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F. Pellegrini : NUCLEAR SPECTROSCOPY OF THE N = 29
ISOTONES. -

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SUMMARY. -

Energy levels in the N=29 isotones (^{51}Ti , ^{52}V , ^{53}Cr , ^{54}Mn , ^{55}Fe) arising from the coupling of protons in the $1f_{7/2}$ shell and the single neutron in the $2p_{3/2}$ shell are calculated by the use of effective interactions. The required two body residual interactions are deduced from the experimental spectra of ^{50}Ti and ^{56}Co . A comparison with available experimental data shows in general good correspondence. However, some low lying states indicate the mixing of configurations in which the single neutron outside the ^{48}Ca core is in the $2p_{1/2}$ and $1f_{5/2}$ orbits.

SOMMARIO. -

Sono stati calcolati per il gruppo degli isotoni ^{51}Ti , ^{52}V , ^{53}Cr , ^{54}Mn e ^{55}Fe i livelli energetici originati dall'accoppiamento del singolo neutrone nella shell $2p_{3/2}$ con i protoni nella shell $1f_{7/2}$. L'effettiva residua interazione tra protone protone e neutrone protone è stata dedotta dagli spettri sperimentali dei nuclei ^{50}Ti e ^{56}Co . Il confronto dei livelli calcolati con quelli osservati sperimentalmente mostra in generale un soddisfacente accordo. Comunque, per alcuni livelli a basse energie di eccitazione risulta evidente l'interazione, oltre a quella considerata, di altre configurazioni in cui il neutrone al di fuori del core ^{48}Ca si trovi nelle orbite $2p_{1/2}$ e $1f_{5/2}$.

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1. INTRODUCTION. -

The intent of this work is to ascertain which properties of the N=29 isotones are consistent with wave functions composed of a ^{48}Ca core plus one neutron in the $2p_{3/2}$ shell and Z-20 protons in the $1f_{7/2}$ shell.

The levels corresponding to the above wave functions are determined by the residual interactions between two protons in the $1f_{7/2}$ shell and by the residual interaction of the $2p_{3/2}$ neutron with the $1f_{7/2}$ proton. The (pp) effective interactions is well known from the spectrum of ^{50}Ti , while the (np) effective interaction can be deduced from the spectrum of ^{56}Co .

Although the N=29 isotones are expected to contain, at low excitation energy, appreciable components of the $2p_{1/2}$ and $1f_{5/2}$ neutron states, our simple model will serve as a basis to recognize which of the experimentally observed levels are well reproduced by the pure $(2p_{3/2})(1f_{7/2})^m$ configuration. Deviations from the results of this model should serve as an indication of the importance of other configurations such as $(2p_{1/2})(1f_{7/2})^m$ and $(1f_{5/2})(1f_{7/2})^m$.

In the literature several theoretical investigations appear of the N=29 isotones. Ramavataram⁽¹⁾ used the unified model, in which the 29th neutron is coupled to a collective core. Maxwell et al.⁽²⁾ performed shell model calculations, using oscillator wave functions and choosing a Gaussian radial dependence for the residual interaction. Vervier⁽³⁾ calculated the levels of the N=29 isotones using for the residual neutron proton interaction a \mathcal{J} force. In the calculations Vervier considered the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ neutron orbits coupled to the possible core states of the protons in the $1f_{7/2}$ shell.

We here extend the analysis to all the N=29 isotones, without doing any assumption for the residual two body interaction, but considering only levels which arise from the pure $(2p_{3/2})(1f_{7/2})^m$ configuration.

2. HAMILTONIAN AND THE WAVE FUNCTIONS. -

The hamiltonian for the $(2p_{3/2})_n(1f_{7/2})_p^m$ configuration outside the ^{48}Ca core will be written as

$$(1) \quad H = H_{\text{core}} + \sum_p H_{\text{p-core}} + H_{\text{n-core}} + \sum_{\text{pairs}} H_{\text{pp}} + \sum_{\text{pairs}} H_{\text{pn}}$$

The first term represents the total interaction between nucleons inside the core, the second and the third terms represent the interactions of the extra core protons and neutron. The fourth and fifth terms represent the residual interactions between the extra core proton and neutron proton pairs.

The basis wave functions for total angular momentum J are chosen such that the protons are in a definite antisymmetric eigenstate of $\sum H_{pp}$, that is

$$(2) \quad |\psi [(J_p, v_p), j_n] J \rangle = \left| \left[(j_p)_{J_p, v_p}^m j_n \right] J \right\rangle .$$

Here $j_p = 7/2$ is the angular momentum of a single proton, $j_n = 3/2$ is the angular momentum of the neutron, m is the number of protons in the $1f_{7/2}$ orbit, J_p is the total angular momentum of the m protons, v_p defines the seniority of the protons group and J is the total angular momentum.

The first three terms of the Hamiltonian contribute to a constant energy to all the states of the $(2p_{3/2})(1f_{7/2})^m$ configuration. The energy matrix can then be written as:

$$(3) \quad E(J) = \text{const} + \left\langle \left[(j_p)_{J_p, v_p}^m j_n \right] J \left| \sum_{\text{pairs}} H_{pp} + \sum_{\text{pairs}} H_{pn} \right| \left[(j_p)_{J'_p, v'_p}^m j_n \right] J \right\rangle =$$

$$= \text{const} + E_p(J, v_p) \delta_{J_p, J'_p} \delta_{v_p, v'_p} + m E_{pn}(J, v_p, J'_p, v'_p, j_n; J)$$

Here E_p is the contribution of proton group and E_{pn} is the contribution of a single neutron proton pair to the matrix elements. More explicitly, E_p and E_{pn} are given by⁽⁴⁾:

$$(4) \quad E_p(J, v_p) = \sum_{I \text{ even}} \left[\frac{m(m-1)}{2} \sum_{vK} (j_p^{m-2} vK; j_p^2 I \left\| j_p^m v_p J_p \right\|^2 \right] \cdot \langle j_p^2 | H_{pp} | j_p^2 \rangle I$$

$$(5) \quad E_{np}(J) = \sum_I \sum_{vK} (j_p^{m-1} vK; j_p \left\| j_p^m v_p J_p \right\| (j_p^{m-1} vK; j_p \left\| j_p^m J'_p v'_p \right\|) \cdot$$

$$\cdot U(K j_p J j_n; J_p I) U(K j_p J j_n; J'_p I) \langle j_p j_n | H_{pn} | j_p j_n \rangle I$$

4.

The quantities in the (... } ...) brackets are the coefficients of fractional parentage for the proton states. The quantities U are the normalized Racah coefficients.

The energy matrix (3) is diagonalized to obtain the eigenvalues and the eigenfunctions. The eigenfunctions can be written as a linear combination of the basic states of equation (2), that is:

$$|\psi, J\rangle = \sum_{\substack{J_p \\ J_n}} C_{J_p J_n}^J \left| \left[(j_p)_{J_p}^m (j_n)_{J_n} \right]_J \right\rangle$$

3. CHOICE OF TWO BODY ENERGIES. -

The two body matrix elements of the residual interaction used for the matrix elements of equation (4) were taken from the experimental spectrum of ^{50}Ti . In Table I is shown the spectrum of $(f_{7/2})_J^2$ protons. The experimental energies and spin assignments are those determined by Sperduto et al. (5)

TABLE I

J^π	E_x (MeV)
0^+	0
2^+	1.555
4^+	2.686
6^+	3.208

TABLE II

J^π	E_x (MeV)
4^+	0
3^+	0.164
5^+	0.560
2^+	0.980

For the matrix elements of equation (5) the residual interactions between the $2p_{3/2}$ neutron and the $1f_{7/2}$ proton were taken from the experimental spectrum of ^{56}Co which is shown in Table II. The energies and spin assignments are those determined by Wells (6).

The experimental spectrum of ^{56}Co gives only in the assumption of a pure configuration the information about the matrix elements between the $2p_{3/2}$ neutron and the $1f_{7/2}$ proton hole. The matrix elements which appear in equation (5) are easily obtained by use of the Pandya relation (7):

$$(6) \quad \langle j_p j_n | H_{pn} | j_p j_n \rangle_I = \text{const} \cdot \sum_{I'} \left[\frac{2I'+1}{2I+1} \right]^{1/2} U(j_p j_n j_n j_p | II')$$

$$\langle (j_p)^{-1} j_n | H_{pn} | (j_p)^{-1} j_n \rangle_{I'}$$

Recently⁽⁸⁾ the spins and levels of ^{58}Co have been experimentally investigated by the $^{59}\text{Co}(p, d)^{58}\text{Co}$ reaction; however, for the calculations we have used the matrix elements derived from the spectrum of ^{56}Co . In fact, comparing the spectra of ^{56}Co and ^{58}Co by applying successively the Pandya relation, it comes out that the less perturbed spectrum is that of ^{56}Co .

4. ENERGY SPECTRA FOR THE N=29 ISOTONES. -

4. 1. - Levels of ^{51}Ti . -

There are 13 possible levels arising from the $(2p_{3/2})(1f_{7/2})^2$ configuration. The calculated energy spectrum is shown in fig. 1 and compared with known experimental levels.

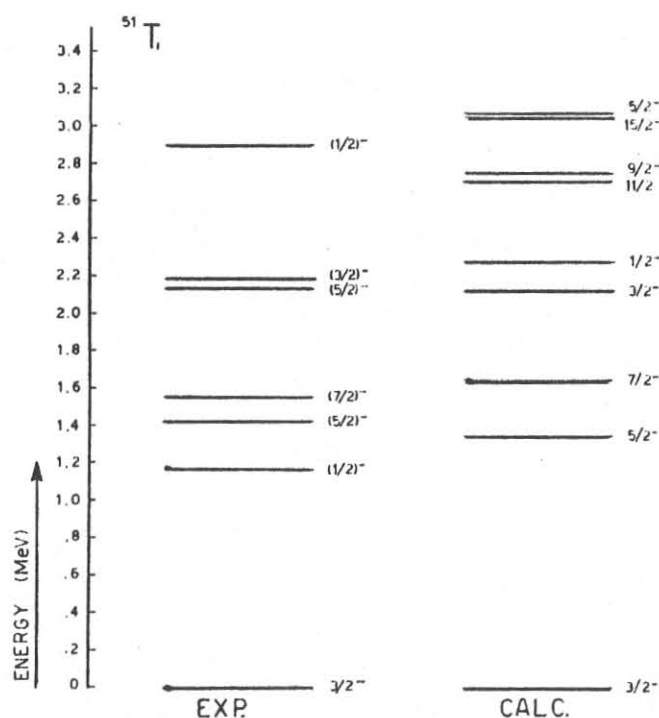


FIG. 1 - Comparison of calculated and experimentally observed levels for the ^{51}Ti nucleus.

6.

The levels with $J^\pi = 7/2^-, 3/2^-$ at 1.55 and 2.18 MeV respectively correspond very well with the calculated ones. The level $J^\pi = 1/2^-$, observed experimentally at 1.16 MeV, is depressed of about 1 MeV to lower excitation energy with respect to the calculated one. This is due to the interaction of the $(2p_{1/2})(1f_{7/2})^2$ configuration. The observed $J^\pi = 5/2^-$ level at 1.42 MeV correspond quite well with the calculated one. However, since this level has been observed in the $^{50}\text{Ti}(d, p)^{51}\text{Ti}$ stripping reaction⁽¹⁾ with a neutron momentum transfer $\ell = 3$, it contains some small admixture of the $(1f_{5/2})(1f_{7/2})^2$ configuration.

We can compare now the calculated spectroscopic factors with the observed ones for the stripping (d, p) reaction on ^{50}Ti .

The spectroscopic factor is given by:

$$S_{\ell=1, J=3/2} = \langle \psi(^{50}\text{Ti})_{J=0} \mid \psi(2p_{3/2})_{J=3/2} \mid \psi(^{51}\text{Ti})_{J=3/2} \rangle^2$$

Taking for the wave function of the ^{50}Ti nucleus a pure $(1f_{7/2})_p^2$ configuration the spectroscopic factor is given simply by

$$S_{13/2} = (C_{00}^{3/2})^2$$

In Table III are shown the calculated spectroscopic factors with the observed ones. (For the coefficients of the eigenfunctions see Appendix)

TABLE III

S_{cal}	S_{exp}	E_x (MeV)	J^π
0.895	0.895	0	$3/2^-$
0	0.134	1.42	$5/2^-$
0	0	1.55	$7/2^-$
0.105	0.075	2.18	$3/2^-$

The experimental values of the spectroscopic factors⁽¹⁾ are known only in their relative values. We have normalized to the calculated one for the transition to the ground state.

4.2. - Levels of ^{52}V -

There are 24 possible levels arising from the $(2p_{3/2})(1f_{7/2})^3$ configuration.

The calculated spectrum compared with the experimental one is shown in fig. 2. The experimental energy positions are those determined by Bjerregaard et al. ⁽⁹⁾.

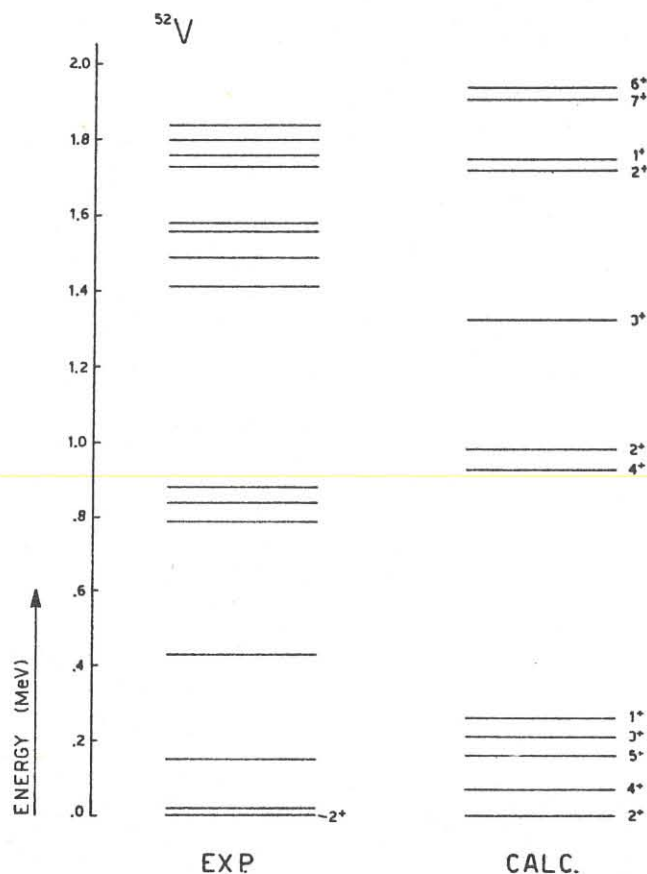


FIG. 2 - Comparison of calculated and experimentally observed levels for the ^{52}V nucleus.

Only the ground state spin of ^{52}V is known which is well reproduced by calculations. The pure $(2p_{3/2})(1f_{7/2})^3$ configuration predicts 5 levels within 360 KeV excitation energy, which are not observed experimentally. However, further information is required about the ^{52}V spectrum in order to comment the experimental levels with the ones calculated in the present work.

We calculate now the spectroscopic factors for the $^{51}\text{V}(d,p)$ ^{52}V stripping reaction for transitions up to 1 MeV excitation energy of our calculated spectrum.

The spectroscopic factor for a $2p_{3/2}$ neutron transfer is given by

$$S_{J\ell} = 1 = (C_{7/2,1}^J)^2 U^2(7/2 \ 0 \ 7/2 \ 3/2 \ | \ 7/2 \ J) = (C_{7/2,1}^J)^2 \frac{2J+1}{32} .$$

The calculated spectroscopic factors are shown in Table IV.

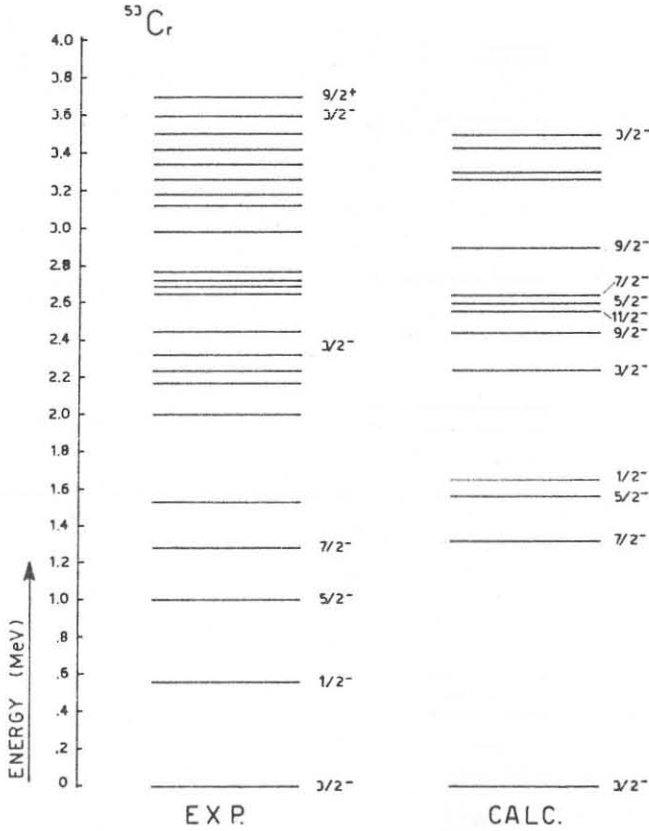


TABLE IV

S_{cal}	E_x (MeV)	J
0.108	0	2^+
0.134	0.07	4^+
0.324	0.16	5^+
0.174	0.21	3^+
0	0.26	1^+
0.122	0.93	4^+
0.018	0.98	2^+

FIG. 3 - Comparison of calculated and experimentally observed levels for the ^{53}Cr nucleus.

4.3. - Levels of ^{53}Cr .

There are 29 possible levels arising from the $(2p_{3/2})(1f_{7/2})^4$ configuration. The calculated spectrum compared with the experimental one is shown in fig. 3. The energy positions and spins of the experimental levels are those determined by Bock et al. (10).

We see the good agreement between the observed and calculated states for the $3/2^-$ (at 0, 2.32 and 3.6 MeV) and for the $7/2^-$ (at 1.28 MeV).

The $1/2^-$ and $5/2^-$ states are strongly depressed to lower excitation energy, due to the interaction of the $(2p_{1/2})(1f_{7/2})^4$ and $(1f_{5/2})(1f_{7/2})^4$ configurations (see Discussion).

The spectroscopic factors for the $^{52}\text{Cr}(d,p)^{53}\text{Cr}$ stripping reaction are given in Table V and are compared with the experimental ones⁽¹⁰⁾

TABLE V

S_{cal}	S_{exp}	E (MeV)	J^π
0.872	0.625	0	$3/2^-$
0	0.055	1.28	$7/2^-$
0.128	0.325	2.32	$3/2^-$
0	0.215	3.61	$3/2^-$

4.4. - Levels of ^{54}Mn . -

There are 24 possible levels arising from the $(2p_{3/2})(1f_{5/2})^5(2p_{1/2})(1f_{7/2})^{-3}$ configuration. The calculated spectrum is shown in fig. 4 and compared with the experimental one. The experimental energies are those determined by Bjenegaard et al. (9).

Little is known about the ^{54}Mn spectrum. It seems that as position energy many of the observed levels are reproduced by calculation. However, further experimental information is required before a comparison between observed and calculated levels can be done.

4.5. - Levels of ^{55}Fe . -

There are 13 possible levels arising from the $(2p_{3/2})(1f_{5/2})^6 \equiv (2p_{3/2})(1f_{5/2})^{-2}$ configuration. In the level calculations for this nucleus as well as for ^{54}Mn we have used equations (4) and (5), where m now stands for the number of proton holes in the $f_{7/2}$ shell, and the matrix elements $\langle j_p j_n | H_{pn} | j_p j_n \rangle_I$ have been replaced by the matrix elements $\langle (j_p)^{-1} j_n | H_{pn} | (j_p)^{-1} j_n \rangle_I$, which appear in equation (6).

The calculated energy spectrum is shown in fig. 5 with the experimental one. The experimental energies and spin assignments are those derived from the study of the $^{54}\text{Fe}(d,p)^{55}\text{Fe}$ stripping reaction⁽¹¹⁾.

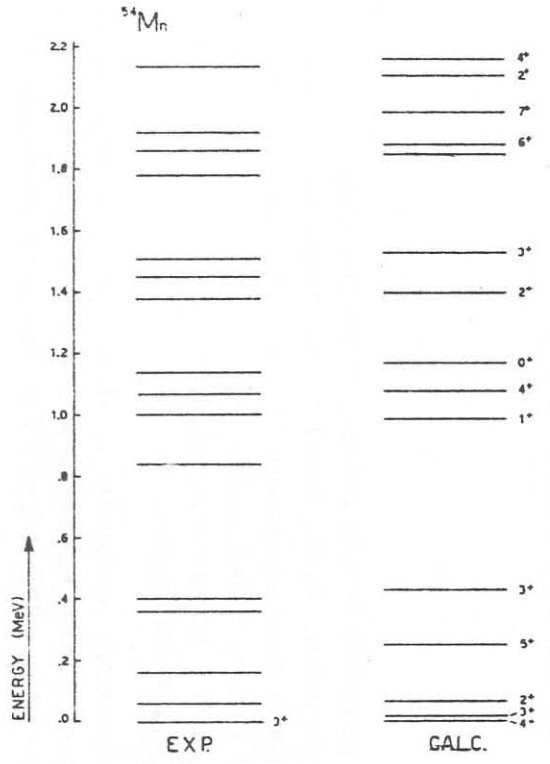


FIG. 4 - Comparison of calculated and experimentally observed levels for the ⁵⁴Mn nucleus.

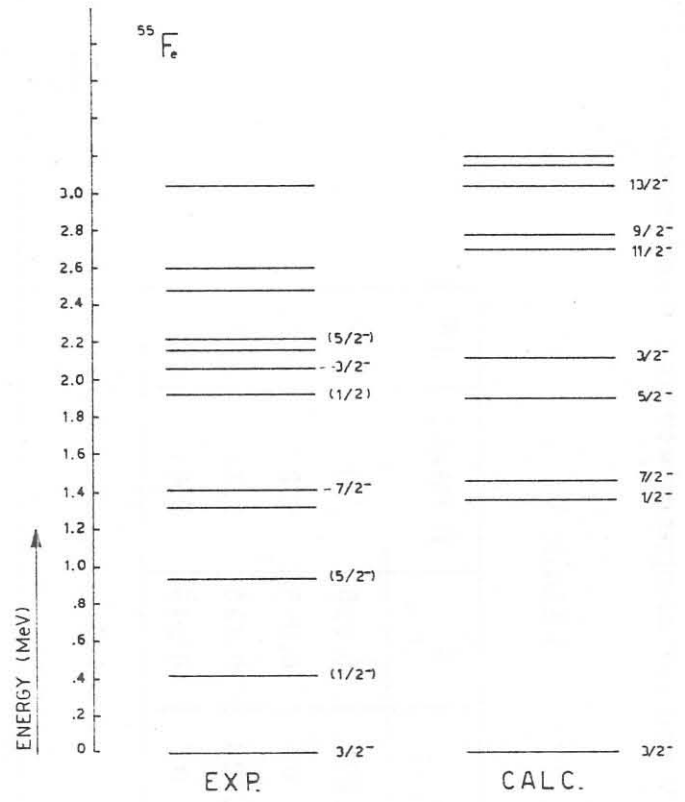


FIG. 5 - Comparison of calculated experimentally observed levels for the ⁵⁵Fe nucleus.

Again we see a good agreement between the observed spectrum and the calculated one for the levels at 0, 1.41 and 2.06 MeV with $J = 3/2^-$, $7/2^-$ and $3/2^-$ respectively. The $1/2^-$ and $5/2^-$ states are more strongly depressed to lower excitation energy compared to ^{51}Ti and ^{53}Cr (see Discussion).

In Table VI are shown the calculated spectroscopic factors with the observed ones for the transitions to the $3/2^-$ and $7/2^-$ states observed in the $^{54}\text{Fe}(d,p)^{55}\text{Fe}$ reaction(12).

TABLE VI

S_{cal}	S_{exp}	E (MeV)	J^π
0.895	0.813	0	$3/2^-$
0	0.017	1.413	$7/2^-$
0.105	0.094	2.061	$3/2^-$

5. MAGNETIC MOMENTS -

Within the framework of our pure $(2p_{3/2})(1f_{7/2})^m$ configuration for the $N=29$ isotones, we can write the z component of the magnetic moment operator as

$$\mu_z = g_p J_{pz} + g_n j_{nz}$$

where J_{pz} and j_{nz} are the z components of the protons and neutron angular momentum operators and g_p and g_n are the corresponding gyromagnetic ratios. The magnetic moment associated with the state (6) is given by:

$$\langle \psi_J | \mu_z | \psi_J \rangle = \frac{g_p + g_n}{2} \sum_{J_p, J_n} (C_{J_p, J_n}^J)^2 \frac{J_p(J_p+1) - j_n(j_n+1)}{(J+1)}$$

In Table VII are shown the calculated magnetic moments with the experimental ones. The best values for the effective gyromagnetic ratio g_p and g_n , as determined from the least square fit are the following:

$$g_p = +1.323$$

$$g_n = -0.507$$

TABLE VII

Nucleus (J^π)	Exp	Calc
^{53}Cr ($3/2^-$)	-0.474	-0.479
^{54}Mn (3^+)	3.296	+3.269
^{56}Co (4^+)	3.804	+3.828

As appears from Table VII, the agreement between the experimental and calculated magnetic moments is very good. The experimental values are reproduced better than 1%.

6. DISCUSSION. -

From the spectra analysis of the $N=29$ isotones group it comes out that the agreement between observed and calculated levels for the low lying states is getting worse near the closure of the $f_{7/2}$ proton shell. What is experimentally observed is that the $1/2^-$ and $5/2^-$ of the odd even isotones are depressed more to lower excitation energy when the $f_{7/2}$ proton shell is filled in. This appears clearly from analysis of fig. 6. In fig. 6 is shown that when we fill protons in the $1f_{7/2}$ shell, the $1/2^-$ and $5/2^-$ states are shifted nearer the ground states of the respective nuclei. This is due to the residual neutron proton interaction, which depresses the $2p_{1/2}$ and $1f_{5/2}$ neutron orbits near the $2p_{3/2}$ neutron state causing an appreciable mixing between the $(2p_{3/2})(1f_{7/2})^m$, $(2p_{1/2})(1f_{7/2})^m$ and $(1f_{5/2})(1f_{7/2})^m$ configurations. So it seems that at the beginning of the $1f_{7/2}$ proton shell it will be enough, in order to have a good description of the low lying states, to consider only the $(2p_{3/2})(1f_{7/2})^m$ and $(2p_{1/2})(1f_{7/2})^m$ configurations. However, for nuclei near the closure of the $1f_{7/2}$ proton shell, it must be included in the above configurations also the $(1f_{5/2})(1f_{7/2})^m$ configuration.

The energy positions of the $7/2^-$ and $3/2^-$ states do not shift appreciably when the $f_{7/2}$ proton shell is filled in, indicating that these states are well described by a pure $(2p_{3/2})(1f_{7/2})^m$ configuration. Another interesting feature is that the $3/2^-$ ground states of ^{51}Ti , ^{53}Cr and ^{55}Fe contain the $(1f_{7/2})^m$ protons which are predominantly in a state with seniority zero, while in the $3/2^-$ and $7/2^-$ excited states of the corresponding nuclei, the $(1f_{7/2})^m$ protons are in a state predominantly with seniority two. The residual neutron proton interaction has little effect in mixing group of protons with different seniority, As appears clearly from the ei

genfunctions tabulated in the Appendix, for the ^{51}Ti , ^{53}Cr and ^{55}Fe nuclei, the residual interaction causes a mixing of protons with different seniority no higher than 16%. For the ^{52}V and ^{54}Mn nuclei in which the number of protons in the $1f_{7/2}$ shell is odd, the seniority mixing caused by the residual neutron proton interaction is more accentuated.

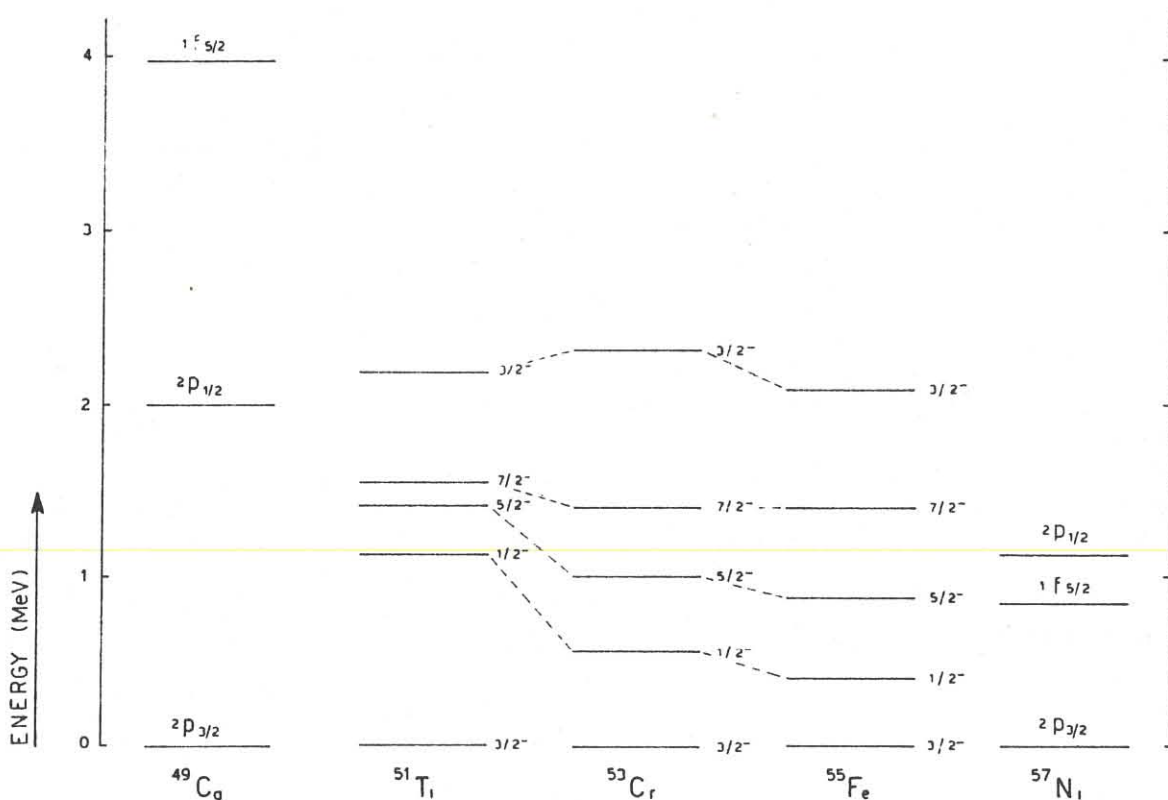


FIG. 6 - Energy positions of the $1/2$, $5/2$, $7/2$ and $3/2$ levels for the odd even $N=29$ isotones. In ^{49}Ca are shown the unperturbed positions of the $2p_{1/2}$ and $1f_{5/2}$ single neutron states, with no protons in the $f_{7/2}$ shell. In ^{57}Ni the perturbed positions of the respective single $2p_{1/2}$ and $1f_{5/2}$ neutron states when the $f_{7/2}$ proton shell is closed are shown.

7. CONCLUSIONS. -

Some of the low lying levels of the $N=29$ isotones are well accounted for in terms of the simple shell model picture of one $2p_{3/2}$ neutron and $1f_{7/2}$ protons outside a ^{48}Ca core. Our simple model serves as a good guide for interpreting the experimentally observed levels and judging the importance of other configurations. The present analysis indicates the importance of the $(2p_{1/2})(1f_{7/2})^m$ configuration for $m \leq 3$, in order to have a good interpretation of the low lying states, while for $m \geq 3$ it is necessary to con

sider also the $(1f_{5/2})(1f_{7/2})^m$ configuration.

However, before a general conclusion can be traced out, much experimental work has to be done.

The most prominent work seems to be high resolution (d, p) stripping reactions accomplished by \mathcal{V} angular correlations experiments. In addition, single proton transfer reactions are required of the type $^{53}\text{Cr}(d, ^3\text{He})^{52}\text{V}$ and $^{53}\text{Cr}(^3\text{He}, d)^{54}\text{Mn}$ or their (d, n) equivalent in order to test the configurations of the $f_{7/2}$ proton group.

Only such experiments performed with sufficient energy resolution can provide good information and would be very desirable for a full analysis of the N=29 isotones.

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APPENDIX

We give here the wave function for the N=29 isotones for those states which have a good correspondence with the experimental ones.

$$|^{51}\text{Ti}J\rangle = \sum_{J_p} \sum_{J_v} C_{J_p J_v}^J | \left[(j_p)_{J_p}^2 j_n \right] J \rangle$$

E (MeV)	J	C ₀₀	C ₂₂	C ₄₂	C ₆₂
0	3/2	0.946	0.325		
1.42	5/2		0.986	0.167	
1.55	7/2		0.930	0.385	
2.89	3/2	0.325	-0.946		

$$|^{52}\text{V}J\rangle = \sum_{J_p} \sum_{J_v} C_{J_p J_v}^J | \left[(j_p)_{J_p}^3 j_n \right] J \rangle$$

E (MeV)	J	C _{7/2,1}	C _{3/2,3}	C _{5/2,3}	C _{9/2,3}	C _{11/2,3}	C _{15/2,3}
0	2	0.833	-0.035	-0.552			
0.075	4	0.691		0.715	-0.041	0.099	
0.156	5	0.971			-0.094	0.221	
0.930	4	0.659		-0.662	0.227	0.276	
0.985	2	0.336	-0.760	0.556			

$$|^{53}\text{Cr}J\rangle = \sum_{J_p} \sum_{J_v} C_{J_p J_v}^J | \left[(j_p)_{J_p}^4 j_n \right] J \rangle$$

E (MeV)	J	C ₀₀	C ₂₂	C ₄₂	C ₆₂	C ₂₄	C ₄₄	C ₅₄	C ₈₄
0	3/2	0.934	0.358						
1.30	7/2		0.913	-0.002		-0.071	-0.401	0.009	
2.32	3/2	-0.358	0.934						
3.60	3/2			1					

$$|^{54}\text{M}_n^J\rangle = \sum_{J_p^v p} C_{J_p^v p}^J | [(j_p)_{J_p^v p}^5 j_n] J \rangle$$

E (MeV)	J	$C_{7/2,1}$	$C_{3/2,3}$	$C_{5/2,3}$	$C_{9/2,3}$	$C_{11/2,3}$	$C_{15/2,3}$
0	4	-0.078	-0.474	0.868	-0.123		
0.019	3	0.753	0.091	0.650	-0.041		
0.070	2	0.718	0.131	0.683			
0.250	5	0.962			0.107	-0.249	
0.430	3	-0.582	-0.327	0.730	0.147		
1.00	1		0.702	0.712			
1.080	4	0.241	0.825	0.435	-0.267		
1.170	0		1				
1.400	2	-0.676	-0.101	0.730			
1.530	3	-0.234	0.912	0.162	0.296		

$$|^{55}\text{Fe}J\rangle = \sum_{J_p^v p} C_{J_p^v p}^J | [(j_p)_{J_p^v p}^6 j_n] J \rangle$$

E (MeV)	J	C_{00}	C_{22}	C_{42}	C_{62}
0	3/2	0.946	-0.325		
1.413	7/2		0.960	-0.280	
2.061	3/2	0.325	0.946		