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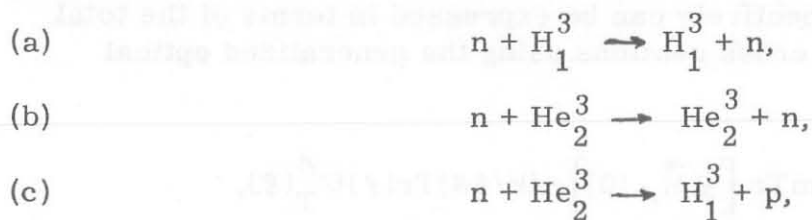
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L. Drigo, G. Moschini^(x) and C. Villi: ON THE SCATTERING OF NEUTRONS
BY H_1^3 AND He_2^3 NUCLEI^(o). -

The increasing demand of nuclear data needed for applicational purposes has widely stimulated precision measurements of neutron cross sections. Extensive programmes are now in progress, in order to co-ordinate and often reconcile conflicting experimental results⁽¹⁾; these time and money consuming efforts are generally based on empirical and phenomenological grounds, the drawback of which in many cases becomes apparent when gaps are filled by interpolation or unknown nuclear parameters are predicted by extrapolation.

The purpose of this note, and that of the subsequent ones, which will be published elsewhere, is to emphasize some critical aspects involved by the analysis of unpolarized neutron data, which are passed without comment in the evaluation work.

We shall begin by examining the following four nucleon processes



which are relevant both for fundamental research and for practical needs⁽²⁾.

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Assuming that the two mirror nuclei H_1^3 and He_2^3 can be described as an isospin doublet, the scattering amplitudes of the two elastic processes (a) and (b) and the exchange process (c) in charge space are

$$\begin{aligned}
 (1a) \quad \vec{f}_a &\equiv \langle nH_1^3 | \vec{\Omega} | nH_1^3 \rangle = \vec{f}_1, \\
 (1b) \quad \vec{f}_b &\equiv \langle nHe_2^3 | \vec{\Omega} | nHe_2^3 \rangle = \frac{1}{2}(\vec{f}_1 + \vec{f}_0), \\
 (1c) \quad \vec{f}_c &\equiv \langle nHe_2^3 | \vec{\Omega} | pH_1^3 \rangle = \frac{1}{2}(\vec{f}_1 - \vec{f}_0),
 \end{aligned}$$

where $\vec{\Omega}$ is a transition operator conserving the total isospin T , and f_T the corresponding amplitude ($T=0,1$). From Eqs (1) it is found

$$(2) \quad \sigma_0(\theta) = |\vec{f}_0|^2 = 2 \{ \sigma_b(\theta) + \sigma_c(\theta) \} - \sigma_a(\theta),$$

where $\sigma_\alpha(\theta) = |\vec{f}_\alpha|^2$ ($\alpha = a, b, c$) are the measured unpolarized angular distributions. The dynamical prescriptions arising from a presumptive conservation of total isospin are somewhat concealed by Eq. (2), because the elastic scattering from a pure isospin state $T=0$ is unobservable, except perhaps as final state interaction in the $D+D \rightarrow He^3+n$ reaction. The condition $\sigma_0(\theta) \geq 0$, which is only necessary for the validity of the charge independence (CI) hypothesis, is implicit in the literature⁽³⁾.

From Eqs (1) and Eq. (2) it is obtained

$$(3) \quad F(k) \equiv (s_a + s_b - s_c) / (R_a R_b + I_a I_b) = 2,$$

where $k^2 \sigma_\alpha(0) \equiv s_\alpha$, $k \text{Re} \vec{f}_\beta(0) \equiv R_\beta$ and $k \text{Im} \vec{f}_\beta(0) \equiv I_\beta$ ($\beta = a, b$). Equation (3) expresses in a necessary and sufficient way the constraints imposed by CI on the observable quantities s_α , R_β and I_β : if the equality (3) turns out to be not fulfilled, either the reliability of the data or the validity of the total isospin conservation, or both, come into question.

The real and imaginary parts of the forward scattering amplitudes (FSA) R_β and I_β respectively can be expressed in terms of the total unpolarized and polarized cross sections using the generalized optical theorem⁽⁴⁾

$$(4) \quad \text{ImTr} \left[\rho \vec{M}_\beta(0) \right] = (k/4\pi) \text{Tr}(\rho) \sigma_T^\beta(\varrho),$$

where ρ is the density matrix representing the initial polarization, $\vec{M}_\beta(0)$ is the forward scattering matrix in spin space, and $\sigma_T^\beta(\varrho)$ is the corresponding total cross section. The general form of $\vec{M}_\beta \equiv \vec{M}_\beta(0)$ is

$$(5) \quad \vec{M}_\beta = A_\beta + B_\beta \sigma_z^{(n)} \sigma_z^{(t)} + C_\beta \vec{\sigma}^{(n)} \cdot \vec{\sigma}^{(t)},$$

where $\vec{\sigma}^{(n)}$ and $\vec{\sigma}^{(t)}$ are the Pauli spin operators for the incident neutron and the spin 1/2 target nucleus, and A_β , B_β , C_β are scalar amplitudes.

For the incident neutrons and target nuclei having independent polarization, the density matrix reads

$$(6) \quad \rho = \frac{1}{4} \left\{ \hat{1} + \vec{P}_n \cdot \vec{\sigma}^{(n)} \right\} \left\{ \hat{1} + \vec{P}_t \cdot \vec{\sigma}^{(t)} \right\}.$$

If $\vec{P}_n = \vec{P}_t = 0$, the unpolarized total cross section $\sigma_T^\beta(\hat{1}) \equiv \sigma_T^\beta$ satisfies the relation

$$(7) \quad k \text{Im} A_\beta = (k^2/4\pi) \sigma_T^\beta \equiv S_\beta,$$

which expresses the optical theorem for particles with spin-dependent FSA. Since for unpolarized incident neutrons one has

$$(8) \quad R_\beta = \frac{1}{2} k \left\{ \text{Tr}(\text{Re} \vec{M}_\beta)^2 \right\}^{\frac{1}{2}}, \quad I_\beta = \frac{1}{2} k \left\{ \text{Tr}(\text{Im} \vec{M}_\beta)^2 \right\}^{\frac{1}{2}},$$

it is found

$$(9) \quad R_\beta = (\Sigma_\beta^2 - \xi_\beta^2)^{\frac{1}{2}} \geq 0, \quad I_\beta = (S_\beta^2 + \xi_\beta^2)^{\frac{1}{2}} > 0,$$

where $\Sigma_\beta^2 = s_\beta - S_\beta^2$ and ξ_β is the contribution to the imaginary parts of FSA arising from spin-dependent terms. From Eq. (5) and Eq. (6) it follows that ($P_{nt} \equiv P_n P_t$)

$$(10) \quad \xi_\beta^2 = (2/P_{nt}^2)(S_\beta' - S_\beta)^2 + (1/P_{nt}^2)(S_\beta'' - S_\beta)^2,$$

where $\sigma_t'^2 \equiv (4\pi/k^2)S_\beta'$ and $\sigma_t''^2 \equiv (4\pi/k^2)S_\beta''$ are the polarized total cross sections measured with \vec{P}_n and \vec{P}_t parallel and transverse to the beam, parallel and along the beam (z -axis) respectively⁽⁵⁾.

The values of s_α , $s_0 \equiv k^2 \sigma_0(0)$, S_β and Σ_β^2 , obtained from the available data in the energy interval $1.0 \text{ MeV} \leq E_n \leq 8.0 \text{ MeV}$ are listed in Table I. No reaction channel is open for the process (a); the reactions competing with the process (b) have Q -values 0.764 MeV ($n + \text{He}_2^3 \rightarrow \text{H}_1^3 + p$) and -3.266 MeV ($n + \text{He}_2^3 \rightarrow \text{D} + \text{D}$). It is seen that, even in the absence of spin-dependent terms ($\xi_\beta = 0$), the Wick's inequality is not satisfied by the data concerning the process (b) at $E_n = 1.0 \text{ MeV}$, 5.0 MeV , 6.0 MeV and 8.0 MeV . Whether such an inconsistency arises from a too small value of the extrapolated forward cross section or from an abnormally large value of

TABLE I

Values of s_a , s_0 , s_b and Σ_a^2 as a function of the incident neutron energy E_n (in MeV) ($k^2=2.7084E_n$ barn $^{-1}$)

E_n	$s_a^{(x)}$	$s_b^{(x)}$	$s_c^{(o)}$	s_0
1.0	0.160+0.030	0.374+0.038	0.160+0.008	0.908+0.084
2.0	0.877+0.184	2.248+0.060	0.455+0.022	4.529+0.224
2.6	-	2.943+0.176	0.514+0.028	-
3.5	3.801+0.426	4.986+0.066	0.502+0.028	7.175+0.447
5.0	-	6.162+0.812	0.379+0.014	-
6.0	7.410+0.260	7.183+0.455	0.309+0.016	7.574+0.946
8.0	-	8.818+1.300	0.325+0.021	-
E_n	$s_a^{(+)}$	$s_b^{(-)}$	Σ_a^2	Σ_b^2
1.0	0.366+0.011	0.619+0.013	0.026+0.032	-0.009+0.041
2.0	0.914+0.021	1.402+0.026	0.042+0.187	0.282+0.095
2.6	1.306+0.028	1.682+0.034	-	0.114+0.208
3.5	1.834+0.038	2.090+0.045	0.437+0.451	0.618+0.198
5.0	2.415+0.054	2.555+0.065	-	-0.367+0.877
6.0	2.626+0.065	2.795+0.078	0.514+0.428	-0.629+0.630
8.0	2.777+0.086	3.053+0.103	-	-0.503+1.442

(x) - From the data of Seagrave et al. at $E_n=1.0$ MeV, 2.0 MeV, 3.5 MeV and 6.0 MeV (Phys. Rev. 119, 1981 (1960)); from the data of Sayres et al. at $E_n=2.6$ MeV, 5.0 MeV and 8.0 MeV (Phys. Rev. 122, 1853 (1961)).

(o) - Calculated by detailed balance from $H_1^3(p, n)He_2^3$ measurements (H. B. Willard et al., Phys. Rev. 90, 865 (1953); M. D. Goldberg et al., Phys. Rev. 122, 1510 (1961)).

(+) - L. Stewart, Report La-3270 (1965). See also Los Alamos Physics and Cryogenics Groups, Nucl. Phys. 12, 291 (1959).

(-) - From the data of Seagrave et al. at $E_n=1.0$ MeV, 2.0 MeV, 3.5 MeV and 6.0 MeV (Phys. Rev. 119, 1981 (1960)); interpolated values at $E_n = 2.6$ MeV, 5.0 MeV and 8.0 MeV (R. Batchelor and K. Parker, AWRE Report 0-78/64).

the unpolarized total cross section, or from both, it is a matter to be ascertained on the basis of renewed measurements.

In this connection, it has to be pointed out that the angular distribution of the elastically scattered neutrons measured in a plane not orthogonal to that in which the neutron production occurs, might be partially polarized, namely

$$(11) \quad \sigma_{\beta}^{\text{pol}}(\theta) = \sigma_{\beta}(\theta) \left\{ 1 + \vec{P}_n \cdot \vec{A}(\theta) \right\}$$

where the asymmetry $A(\theta)$ can be determined by means of a double scattering experiment. It is then clear that the term $\vec{P}_n \cdot \vec{A}(\theta)$ produces an overall angular distortion of the observed cross section as compared to $\sigma_{\beta}(\theta)$ and this may well lead to the paradoxical result $\Sigma_{\beta}^2 < 0$ if the latter is arbitrarily identified with the former: this circumstance might come into play even if the neutron beam were produced in the forward direction because of the finite dimension of the solid angles involved in the experiment.

The above remark, however trivial it may appear, it might nevertheless give the clue for understanding a possible physical, and not instrumental, cause of the conflicting results so frequently obtained in neutron measurements, which evaluators endeavour to reconcile by resorting to fitting subtleties and personal guessings.

Recent technical achievements⁽⁶⁾ have opened the way - at least for the process (b) - to evaluate ξ_{β}^2 directly from Eq. (10); at the moment, however, the quantities $\sigma_T^{\prime\prime\beta}$ and $\sigma_T^{\prime\prime\prime\beta}$ are still unknown and the inequality $\Sigma_{\beta}^2 \geq \xi_{\beta}^2$ cannot be proved to be satisfied by the available data at the energies where the condition $\Sigma_{\beta}^2 \geq 0$ is fulfilled. Using Tombrello's phaseshifts⁽⁷⁾ it is found that the quantity ξ_a^2 satisfies the condition $\Sigma_a \geq \xi_a^2$ (Table II), but nothing can be said about the quantity ξ_b^2 because the cross section $\sigma_0(\theta)$ has never been analyzed in terms of isot singlet phaseshifts.

TABLE II

Values of ξ_a^2 calculated in S and P wave approximation from Tombrello's phaseshifts.

E_n (MeV)	1.0	2.0	3.5	6.0
ξ_a^2	0.007	0.030	0.178	0.481

The close connection existing between CI prescriptions and the spin dependence of the FSA arising from spin-orbit and tensor forces can be readily brought into view by noting that Eq. (3) follows from the system of equations ($\mathcal{J}=0, c$)

$$(12) \quad r_{\mathcal{J}}^2 = (R - X_{\mathcal{J}})^2 + (I - Y_{\mathcal{J}})^2,$$

where $2r_0 = \sqrt{s_0}$, $r_c = \sqrt{s_c}$, $2X_0 = R_a = X_c$ and $2Y_0 = I_a = Y_c$; then, in the (R, I) -plane, the physically meaningful intersection of the two circles of radii $r_{\mathcal{J}}$ and centres located at the point $Q_{\mathcal{J}} = (X_{\mathcal{J}}, Y_{\mathcal{J}})$ has co-ordinates $R=R_b$ and $I=I_b$. This is shown in Fig. 1 for the two limiting cases $\xi_a=0$ and $\xi_a = \Sigma_a$. Due to the experimental uncertainties of the input data s_{α} and S_{β} , the intersection spans the shadowed area. The dotted line is the locus of the point $P_b = (R_b, I_b)$, distant s_b from the origin 0, as a function of ξ_b ; due to the sign ambiguity of the real part of the FSA, the two limiting points $P'_b = (\Sigma_b, S_b)$ are symmetric with respect to $R_b=0$. It is seen that if $\xi_{\beta}=0$ the data at $E_n=3.5$ MeV simulate a violation of CI; in this case it is found $F(k)=1.903 \pm 0.015$ and $F(k)=2.500 \pm 0.365$ according as $\Sigma_a \Sigma_b < 0$ or $\Sigma_a \Sigma_b > 0$.

In conclusion, the lack of experimental knowledge of the quantities ξ_{β} and s_c prevents one from testing the internal consistency of the available data concerning the processes under consideration: this implies that the adjusted data nowadays recommended by evaluators should be taken with caution. Furthermore, the conservation of total isospin and time reversal invariance in the scattering of nucleons by He_2^3 and H_1^3 is still an extrapolated hypothesis not supported by clear-cut experimental evidence: measurements with polarized beams and polarized targets are required in order to evaluate charge breaking effects⁽⁸⁾ and check the reciprocity properties of the S-matrix in four-nucleon processes. Efforts in this direction are now being made by the experimentalists of the Laboratory of Legnaro.

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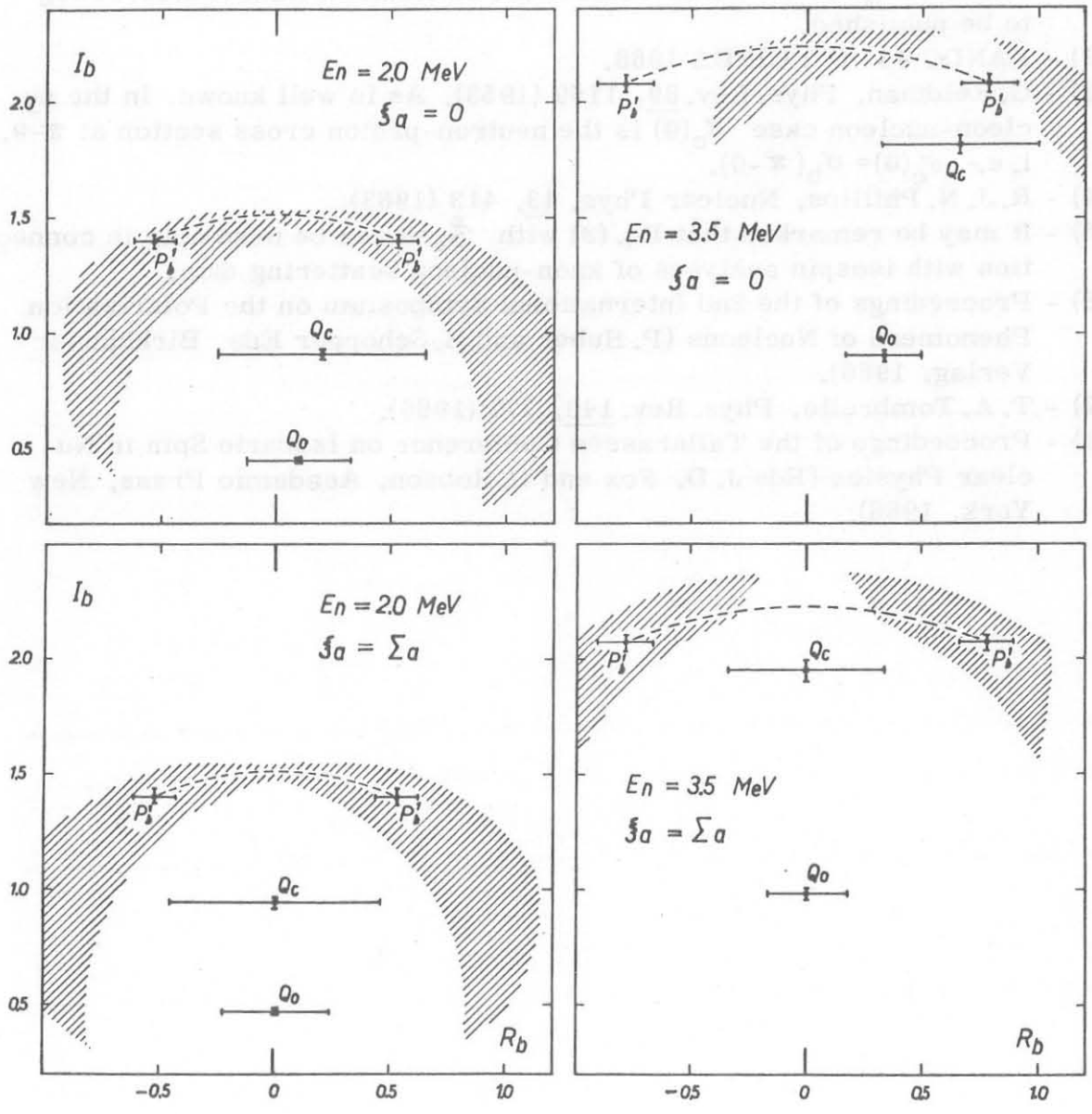


FIG. 1 - Correlation between ξ_a and ξ_b according to the CI hypothesis.

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