Comitato Nazionale per L'Energia Nucleare
ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Milano
67/2

INFN/BE - 67/3
20 Marzo 1967.

E. Gadioli and L. Zetta: REMARKS ON THE ANALYSIS OF STATISTICAL REACTIONS SPECTRA. -

Reparto Tipografico
dei Laboratori Nazionali di Frascati

Istituto Nazionale di Fisica Nucleare
Sezione di Milano

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INTRODUCTION -

The great importance of the level density in the statistical model of nuclear reactions is well known.

In fact, all the quantitative predictions of this model depend in an essential way on the knowledge of the level density.

Experimental information about the level density can be obtained by analysing:

a) the levels of the residual nuclei of many reactions (mainly (p, p'), (d, p), (d, α), (t, p) reactions)[1];
b) the shape of the spectra of particles emitted in statistical reactions. Generally, these reactions are induced by incident particles having an energy $E_i \leq 20$ MeV (see for instance ref. (2) and (3));
c) the slow neutron resonances[2][3][4];
d) the radiation widths[5];
e) the level widths of nuclei in the continuum energy region[6][7].

Some of these experimental information are direct information e.g. they directly give the value of the level density, the other ones are indirect information: in this case the value of the level density extracted from the experimental data can be influenced by an incorrect estimation

of other nuclear quantities such as the transmission functions of the particles emitted from the Compound Nucleus (C. N.).

In the same case other sources of uncertaincy on the obtained value of the level density can be the rather rough approximations one usually does in order to simplify the analysis of experimental results.

In this work we want to test the assumptions usually made in the analysis of the shape of spectra of particles emitted in C. N. reactions, that we think are not strictly necessary and influence the values of the characteristic parameters of level density one deduces, especially in the case of reactions involving light nuclei.

THE THEORETICAL EXPRESSION OF THE LEVEL DENSITY -

One of the most commonly used expressions of the level density is the following due to Lang and Le Couteur[8].

$$\rho(U, J) = \frac{\hbar^3}{24\sqrt{8}} (2J + 1) \exp\left[- \frac{J(J+1)}{2\sigma^2} \right] a^{1/2} \mathcal{J}^{-3/2} \cdot \frac{\exp\left[2\sqrt{aU} \right]}{(U + t)^2} \quad (1)$$

which gives the density of levels of spin J and a given parity of a nucleus excited to an energy U.

This formula is derived in the framework of Fermi gas model of the nucleus. $\mathcal{J}$ is the moment of inertia of the nucleus, t the thermodynamic temperature, $\sigma^2 = (\mathcal{J}t/\hbar^2)$ is called the spin cut-off factor. a is a characteristic parameter related to the spacing of single nucleon states near the top of the Fermi distribution (for a discussion of the theoretical basis of formula (1) see for instance ref. (9)).

Recently[6] it has been shown that this formula, substituting the usual excitation energy U with the effective excitation energy $U' = (U + (70/A) - \Delta)$ MeV, where Δ is the pairing energy[3, 4, 10], assuming for the radius of the nucleus $R \simeq 1.5\ A^{1/3}$ fm., for the moment of inertia $\mathcal{J} = 0.7\ \mathcal{J}_{rig}$, and $a = (0.127\ A)\ \text{MeV}^{-1}$, seems to predict correctly the slope and the absolute value of the level density of light nuclei ($20 \leq A \leq 70$) for excitation energies ranging from ~7 to ~20 MeV.

For nuclei with $A \geq 70$ the trend in the experimental value of the level density, at the excitation energy corresponding to slow neutron resonances, is less regular and shows sudden changes for $A \sim 140$ and $A \sim 210$ that seems to be due to shell effects. It is not yet completely

established if these shell effects can be explained by the use of a simple formula like (1) and in this case if they can be taken into account by a sudden change in the value of the a parameter[2][3][4] or more simply by an increased value of the pairing energy for magic shell nuclei.

THE SHAPE OF THE SPECTRUM OF THE PARTICLES EMITTED IN STATISTICAL REACTIONS -

A) Let us first consider the case of integrated cross-sections.

The statistical model predicts the following expression for the average cross-section of a reaction which proceeds from an initial channel α to a final channel α', through the formation of a highly excited C.N. whose levels are overlapped:

$$(2) \qquad \sigma_{\alpha,\alpha'} = \frac{\pi \lambda_\alpha^2}{(2i+1)(2I+1)} \sum_J (2J+1) \frac{\sum_{lsl's'} T^J_{\alpha ls} T^J_{\alpha' l's'}}{\sum_c T^J_c}$$

i and I are the spin of incident particle and target nucleus; J is the spin of C.N.; T are the transmission functions. The primed and unprimed quantities refer to the initial and final channel respectively. (For a discussion of the assumptions according to which formula (2) has been derived see ref. (11)).

When the excitation energy of the residual nucleus increases, if the experimental energy resolution of the detecting apparatus is not too high, the peaks of the spectrum corresponding to the different transitions overlap and the energy distribution of the particles emitted in the reaction becomes a continuous distribution.

In this case one makes the assumption that the energy levels of the residual nucleus can be approximately described by a level density function $\rho(U, j)$ and the average cross-section for unit energy becomes:

$$\sigma_\alpha(E) = \frac{\pi \lambda_\alpha^2}{(2i+1)(2I+1)} \sum_J (2J+1) \frac{\sum_{ls} T^J_{\alpha ls}}{\sum_c T^J_c} \sum_{l'=0}^{\infty} \sum_{|J-l'|}^{J+l'} \sum_{s'} T^J_{l's'}(E) \cdot$$

$$(3) \qquad \cdot \sum_{|s'-i'|}^{s'+i'} \sum_j \rho(U)(2j+1) \exp\left[-j(j+1)/2\sigma^2\right]$$

where E is the disintegration energy of the final channel and U is the effective excitation energy of the residual nucleus.

The quantity $\Sigma_c T^J_c$ which takes into account all the possible decay modes of the C.N. can be approximated by an integral introducing

the level density of the residual nuclei to which the C.N. can decay. Expression (3) is quite involved and usually one makes some rough approximations in order to simplify it. The most usual assumptions are the following:

1) one assumes that the spin cut-off factors which appear in all the level densities are infinitely large;
2) one assumes that $T^J_{l's'}$, T^J_{ls}, and all the T^J_c does depend only on the angular momentum 1.

In this case it is easy to show that, utilising formula (1),

$$(4) \qquad \sigma_\alpha(E) \propto E\,\sigma_c(E)\,\frac{e^{2\sqrt{aU}}}{(U+t)^2}$$

$\sigma_c(E)$ is the inverse cross-section at the energy E.

The plot of

$$\log_e\left\{\frac{\sigma_\alpha(E)}{E\,\sigma_c(E)}\,(U+t)^2\right\}$$

against $\sqrt{U}$ is a straight line of slope $2\sqrt{a}$.

Approximation 2) is probably a quite good approximation but approximation 1) is surely a bad approximation especially in the case of light nuclei and not too high excitation energies.

It should be noted that expression (4) can be obtained also assuming that the C.N. decays only by emitting particles of zero angular momentum (for a discussion of this point see Lang (3) and ref. (13)). Also so this one seems to be in many cases an extreme approximation.

Most of the analysis of experimental spectra in order to obtain empirical a values are based on expression (4) (for further references see ref. (14) and (15)).

It seems to us that it is possible to simplify expression (3) also by making less drastic approximations. We will discuss here in some length the subject.

a) Starting from expression (3) we can note that

$$(5) \qquad \sum_{|s'-i'|}^{s'+i'} (2j+1)\exp\left[-j(j+1)/2\sigma^2\right] \approx A\,(2s'+1)\,e^{-\frac{s'(s'+1)}{2\sigma^2}}$$

A is a coefficient of proportionality equal to 1 for α particle emission and equal to about 2 for proton and neutron emission.

b) We can assume that $T^J_{l's'}$ does depend only on l'. Taking into account (5) we have then

$$\sum_{l'=0}^{\infty} \sum_{|J-l'|}^{J+l'} s' \; T^{J}_{l's'} \sum_{|s'-i'|}^{s'+i'} j \rho(U)(2j+1)\exp\left[-j(j+1)/2\sigma^2\right] \approx$$

(6)

$$\approx A\rho(U) \sum_{l'=0}^{\infty} T_{l'} \sum_{|J-l'|}^{J+l'} s' (2s'+1)\, e^{-\frac{s'(s'+1)}{2\sigma^2}}$$

c) We can now substitute each term $(2s' + 1)\exp\left[-s'(s'+1)/2\sigma^2\right]$ in the sum (6) with its average value that can be assumed, at each energy, simply proportional to σ^2 (1), and (in the sum over l') we can substitute ∞ with l'_{max}. We finally obtain:

$$\sum_{l'=0}^{\infty} \sum_{s'} T^{J}_{l's'} \sum_{j} \rho(U)(2j+1)\exp\left[-j(j+1)/2\sigma^2\right] \propto$$

(7)

$$\propto \rho(U) \sum_{l'=0}^{l'_{max}} T_{l'} \sum_{|J-l'|}^{J+l'} \sigma^2$$

The sum $\sum_{s'} \sigma^2$ can be easily computed in two extreme cases:

I) to the reaction contribute mostly J values greater than the outgoing angular momenta, e. g., at each energy E, on the average $J \geq l'_{max}$. This will be approximately the case for (n, n'), (p, p') reactions; for (n, p) and (p, n) reactions if we consider energy ranges of the emitted particles corresponding to which $E_{outgoing} < E_{ingoing}$; for reactions (heavy ions, light particles).

In this case we obtain

$$\rho(U) \sum_{l'=0}^{l'max} T_{l'} \sum_{|J-l'|}^{J+l'} s' \sigma^2 \approx \rho(U)\sigma^2 \sum_{l'=0}^{l'max} T_{l'}(2l' + 1) \propto$$

(8)

$$\propto \rho(U)\sigma^2 \sigma_c(E)E$$

and finally, taking into account that $\sigma^2 = \frac{\mathcal{J} t}{\hbar^2} = \frac{\mathcal{J}}{\hbar^2}\sqrt{\frac{U+t}{a}}$:

$$\sigma_\alpha(E) \propto \rho(U)\sigma^2 \sigma_c(E)E \propto \frac{e^{2\sqrt{aU}}}{(U+t)^2} (\sqrt{U+t})\, \sigma_c(E)E =$$

(9)

$$= \frac{e^{2\sqrt{aU}}}{(U+t)^{3/2}} \sigma_c(E)E$$

In this case the plot of

$$\log_e \left\{ \frac{\sigma_\alpha(E)}{E\,\sigma_c(E)} \, (U+t)^{3/2} \right\}$$

against $\sqrt{U}$ is a straight line of slope $2\sqrt{a}$.

II) One can suppose, on the contrary, that on the average $J \leq l'$.

This case seems a priori that corresponding to (n, α) and (p, α) reactions.

In this case we obtain

$$(10) \qquad \rho(U) \sum_{l'=0}^{l'_{max}} T_{l'} \sum_{|J-l'|}^{J+l'} s \sigma^2 \approx \rho(U)\,\sigma^2 (2J+1) \sum_{l'=0}^{l'_{max}} T_{l'}$$

We can now introduce a sharp cut-off approximation for the transmission functions so that they are either 0 or $\bar{T}$:

$$\sum_{l'=0}^{l'_{max}} T_{l'} \sim (l'_{max} + 1)\bar{T}$$

But in the same approximation $\bar{T}(l'_{max} + 1)^2 \propto E\,\sigma_c(E)$, and assuming $l'_{max} \propto \sqrt{E}$

$$\bar{T}(l'_{max} + 1) \propto \frac{E\,\sigma_c(E)}{(l'_{max}+1)} \propto \frac{E\,\sigma_c(E)}{\sqrt{E}} = \sqrt{E}\;\sigma_c(E)$$

In this second case we have then

$$(11) \qquad \sigma_\alpha(E) \propto \rho(U)\,\sigma^2 \sqrt{E}\;\sigma_c(E) \propto \frac{e^{2\sqrt{aU}}}{(U+t)^{3/2}} \sqrt{E}\;\sigma_c(E)$$

The plot of

$$\log_e \left\{ \frac{\sigma_\alpha(E)}{\sqrt{E}\,\sigma_c(E)} \, (U+t)^{3/2} \right\}$$

against $\sqrt{U}$ is a straight line of slope $2\sqrt{a}$.

The approximations introduced to derive expressions (9) and (11) seem to be, in the considered cases, much more reliable, then the ones leading to expression (4). Moreover, they give very simple formulas too, that can be easily utilized to derive the a parameters characterizing the level densities of the residual nucleus.

To test this point we have calculated $\sigma_\alpha(E)$ by means of formula (3) for hypothetical (n, p) and (n, α) reactions induced by ~14 MeV neutrons on target nuclei ranging from A = 28 to A = 210. The chosen target nuclei are: Si^{28}, Sc^{44}, Co^{56}, Ga^{68}, Zr^{89}, Ba^{133}, W^{182}, Bi^{210}. The calculations have been done by assuming in (3) for nuclei with A $\leq$ 90, U = = (Emax - E - Δ + (70/A)) MeV; for nuclei with larger A values we have assumed U = (Emax - E - Δ) MeV; in both the preceding formulas is Emax = Einc + Q. For the calculation of $\Sigma_c T_c^J$ we have considered the decay of C.N. by neutron, proton and α particles emission. For all the considered reactions, except the reactions induced on Zr^{89} and Bi^{210}, the a values necessary for calculating (3) have been taken as given by the law a = (0.127 A) MeV^{-1}. In the case of the reactions induced on Zr^{89} and Bi^{210}, to simulate shell effects, we have taken a = 9 and a = = 10 respectively for all the nuclei to which the compound nucleus can decay by neutron, proton and α emission. For the nuclear radius R we have assumed R = 1.5 $A^{1/3}$ fm and for the moment of inertia $\mathcal{J}$ = 0.7 $\mathcal{J}_{rig}$. The transmission functions necessary for the calculations have been taken from ref. (16) for neutrons and protons and from ref. (17) for α-particles.

In fig. 1 and 2 we report some of the calculated spectra.

The calculated spectra have been subsequently analyzed, as one usually does with experimental spectra, by utilizing formulas (11), (9) and (4).

In table I the comparison between the value of a we attributed in the theoretical calculation to the residual nucleus (in the case of (n, p) reactions) and the a values obtained by means of the reduced spectra utilizing formulas (9) and (4) is shown.

In table II the same comparison is made, in the case of (n, α) reactions, between the value of a we attributed to the residual nucleus and the a values obtained by means of formulae (11), (9) and (4). The results corresponding to such tables are also reported in fig. 3 and 4 respectively, in a somewhat different way: here the ratios between the deduced and the correct a values are shown against the mass number of residual nuclei.

In analysing the calculated spectra with the various formulae we reported, we followed an iterative procedure, calculating first a without introducing the temperature t; these approximate a are then used to calculate approximate temperature t and to deduce new more approximate a values.

We have seen that these last a values are already an accurate estimation of the a we can obtain by utilizing the reported formulas with the correct value of the temperature.

As one can see from Table I and fig. 3, formula (9) in all the mass range we investigated allows an estimation of a correct within about 4% in the case of (n, p) reactions. On the other hand the experi-

TABLE I

Comparison between the value $\underline{a}_c$ we attributed to the residual nucleus in the calculation of (n, p) reactions and the values $\underline{a}_1$ and $\underline{a}_2$ we deduced by analysing the calculated spectra with formulae (9) and (4) respectively. In first column the reactions we considered are reported; in second column the interval of the square root of the effective excitation energy, over which the theoretical spectrum extends, is shown.

Reactions	$\Delta(U)^{1/2}$	$\underline{a}_c$	$\underline{a}_1$	$\underline{a}_2$
$Si^{28}(n, p)Al^{28}$	2.20-3.29	3.556	3.81	4.48
$Sc^{44}(n, p)Ca^{44}$	1.94-3.71	5.588	5.75	6.58
$Co^{56}(n, p)Fe^{56}$	1.27-3.95	7.112	7.28	8.34
$Ga^{68}(n, p)Zn^{68}$	1.46-3.73	8.636	8.64	9.77
$Zr^{89}(n, p)Y^{89}$	1.08-3.61	9.000	9.38	10.16
$Ba^{133}(n, p)Cs^{133}$	0.69-3.22	16.891	16.19	18.34
$W^{182}(n, p)Ta^{182}$	1.06-2.66	23.114	22.02	24.57
$Bi^{210}(n, p)Pb^{210}$	0.48-2.68	10.000	9.17	11.19

TABLE II

Comparison between the value $\underline{a}_c$ we attributed to the residual nucleus in the calculations of (n, α) reactions and the values $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ we deduced by analysing the calculated spectra with formulae (11), (9) and (4) respectively. In first column the reactions we considered are reported; in second column, the interval of the square root of the effective excitation energy, over which the theoretical spectrum extends, is shown.

Reactions	$\Delta(U)^{1/2}$	$\underline{a}_c$	$\underline{a}_1$	$\underline{a}_2$	$\underline{a}_3$
$Si^{28}(n, \alpha)Mg^{25}$	1.80-3.04	3.175	3.41	4.24	5.04
$Sc^{44}(n, \alpha)K^{41}$	1.51-3.36	5.207	5.61	6.19	7.18
$Co^{56}(n, \alpha)Mn^{53}$	1.39-3.45	6.858	7.22	7.86	9.02
$Ga^{68}(n, \alpha)Cu^{65}$	1.80-3.35	8.255	8.76	9.43	10.60
$Zr^{89}(n, \alpha)Sr^{86}$	1.16-2.89	9.000	8.98	9.48	10.97
$Ba^{133}(n, \alpha)Xe^{130}$	0.83-2.95	16.510	15.92	16.45	18.66
$W^{182}(n, \alpha)Hf^{179}$	0.62-2.72	22.733	21.95	22.42	25.42
$Bi^{210}(n, \alpha)Tl^{207}$	0.52-2.70	10.000	9.51	9.75	11.76

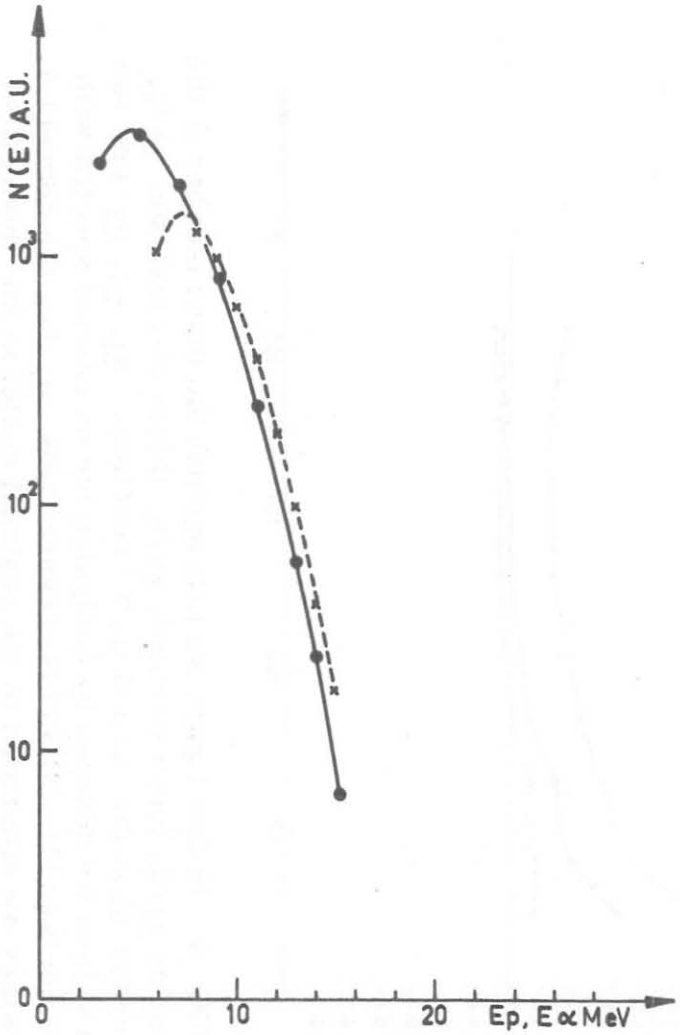

FIG. 1 - Theoretical spectra of the reactions $Sc^{44}(n, p)Ca^{44}$ (black circles) and $Sc^{44}(n,\alpha)K^{41}$ (crosses). The energies are calculated in the C. M. system.

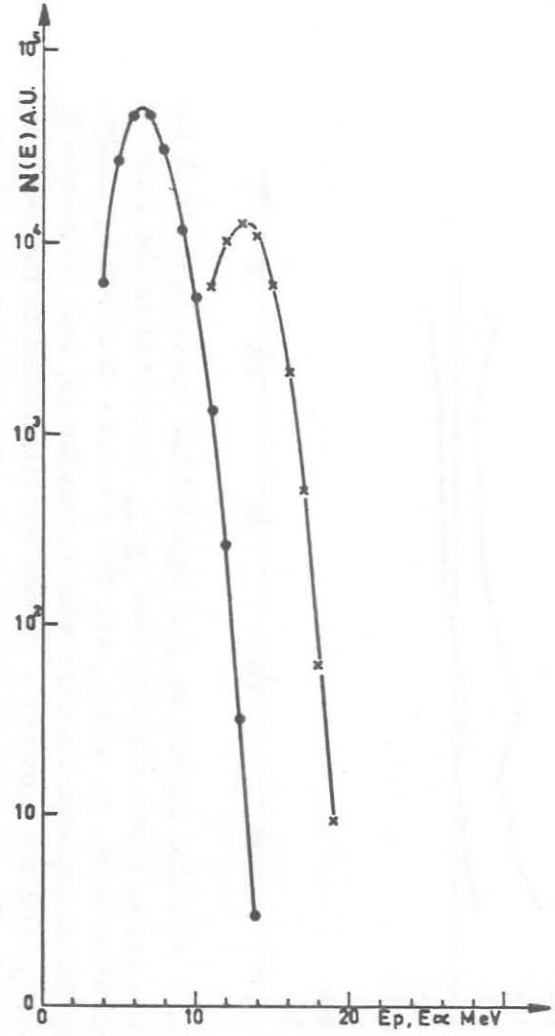

FIG. 2 - Theoretical spectra of the reactions $Ba^{133}(n, p)Cs^{133}$ (black circles) and $Ba^{133}(n,\alpha)Xe^{130}$ (crosses). The energies are calculated in the C. M. system.

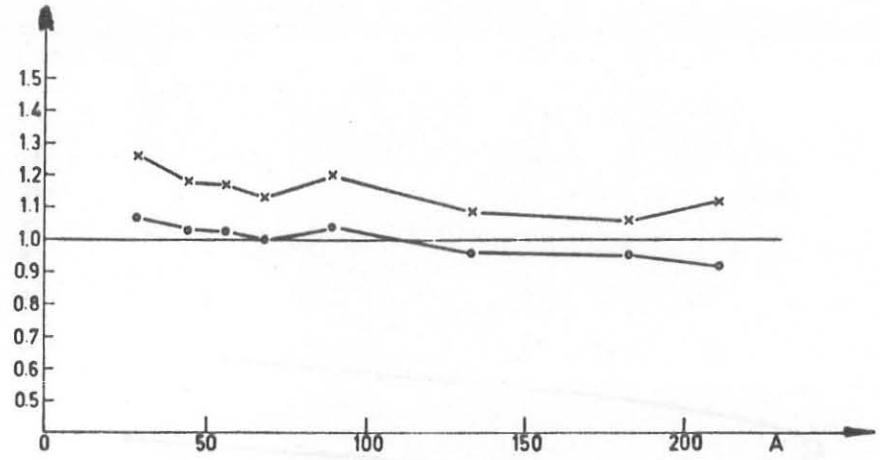

FIG. 3 - In this figure we plot against the mass number A the ratios $\underline{a}_1/\underline{a}_c$ (open circles) and $\underline{a}_2/\underline{a}_c$ (crosses) in the case of (n, p) reactions. $\underline{a}_1$ and $\underline{a}_2$ are the $\underline{a}$ values deduced by analysing the calculated spectra with formulae (9) and (4) respectively; $\underline{a}_c$ are the correct $\underline{a}$ values we attributed to the residual nuclei in the theoretical calculations. As one can see, formula (9) allows a much better estimation of the correct $\underline{a}$ values.

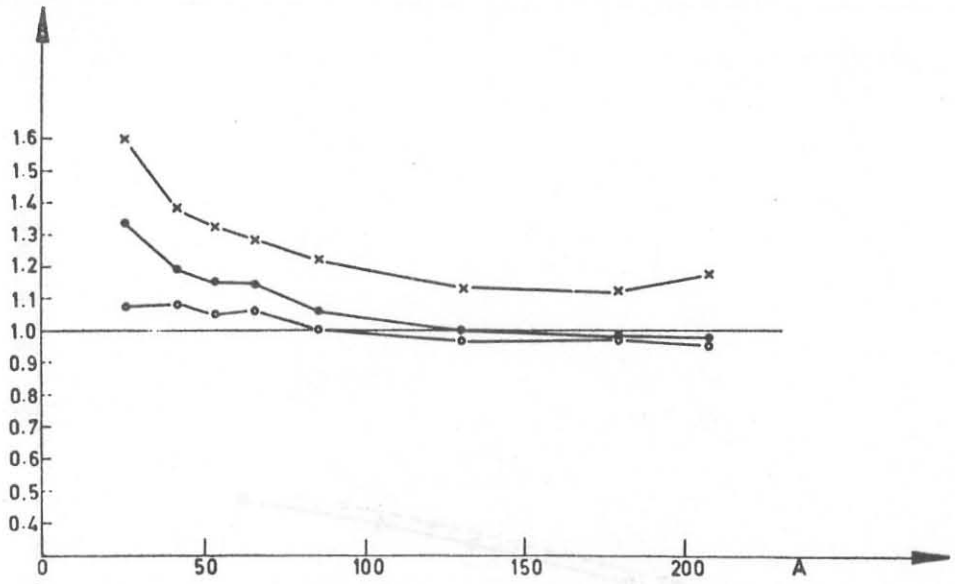

FIG. 4 - In this figure we plot against the mass number A the ratios $\underline{a}_1/\underline{a}_c$ (open circles), $\underline{a}_2/\underline{a}_c$ (black circles) and $\underline{a}_3/\underline{a}_c$ (crosses) in the case of (n, α) reactions. $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ are the a values we deduced by analysing the calculated spectra with formulas (11), (9) and (4) respectively. $\underline{a}_c$ are the correct a values we attributed to the residual nuclei in the theoretical calculations. As one can see, formula (11) allows an almost correct estimation of the $\underline{a}_c$ values in all the mass range we examined.

mental errors and the theoretical uncertainties concerning the transmission coefficients (for a discussion of this point see § 2 of ref. (14)) do not certainly allow an estimation of a from experimental results, also using formula (3), characterized by a greater accuracy than this one; the use of formula (9) seems then to be justified in the analysis of experimental results.

Formula (4) gives less accurate results expecially in the case of light nuclei. One can slightly improve the results obtained by using this formula, by choosing less extended ranges of the residual nucleus excitation energy.

This improvement is rather small and at the same time the reduction of the energy spectrum of emitted particles introduces a new source of uncertaincy in the analysis of experimental results.

We think that these conclusions can be extended also to the case of (n, n') reactions and reactions of the type (heavy ions, light particle).

Table II and fig. 4 show that the analysis of statistical nuclear spectra with formula (11) allows an estimation of a, in the case of (n, α) reactions, in all the mass range we investigated, correct within 5%. The same conclusions of the preceding discussion hold and the use of formula (11) seems justified in the analysis of experimental results.

Formula (9) in the case of (n, α) reactions does not give correct results for target nuclei having $A \leq 100$. For heavier nuclei, it gives the a values with very good accuracy.

Formula (4) gives much less accurate results for all nuclei we studied. In the case of light nuclei the disagreement between the obtained a values and the correct ones is expecially big.

It is important to note that the results we obtained are independent from the expression of the effective excitation energy we introduced in the level density formula as one can see form the derivation of formulae (9) and (11).

B) We can now examine the case of differential spectra, e.g., the spectra of the particles emitted, in a given reaction, to a given angle with respect to the incident beam.

This case is experimentally the most frequent one.

The shape of the average angular distribution of the particles emitted in a statistical reaction is due to angular momentum effects. One can take into account these effects obtaining the following formula[11, 18, 19]:

$$\sigma_{\alpha\alpha'}(\theta, E) = \frac{\lambda^2}{4} \frac{1}{(2i+1)(2I+1)} \sum_{\substack{ls \\ l's' \\ JL}} (-)^{s'-s} \bar{Z}(1Jl'J;sL)\bar{Z}(l'Jl'J;s'L) \cdot$$

(12)

$$\cdot \frac{T^J_{\alpha ls}\, T^J_{\alpha' l's'}}{\sum_c T^J_c} P_L(\cos\theta)$$

which gives the angular distribution of the particles emitted in $\alpha \rightarrow \alpha'$ reactions, that is, reactions proceeding from an initial channel α to a final channel α'. The $\bar{Z}$ are kinematical coefficients.

When one considers the spectrum of the particles leading to different levels of the residual nucleus one obtains a formula like (3) introducing the level density of the final nucleus.

The angular distributions are always symmetric with respect to 90^o, and generally they have a rise in forward and backward directions; in some particular cases dynamical effects can give rise to a maximum at 90^o (for a discussion of this point see ref. (20)).

The use of complicated formulas like (12), in the analysis of differential spectra does not seem very useful in many cases.

Let us make some simple considerations:

a) in the approximation that all the spin cut-off factors are infinitely large the angular distribution of particles emitted in a statistical reaction becomes isotropic[11][21];
b) if in the reactions many levels of the compound and residual nuclei are excited it is possible to introduce the random phase approximation and in the expression of the cross-section all interference terms vanish.

This fact ensures the possibility of introducing a classical approximation in the description of the reaction and obtaining a very simple formula for the angular distribution when incident particles, outgoing particles and target nuclei are spinless[22][14].

This approximation seems to be rather good for (n, n'), (p, p'), (p, n), (n, p), (n, α), (p, α) reactions at energy $E_i \leq 20$ MeV on even-even nuclei; and becomes better as the mass of target nuclei increases. If the classical approximation holds we can write :

$$\sigma(\theta, E) = \frac{\sigma(E)}{4\pi} \frac{1 + (\eta/2)\cos^2\theta}{1 + \eta/6}$$

(13)

$$\eta = \frac{\overline{J^2}\,\overline{l^2}}{(2\sigma^2)^2}$$

J is the spin of C.N., l the angular momentum of outgoing particle, σ^2 the spin cut-off factor of the residual nucleus.

The angular distributions are anisotropic and symmetric with respect to 90°; the anisotropy increases as η increases.

If the spin of the target nucleus is different from zero the angular distributions become more isotropic.

It is interesting to note that, in the case of the classical approximation, for $\cos^2\theta = 1/3$, ($\theta \approx 125°$), $\sigma(\theta, E) = (\sigma(E))/4\pi$ and it is possible to analyse the spectrum of the emitted particles with the formulae reported in A.

If $\theta \neq 125°$ in the expression of $\sigma(\theta, E)$ we must take into account the energy dependent term

$$R = \frac{1 + \frac{\eta}{2}\cos^2\theta}{1 + \frac{\eta}{6}} \tag{14}$$

The energy variation of this term is stronger for (n, α) and (p, α) than for (n, n'), (n, p), (p, p'), (p, n) reactions, and it is particularly important for light nuclei.

In fig. 5 and 6 we plot against the disintegration energy of the final channel, the ratios

$$R_1 = \frac{1}{1 + \eta/6} \quad \text{and} \quad R_2 = \frac{1 + \eta/2}{1 + \eta/6} \ ,$$

corresponding to the emission angles $\theta_1 = 90°$ and $\theta_2 = 180°$ for (n, p) and (n, α) reactions on various spinless target nuclei. The nuclei we have considered are: Si^{28}, Ca^{42}, Ni^{60}, in the case of (n, p) and Si^{28}, Ca^{42}, Ni^{60}, Zr^{90}, Pd^{106}, Te^{124} in the case of (n, α) reactions. The calculations have been performed by introducing the sharp cut-off approximation on the transmission coefficients reported by ref. (16) and (17) and calculating σ^2 with formula $\sigma^2 = (\mathcal{J}t/\hbar^2)$, where t, the thermodynamic temperature is connected to the effective excitation energy U by: $U = at^2 - t^{(8)}$. $\mathcal{J}$ is calculated as sketched out in section 2.

As one can see, expecially for angles near 90°, the energy dependence of (14) can be neglected in the case of (n, p) reactions for target nuclei with $A \geq 40$; in the case of (n, α) reactions it can be neglected for target nuclei with $A \geq 100$, taking also into account the strong variation of $\sigma(E)$ with E.

For light nuclei, expecially for (n, α) reactions, in many cases it is not possible to neglect the energy variation of (14), that can be easily taken into account by keeping in mind the definition of η, with the help of transmission coefficients tables.

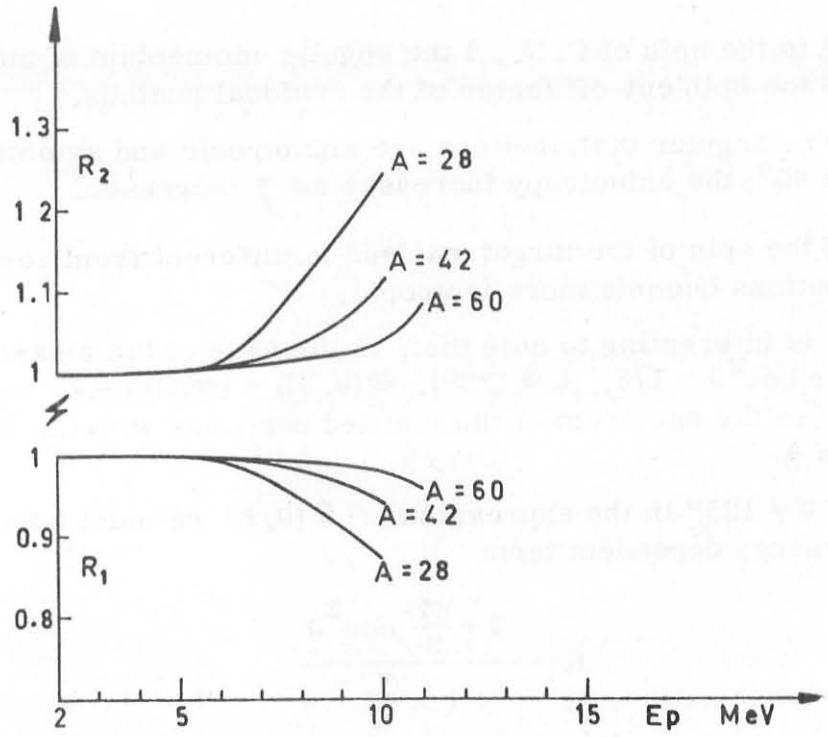

FIG. 5 - Plot against the disintegration energy of the final channel (in the C. M. system) of the ratios R_1 and R_2 described in the text, in the case of (n, p) reactions on various spinless target nuclei.

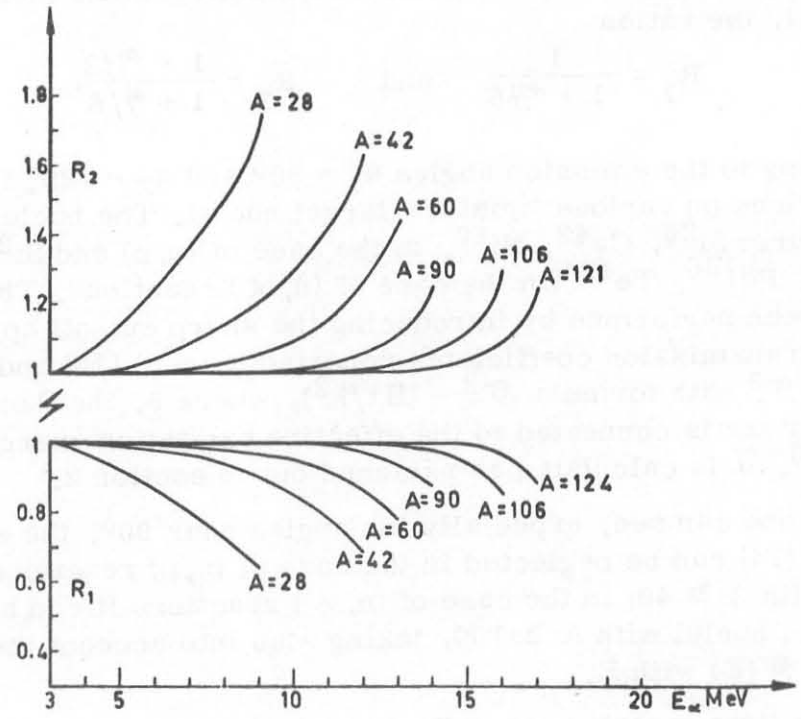

FIG. 6 - Plot against the disintegration energy of the final channel (in the C. M. system) of the ratios R_1 and R_2 described in the text, in the case of (n, α) reactions on various spinless target nuclei.

The goodness of the classical approximation can be tested by fitting the angular distribution of groups of particles emitted in statistical reactions.

The small anisotropy of (n, p) and (n, n') angular distribution can not generally be detected, owing to the great experimental errors. See for instance[23]. In the case of (n, α) reactions the results are lacking but in the few cases we could examine, the classical approximation seems to be quite good. See for instance ref. (22) and fig. 7 and 8.

The angular dependence of the spectra slope should become less important as the spin of target nucleus increases.

CONCLUSIONS -

In this work we have tested the assumptions usually made in the analysis of the shape of the spectra of particles emitted in statistical reactions.

Our conclusions are that formula (4) commonly used for deducing the value of the parameter a characteristic of the Fermi gas level density (1), gives not very accurate results expecially in the case of (n, α) reactions on light nuclei, and in general overestimates the correct a values.

By making less drastic approximations, we have, then, derived two other simple expressions (expressions (9) and (11)) suitable for (n, p) (or (n, n')) and (n, α) reactions respectively, that relate the integrated spectra of particles emitted in statistical reactions to the level density of residual nucleus and allow much more correct estimations of the a values.

We have discussed also the case of differential spectra; we have shown that expecially for (n, α) reactions on light nuclei, in many cases, it is not possible to neglect, as one usually does, the presence of an angle and energy dependent factor (see (14)) in the expression which relate the experimental $\sigma(\theta, E)$ to the level density of residual nucleus.

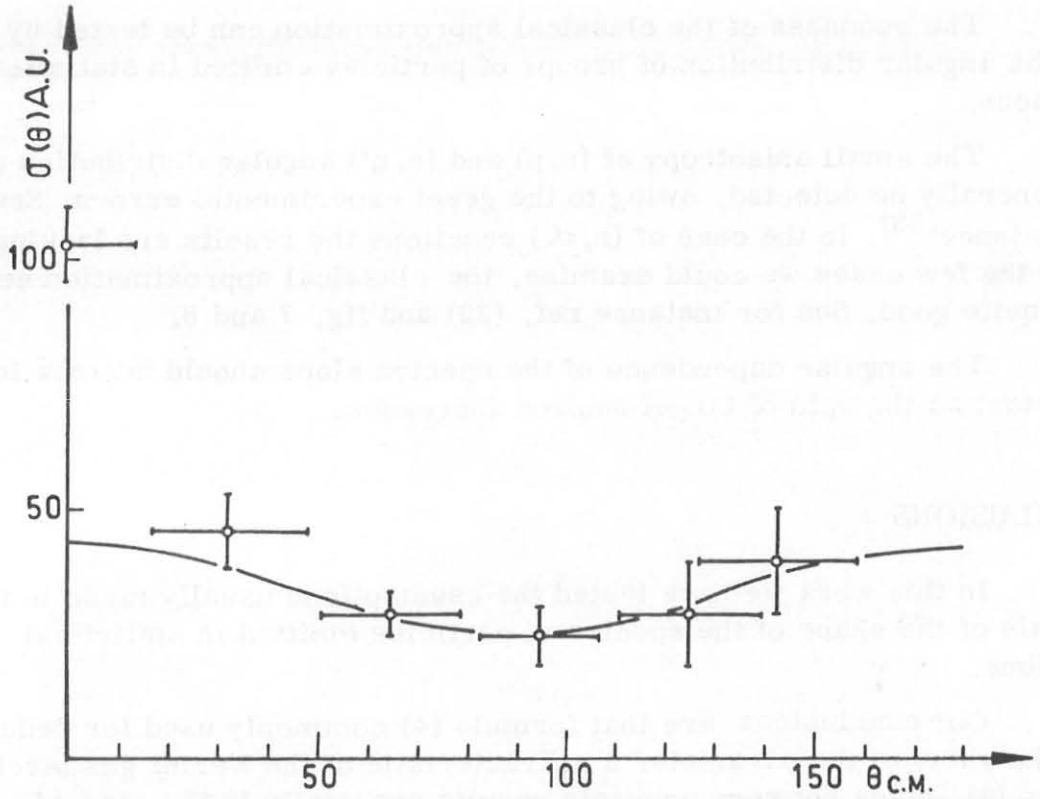

FIG. 7 - Angular distribution of $P^{31}(n,\alpha)^{28}$ Al alpha particles emitted with an energy of 5.5 - 10 MeV[24]. The solid line is the theoretical angular distribution given by (13); η is calculated as described in the text.

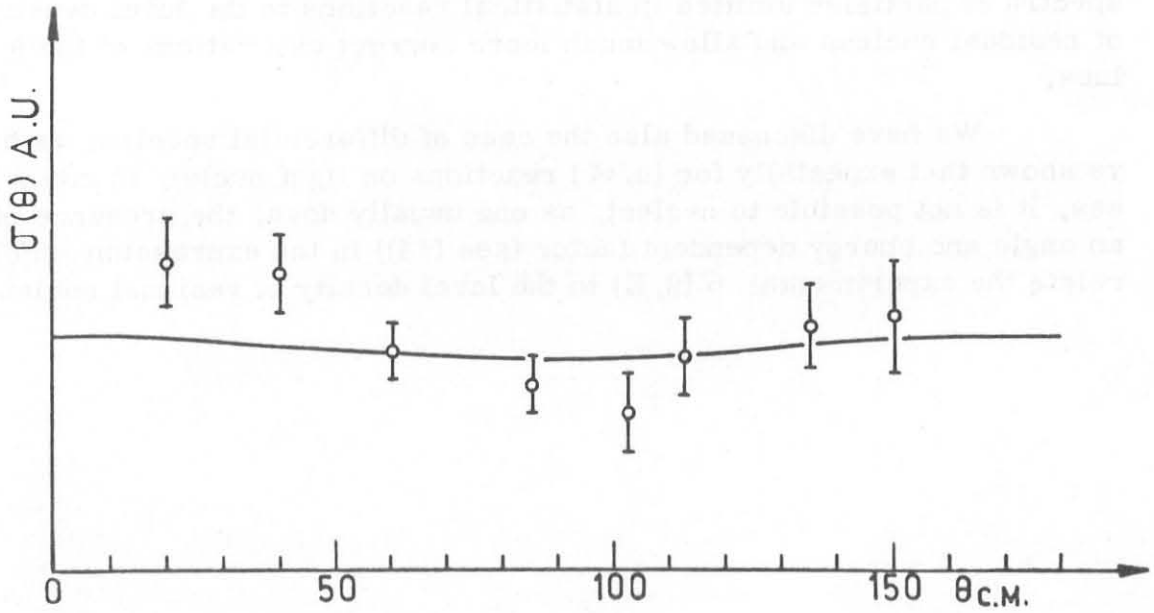

FIG. 8 - Angular distribution of $Ni^{58}(n,\alpha)Fe^{55}$ alpha particles[25]. The solid line is the theoretical angular distribution given by (13); η is calculated as described in the text.

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