in the theory.

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Fig. 1 - Experimental points of ref. (2) for $\mathrm{d} \sigma / \mathrm{dK}$ in arbitrary units. Theoretical curves $I, ~ I I$ and III correspond to the value $2.5,2.0$ and 1.5 F of the pa rameter $R$ in ref. $\left({ }^{6}\right)$.


Fig. 2 - Circles are the calculated point for $d \sigma / d K d \Omega_{1} d \Omega_{2} \left\lvert\, \begin{aligned} & \cos \vartheta_{1}=1 \\ & \cos \vartheta_{2}=-1\end{aligned}\right.$ from the data of Fig. 1, using formula (5) in the text. Triangles are the data of ref. ( ${ }^{1}$ ). Theoretical curves $I$ and II correspond respectively to the value 0.0 and 3.0 $F$ of the parameter $R$ in ref. ( ${ }^{6}$ ).

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