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L. Taffara and V. Vanzani : INVESTIGATION OF FEYNMAN DIAGRAM MECHANISMS IN (^{3}He,α). -

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L. Taffara[(o)] and V. Vanzani: INVESTIGATION OF FEYNMAN DIAGRAM MECHANISMS IN (^{3}He,α).[(x)]

SUMMARY. -

It is pointed out that the difficulties arising from the interpretation of the (^{3}He,α) reactions can be overcome by applying the Feynman diagram technique. The cluster triangle graph contributions are studied in order to disclose the reaction mechanisms. Angular distributions up to 90° for the reaction ^{10}B(^{3}He,α)^{9}B at 5.5 MeV and excitation function at 30° are calculated. Satisfactory agreement with experimental data is obtained.

1. - INTRODUCTION. -

Studies of the (^{3}He,α) reaction have pointed out that in general the reaction does not proceed via the simple pick-up of a neutron from the target nucleus in analogy with the (p, d) reaction[(1)]. Other types of direct interaction mechanisms, such as the so-called knock-out, heavy-particle stripping and nucleon cluster substitution, have been suggested to explain the (^{3}He,α) reaction features[(2, 3)]. It has been found that these mechanisms are relevant for light target nuclei, whereas the simple pick-up becomes predominant for heavier nuclei: in fact, in the latter case the data have been successfully interpreted by assuming a pick-up mechanism and applying the distorted-wave theory[(4)].

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Such a theory applied to (^{3}He, α) reactions on light nuclei has obtained less success[5]; the observed discrepancies are brought about by the inadequacy of the distorted-wave pick-up theory used for these processes.

A new approach to direct reactions, based on the polology of the Feynman diagram amplitudes, is suitable for taking into account mechanisms which are more complicated than the simple pick-up or stripping ones[6, 7, 8]: according to this method, direct processes are described by one or several Feynman diagrams whose singularities are close to the physical region of the kinematic variables. The different role played in the (p, d) and (^{3}He, α) reactions by the simple pick-up mechanism is qualitatively explained by the location of the singularities of the pole graph; this is true also for more complex reaction mechanisms.

Owing to the low deuteron binding energy, in the (p, d) reactions the pole graph singularity in momentum transfer is rather close to the boundary of the physical region, whereas the nearest singularities corresponding to other possible graphs (triangle graphs) are generally located approximately from 30 to 50 times farther than the pole from the physical region. On the contrary, owing to the tightly bound nature of the alpha particle, in the (^{3}He, α) reactions the pole is located much farther from the physical region than in the (p, d) reaction. Furthermore, the nearest triangle singularities are relatively close to the pole, especially for some light nuclei. Some illustrative examples, concerning the triangle graph represented in Fig. 1, are shown in Table I: it is seen that, if the virtual nucleus "3" has an α-particle structure, the singularity is rather close to the pole. For heavier nuclei, the triangle singularities are in general far from the pole. However, the most important direct mechanisms are not only selected on the basis of the positions of the singularities, but also by the magnitude of the vertex functions[3, 9].

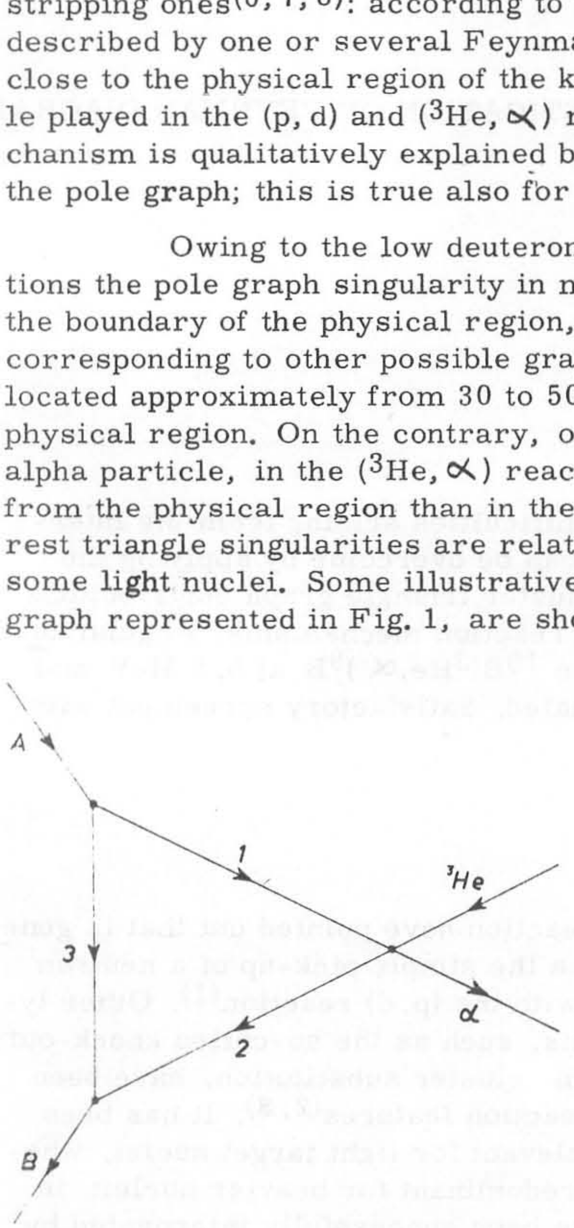

FIG. 1 - Triangle graph mechanism for the A(^{3}He, α)B reaction.

2. - TRIANGLE DIAGRAM AMPLITUDE. -

The aim of the present work is to calculate the amplitude of the triangle graph represented in Fig. 1, by using the simplified one-particle model for the three-ray vertices and assuming that the virtual reaction "1"(^{3}He, α)"2" proceeds by means of a simple neutron transfer. Under these assumptions the diagram turns out to be a pole graph with a cluster triangle graph describing the vertex $A \to B + n$. This brings to view that one needs to evaluate a nuclear cluster contribution to the simple pick-up amplitude. The results will be compared with the experimental data, for not too much large angles, where exchange contributions may become important.

TABLE I

Nearest triangle singularities to the physical region boundary in the variable t for some ($^3He,\alpha$) reactions on light nuclei. The physical region is $t \leq 0$. The units for t are MeV· atomic mass - unit. The pole position is $t_0=41.2$ MeV· AMU.

Target A	Virtual particles 1	2	3	Singularity $\vert t_\Delta \vert$
Li^7	t	d	α	89.4
Be^9	He^5	α	α	88.6
B^{10}	d	p	Be^8	51.4
C^{12}	d	p	B^{10}	231.6
N^{14}	d	p	C^{12}	97.5
O^{16}	d	p	N^{14}	192.3
F^{18}	d	p	O^{16}	69.3

According to the Feynman diagram rules, the pole graph amplitude M_{fi} for the process A(a, b)B, in which a neutron is transferred from the nucleus "A" to the nucleus "a" has the form

$$M_{fi} = 2m_n V \frac{\sum_{M_n} M_A^{Bn}(\Delta) M_{na}^{b}}{k_n^2 - 2m_n E_n} \tag{1}$$

and the triangle vertex amplitude M_A^{Bn} is expressed as

$$M_A^{Bn}(\Delta) = -\frac{i}{(2\pi)^4} 8 m_1 m_2 m_3 V^3 \times$$

$$\sum_{M_1 M_2 M_3} \int \frac{M_A^{13} M_{23}^{B} M_1^{2n} d\bar{k}_3 dE_3}{(k_1^2 - 2m_1 E_1)(k_2^2 - 2m_2 E_2)(k_3^2 - 2m_3 E_3)} \tag{2}$$

In eqs. (1) and (2) M_i denote the z-components of the spins s_i, m_i the masses, $\bar{k}_i$ and E_i the moments and energies and V the normalizing volume. The three-ray vertex amplitudes M_z^{xy} in the one-particle model can be written as

(3)
$$M_z^{xy} = \frac{\sqrt{2}\pi\chi_z}{\mu_{xy} V^{3/2}} \sum_{s\ell} F_{s\ell}(k_{xy}) \gamma_{s\ell} \langle s_x s_y, M_x M_y | s M_s \rangle \times$$
$$\times \langle s\ell, M_s M_\ell | s_z M_z \rangle Y_\ell^{M_\ell}(\hat{k}_{xy}) ,$$

where s, ℓ are the total spin and relative orbital moment of the pair of the particle x and y, $\bar{k}_{xy}$ their relative linear moment, μ_{xy} their reduced mass and $\chi_z = \sqrt{2\mu_{xy}\varepsilon_z}$, with ε_z the binding energy of the x or y particle in the nucleus z. The reduced vertex parts $\gamma_{s\ell}$ are related to the spectroscopic factors. The form factor $F_{s\ell}(k_{xy})$ is normalized to the unity as in ref. (10).

Let us express the relative moments $\bar{k}_{xy}$ in the laboratory system ($\bar{k}_A = 0$) in terms of the integration variables $\bar{k}_3$

(4)
$$\bar{k}_{13} = \bar{k}_3, \quad \bar{k}_{23} = \bar{k}_3 - \frac{m_3}{m_B}\bar{k}_B, \quad \bar{k}_{2n} = \frac{m_n}{m_1}\bar{k}_3 - \bar{k}_B.$$

The integration over E_3 in eq. (2) can be easily performed by using the residue theorem and the integration over the azimuthal angle of the moment $\bar{k}_3$ by choosing the polar axis along the direction of the moment $\bar{k}_B$. The integral over the remaining variables reads ($u = \cos\theta_{13}$, $v = \cos\theta_{23}$, $w = \cos\theta_{2n}$)

(5)
$$I_{\ell_{13}\ell_{23}\ell_{2n}}^{M_{13}M_{23}M_{2n}} = \int_0^\infty k_3^2\, dk_3 \quad \times$$
$$\times \int_{-1}^{1} \frac{P_{\ell_{13}}^{M_{13}}(u) P_{\ell_{23}}^{M_{23}}(v) P_{\ell_{2n}}^{M_{2n}}(w) F_{\ell_{13}}(k_{13}) F_{\ell_{23}}(k_{23}) F_{\ell_{2n}}(k_{2n}) du}{(k_{13}^2 + \chi_A^2)(k_{23}^2 + \chi_B^2)}$$

and is evaluated numerically assuming suitable expressions for the form factors. The differential cross section in the centre of mass system is connected with the amplitude M_{fi} by the relation

$$\frac{d\sigma_{fi}}{d\Omega} = V^4 \frac{\mu_i \mu_f}{(2\pi)^2} \frac{p_f}{p_i} \frac{1}{(2s_a+1)(2s_A+1)} \sum_{M_a M_A M_b M_B} |M_{fi}|^2 , \tag{6}$$

where $\bar{p}_i$ and $\bar{p}_f$ are the relative moments in the initial and final channel. In the present calculations the initial and final state interactions are neglected.

3. - NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTAL DATA FOR ^{10}B TARGET. -

A tentative calculation has been made for the reaction ^{10}B $(^3He,\alpha)^9B$. Taking as virtual particles "1"=d, "2"=p, "3"=^8Be, one obtains a triangle singularity very near to the pole (see Table I). The relative orbital moments taken into account are $\ell_{13}=2$, $\ell_{23}=1$, $\ell_{2n}=$ $=\ell_{n^3He}=0$. The form factors of the deuteron and alpha particle are assumed to be constant, while for the form factor of the ^{10}B and ^9B use has been made of the crude approximation $F_\ell(k_{xy}) \propto (k_{xy}^2 + \chi_z^2) j_\ell(k_{xy} R_z)$, R_z being the nuclear cluster parameter. The angular distribution is calculated for a bombarding energy of 5.5 MeV in the laboratory system. A suitable constant has to be added to the calculated cross section, in order to take into account the corresponding compound nucleus isotropic background(10). In Fig. 2 the theoretical results are compared with the experimental data, taken from ref. (11). The position of the maxima and the ratio between the 0^o cross section and the main peak are reproduced for R_A = 4.1 fm, R_B = 3.5 fm. The small oscillations which appear with the increase of the angle should be damped by the contributions of the exchange mechanisms. The relatively small value obtained for the cluster parameters can reflect the less peripherical nature of the triangle graph with respect to the simple pole graph.

In Fig. 3 the excitation function calculated at 30^o (c. m.) is compared with the experimental results, taken from ref. (12); we have normalized to the experimental value for 5.5 MeV. It appears that the cluster triangle graph mechanism gives a satisfactory fit, whereas the simple pick-up calculation (R=6 fm) disagrees with the data. At low energy poor agreement is obtained, because in this region one cannot neglect the contributions from graphs which are more complex than the triangle ones, and describe quasi-compound nucleus effects.

In conclusion, the obtained results show that the direct mechanisms differing from the simple pick-up must be taken into account in the $(^3He,\alpha)$ reactions. The contribution of the knock-out process and other exchange mechanisms can be evaluated in similar way, by introducing suitable representative triangle graphs. Finally, it seems worthwhile to remark that the difficulties connected with the interpretation of

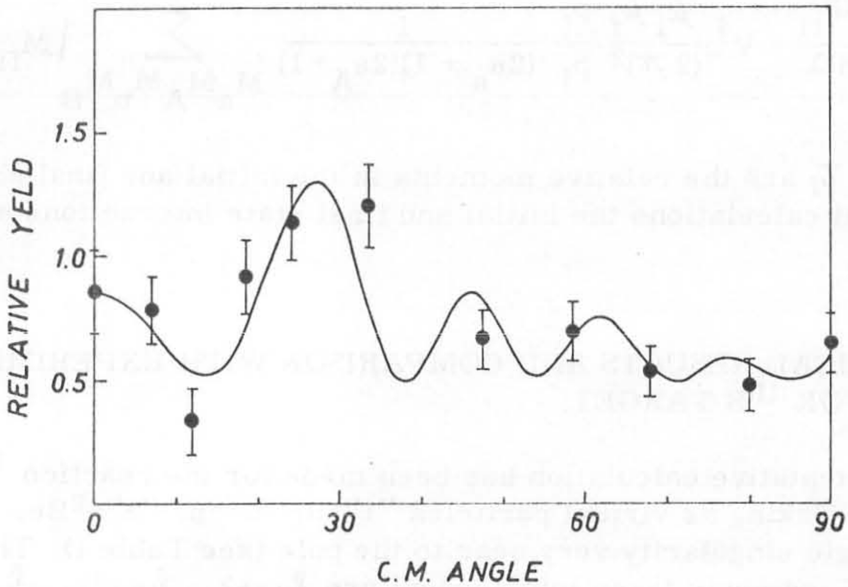

FIG. 2 - Angular distribution of ground state alpha particles from $^{10}B(^{3}He,\alpha)^{9}B$ at $E_{3He} = 5.5$ MeV. Experimental data are taken from ref. (11). Solid curve corresponds to cluster triangle graph calculation.

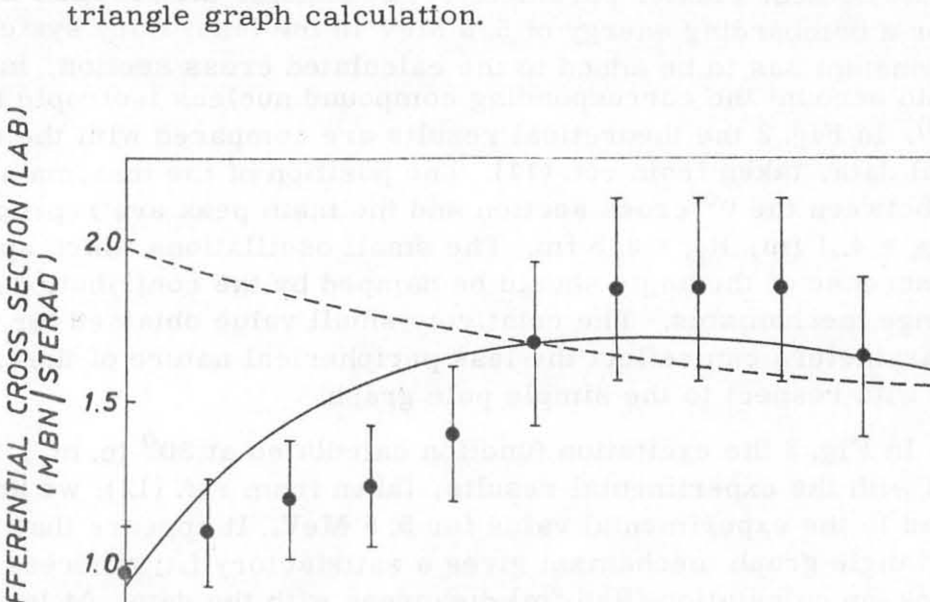

FIG. 3 - Excitation function for ground state alpha particles from $^{10}B(^{3}He,\alpha)^{9}B$ at 26° (Lab). Experimental data are taken from ref. (12). Solid curve corresponds to the cluster triangle graph calculation, dashed curve to the simple pick up calculation.

the (^{3}He,α) reactions can be explained by means of an appropriate analysis of the role of different mechanisms in terms of the corresponding Feynman diagrams.

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