Comitato Nazionale per L'Energia Nucleare
ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Milano
66/11

INFN/BE-66/11
16 Novembre 1966

E. Gadioli and I. Iori: INFORMATIONS ABOUT THE LEVEL DENSITY OF LIGHT NUCLEI ($20 \lesssim A \lesssim 70$) OBTAINED BY ANALYZING FLUCTUATING EXCITATION FUNCTIONS. -

Reparto Tipografico
dei Laboratori Nazionali di Frascati

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INTRODUCTION -

Various formulae have been proposed to describe the level density of atomic nuclei. One of the most used expressions for the density of levels of spin J and parity π is the following due to Lang and Le Couteur(1):

$$(1) \qquad \varrho(U,J) = \frac{\hbar^3}{24\sqrt{8}} (2J+1) \exp\left[\frac{-J(J+1)}{2\sigma^2}\right] a^{1/2} \mathcal{J}^{-3/2} \frac{\exp(2\sqrt{aU})}{(U+\underline{t})^2}$$

σ^2 is the spin cut-off factor, $\underline{t}$ is the thermodynamic temperature, $\underline{a}$ is a parameter related to the spacing of the single particle states at the top of the Fermi distribution; J is the moment of inertia of the nucleus(2). σ^2, $\mathcal{J}$ and $\underline{t}$ are related through the expression $\sigma^2 = (\mathcal{J}\underline{t}/\hbar^2)$; $\underline{t}$ depends on the excitation energy U by means of the formula:

$$a\,\underline{t}^2 - \underline{t} = U.$$

Essentially three parameters are contained in formula (1):

1) - $\underline{a}$;
2) - the spin cut-off factor σ^2;

(x) - Work supported by the Research Contract Euratom-CNEN/INFN.

3) - the zero of the energy scale from which the excitation energy U has to be evaluated.

ANALYSIS OF SLOW NEUTRON RESONANCES BY MEANS OF THE LANG AND LE COUTEUR FORMULA -

Looking at the experimental level densities obtained by analyzing the slow neutron resonances, empirical values of $\underline{a}$ have been obtained by E. Erba et al. [3] utilizing formula (1). These authors have assumed for $\mathcal{J}$ the rigid body value corresponding to a spherical nucleus with radius $R = 1.4\ A^{1/3}$ fm; they have taken for the zero of the energy scale the ground state of the nuclei in the case of odd-odd nuclei, whereas, for even-odd and even-even nuclei, the zero of the energy scale has been displayed over the ground state by a quantity equal to the pairing energy Δ as given by Cameron[4].

In fig. 1 the $\underline{a}$ values obtained in ref. 3 are plotted versus A for nuclei with $20 \leq A \leq 70$ (big black points). These values seem to show a linear dependence on A, being distributed around the straight line of equation $\underline{a} \simeq (0.127\ A + 1.13)\ \text{Mev}^{-1}$ with fluctuations of the order of 10-15% (with the only exception of Al^{28}).

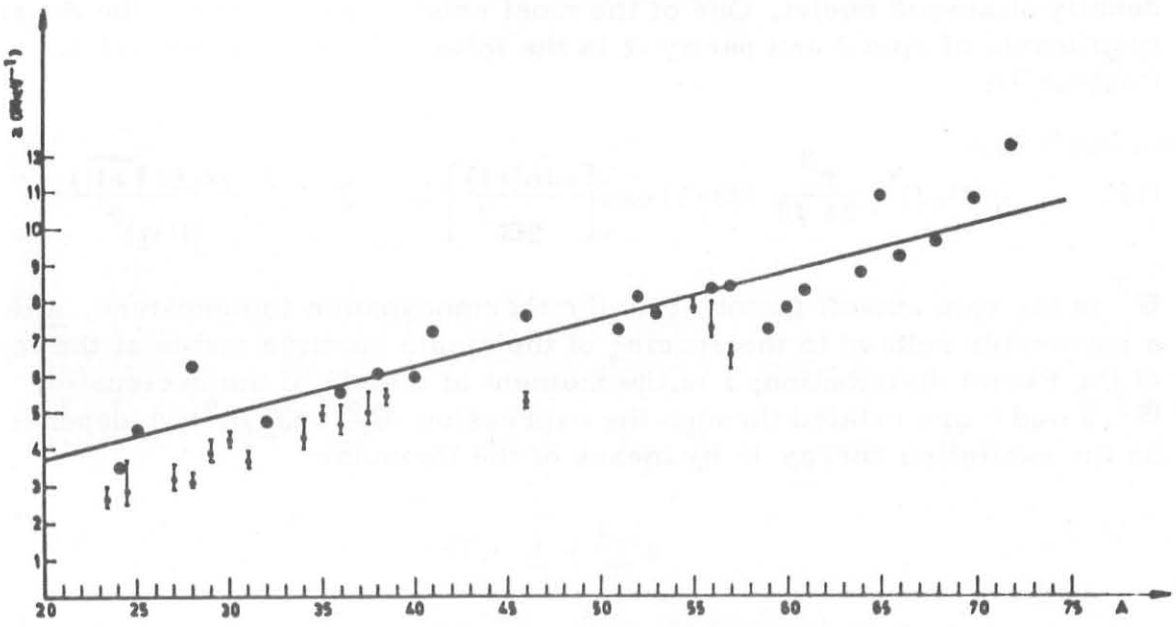

FIG. 1 - Comparison between low energy $\underline{a}$ values (big black points) deduced from the analysis of slow neutron resonances by E. Erba et al., (ref. 3) using formula (1), and high energy $\underline{a}$ values (small black points) obtained from the analysis of coherence energies characterizing fluctuating cross-sections.

The energy dependence of formula (1) could not be tested: in fact the $\underline{a}$ parameters have been obtained for a fixed excitation energy of the order of 7-8 Mev.

It has to be noticed that the Fermi gas model predicts a dependence of $\underline{a}$ on A of the type $\underline{a} = kA$. The empirical $\underline{a}$ values of ref. 3, on the other hand, seem to follow a law of the type $\underline{a} = kA + h$. In the Fermi gas model the value of k corresponds to a radius of the nucleus $R = 1.57\ A^{1/3}$ fm, that is not too unrealistic. This result may suggest that, if the Fermi gas model works, the term h appears because the two other parameters, appearing in (1), have been uncorrectly choosen. It is important to notice, however, that at excitation energies of the order of 7-8 Mev, formula (1) with the $\underline{a}$ values of ref. 3, fits correctly the experimental data, just because the $\underline{a}$ values are empirical.

ANALYSIS OF FLUCTUATING CROSS-SECTIONS AND COMPARISON WITH THE SLOW NEUTRON RESONANCES RESULTS -

Recently other experimental data have become available, permitting to study the level density at an excitation energy of the order of 15-20 Mev.

This source of information is the phenomenon of the fluctuations[5] of the nuclear reactions cross-sections at excitation energies in the continuum energy region.

Recently we have shown[6] how the "coherence energy" Γ characterizing these fluctuations can be expressed by means of the widths of the levels of the compound nucleus involved in the reaction.

In short notation one can write:

$$\Gamma = f\left[\Gamma_J^{\pi}(U)\right] \tag{2}$$

Γ is the coherence energy determined experimentally, $\Gamma_J^{\pi}(U)$ is the width of the levels of the C. N. of spin J and parity π. The function f indicates a weighted average of the $\Gamma_J^{\pi}(U)$ over J, π and the energy U.

The $\Gamma_J^{\pi}(U)$ are expressed by the formula[5]:

$$\Gamma_J^{\pi}(U) = \frac{1}{2\pi\rho(U,J)} \sum_{\nu} \int_0^{U_\nu^{max}} dU_\nu\, \rho_{o\nu}(U_\nu^{*}) \sum_{l'} T_{l'}^{\nu}(U_\nu) \sum_{s'} \sum_{j} (2j+1)\, e^{-\frac{j(j+1)}{2\sigma^2}} \tag{3}$$

ν labels the various residual nuclei to which the C. N. can decay, $T_{l'}^{\nu}(U_\nu)$

are the transmission functions of the corresponding emitted particles, j is the spin of the various residual nuclei, $\varrho(U,J)$ is the level density of the C.N., $\varrho_{o\nu}(U_\nu^x)$ are the level densities of the residual nuclei ν, U_ν^{Max} is given by the following expression:

$$U_\nu^{Max} = E_{inc} + Q^{(\nu)} - \Delta \tag{4}$$

where E_{inc} is the kinetic energy, in the center of mass system of the entrance channel, $Q^{(\nu)}$ are the Q values of the various possible reactions; Δ is the Cameron[4] pairing energy of the residual nucleus ν,

$$U_\nu^x = U_\nu^{Max} - U_\nu \tag{5}$$

The excitation energy U of the C.N. is given by

$$U = E_{inc} + B - \tag{6}$$

where B is the binding energy of the incident particle in the C.N.

The integrands in (3) have a Maxwellian shape. In the case of light nuclei, for statistical reactions proceeding through the formation of a C.N. excited at an energy $U \approx 15$-20 Mev, the main contribution to the sum, in (3), is that corresponding to the neutron and proton decay and the principal contribution to this term is due to values of the integrand for which $U_\nu^x = (U_\nu^{Max} - U_\nu) \approx 4$-$7$ Mev. These values of the energy are not very different from the energy at which the empirical a values of ref. 3 have been evaluated. Then, by using the optical model transmission functions and the Lang and Le Couteur level density formula with such a values, we believe that the sum in (3) can be evaluated with a reasonable accuracy.

After having performed the sum, in the expression of $\Gamma_J^{\pi}(U)$ the only unknown quantity which remains is the level density of the C.N. As Γ is a quantity deduced experimentally and the functional dependence of Γ on the $\Gamma_J^{\pi}(U)$ is known, it is possible to obtain, from formula (2), semi-empirical values of the C.N. level density.

By using the same σ^2 and the same zero of the energy scale that Erba et al. used, we obtain a values corresponding to an excitation energy of 15-20 Mev. A comparison of the a values so obtained with the same quantities at low energy, gives a good test of the energy dependence of formula (1) and of the parameters used(x).

(x) - We have performed these calculations by using neutrons and protons transmission functions given in ref. 7, and α particles transmission functions given in ref. 8. The deuterons transmission functions, in the case of the (d, p) and (d, α) reactions on target nuclei having $A \approx 28$ have been calculated with the parameters reported in ref. 6.

In fig. 1 the small black points are the a values obtained at high excitation energy. To perform a coherent analysis, we have reexamined all the published results on the fluctuation phenomena following the method outlined in ref. 9. In some cases the values of Γ we obtained are different from the values given by the authors. In Table I, row V gives the Γ values we used. The errors affecting the a values we deduced, are due to the errors affecting the Γ values.

The errors on Γ are evaluated, in the case of nuclei with $A < 50$, by means of the formulas derived in ref. 10. For these nuclei the uncertainty on the knowledge of the experimental energy resolution is unimportant as far as concerns the coherence energy.

For nuclei with $A > 50$ the coherence energy becomes small and of the order of the experimental resolution. For these nuclei, the error affecting the experimental value of Γ is mainly due to the uncertainty on the knowledge of the energy resolution δ (x). Some published experimental results have been omitted from our analysis mainly for two reasons: either the energy interval explored was too narrow or the excitation energy of the C.N. was too low to assure the overlapping of the C.N. levels.

In order to semplify our analysis for the evaluation of the a values, we used the following formula(6) which relates Γ to the various $\Gamma^{\pi}_{J}(U)$:

$$(7) \qquad \frac{1}{\Gamma^2(U)} \simeq \frac{\sum_J \frac{1}{(\Gamma^{\pi}_J(U))^2} \exp(J(J+1)/\sigma^2) \sum_{sl} T_l^2(\alpha, s, J) \sum_{s'l'} T_{l'}^2(\alpha', s', J)}{\sum_J \exp(J(J+1)/\sigma^2) \sum_{sl} T_l^2(\alpha, s, J) \sum_{s'l'} T_{l'}^2(\alpha', s', J)}$$

This formula strictly holds for integrated excitation functions and purely statistical reactions. The use of this formula however, should be quite correct also in the case of differential excitation functions as long as one considers reactions characterized by not too low values of the spin in the entrance and exit channels and the differential excitation functions are measured at angles where the direct contribution is certainly small (for a discussion of this point see section 4 of ref. 6).

In the case of the reaction $^{12}C(^{12}C,\alpha_o)^{20}Ne$, Γ was obtained as an average value of several $\Gamma(\theta)$ obtained at different angles using the

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(x) - The values assumed for δ in the reactions on nuclei with $A > 50$ we examined are as follows:

$^{54}Cr\ (p,\alpha_o)^{51}V$	27)	$\delta = 3.5 \pm 0.5$ kev
$^{55}Mn\ (p,\alpha_o)^{52}Cr$	28)	$\delta = 4.0 \pm 1.0$ kev
$^{56}Fe\ (p,p_o)^{56}Fe$	29)	$\delta = 3.0 \pm 0.5$ kev

TABLE I

Reaction	C.N.	U	U'	Γ (keV)	Ref.
$^{23}Na(p,\alpha_o)^{20}Ne$	^{24}Mg	16.692	19.608	108±56	(11)
$^{12}C(^{12}C,\alpha_o)^{20}Ne$	^{24}Mg	20.815	23.731	111±94	(12)
$^{26}Mg(p, p_2)^{26}Mg^*$	^{27}Al	15.491	18.084	69±29	(13)
$^{26}Mg(p,\alpha_o)^{23}Na$	^{27}Al	15.722	18.315	53±27	(14)
$^{25}Mg(d, p_o)^{26}Mg$	^{27}Al	17.056	19.648	67±38	(15)
$^{27}Al(p,\alpha_o)^{24}Mg$	^{28}Si	14.994	17.494	21±4.5	(16)
$^{27}Al(p,\alpha_1)^{24}Mg^*$	^{28}Si	14.994	17.494	18±3	(16)
$^{27}Al(p,\alpha_2)^{24}Mg^*$	^{28}Si	15.717	18.217	26.5±8	(16)
$^{27}Al(p,\alpha_3)^{24}Mg^*$	^{28}Si	15.717	18.217	29±9	(16)
$^{27}Al(p, p_o)^{27}Al$	^{28}Si	16.200	18.700	23±7	(17)
$^{27}Al(p,\alpha_o)^{24}Mg$	^{28}Si	16.923	19.423	46±13	(18)
$^{27}Al(p,\alpha_1)^{24}Mg^*$	^{28}Si	16.923	19.423	41±10	(18)
$^{27}Al(p,\alpha_o)^{24}Mg$	^{28}Si	17.357	19.857	22±6	(19)
$^{27}Al(p,\alpha_1)^{24}Mg^*$	^{28}Si	17.357	19.857	40±11	(19)
$^{27}Al(d,\alpha_o)^{25}Mg$	^{29}Si	17.425	19.838	31.5±15	(20)
$^{28}Si(d,\alpha_o)^{26}Al$	^{30}P	19.645	21.978	45±16	(21)
$^{27}Al(\alpha, p_1)^{30}Si^*$	^{31}P	14.636	16.894	16±6	(22)
$^{27}Al(\alpha, p_1)^{30}Si^*$	^{31}P	14.745	17.003	17±5	(22)
^{32}S (d, $p_1)^{33}S^*$	^{34}Cl	13.518	15.576	32±18	(23)
$^{31}P(\alpha, p_1)^{34}S^*$	^{35}Cl	17.961	19.961	24±7	(24)
$^{35}Cl(p,\alpha_o)^{32}S$	^{36}A	15.068	17.012	18±11	(25)
$^{37}Cl(p,\alpha_o)^{34}S$	^{38}A	16.866	18.708	19±13	(25)
$^{37}Cl(p,\alpha_o)^{34}S$	^{38}A	18.269	20.111	14±4.5	(25)
$^{37}Cl(p,\alpha_1)^{34}S^*$	^{38}A	18.269	20.111	16±5	(25)
$^{45}Sc(p,\alpha_1)^{42}Ca^*$	^{46}Ti	15.815	17.336	3.4±0.8	(26)
$^{54}Cr(p,\alpha_o)^{51}V$	^{55}Mn	17.638	18.911	$3.4^{+0.4}_{-0.7}$	(27)
$^{55}Mn(p,\alpha_o)^{52}Cr$	^{56}Fe	16.952	18.202	$3.2^{+1.0}_{-2.2}$	(28)
$^{56}Fe(p, p_o)^{56}Fe$	^{57}Co	14.136	15.364	$2.0^{+1.7}_{-0.2}$	(29)

TABLE I - Summary of the reactions examined to evaluate the high energy a values.

formula reported in section 4 of ref. 6.

Table II summarizes the a values reported in fig. 1.

As it is possible to see from fig. 1 the a values we obtained at high excitation energy, are always smaller than the low energy values. In fact, using the low energy a values to calculate Γ we should sistematically obtain Γ values much smaller than the experimental ones.

However an interesting point has to be noticed: the high energy a values reported as functions of A, still lie approximately on a straight line. The slope of this straight line is approximately the same as the one obtained at low energy, but now is $h \approx 0$.

This is the main point: the experimental results seem to suggest that the slope of the straight line fitting the low energy a values is quite correct; the term h may be due to the influence of other uncorrectly choosen parameters. However their influence disappears at high energy as the values of $\varrho(U, J)$ there, depend almost completely on the values of a.

In fact it is easy to show that a systematic variation of both σ^2 and of the zero of the energy scale, causes a translation of all the low energy a values which, as a consequence, causes h to increase or to decrease.

Our interest is now devoted to see wether it is possible to obtain new values of the spin cut-off factor σ^2 and of the zero of the energy scale corresponding to which the a values, also at low energy, vary according to the law $a = kA$. In such case the same set of parameters should allow a correct estimation of $\varrho(U, J)$ in the energy range from about 7 Mev up to ~ 20 Mev.

Let us consider first the spin cut-off factor σ^2.

If one supposes the nucleons of the nucleus to move into an infinite square well, one may show that $\mathcal{J} = 2/5\ A\ R^2$, that is $\mathcal{J}$ equals the rigid body value(30). However this result depends strongly on the shape of the nuclear potential assumed. In the case of an harmonic oscillator potential, the value of $\mathcal{J}$ is reduced to 62.5% of the rigid body value(30). Moreover it is well known that σ^2 is connected to $\langle m^2 \rangle$, the average, extended to the excited nucleons, of the square of the projection of the total angular momentum j of each nucleon on the Z axis. If one makes the assumption that this value does not differ sensitively from that corresponding to all nucleons in a shell model potential and one uses the estimate of this quantity as given by Jensen and Luttinger(31), one obtains a value of the moment of inertia of the nucleus reduced by a factor 0.5 with respect to the rigid body value, assuming $R = 1.5\ A^{1/3}$fm.

We decided to assume $R = 1.5\ A^{1/3}$fm and a value of $\mathcal{J}$ lying between the mentioned values: $\mathcal{J} = 0.7\ \mathcal{J}_{RIG}$.

The ricalculated a values at low energy, corresponding to such

TABLE II

C.N.	U (Mev)	a (Mev^{-1})	C.N.	U (Mev)	a (Mev^{-1})
^{24}Na	6.959	3.48	^{38}A	16.866	$4.94^{-0.27}_{+0.64}$
^{24}Mg	16.692	$2.64^{-0.16}_{+0.29}$	^{38}A	18.269	$5.39^{-0.14+}_{+0.19}$
^{24}Mg	20.815	$2.86^{-0.26}_{+0.87}$	^{40}K	7.798	5.93
^{25}Mg	5.231	4.52	^{41}Ca	6.850	7.14
^{27}Al	15.491	$3.04^{-0.15}_{+0.23}$	^{46}Sc	8.766	7.57
^{27}Al	15.722	$3.17^{-0.17}_{+0.32}$	^{46}Ti	15.815	$5.27^{-0.12}_{+0.13}$
^{27}Al	17.056	$3.36^{-0.19}_{+0.36}$	^{51}V	9.570	7.18
^{28}Al	7.724	6.25	^{52}V	7.304	8.06
^{28}Si	14.994	$2.88^{-0.08+}_{+0.09}$	^{53}Cr	6.503	7.62
^{28}Si	15.717	$3.01^{-0.11+}_{+0.15}$	^{55}Mn	17.638	$7.82^{-0.05}_{+0.18}$
^{28}Si	16.200	$3.28^{-0.11}_{+0.17}$	^{56}Mn	7.270	8.28
^{28}Si	16.923	$3.12^{-0.10+}_{+0.13}$	^{56}Fe	16.952	$7.22^{-0.17}_{+0.74}$
^{28}Si	17.357	$3.37^{-0.10+}_{+0.14}$	^{57}Fe	6.192	8.30
^{29}Si	17.425	$3.91^{-0.17}_{+0.30}$	^{57}Co	14.136	$6.47^{-0.40}_{+0.09}$
^{30}P	19.645	$4.28^{-0.14}_{+0.20}$	^{59}Ni	7.631	7.17
^{31}P	14.636	$3.65^{-0.14}_{+0.20}$	^{60}Co	7.947	7.74
^{31}P	14.745	$3.67^{-0.13}_{+0.19}$	^{61}Ni	6.453	8.30
^{32}P	7.937	4.76	^{64}Cu	7.916	8.65
^{34}Cl	13.518	$4.33^{-0.23}_{+0.46}$	^{65}Zn	6.899	10.76
^{35}Cl	17.961	$4.95^{-0.13}_{+0.15}$	^{66}Cu	7.061	9.10
^{36}Cl	8.577	5.47	^{68}Zn	7.589	9.51
^{36}A	15.068	$4.65^{-0.25}_{+0.50}$	^{70}Ga	7.730	10.64
^{38}Cl	6.110	6.03	^{72}Ga	6.960	12.06

TABLE II - Summary of the low energy a values given by E. Erba et al.[3] and the high energy a values obtained analyzing fluctuating cross-sections. For the level density it has been assumed expression (1). The values marked with a cross are an average of two a values corresponding to coherence energies characterizing excitation functions leading to different final states.

value of σ^2, are distributed around a straight line of equation $\underline{a} = (0.125\ A + 1.03)\ Mev^{-1}$. The new choice of $\mathcal{J}$ caused a very small variation of the slope and a more sensible variation of h. We examine now if it is possible to choose a zero of the energy scale for each nucleus corresponding to which the $\underline{a}$ values giving the correct estimate of the level density both at 7 and 15--20 Mev, are distributed around a straight line of equation $\underline{a} = 0.127\ A\ Mev^{-1}$.

This goal is attained substituting, in expression (1), the value of the energy U with the value $U' = U + \Delta U$, being $\Delta U = (70/A)$ Mev. In fig. 2 the open circles are the new low energy $\underline{a}$ values we obtained, the black points are the $\underline{a}$ values deduced from the analysis of the coherence energy Γ using the low energy $\underline{a}$ values. These values are not so different from the ones previously obtained for the following reasons:

a) - at high energy the absolute value of the level density depends almost entirely on $\underline{a}$ and small variations of this parameter compensate stronger variations of the other parameters appearing in formula (1).

b) - In the cases we examined, the value of the numerator in expression (3) calculated with $\varrho(U', J)$, $\mathcal{J} = 0.7\,\mathcal{J}_{RIG}$ ($R_o = 1.5$ fm) and $\underline{a} = (0.127\ A)\ Mev^{-1}$, is a bit larger than the value calculated using $\varrho(U, J)$ with $\mathcal{J} = \mathcal{J}_{RIG}$ ($R_o = 1.4$ fm) and $\underline{a} = (0.127\ A + 1.13)\ Mev^{-1}$. This increase, due to the fact that at low U values $\varrho(U, J)$ as absolute value is smaller than $\varrho(U', J)$, reduces the lowering of the high energy $\underline{a}$ values due to the introduction of the term ΔU in the level density expression.

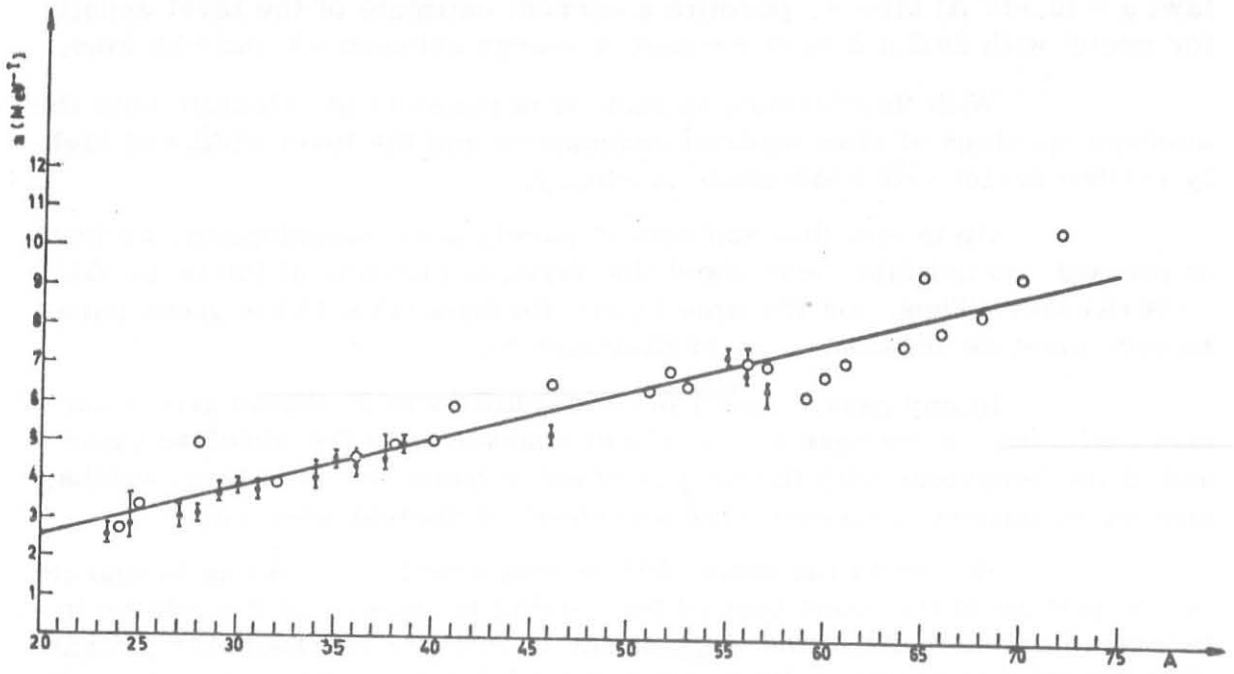

FIG. 2 - Comparison between low energy $\underline{a}$ values (open circles) deduced from slow neutron resonances and high energy $\underline{a}$ values (black points), obtained analyzing coherence energies characterizing fluctuating cross-sections, utilizing the level density $\varrho(U', J)$ described in the text.

In Table III are reported the data for the low and high energy a values.

One can see that the agreement between the low and high energy a values is quite satisfactory. The same set of fixed parameters gives now the correct estimate of the level density from ~7 to ~20 Mev. This fact guarantees the possibility of evaluating, with a reasonable accuracy, nuclear quantities such as the level widths of nuclei.

It should be noticed that, for $A > 40$ the deviations of the single a values from the straight line can be due to experimental errors we did not take correctly into account.

Moreover, due to the quite strong variation of the $\Gamma^{\pi}_{J}(U)$ with J[6] deviations of single a values may be due to the uncertainty in the calculated transmission functions. Other sources of errors may be the approximations we did in calculating $\Gamma^{\pi}_{J}(U)$ and Γ.

CONCLUDING REMARKS -

The conclusions of this analysis are the following: formula (1) with $\gamma = 0.7\ \gamma_{RIG}$, $R = 1.5\ A^{1/3}$fm, substituting to the excitation energy U the value $U' = (U + 70/A)$ Mev, and using the a values given by the law: $a = (0.127\ A)\ \text{Mev}^{-1}$, permits a correct estimate of the level density for nuclei with $20 \leqq A \leqq 70$ at excitation energy between ~7 and ~20 Mev.

With this formula in fact, it is possible to calculate both the average spacings of slow neutron resonances and the level widths of highly excited nuclei with reasonable accuracy.

Up to now this analysis is purely phenomenological: we have, at present, no definite ideas about the physical meaning of the term $\Delta U = (70/A)$ Mev. Thus, for the time being, formula (1) with the given parameters must be considered as semiempirical.

In any case it has a practical utility as it should give a correct estimate, in the case of statistical reactions, of the absolute value and of the behaviour with the energy of some quantities like level widths, nuclear reactions cross-sections and shape of nuclear spectra.

As far as the term ΔU is concerned, our feeling is that its origin may be of the same type of the pairing energies and it could be interpreted on the basis of the suggestions of Hurwitz and Bethe[32]; regarding the pairing energies the correctness of these suggestions is already proved.

Examining the parameters used in this analysis, what follows can be said. The agreement between the low and high energy a values seems to suggest that γ should not exceed the value we used. In fact a higher va-

TABLE III

C.N.	U'(Mev)	a (Mev^{-1})	C.N.	U'(Mev)	a (Mev^{-1})
^{24}Na	9.906	2.70	^{36}A	17.012	$4.37^{-0.23}_{+0.45}$
^{24}Mg	19.608	$2.55^{-0.14}_{+0.27}$	^{38}Cl	7.950	4.88
^{24}Mg	23.731	$2.79^{-0.25}_{+0.83}$	^{38}A	18.708	$4.62^{-0.25}_{+0.59}$
^{25}Mg	8.031	3.36	^{38}A	20.111	$4.93^{-0.13+}_{+0.17}$
^{27}Al	18.084	$2.90^{-0.14}_{+0.21}$	^{40}K	9.548	4.98
^{27}Al	18.315	$3.02^{-0.16}_{+0.28}$	^{41}Ca	8.577	5.89
^{27}Al	19.648	$3.10^{-0.16}_{+0.33}$	^{46}Sc	10.287	6.53
^{28}Al	10.224	4.93	^{46}Ti	17.336	$5.19^{-0.11}_{+0.12}$
^{28}Si	17.494	$3.00^{-0.07+}_{+0.08}$	^{51}V	10.942	6.36
^{28}Si	18.217	$3.02^{-0.11+}_{+0.13}$	^{52}V	8.650	6.92
^{28}Si	18.700	$3.20^{-0.11}_{+0.15}$	^{53}Cr	7.823	6.47
^{28}Si	19.423	$3.03^{-0.09+}_{+0.11}$	^{55}Mn	18.911	$7.23^{-0.05}_{+0.16}$
^{28}Si	19.857	$3.22^{-0.09+}_{+0.13}$	^{56}Mn	8.520	7.16
^{29}Si	19.838	$3.59^{-0.16}_{+0.27}$	^{56}Fe	18.202	$6.81^{-0.16}_{+0.69}$
^{30}P	21.978	$3.77^{-0.12}_{+0.19}$	^{57}Fe	7.420	7.07
^{31}P	16.894	$3.72^{-0.14}_{+0.19}$	^{57}Co	15.364	$6.39^{-0.38}_{+0.09}$
^{31}P	17.003	$3.72^{-0.13}_{+0.18}$	^{59}Ni	8.817	6.26
^{32}P	10.126	3.90	^{60}Co	8.663	6.77
^{34}Cl	15.576	$4.03^{-0.22}_{+0.41}$	^{61}Ni	7.600	7.15
^{35}Cl	19.961	$4.51^{-0.12}_{+0.14}$	^{64}Cu	9.054	7.61
^{36}Cl	10.521	4.59	^{65}Zn	7.975	9.37
			^{66}Cu	8.121	7.96
			^{68}Zn	8.618	8.40
			^{70}Ga	8.730	9.44
			^{72}Ga	7.932	10.62

TABLE III - Summary of the a values obtained from slow neutron resonances and the a values obtained analyzing fluctuating cross-sections. For the level density, the expression $\rho(U', J)$ as given in the text, is assumed. The values marked with a cross are an average of two a values corresponding to coherence energies charactarizing excitation functions leading to different final states.

lue of γ in the calculation of a from the slow neutron resonances with the Lang and Le Couteur formula without the term ΔU, gives an increase of h though k remains unchanged. This can be seen directly calculating a with a change in the parameter γ. In our procedure an increase of h causes ΔU to increase if a is still given by the law $a = (0.127\ A)\ Mev^{-1}$. Such an increase of ΔU, however, does not modify the $\xi(U, J)$ value of slow neutron resonances which is the experimental one, but causes a slight decrease of the high energy a values, making worse the agreement between the low and high energy a values.

The pairing energies we used are taken from ref. 4. Other values of these quantities are given by Newton[33] and in a recent paper by Cameron[34]. The differences between different values are not appreciable, of the order of 100-200 kev; as a consequence our results are largely independent from the choice of the pairing energies.

The transmission coefficients we used are from ref. 7 for neutrons and protons and from ref. 8 for α particles. For low energy neutrons and protons they are obtained by extrapolating, at low energy, the optical model potential parameters fitting experimental data at higher energy (≥ 4 Mev). Other calculations of transmission functions have appeared in the literature for neutrons and protons[35]. For low energy neutrons ($E_n < 4$ Mev), these transmission functions differ somewhat from ours, being larger: the disagreement is smaller as the nuclear mass increases. For protons there are not appreciable differences. As far as our analysis is concerned, however, the difference in the neutron transmission functions has a small influence on the results. We estimated that the use of the transmission functions of ref. 35 could increase some of the high energy a values (for nuclei with $A \approx 20\text{-}30$) of a maximum of 10%. Such a change does not alter at all our conclusions: in fact it could increase the agreement we obtained between the low and high energy a values.

We have used the level density formula (1) substituting to U, $U' = U + \Delta U$. It is a question wether, from a theoretical point of view, in the level density expression it is better to use $(U + t)^2$ or U^2 in the denominator(1)(34). In the considered energy region t is always much smaller than U so the use of one or the other term do not modify the conclusions we have drawn.

Just to conclude, our opinion is that more work is necessary to establish with greater accuracy the coefficient k in the equation $a = kA$ and the term ΔU. At present the experimental results, we have utilised, are not sufficient to evaluate these terms with a very good accuracy. An improvement in the knowledge of these values could be obtained by analyzing a greater number of results concerning the slow neutron resonances and the fluctuation phenomena.

Thanks are due to Dr. T. Ericson for helpfull discussions and to Dr. A. Marini for continuous help.

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