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E. Gadioli, I. Iori, A. Marini and M. Sansoni: THE LEVEL  
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PREVISIONS. -

E. Gadioli, I. Iori, A. Marini and M. Sansoni: THE LEVEL WIDTH OF ATOMIC NUCLEI IN THE CONTINUUM ENERGY REGION: COMPARISON WITH THE STATISTICAL MODEL PREVISIONS. (x)

## INTRODUCTION -

The value of the coherence energy  $\Gamma$  is one of the most important informations one can draw from the analysis of the fluctuating excitation function (studied with good energy resolution) of a reaction in which the compound nucleus is excited in the continuum energy region. (1, 2)

A method to obtain a correct value of  $\Gamma$ , from an analysis of experimental results, has been already shown<sup>(3, 4)</sup>.

In this paper we now derive the theoretical expression of  $\Gamma$  (in the framework of the statistical model) that must be compared with the experimental value for a given reaction.

In Section 1 we report the theoretical expression of  $\Gamma$ , usually compared with the experimental values, pointing out the approximations introduced.

In Section 2 we discuss the validity of such approximations and derive more correct expressions for  $\Gamma$ , in the case of integrated cross-sections and purely statistical reactions. We show that  $\Gamma$  is a weighted average over spins, energy and parity, of the widths of the C.N. levels involved in the reaction.

In Section 3 the influence of non statistical effects on the value of  $\Gamma$  is studied.

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2.

In Section 4, at last, we examine the dependence on the emission angle  $\theta$  of the  $\Gamma$  deduced by the analysis of a differential excitation function.

## SECTION 1 -

Let us consider a reaction which proceeds, at least partially, through the formation of a C.N. in the continuum energy region.

All the C.N. levels involved in this reaction are characterized by a width  $\Gamma_J^\pi(E)$  function of their energy, spin and parity. The coherence energy  $\Gamma$  characterizing the fluctuations of the excitation function, is given by a weighted average over energy, spin and parity of all the  $\Gamma_J^\pi(E)$ . The statistical model gives for  $\Gamma_J^\pi(E)$  the following expression:

$$(1) \quad \Gamma_J^\pi(E) = \frac{D_J(E)}{2\pi} \sum_{\nu} \int_0^{E_{\nu}^{\text{Max}}} dE_{\nu} \rho_{0\nu}(E_{\nu}^{\text{x}}) \sum_{l'} T_{l'}^{(\nu)}(E_{\nu}) \sum_{s'} \sum_j F(j)$$

$D_J(E)$  is the spacing, at the energy  $E$ , of the C.N. levels of definite parity and spin  $J$ ,  $\rho_{0\nu}(E_{\nu}^{\text{x}})$  gives the energy dependence of the density of the levels of definite parity of the residual nuclei  $\nu$  to which the C.N. may decay,  $T_{l'}^{(\nu)}$  are the transmission functions of the particles emitted in each of the allowed final channels,  $F(j) = (2j+1) \exp(-j(j+1)/2\zeta_{\nu}^2)$  gives the spin dependence of the residual nuclei level densities,  $\zeta_{\nu}^2$  is the spin cut-off factor.

In the analysis of the fluctuations, a great simplification is obtained making the following approximations:

a) The spin cut-off factors are considered infinitely large. In this approximation all the exponential terms appearing in the spin distribution of the level densities are equal to unity and  $\Gamma_J^\pi(E) \simeq \Gamma^{\text{x}}(E)$ , being

$$(2) \quad \Gamma^{\text{x}}(E) = \frac{D_0(E)}{2\pi^2} \sum_{\nu} (2i_{\nu} + 1) \int_0^{E_{\nu}^{\text{Max}}} K_{\nu}^2 \rho_{0\nu}(E_{\nu}^{\text{x}}) \zeta_{\nu}^{\text{x}}(E_{\nu}) dE_{\nu}$$

where  $\zeta_{\nu}^{\text{x}}(E_{\nu})$  is the inverse cross-section.

b) Be  $E_2 - E_1$  the energy interval in which the excitation functions are measured. In this interval  $\Gamma^{\text{x}}(E)$  is assumed to depend weakly on  $E$  and to be approximately equal to  $\Gamma^{\text{x}}(\bar{E})$  with  $\bar{E} = (E_1 + E_2)/2$ .

If, as it is usually done in the analysis of the experimental results<sup>(5)</sup>, the approximations a) and b) are assumed to be valid, it is:

$$\Gamma \simeq \Gamma^{\text{x}}(\bar{E})$$



## SECTION 2 -

In this Section we examine the extent of validity of the approximations a) and b): at first we consider only integrated cross sections and pu rely statistical reactions.

The approximation a) is rather rough, particularly for light nuclei ( $A \leq 50$ ).

Let us consider a nucleus with  $A \simeq 50$ ,  $\mathcal{J}$  be the moment of inertia of the rigid nucleus (at present experimental data seem to indicate that this is a limit value for the moment of inertia.)<sup>(6)</sup> and  $t=1.5$  MeV the nuclear temperature<sup>(x)</sup> (we assume an excitation energy  $E \simeq 16$  MeV, reasonable value for the excitation energy of the C.N.). Taking  $R = 1.4 A^{1/3}$  fm., we get

$$\mathcal{G}^2 = \frac{\mathcal{J} t}{\hbar^2} = 19.2.$$

With such a value of  $\mathcal{G}^2$ , varying  $J$  from 0 to 5, the term  $\exp(-J(J+1)/2\mathcal{G}^2)$

appearing in the expression of  $D_J(E)$ , varies from 1 to 0.46. The approximation is really poor and it becomes worse as  $A$  and  $E$  decrease. Taking into account all the exponential terms which give the spin distributions,  $\Gamma_J^\pi(E)$  depends on  $J$ , decreasing as  $J$  increases. For example, in the case of the C.N.  $\text{Sc}^{45}$  at an effective excitation energy<sup>(+)</sup>  $E = 16.25$  MeV, there is a factor 2 in the  $\Gamma_J^\pi(E)$  values for  $J = 1/2$  and  $J = 11/2$ . (Here and in the following we use the level density expression given by Lang and LeCouteur<sup>(8)</sup> with the "a" parameters of ref. 7).

For lighter nuclei the dependence of  $\Gamma_J^\pi(E)$  on  $J$  still increases: for instance in the case of  $\text{Si}^{29}$  at  $E \simeq 17.5$  MeV,  $\Gamma_J^\pi(E)$  changes of a factor 3 going from  $J=1/2$  to  $J=11/2$ .

The absolute values of  $\Gamma_J^\pi(E)$  and  $\Gamma^x(E)$  so calculated may be no correct: in fact while the level density  $\rho(E)$  we have used, appears to be correct for  $E \leq 10$  MeV, it seems to increase too much with the energy for  $E \geq 10$  MeV<sup>(10)</sup>. However, at a given energy, expression (1) should allow to evaluate the behaviour of  $\Gamma_J^\pi(E)$  with  $J$ , with a small error.

Going back to the examples cited before, the value of  $\Gamma^x(E)$  results to be larger than  $\Gamma_{1/2}^\pi(E)$ ; in general it is larger than the  $\Gamma_J^\pi(E)$  corresponding to the minimum  $J$  allowed, the largest of all the  $\Gamma_J^\pi(E)$ . It is so an over-estimation of the correct theoretical value of the coherence energy  $\Gamma$  which, indeed, is a proper average value of the different  $\Gamma_J^\pi(E)$  given by the statistical model.

(x) - We have calculated  $t$  from the relation  $E = at^2 - t$  assuming the value of the parameter "a" given by Erba et al.<sup>(7)</sup>.

(+) - The effective excitation energy  $E$  is given by the relation  $E = U - \Delta$  where  $U$  is the usual excitation energy and  $\Delta$  the pairing energy as given by Cameron<sup>(9)</sup>.

Approximation b): the expression (1) gives the energy dependence of  $\Gamma_J^\pi(E)$ . We limit our considerations to the case of a quite slow variation of  $\Gamma_J^\pi(E)$  with the energy. If  $\Delta E$  is the energy interval in which the excitation function is studied, in such a case, it is reasonable to assume that  $\Delta E$  can be divided in  $i$  sub-intervals  $\Delta'E$  into which, although the excitation function shows many fluctuations, all the  $\Gamma_J^\pi(E)$  may be considered approximately constant<sup>(x)</sup>. In the energy interval  $\Delta E$  together with the  $\Gamma_J^\pi(E)$ , in general will vary with  $E$  the average statistical cross-section  $\sigma_c(E)$  around which the fluctuations occur<sup>(11)</sup> and the number  $N(E)$  of uncorrelated channels equally contributing to the reaction:

$$N(E) = \frac{\left\{ \sum_{Jlsl's'} \frac{T_1(\alpha)T_{1,(\alpha')}e^{J(J+1)/2\sigma^2}}{\Gamma_J} \right\}^2}{\sum_{Jlsl's'} \left\{ \frac{T_1(\alpha)T_{1,(\alpha')}e^{J(J+1)/2\sigma^2}}{\Gamma_J} \right\}^2}$$

Let us assume both  $\sigma_c(E)$  and  $N(E)$  almost constant into each sub-interval  $\Delta'E$ . We consider now only the energy dependence of the theoretical expression of the coherence energy  $\Gamma$  (the average over spins and parity is supposed to be already done): let  $\Gamma(E)$  be the averaged value of the  $\Gamma_J^\pi(E)$  at a given energy  $E$ .

In the case of the integrated cross-section, it is possible to show, in a simple way, that

$$(3) \quad F_{\text{tot}}(\varepsilon) = \langle (\sigma(E) - \sigma_c(E))(\sigma(E+\varepsilon) - \sigma_c(E+\varepsilon)) \rangle \propto \sum_{k=1}^i \frac{\Gamma^2(E_k)}{\varepsilon^2 + \Gamma^2(E_k)} \frac{[\sigma_c(E_k)]^2}{N(E_k)}$$

(x) - The ideal case we examine approaches more and more real situations, as the mass number of the examined C, N, increases; however our assumption becomes already correct for  $A \simeq 30$ . In the case of  $\text{Si}^{29}$ , for example, for  $17.5 < E < 19.5$  MeV the various  $\Gamma_J^\pi(E)$  are of the order of some ten KeV (10-50 KeV) and in an energy interval of about 1 MeV their relative variation is of the order of 20 - 30%; the relative changes of the  $\Gamma_J^\pi(E)$  are observed on the values calculated with the expression (1) and the level density used before. If, for  $E > 10$  MeV the true level density increases slower than the one we used, the relative change of the  $\Gamma_J^\pi(E)$  decreases and our conclusions are still more valid. By increasing  $E$  the absolute values of the  $\Gamma_J^\pi(E)$  increase, but their relative change decrease; for example for  $19.5 < E < 22.5$  MeV the values of  $\Gamma_J^\pi(E)$  range between 20 and 90 KeV, but over a larger energy interval of about 1.5 MeV, their relative variation is still of the order of 25%.

where the brackets have the meaning of an average over the complete energy interval  $\Delta E$ .

An approximate method to relate  $\Gamma$  to the various  $\Gamma(E_k)$  is the following: in the hypothesis of a slow variation of the  $\Gamma_j^\pi(E)$  with  $E$ ,  $F_{\text{tot}}(\xi)$  has approximately the shape of a Laurentzian curve of half width  $\Gamma$ .

Then we may write:

$$(4) \quad F_{\text{tot}}(n\Gamma) \simeq \frac{1}{n^2+1} F_{\text{tot}}(0)$$

Writing explicitly this expression, with  $F_{\text{tot}}(\xi)$  given by (3), neglecting  $\Gamma^2(E_k)$  with respect to  $(n\Gamma)^2$ , when  $n \rightarrow \infty$ , one gets

$$(5) \quad \Gamma^2 \simeq \frac{\sum_{k=1}^i \Gamma^2(E_k) \frac{[\sigma_c(E_k)]^2}{N(E_k)}}{\sum_{k=1}^i \frac{[\sigma_c(E_k)]^2}{N(E_k)}}$$

This method of expressing  $\Gamma$  in terms of the  $\Gamma(E_k)$  is a very good approximation when expression (3) is valid. As an example of this statement, in fig. 1 are plotted the functions  $W(\xi)$  and  $V(\xi)$  in a limit case.

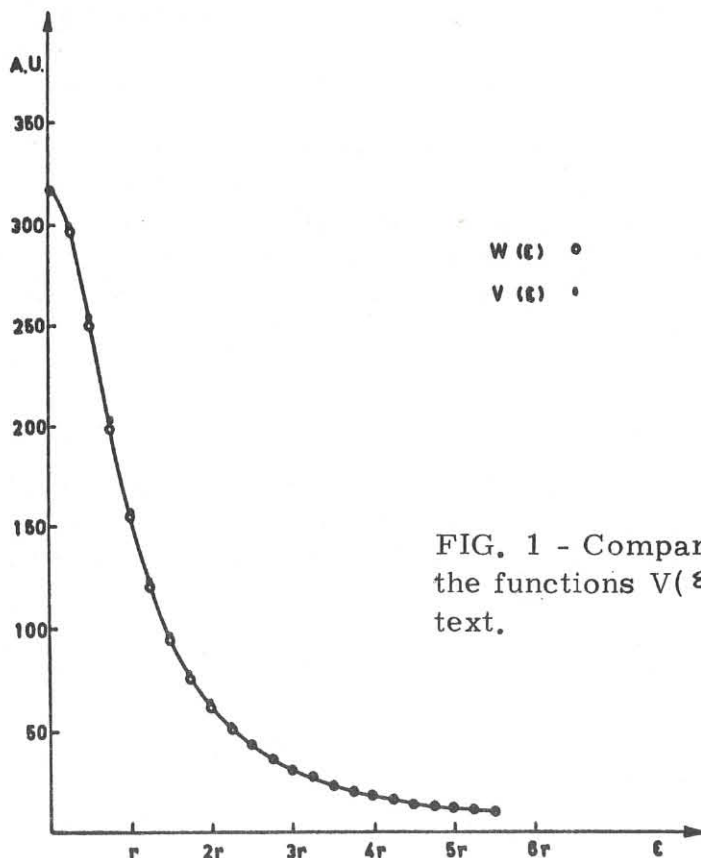


FIG. 1 - Comparison, as a function of  $\xi$ , of the functions  $V(\xi)$  and  $W(\xi)$  described in the text.

They are given by

$$W(\xi) = \sum_{k=1}^i \frac{\Gamma^2(E_k)}{\xi^2 + \Gamma^2(E_k)} \frac{[\sigma_c(E_k)]^2}{N(E_k)} \quad \text{and}$$

$$V(\xi) = \frac{\Gamma^2}{\xi^2 + \Gamma^2} \sum_{k=1}^i \frac{[\sigma_c(E_k)]^2}{N(E_k)}$$

with  $\Gamma^2$  as from expression (5). The assumptions we made are the following: a change in the  $\Gamma(E_k)$  of a factor 3 in the interval  $\Delta E$  divided into the  $i$  sub-intervals  $\Delta'E$ ; in the same interval we have supposed that the terms  $[\sigma_c(E_k)]^2/N(E_k)$  vary with a quadratic law:  $[\sigma_c(E_k)]^2/N(E_k) = K E_k^2$ , with  $E_k$  ranging, with discrete values, from  $\Delta'E/2$  to  $\Delta E - (\Delta'E/2)$  being  $\Delta E$  and  $\Delta'E$  the intervals already defined;  $i$  has been chosen equal to 9. The experimental data seem to indicate that a variation like the one we have chosen for the quantities defining  $W(\xi)$  and  $V(\xi)$  is possible only over an energy interval larger than 5 MeV for nuclei with  $A \simeq 30$ , larger than 2.5 MeV for  $A \simeq 45$  and larger than 2 MeV for  $A \simeq 60$ . In the case we examined  $\Gamma(\bar{E})$  approximates  $\bar{\Gamma}$  (being  $\bar{E} = \Delta E/2$ ) within 30%. When the relative change of the  $\Gamma(E_k)$  and the  $[\sigma_c(E_k)]^2/N(E_k)$ , in the considered energy interval, decreases, the function  $V(\xi)$  is still a very good approximation of the function  $W(\xi)$  and the approximation  $\Gamma \simeq \Gamma(\bar{E})$  becomes better. For example if the  $\Gamma(E_k)$  vary as we have already assumed, but the terms  $[\sigma_c(E_k)]^2/N(E_k)$  change only of a factor 2 in the energy interval  $\Delta E$ , the preceding approximation would give an error of the order of 10%.

From the preceding considerations one deduces that the approximation a) is no good indeed, while approximation b) may, in some cases, be correct enough (in general for high values of the incident energy). In general however, once the average over the spins and parity is made, one should calculate  $\Gamma$  with expression (5) before comparing the theoretical previsions of the statistical model with the value obtained from an analysis of the experimental data.

Average over spins and parity.

The  $\Gamma(E_k)$  in formula (3) are given by a weighted average over the spins and parity of the different  $\Gamma_J^\pi(E)$  of the C.N. levels interested in the reaction. In an energy interval  $E - (\Delta'E/2)$ ,  $E + (\Delta'E/2)$  the autocorrelation function  $F_{\alpha\alpha'}(E, \xi)$  has the expression<sup>(1)</sup>:

$$(6) \quad F_{\alpha\alpha'}(E, \xi) = \left[ \frac{\pi \chi_\alpha^2}{(2i+1)(2I+1)} \right]^2 \sum_{J|s|s'} (2J+1)^2 \frac{[\Gamma_J^\pi(E)]^2}{\xi^2 + [\Gamma_J^\pi(E)]^2} \cdot \left\{ \psi_{\alpha s l, \alpha' s' l'}^J(E) \right\}^2$$



$\alpha$  and  $\alpha'$  stand for the initial and final channel,  $i$  and  $I$  are the spins of the incident particle and of the target nucleus,  $J$  is the spin of the C.N. levels,  $l$  and  $l'$ ,  $s$  and  $s'$  are the orbital angular momenta and the channel spins in the entrance and exit channels. For a fixed value of  $l$  the possible values of  $l'$  are limited by the spin and parity conservation laws. In (6)  $\mathcal{C}_{\alpha s l, \alpha' s' l'}^J(E)$  is connected to the average statistical cross-section  $\overline{\sigma}_c(E)$  through the relation

$$(7) \quad \overline{\sigma}_c(E) = \frac{\pi \chi_\alpha^2}{(2i+1)(2I+1)} \sum_{J l s l' s'} (2J+1) \mathcal{C}_{\alpha s l, \alpha' s' l'}^J(E)$$

and is given by

$$(8) \quad \mathcal{C}_{\alpha s l, \alpha' s' l'}^J(E) = \frac{2\pi}{D_J(E)} \frac{\langle \Gamma_{\alpha s l}^J(E) \rangle \langle \Gamma_{\alpha' s' l'}^J(E) \rangle}{\Gamma_J^\pi(E)}$$

The partial widths  $\langle \Gamma_{\alpha s l}^J(E) \rangle$  and  $\langle \Gamma_{\alpha' s' l'}^J(E) \rangle$  can be expressed by means of the transmission functions  $T_1(\alpha, s, J)$  and  $T_{1'}(\alpha', s', J)$  through the relations

$$(9) \quad \langle \Gamma_{\alpha s l}^J(E) \rangle = \frac{D_J(E)}{2\pi} T_1(\alpha, s, J) \quad \text{and}$$

$$\langle \Gamma_{\alpha' s' l'}^J(E) \rangle = \frac{D_J(E)}{2\pi} T_{1'}(\alpha', s', J)$$

$$(10) \quad D_J(E) = 1/\rho(E, J), \quad \text{with}$$

$$(11) \quad \rho(E, J) = \rho(E) (2J+1) \exp \left\{ -J(J+1)/2 \theta^2 \right\}.$$

$\Gamma_J^\pi(E)$  is given by (1).

By introducing expressions (8) (9) (10) (11) in (6) one obtains:

$$(12) \quad F_{\alpha\alpha'}(E, \xi) = \left[ \frac{\chi_\alpha^2}{2(2i+1)(2I+1)\rho(E)} \right]^2 \sum_J \frac{1}{\xi^2 + [\Gamma_J^\pi(E)]^2} \cdot \\ \cdot \exp \left[ J(J+1)/6^2 \right] \sum_{s l s' l'} T_1^2(\alpha, s, J) T_{1'}^2(\alpha', s', J).$$



$F_{\alpha\alpha'}(E, \epsilon)$  has the approximate shape of a Laurentzian curve of half width  $\Gamma(E)$ ; for each value of  $n$ ,  $F_{\alpha\alpha'}(E, n\Gamma(E)) \approx (1/(n^2 + 1))F_{\alpha\alpha'}(E, 0)$ . By imposing such a condition into (12), in the limit of  $n \rightarrow \infty$ , neglecting  $[\Gamma_J(E)]^2$  with respect to  $n^2 \Gamma^2(E)$  one obtains:

$$(13) \quad \frac{1}{\Gamma^2(E)} \approx \frac{\sum_J \frac{1}{[\Gamma_J(E)]^2} \exp\{J(J+1)/6^2\} \sum_{s1} T_1^2(\alpha, s, J) \sum_{s'1'} T_1^2(\alpha', s', J)}{\sum_J \exp\{J(J+1)/6^2\} \sum_{s1} T_1^2(\alpha, s, J) \sum_{s'1'} T_1^2(\alpha', s', J)}$$

We have tested the goodness of this method to derive  $\Gamma(E)$  as a function of the  $\Gamma_J(E)$  in the case of various fictitious reactions on nuclei with  $24 \leq A \leq 60$ , for different excitation energies of the C.N. (from 13 to 20 MeV) and we have got good results in every case.

One sees that the weighting factors of the different terms  $1/[\Gamma_J(E)]^2$  depend not only on the spin distribution of the C.N. levels involved in the reaction, but also on the transmission functions of the incident and emitted particles. In fig. 2 the behaviour vs.  $J$  of the following quantities is reported:

$$A(J) = \exp(J(J+1)/6^2) \sum_{s1} T_1^2(\alpha, s, J) \sum_{s'1'} T_1^2(\alpha', s', J)$$

$$B(J) = \sum_{s1} T_1^2(\alpha, s, J) \sum_{s'1'} T_1^2(\alpha', s', J)$$

$$C(J) = (2J+1) \exp\{-J(J+1)/2 \cdot 6^2\}$$

for the reactions:

a)  $Mg^{26}(p, p_2)Mg^{26x}$  at  $(E_p)_{lab.} = 9.4$  MeV,

b)  $Cl^{37}(p, \alpha_1)S^{34x}$  at  $(E_p)_{lab.} = 11.5$  MeV and

c)  $Ge^{76}(\alpha, \alpha_o)Ge^{76}$  at  $(E_\alpha)_{lab.} = 12$  MeV.

In each case the curves are normalized to give the same area.

We conclude that, in the case of pure statistical reactions, before comparing the theoretical level widths with the coherence energy deduced from an analysis of the experimental integrated excitation functions, one has to average them over the spins, parity and energy by means of the formulas (13) and (5).

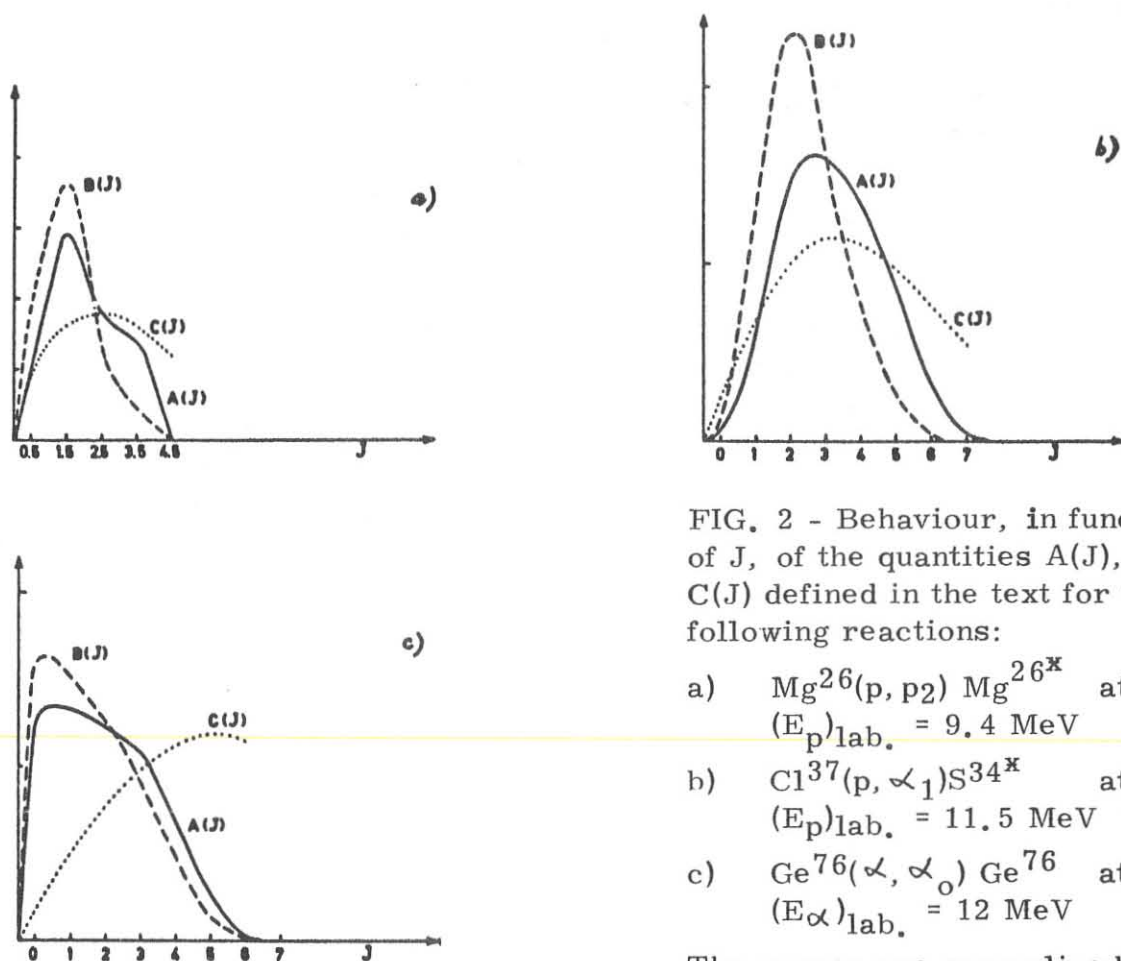


FIG. 2 - Behaviour, in function of  $J$ , of the quantities  $A(J)$ ,  $B(J)$ ,  $C(J)$  defined in the text for the following reactions:

- a)  $Mg^{26}(p, p_2) Mg^{26x}$  at  $(E_p)_{lab.} = 9.4$  MeV  
 b)  $Cl^{37}(p, \alpha_1) S^{34x}$  at  $(E_p)_{lab.} = 11.5$  MeV  
 c)  $Ge^{76}(\alpha, \alpha_0) Ge^{76}$  at  $(E_\alpha)_{lab.} = 12$  MeV

The curves are normalized to the same area.

### SECTION 3 -

When a non statistical effect contributes to the considered reaction expressions (6) and (3) we started from, to average over spins and energy, are to be modified. An interference term between the two effects is introduced<sup>(1)</sup>.

In the case of integrated cross-sections, expression (6) is modified in the following:

$$F_{\alpha\alpha'}(E, \xi) = \left\{ \frac{\pi \chi_\alpha^2}{(2i+1)(2I+1)} \right\}^2 \sum_{J s l s' l'} (2J+1)^2 \frac{(\Gamma_J^\pi(E))^2}{\xi^2 + (\Gamma_J^\pi(E))^2} \quad (14)$$

$$\varphi_{\alpha s l, \alpha' s' l'}^J \left\{ 2 \left| \langle S_{\alpha s l, \alpha' s' l'}^J \rangle \right|^2 + \varphi_{\alpha s l, \alpha' s' l'}^J \right\}$$

10.

The terms  $|\langle S_{\alpha s l, \alpha' s' l'}^J \rangle|^2$  are connected to the non statistical process cross-section  $\sigma_{\alpha\alpha'}^{DI}(E)$  by the relation:

$$(15) \quad \sigma_{\alpha\alpha'}^{DI}(E) = \frac{\pi \lambda_{\alpha}^2}{(2i+1)(2I+1)} \sum_{J s l s' l'} (2J+1) |\langle S_{\alpha s l, \alpha' s' l'}^J \rangle|^2$$

With the same technique already used to average over spins and parity, the analogue expression of (13) is

$$(16) \quad \frac{1}{\Gamma^2(E)} = \frac{\sum_{J s l s' l'} (2J+1)^2 [2 |\langle S_{\alpha s l, \alpha' s' l'}^J \rangle|^2 e_{\alpha s l, \alpha' s' l'}^J + (\psi_{\alpha s l, \alpha' s' l'}^J)^2]}{\sum_{J s l s' l'} (2J+1)^2 (\Gamma_J^{\pi}(E))^2 [2 |\langle S_{\alpha s l, \alpha' s' l'}^J \rangle|^2 e_{\alpha s l, \alpha' s' l'}^J + (\psi_{\alpha s l, \alpha' s' l'}^J)^2]}$$

while (3) is substituted by

$$(17) \quad F_{\text{tot}}(\varepsilon) \propto \sum_{k=1}^i \frac{\Gamma^2(E_k)}{(\varepsilon^2 + \Gamma^2(E_k))} A(E_k)$$

with

$$A(E_k) = \left\{ \frac{\pi \lambda_{\alpha}^2}{(2i+1)(2I+1)} \right\}^2 \sum_{J s l s' l'} (2J+1)^2 [2 |\langle S_{\alpha s l, \alpha' s' l'}^J(E_k) \rangle|^2 e_{\alpha s l, \alpha' s' l'}^J(E_k) + (\psi_{\alpha s l, \alpha' s' l'}^J(E_k))^2]$$

and expression (5) becomes

$$(18) \quad \Gamma^2 \approx \frac{\sum_{k=1}^i \Gamma^2(E_k) A(E_k)}{\sum_{k=1}^i A(E_k)}$$

In expressions (14) (16) (17) (18)  $\psi_{\alpha s l, \alpha' s' l'}^J$  is still given by (8), while  $|\langle S_{\alpha s l, \alpha' s' l'}^J \rangle|^2$  can be calculated in the framework of a particular model. In Appendix  $|\langle S_{\alpha s l, \alpha' s' l'}^J \rangle|^2$  is calculated for stripping and pick-up reactions, (d, p), (d, n), (p, d), (n, d), with the DWBA; the method can be generalized to other types of reactions always within the DWBA, for example to inelastic scattering, knock-on reactions etc.

The interference between statistical and non statistical processes appears only when they proceed through the same channels. In some cases the non statistical effects can proceed, at least partially, through channels which are different from the effective ones through which the statistical effects proceed. In fact with good approximation, in most cases, one can think that only  $N$  uncorrelated channels contribute equally to a statistical reaction<sup>(12)</sup>. It is possible to neglect the contribution to the reaction of the channels with negligible transmission functions (corresponding to high values of the angular momenta of the incident and emitted particles). On the other hand, the particular mechanism of the non statistical effect, may favour channels characterized by angular momentum values larger than the ones of the statistical effect. In such a case the interference between the two effects will be small and, with good approximation, expression (3) and (6) will still be valid.

We want to show now the use of formula (16) in the case of the reaction  $Al^{27}(d, p_0)Al^{28}$ ; the terms  $|\langle S_{\alpha sl}^J, \alpha 's' l' \rangle|^2$  have been calculated in the DWBA following the method outlined in Appendix.

In Tab. I the optical model parameters used are reported; in Tab. II, for different values of the incident particle energy and for different  $J$ , are

TABLE I

DEUTERON	NEUTRON	PROTON
$V_R^{(x)}$ (MeV) = 94.83	$V_R^{(x)}$ (MeV) = 95	$V_R^{(x)}$ (MeV) = 50
$r_R$ (fm) = 1.15	$r_R$ (fm) = 1.15	$r_R$ (fm) = 1.25
$a_R$ (fm) = 0.81	$a_R$ (fm) = 0.81	$a_R$ (fm) = 0.45
$W_I^{(+)}$ (MeV) = 20		$W_I^{(x)}$ (MeV) = 12
$r_I$ (fm) = 1.34		$r_I$ (fm) = 1.25
$b_I$ (fm) = 1.469		$a_I$ (fm) = 0.45
$r_C$ (fm) = 1.15		$r_C$ (fm) = 1.30

Optical model parameters used in the calculation of the terms  $|\langle S_{\alpha sl}^J, \alpha 's' l' \rangle|^2$ . For deuterons they are within the range of values suggested by ref. (13); for protons they are similar to values given in the literature<sup>(14)</sup>; for neutrons are similar to the deuterons parameters as suggested in ref. (15).

(x) - Potential well of Saxon-Wood shape.

(+) - Potential well of Gaussian shape.



TABLE II

E \ J	1/2	3/2	5/2	7/2	9/2	11/2	13/2	15/2
2	0, 878	1, 940	0, 685	0, 701	0, 803	0, 160		
3	0, 439	0, 614	0, 656	0, 618	0, 460	0, 137	0, 147	
5	0, 112	0, 200	0, 284	0, 274	0, 186	0, 081	0, 081	
7	0, 045	0, 095	0, 130	0, 130	0, 095	0, 053	0, 035	0, 004
9	0, 025	0, 067	0, 067	0, 074	0, 049	0, 032	0, 021	0, 004

x The energies are in MeV.

reported the ratios between the values of the statistical term

$$(19) \quad S.T. J(E) = \left[ \frac{\pi \chi^2}{(2i+1)(2I+1)} \right]^2 (2J+1)^2 (\psi_{\alpha sl, \alpha' s' l'}^J)^2$$

and the interference term

$$(20) \quad I.T. J(E) = 2 \frac{\pi \chi^2}{(2i+1)(2I+1)} (2J+1)^2 \left| \langle S_{\alpha sl, \alpha' s' l'}^J \rangle \right|^2 \psi_{\alpha sl, \alpha' s' l'}^J$$

In order to obtain the absolute value of the direct effect contribution to this reaction we proceeded as follows: the differential statistical cross-section  $\sigma_c(\theta, E)$  for the processes  $Al^{27}(d, p_0)$  and  $Al^{27}(d, p_1)$  at  $(E_d)_{lab.} = 2$  MeV has been calculated in shape and absolute value by using the Blatt and Biedernharn formula<sup>(16)</sup>: the transmission functions were calculated with the same optical model parameters of Tab. I and the level densities with the Lang and Le Couteur formula and the "a" parameters of ref. 7). The direct effect differential cross-section  $\sigma^{DI}(\theta, E)$  for the two processes has been calculated in the DWBA using the code TOBIA 2 developed at the University of Milano. The absolute value of  $\sigma^{DI}(\theta, E)$  is not known unless the reduced width of the captured neutron is given.

With an oportune mixing of direct and statistical contribution, the experimental average angular distribution of the reaction

$$Al^{27}(d, p_0 + p_1)Al^{28} \text{ at } (E_d)_{lab.} = 2 \text{ MeV}^{(17)}$$

has been then reproduced in shape and absolute value<sup>(+)</sup>. In this way the abso

(+) - The actual energy resolution do not allow to separate the doublet of  $Al^{28}$ .

lute value of the direct contribution to these reactions has been derived. The absolute value of  $\sigma^{DI}(\theta)$  at the same energy, for the  $(d, p_0)$  transition has then been deduced taking into account the non interference between the transitions to the two final states of  $Al^{28}$  and assuming the same reduced width for the captured neutron in the different final states. The absolute value of  $\sigma^{DI}(\theta)$  for the  $(d, p_0)$  transition at higher energies, has been obtained by normalization.

In fig. 3 the experimental points and the calculated curve, are plotted for the differential cross-section of the reaction  $Al^{27}(d, p_0 + p_1) Al^{28}$  leading to the doublet  $3^+$  and  $2^+$  of  $Al^{28}$  at  $(E_d)_{lab.} = 2$  MeV.

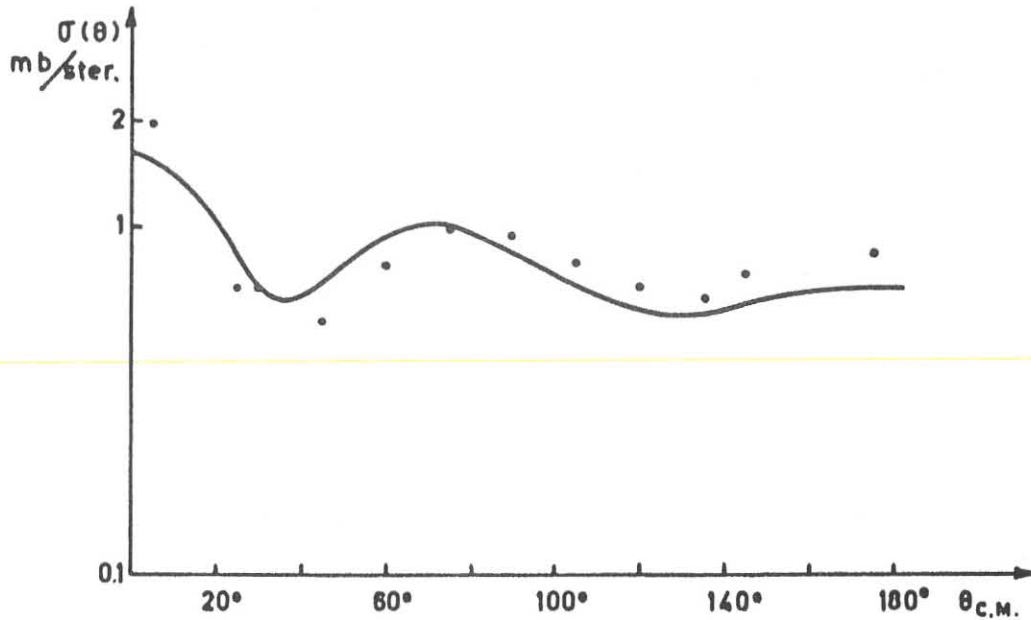


FIG. 3 - Angular distribution of protons emitted in the reaction  $Al^{27}(d, p_0 + p_1) Al^{28}$  at  $(E_d)_{lab.} = 2$  MeV. The experimental points are from ref. 17); the curve is calculated with a mixture of 53% of statistical effect and 47% of direct effect.

Tab. II shows that, in this case, the interference term is always larger than the statistical one. In fig. 4, at different deuteron energies, the weight factors  $S. T. J$  and  $(I. T. J + S. T. J)$  as a function of  $J$  are reported; (the histograms are normalized to the same area). The interference term tends to weigh more, in the average, the  $\Gamma_J^\pi(E)$  corresponding to the highest  $J$  values: this behaviour seems to be emphasized as the energy increases.

TABLE III

E (MeV)	2	3	5	7	9
$\frac{\Gamma_{Di. + C. N.}}{\Gamma_{C. N.}}$	0.985	0.931	0.937	0.888	0.895

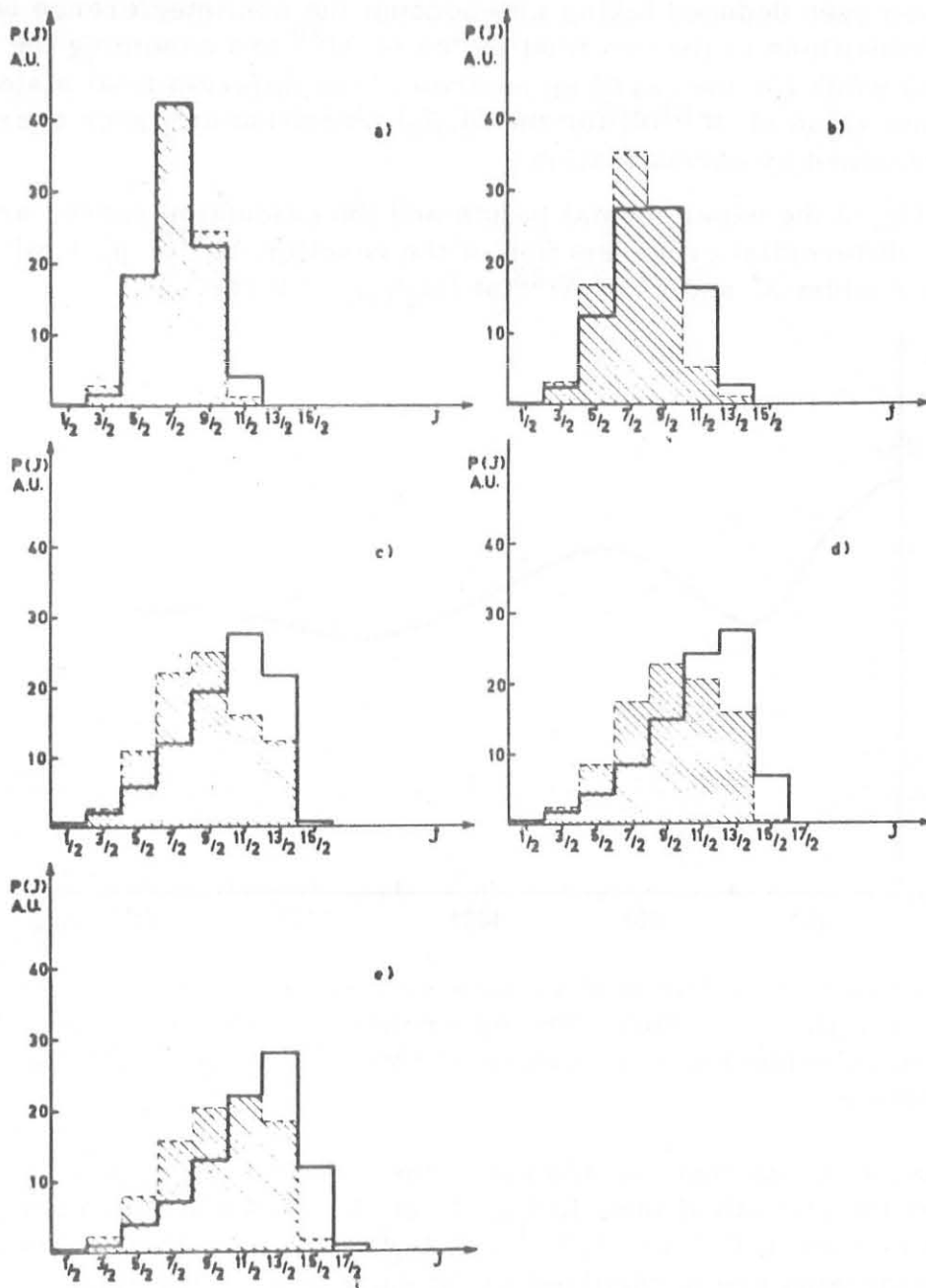


FIG. 4 - Behaviour, as a function of  $J$ , of the weight factors  $S, T, J$  and  $S, T, J + I, T, J$  for the reaction  $Al^{27}(d, p_0)Al^{28}$  for various values of the deuteron energy. The dotted line histograms are for the terms  $S, T, J$ ; the full line histograms are for the terms  $S, T, J + I, T, J$ . They are all normalized to the same area. The different cases are: a)  $E_d = 2$  MeV, b)  $E_d = 3$  MeV, c)  $E_d = 5$  MeV, d)  $E_d = 7$  MeV, e)  $E_d = 9$  MeV.

In Tab. III, at the same deuteron energies, the ratios between  $\Gamma_{DI+C, N}(E)$  calculated with (16) and  $\Gamma_{CN}(E)$  calculated with (13) are reported: although the interference is large, it weakly affects the values of  $\Gamma(E)$ . The case examined is a rather typical one so that it is plausible that for light nuclei ( $A \leq 30$ ) the presence of a non statistical effect, has a small influence on  $\Gamma$ ; however its importance should increase for heavier nuclei.

#### SECTION 4 -

For experimental reasons, the excitation functions are often measured for differential cross-sections, that is at some particular angle  $\theta$ . Be  $\Gamma(\theta, E)$  the coherence energy determined from the analysis of a differential excitation function. It is an obvious question to ask whether this quantity does depend on  $\theta$  and how the  $\Gamma(\theta, E)$  are related to the  $\Gamma$  deduced by the analysis of the corresponding integrated excitation function

Let confine our analysis to the case of pure statistical reactions. Starting from formula (31) of ref. 1) one obtains the following expression for  $\Gamma(\theta, E)$  averaged over the spins and parity:

$$\frac{1}{\Gamma^2(\theta, E)} = \frac{\sum_{L_1 L_2} \sum_{(-)}^{2(s'-s)} \mathcal{H} \frac{1}{g(J_1)g(J_2)} P_{L_1}(\cos\theta) P_{L_2}(\cos\theta)}{\sum_{L_1 L_2} \sum_{(-)}^{2(s'-s)} \mathcal{H} \frac{1}{g(J_1)g(J_2)} \left[ \Gamma_{J_1}^{\pi}(E) \Gamma_{J_2}^{\pi}(E) \right] P_{L_1}(\cos\theta) P_{L_2}(\cos\theta)}$$

(21)

where

$$\mathcal{H} = \bar{Z}(1_{1J_1} 1_{2J_2}; sL_1) \bar{Z}(1'_{1J_1} 1'_{2J_2}; s'L_1) \bar{Z}(1_{1J_1} 1_{2J_2}; sL_2) \bar{Z}(1'_{1J_1} 1'_{2J_2}; s'L_2)$$

$$\mathcal{L} = T_{1_1}(\alpha, s, J_1) T'_{1_1}(\alpha', s', J_1) T_{1_2}(\alpha, s, J_2) T'_{1_2}(\alpha', s', J_2)$$

$$g(J) = 2\pi \left[ \Gamma_J^{\pi} \right] / D_J$$

Here the non primed and primed quantities are respectively for the entrance and exit channels;  $\bar{Z}$  are the Blatt-Biedernharn and Huby coefficients.

The value of  $\Gamma(\theta, E)$  averaged over the energy, is straight obtained applying the same method as before.



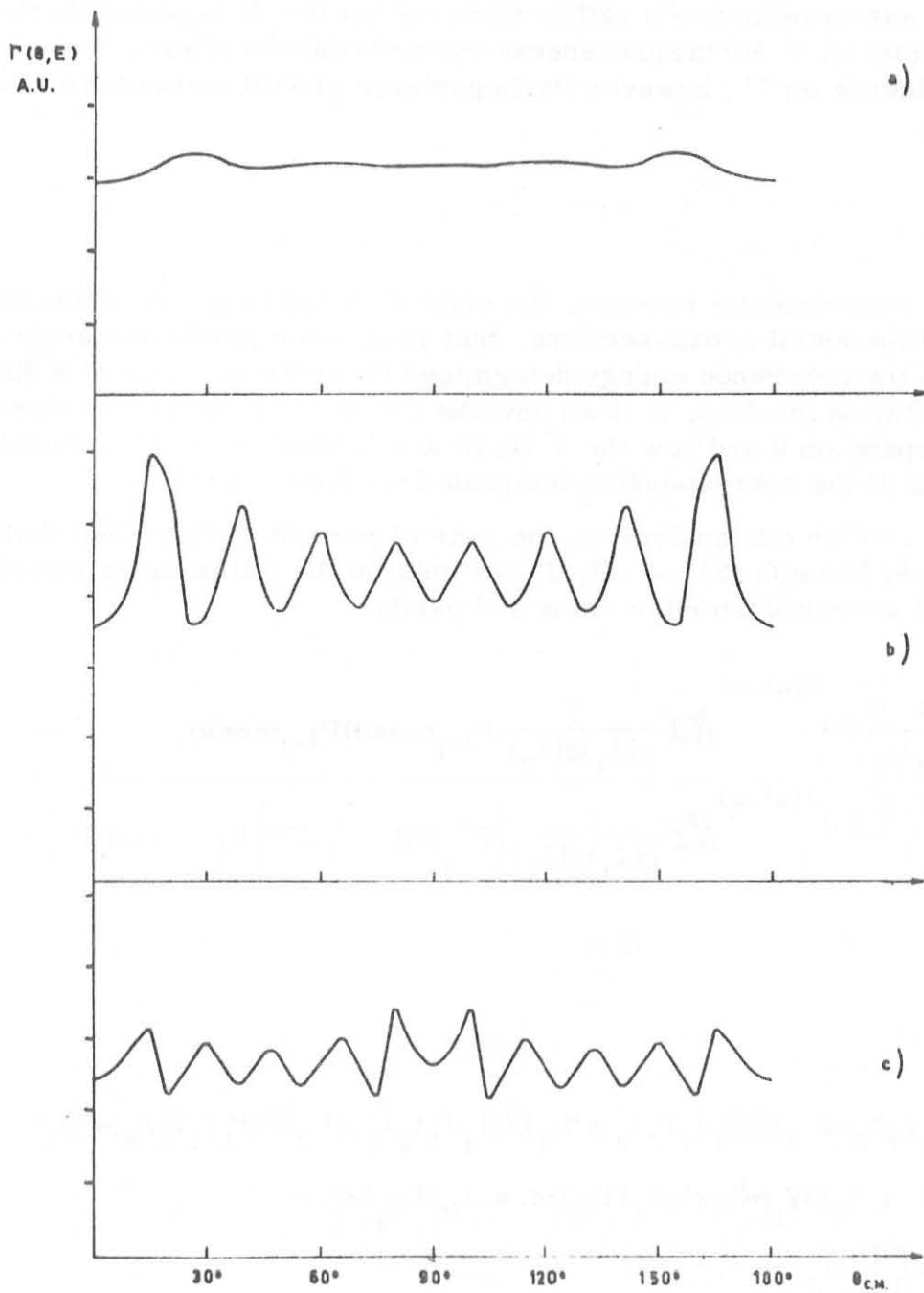


FIG. 5 - Plot of  $\Gamma(\theta, E)$  vs.  $\theta$  for the reactions:

a)  $Ge^{76}(\alpha, \alpha_0) Ge^{76}$  at  $(E_\alpha)_{lab.} = 12$  MeV

b)  $S^{32}(\alpha, \alpha_0) S^{32}$  at  $(E_\alpha)_{lab.} = 13.35$  MeV

c)  $C^{12}(C^{12}, \alpha_0) Ne^{20}$  at  $(E_{C^{12}})_{lab.} = 23$  MeV

By examining expression (21) it is evident that, when the incident and outgoing particles have zero angular momenta,  $\Gamma(\theta, E)$  does not depend on  $\theta$ . When either the orbital angular momenta and the channel spins, may reach large values, there is a very large number of terms summed up in (21) and the coefficients of the Legendre polynomials can't behave in such a way to stress the importance of some particular couple  $(L_1, L_2)$ . Therefore, as the Legendre polynomials are oscillating functions of  $\theta$  and vary strongly with  $L$ , for a fixed  $\theta$ , it seems reasonable to think that  $\Gamma(\theta, E)$  does not depend very much on  $\theta$ .

More difficulties arise in examining the case in which high values of the ingoing and outgoing orbital angular momenta are allowed, while the initial and final channel spins are zero, because the number of terms in expression (21) is strongly reduced. However this simplification allows a numerical calculation of the behaviour of  $\Gamma(\theta, E)$  with  $\theta$  for a given reaction. We have examined some reactions of this type. In general  $\Gamma(\theta, E)$  does not show strong systematic variations with  $\theta$ ; in some cases however, it shows oscillations which may be of the order of 20-25% around a constant value. As an example in fig. 5 is plotted  $\Gamma(\theta, E)$  vs.  $\theta$  for the reactions: a)  $\text{Ge}^{76}(\alpha, \alpha_0)\text{Ge}^{76}$  at  $(E\alpha)_{\text{lab.}} = 12$  MeV, b)  $\text{S}^{32}(\alpha, \alpha_0)\text{S}^{32}$  at  $(E\alpha)_{\text{lab.}} = 13.35$  MeV, c)  $\text{C}^{12}(\text{C}^{12}, \alpha_0)\text{Ne}^{20}$  at  $(E_{\text{C}^{12}})_{\text{lab.}} = 23$  MeV. Let us note explicitly that  $\Gamma(\theta, E)$  is always symmetric with respect to  $\theta=90^\circ$ . Fig. 5 shows how, in the case of small channel spins, the  $\theta$  dependence of  $\Gamma(\theta, E)$  may be important. Values of  $\Gamma(\theta, E)$  determined at different angles, with good angular resolution measurements, may be different also by a factor 1.5. In these cases one should compare the experimental values of  $\Gamma(\theta, E)$  with the theoretical expression given by (21), averaged over the energy.

It is possible to show that the value of  $\Gamma(E)$  characterizing the fluctuations of the integrated excitation function at an energy  $E$ , is an opportune average of the  $\Gamma(\theta, E)$ . In fact, if  $N(\theta_i)$  is the quantity defined by Brink and al. (18) as the inverse of the normalized variance of the differential cross-section and  $\sigma_c(\theta_i)$  the average differential statistical cross-section at an angle  $\theta_i$ , the value of  $\Gamma(E)$  determined from the integrated excitation function, is related to the  $\Gamma(\theta, E)$  through the relation:

$$\Gamma^2(E) \approx \frac{\sum_{\theta_i} \left\{ \frac{\text{sen}^2 \theta_i \sigma_c^2(\theta_i)}{N(\theta_i)} \right\} \Gamma^2(\theta_i, E)}{\sum_{\theta_i} \left\{ \frac{\text{sen}^2 \theta_i \sigma_c^2(\theta_i)}{N(\theta_i)} \right\}}$$

The interval separating two successive angles  $\theta_i$  in the sums, is the coherence angle  $\Delta\theta$ .

When  $\Gamma(\theta, E)$  is almost independent from  $\theta$ , the coherence energies characterizing the integrated and all the differential excitation func-

tions, are the same. So it is convenient to analyze the coherence energy determined from a differential excitation function by using formulas (5) and (13).

The situation is more complicated when the presence of non statistical terms is taken into account in the expression of the auto-correlation function for the differential cross-sections (see formulas (30) and (31) of ref. 1)). Also in this case one can follow the mathematical methods outlined before to evaluate the different types of averages. The expression obtained for  $\Gamma(\theta, E)$  is very complicated and it is difficult to predict its behaviour with  $\theta$ ; moreover a numerical calculation is practically impossible due to the very large number of kinematics coefficients which should be taken into account. So, when a non statistical effect is surely present, in order to determine the coherence energy  $\Gamma$ , we suggest to analyze the integrated excitation functions utilizing the formulas and the methods previously shown, or to analyze differential excitation functions at angles where the non statistical contribution is surely small.

#### APPENDIX -

Let us consider a stripping reaction in which, due to the selection rules governing the process, the captured particle of spin  $i_c$ , is in a state characterized by the quantum numbers  $l_c$  and  $j_c$  (19).

In the DWBA with the zero range approximation, the differential cross-section for such a process, neglecting spin orbit interactions, can be written (20):

$$(A1) \quad \sigma_{l_c i_c j_c}(\theta) = A \sum_{\lambda_c = -l_c}^{l_c} \left| B_{l_c \lambda_c}^{i_c j_c}(\theta) \right|^2$$

A being a coefficient depending on the energies and the masses of the particles involved in the reaction, but independent from the emission angle  $\theta$ ;  $B_{l_c \lambda_c}^{i_c j_c}(\theta)$  are matrix elements given by

$$(A2) \quad B_{l_c \lambda_c}^{i_c j_c}(\theta) = \sum_{l_i l_e} C_{l_i l_e l_c}^{i_c j_c} Y_{l_e \lambda_c}(\theta, 0)$$

with

$$(A3) \quad C_{l_i l_e l_c}^{i_c j_c} = (-1)^{\frac{l_i - l_e - l_c}{2}} \sqrt{2l_e + 1} (l_e - \lambda_c \ l_c \ \lambda_c \ | \ l_i \ 0) (l_e \ 0 \ l_c \ 0 \ | \ l_i \ 0) \cdot \int_0^{\infty} r^2 dr f_{l_i}^{(i)}(r) f_{l_c}^{i_c j_c}(r) f_{l_e}^{(e)}\left(\frac{MT}{MR} r\right)$$

Here  $M_T$  and  $M_R$  are the masses of the target and residual nuclei;  $l_i$  and  $l_e$  the orbital angular momentum quantum numbers of the incident and emitted particles;  $f_{l_i}^{(i)}(r)$ ,  $f_{l_c}^{i_c j_c}(r)$ ,  $f_{l_e}^{(e)}(M_T r / M_r)$  are the incident, captured and emitted particles radial wave functions calculated taking into account the distortions introduced by the optical model central potentials.

The integrated cross-section is given by the expression:

$$(A4) \quad \sigma_{l_c i_c j_c} = \iint d\theta \sin\theta d\psi \sigma_{l_c i_c j_c}(\theta) = A \sum_{\lambda_c} \iint d\theta \sin\theta d\psi |B_{l_c \lambda_c}^{i_c j_c}(\theta)|^2$$

Substituting (A. 2) in (A. 4), with a simple calculation, one gets:

$$(A5) \quad \sigma_{l_c i_c j_c} = A \sum_{l_i, l_e} \left\{ \sum_{\lambda_c} \left[ \text{Re} C_{l_i l_e l_c \lambda_c}^{i_c j_c} \left( \sum_{l_i'} \text{Re} C_{l_i' l_e l_c \lambda_c}^{i_c j_c} \right) + \text{Im} C_{l_i l_e l_c \lambda_c}^{i_c j_c} \times \right. \right. \\ \left. \left. \times \left( \sum_{l_i'} \text{Im} C_{l_i' l_e l_c \lambda_c}^{i_c j_c} \right) \right] \right\} = A \sum_{l_i, l_e} F_{l_i l_e}^{i_c j_c}$$

Expression (A5) has to be compared with the cross-section expression given by the general theory of nuclear reactions(1):

$$(A6) \quad \sigma_{l_c i_c j_c} = \frac{\pi \lambda_i^2}{(2i_i+1)(2I_T+1)} \sum_{s_i s_e} (2J+1) \left| \langle S_{\alpha_i s_i l_i, \alpha_e s_e l_e}^J \rangle \right|^2$$

where  $\lambda_i$  is the wave length of the incident particle,  $J$  is the total angular momentum quantum number for the incident particle plus target nucleus system; all the other symbols have the usual meaning for the initial (i) and final (e) systems.

Comparing expressions (A. 5) and (A. 6) it comes out

$$(A7) \quad A F_{l_i l_e}^{i_c j_c} = \frac{\pi \lambda_i^2}{(2i_i+1)(2I_T+1)} \sum_{s_i s_e} (2J+1) \left| \langle S_{\alpha_i s_i l_i, \alpha_e s_e l_e}^J \rangle \right|^2$$

By means of (A7) the quantities  $\left| \langle S_{\alpha_i s_i l_i, \alpha_e s_e l_e}^J \rangle \right|^2$  can be calculated assuming that they are almost independent on  $J$ ,  $s_i$ ,  $s_e$ , assumption quite reasonable in the case of stripping reactions which are well described by the DWBA neglecting spin orbit interactions. In such a case one obtains:



$$(A8) \quad \left| \langle S_{\alpha_i s_i l_i, \alpha_e s_e l_e}^J \rangle \right|^2 = \frac{A(2i_i+1)(2I_T+1)}{\pi \lambda_i^2} \cdot \frac{F_{l_i l_e l_c}^{i_c j_c}}{\sum_{J s_i s_e} (2J+1) \eta_{\alpha_i \alpha_e l_i l_e}(J, s_i, s_e)}$$

where  $\eta_{\alpha_i \alpha_e l_i l_e}(J, s_i, s_e)$  gives the number of terms  $|\langle S \rangle|^2$  which, in the transition  $\alpha_i \rightarrow \alpha_e$  correspond to a given  $J$  value for a fixed couple of values  $l_i$  and  $l_e$ .

The coefficient  $A$  in (A5) (A7) and (A8) depends on kinematic factors and on the reduced width<sup>(19)</sup> of the captured particle: this can be calculated assuming particular models describing the structure of the residual nucleus (for instance shell model with  $j$ - $j$  coupling).

In a more simple way the value of  $A$  can be derived empirically by imposing to the expression (A5) to give the experimental absolute value. This is the method we followed.

This method for calculating the terms  $|\langle S_{\alpha_i s_i l_i, \alpha_e s_e l_e}^J \rangle|^2$  can be generalized to the case in which, due to configuration mixing, the quantum numbers  $l_c$  and  $j_c$  of the captured particle can assume different values: in such a case the stripping cross-section is given by:

$$\sigma_{\text{tot}} = \sum_{l_c j_c} \sigma_{l_c i_c j_c}$$

This type of calculation can be immediately extended to pick-up reactions and to other direct reactions, as inelastic scattering, knock-out, etc. whose differential cross-section can be expressed by a formula formally analogue to (A1).

The numerical calculation of the terms  $AF_{l_i l_e l_c}^{i_c j_c}$  has been done by using the code TOBIA 2.

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