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L. Taffara e V. Vanzani : FEYNMAN DIAGRAM APPROACH IN HEAVY ION INTERACTIONS. -

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L. Taffara[(o)] e V. Vanzani: FEYNMAN DIAGRAM APPROACH IN HEAVY ION INTERACTIONS[(x)]. -

The application of polology methods to direct nuclear reactions has proved useful in the analysis of the transfer of a single particle by a system of particles (e. g. nucleon-nucleus interactions)[(1)]. This new approach provides additional insight into the limitations of the usual approximation schemes (e. g. the Butler stripping theory, the Distorted-Wave Born Approximation) and has suggested new calculation schemes and experiments. Recently, attempts have been made to reformulate the usual direct interaction theories for rearrangement scattering of nuclei by nuclei[(2, 3)], but the results so far obtained are not always satisfactory.

Our purpose is to extend the Feynman diagram techniques, which are successful in the nucleon-nucleus case, to the transfer reactions between two complex nuclei.

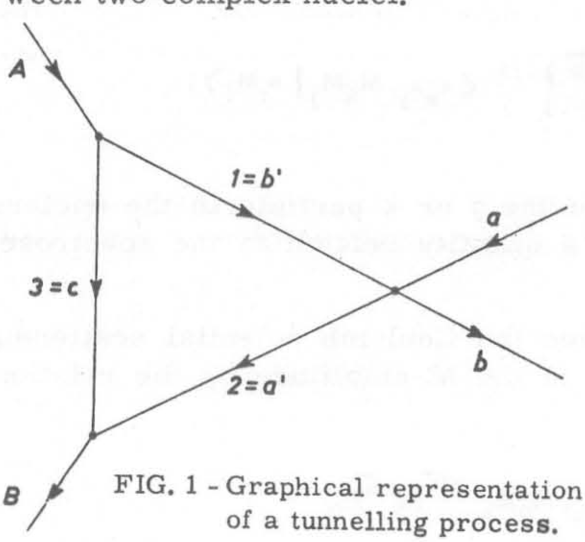

FIG. 1 - Graphical representation of a tunnelling process.

The triangular graph description of a rearrangement process with a neutron transfer represented in Fig. 1 is a very reasonable physical way of representing the well known tunnelling process[(2)].

The rearrangement process

(1) $a + (b + c) \rightarrow b + (a + c)$

is assumed for the reaction A(a, b)B: the reaction proceeds by the tunnel-

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ling of the particle c from the bound state b + c to the other a + c, while the "cores" a and b scatter via the Coulomb potential. In the simplest formulation of this process(2), the particle c is a neutron and the low energy case is considered, so that the scattering effects of all potentials except the Coulomb interaction can be neglected. Therefore, in our approach the nuclear amplitude vertex parts (form factors) describing the virtual decay of the nucleus A and the virtual synthesis of the nucleus B are assumed constant (zero range interaction).

According to the Feynman diagram rules, the amplitude M_{fi} of the process represented in Fig. 1 has the form

$$M_{fi} = -\frac{i}{(2\pi)^4} 8m_1m_2m_3V^3 \sum_{M_1M_2M_3} \int \frac{M_A^{13}\, M_{23}^{B}\, M_{1a}^{2b}\, d\bar{k}_3\, dE_3}{(k_1^2 - 2m_1E_1)(k_2^2 - 2m_2E_2)(k_3^2 - 2m_3E_3)} \tag{2}$$

and is connected with the differential cross section in the centre of mass system by the relation

$$\frac{d\sigma_{fi}}{d\Omega} = V^4 \frac{\mu_i \mu_f}{(2\pi)^2} \frac{p_f}{p_i} \frac{1}{(2s_a+1)(2s_A+1)} \sum_{M_aM_AM_bM_B} |M_{fi}|^2 , \tag{3}$$

where s_i denote spins, M_i their z - components, m_i the masses, μ_i the reduced masses, $\bar{k}_i$ and E_i the moments and energies, $\bar{p}_i$ and $\bar{p}_f$ the moments in the initial and final channel, V the normalizing volume. The vertex amplitudes M_i^{jk}, calculated in the usual one-particle model with the assumed zero-range interactions, are the following constant quantities

$$M_i^{jk} = \frac{\gamma_i}{\mu_{jk} V^{3/2}} \left\{ \frac{\pi}{2} \sqrt{2\mu_{jk}\varepsilon_i^{jk}} \right\}^{1/2} \langle s_k s_j, M_k M_j | s_i M_i \rangle , \tag{4}$$

where ε_i^{jk} is the binding energy of the j or k particle in the nucleus i and γ_i the reduced vertex part, a quantity related to the spectroscopic factor of the nucleus i.

It is convenient to introduce the Coulomb potential scattering amplitude f $(\bar{k}_{1a}, \bar{k}_{2b}, E)$ connected to the M-amplitude by the relation

$$M_{1a}^{2b} = \frac{2\pi}{V^2 \mu_{ab}} \delta_{M_1M_b, M_2M_a} f(\bar{k}_{1a}, \bar{k}_{2b}, E), \tag{5}$$

$\bar{k}_{1a}$ and $\bar{k}_{2b}$ being the relative moments before and after the core-core scattering, and E the centre of mass kinetic energy for the scatte ring process. The Coulomb amplitude f satisfies the following integral equations

(6a) $$f(\bar{k}_{1a},\bar{k}_{2b},E) = -\frac{2\mu_{ab}Z_aZ_be^2}{|\bar{k}_{1a}-\bar{k}_{2b}|^2} + \frac{Z_aZ_be^2}{2\pi^2}\int\frac{f(\bar{k},\bar{k}_{2b},E)d\bar{k}}{|\bar{k}_{1a}-\bar{k}|^2(E-\frac{k^2}{2\mu_{ab}}+i\varepsilon)}$$

(6b) $$f(\bar{k}_{1a},\bar{k}_{2b},E) = -\frac{2\mu_{ab}Z_aZ_be^2}{|\bar{k}_{1a}-\bar{k}_{2b}|^2} + \frac{Z_aZ_be^2}{2\pi^2}\int\frac{f(\bar{k}_{1a},\bar{k},E)d\bar{k}}{|\bar{k}-\bar{k}_{2b}|^2(E-\frac{k^2}{2\mu_{ab}}+i\varepsilon)}$$

If the particles "1" and "2" were on the mass shell (i.e., the moments would be connected with the kinetic energies by the relations $E_i = k_i^2/2m_i$), one obtains the well-known on-energy-shell ($k_{1a}^2 = k_{2b}^2)(2\mu_{ab}E)$ expression for the Coulomb amplitude. The exact off-energy-shell solution of Eqs. (5) can be written in terms of complicated hypergeometric functions[4].

Let us consider the integral

(7) $$I = \int\frac{f(\bar{k}_{1a},\bar{k}_{2b},E)d\bar{k}_3dE_3}{(k_1^2-2m_1E_1)(k_2^2-2m_2E_2)(k_3^2-2m_3E_3)}$$

and express the moments $\bar{k}_{13}$, $\bar{k}_{23}$, $\bar{k}_{1a}$, $\bar{k}_{2b}$ of the relative motion of particles "1" and "3", "2" and "3", etc., and the energy E in terms of the integration variables $\bar{k}_3$, E_3 and the channel moments $\bar{p}_i$, $\bar{p}_f$ (c.m. system)

(8a) $$\bar{k}_{13} = \bar{k}_3 + \frac{m_3}{m_A}\bar{p}_i \qquad \bar{k}_{23} = \bar{k}_3 + \frac{m_3}{m_B}\bar{p}_f$$

(8b) $$\bar{k}_{1a} = \bar{p}_i + \frac{m_a}{m_a+m_b}\bar{k}_3 \qquad \bar{k}_{2b} = -\bar{p}_f - \frac{m_b}{m_a+m_b}\bar{k}_3$$

(8c) $$E = \frac{p_i^2}{2\mu_i} - \left[\varepsilon_A^{13} + E_3 + \frac{k_3^2}{2(m_a+m_b)}\right]$$

The integration over E_3 can easily be performed by considering only the on-energy-shell contribution to Coulomb amplitude. By using the residue method and trivial kinematic relations, one obtains

4.

$$\int \frac{dE_3}{(k_1^2 - 2m_1E_1)(k_2^2 - 2m_2E_2)(k_3^2 - 2m_3E_3)} = -\frac{i\pi\mu_{13}\mu_{23}}{m_1m_2m_3} \times$$

(9)

$$\times \frac{1}{(k_{13}^2 + 2m_{13}\varepsilon_A^{13})(k_{23}^2 + 2m_{23}\varepsilon_A^{23})}$$

Therefore, the amplitude M_{fi} in the on-energy-shell approximation takes the form

(10)
$$M_{fi} = -\frac{\gamma_A \gamma_B C_A^{13} C_B^{23}}{V^2 \pi^2 \mu_{ab}} \int \frac{f(|\bar{k}_3 - \bar{\Delta}|)d\bar{k}_3}{(k_3^2 + 2m_{13}\varepsilon_A^{13})(k_3^2 + 2m_{23}\varepsilon_B^{23})}$$

where $C_i^{jk} = \left(\frac{\pi}{2}\sqrt{2\mu_{jk}\varepsilon_i^{jk}}\right)^{1/2} \langle s_k s_j, M_k M_j | s_i M_i \rangle$ and the momentum transfer in the core-core scattering is expressed as $|\bar{k}_{1a} - \bar{k}_{2b}| = \bar{k}_3 + \bar{p}_i + \bar{p}_f = \bar{k}_3 - \Delta$. The amplitude $f(|\bar{k}_3 - \bar{\Delta}|)$ is the usual Coulomb amplitude. It is assumed $m_A \gg m_3$, $m_B \gg m_3$ (the particle "3" is a neutron). "The amplitude of the triangle graph in Fig. 1 expressed on-energy-shell by Eq. (10) corresponds to the amplitude obtained by Greider (Ref. (2) formula (33)) in the T-matrix formalism; the integral (10) can be evaluated in the same way".

The on-energy-shell approximation is supported by the satisfactory results so far achieved by the Greider method. However, it is worthwhile to look for a rigorous justification of this approximation and provide a mathematical estimation of the off-energy-shell contributions. Now we attempt to carry out such an investigation, by taking into account the exact expressions (6) for the Coulomb amplitude in the integral (7).

The success of the description based on the triangle graph of the Fig. 1 points out the dominance of this mechanism with respect to the pole graph corresponding to a simple neutron transfer, without Coulomb scattering. The predominant role of the triangle graph is mainly due to the large contribution of the four-ray Coulomb vertex amplitude. It occurs in spite of the considerable remoteness of the triangle singularity in comparison with the pole from the physical region boundary and shows that the relative role of different mechanisms is determined not only by the position of the singularities, but also by the magnitude of the vertex functions.

This preliminary attempt to calculate the direct heavy-ion amplitudes by using the Feynman diagrams points out the possibility of applying the general dispersion relation methods to the new field of research concerning the complex nuclei interactions, which has been opened by Tandem accelerators.

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