Comitato Nazionale per L'Energia Nucleare
ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Pisa
66/1

INFN/BE - 66/1
21 Gennaio 1966

L. Lovitch and S. Rosati : THE TWO-NUCLEON SCHRODINGER EQUATION WITH TENSOR FORCES. A FORTRAN PROGRAM FOR THE $J=1^+$ BOUND AND ZERO ENERGY EIGENSOLUTION.

Reparto Tipografico
dei Laboratori Nazionali di Frascati

1

Istituto Nazionale di Fisica Nucleare
Sezione di Pisa

INFN/BE-66/1
21 Gennaio 1966

L. Lovitch[(x)] and S. Rosati: THE TWO-NUCLEON SCHRODINGER EQUATION WITH TENSOR FORCES. A FORTRAN PROGRAM FOR THE $J=1^+$ BOUND AND ZERO ENERGY EIGENSOLUTIONS[(+)]. -

ABSTRACT. -

A Fortran program is presented for determining the bound state and zero energy eigensolutions, and associated properties, of the neutron-proton $J=1^+$ system when the interaction between the particles is described by a given combination of central, tensor and spin-orbit potentials.

1. INTRODUCTION. -

The solution of the deuteron Schrödinger equation in the presence of tensor forces, that is to say, the calculation of the eigenvalue, wave functions and associated properties, in correspondence to some given potential, does not lend itself in any case to an analytic solution in closed form. A perturbation solution when the tensor force is not small is not practical, while the use of trial wave functions accompanied by a standard variational calculation of energy suffers the usual difficulties, especially when a repulsive core is present.

(x) - Department of Physics, Columbia University, New York, N. Y.

(+) - This research was supported in part by the U. S. Atomic Energy Commission.

The variation-iteration procedure developed by Feshbach and Schwinger[1] has been used quite extensively in recent years, especially following the coding by Kalos and Blatt[2] originally devised for the Illinois computer. Apart form being and indirect method, it does not always, of itself, converge rapidly and when modified accordingly results in quite extensive computations. A less essential criticism is that the latter code is written in the symbolic language of the ILLIAC and so is neither readily understood nor easily adapted for other electronic computers.

This report describes a computer program written by us in FORTRAN, which, when the neutron-proton potential is assigned, evaluates:

a) - for the bound state, the eigenvalue, the associated wave functions, the percentage of the D component, the quadrupole moment and the deuteron effective range,

b) - for zero energy, the corresponding wave functions, the scattering length, the effective range, and the first two shape-dependent parameters.

The method adopted, described in refs. 3) and 4), is a direct numerical integration of the equations employing middle-point matching conditions which for the zero energy case determines the solutions uniquely, while for the bound-state deuteron equations it yields a corrector formula to a trial eigenvalue giving quadratic convergence when used iteratively.

The program is composed of three subprograms. The main subprogram includes the input and output instructions and the principle calculations involved. It calls on two subprograms: the first of these is a short program which calculates the potentials at the set of net-points of the integration (allowing four possible combinations of potential shape), while the second solves the set of simultaneous linear equations that result when the continuity conditions for the inward and outward solutions are imposed at the matching-point.

2. MATHEMATICAL FORMULATION. -

Throughout, we choose 1 MeV, 1 fm and $\hbar$ as the units of energy, length and action. In terms of these the velocity of light is 197.32 MeV fm $\hbar^{-1}$, and M, which is twice the reduced neutron-proton mass, is 0.024114 $\mathrm{MeV}^{-1}\mathrm{fm}^{-2}\hbar^{2}$.

The neutron-proton potential is taken to be of the form

$$V(\vec{r}) = V_C(r) + V_T(r)\, S_{12} + V_{LS}(r)\, \vec{L}\, \vec{S} \quad , \tag{1}$$

where

$$S_{12} = 3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) / r^2 - \vec{\sigma}_1 \cdot \vec{\sigma}_2 .$$

The $J = 1^+$ wave function is

$$\psi = {}^3S_1 + {}^3D_1 = r^{-1}\left\{ u(r)\mathcal{Y}^{J=1}_{L=0,\,S=1} + w(r)\mathcal{Y}^{J=1}_{L=2,\,S=1} \right\}, \tag{2}$$

where the angular momentum functions

$$\mathcal{Y}^J_{L,S} = \sum_{M_L,\,M_S} (LSM_L M_S / JM)\, Y_{L,M_L}(\theta, \phi)\, \chi_{S,M_S}$$

are proper combinations of spherical harmonics and spin functions, and in the case of the bound state, the radial functions satisfy the normalisation condition

$$\int_0^\infty dr \left\{ u^2(r) + w^2(r) \right\} = 1 . \tag{3}$$

The Schrödinger equation gives rise to the following coupled equations for $u(r)$ and $w(r)$:

$$\begin{aligned} \frac{d^2u}{dr^2} &= \left\{ -\mathcal{E} + f_1(r) \right\} u + f_2(r)\, w , \\ \frac{d^2w}{dr^2} &= f_2(r)\, u + \left\{ -\mathcal{E} + \frac{b}{r^2} + f_3(r) \right\} w , \end{aligned} \tag{4}$$

where

$$\mathcal{E} = ME, \tag{5a}$$

and

$$\begin{aligned} f_1(r) &= MV_C(r) , \qquad f_2(r) = 8^{1/2}\, MV_T(r) , \\ f_3(r) &= M\left[V_C(r) - 2\,V_T(r) - 3\,V_{LS}(r) \right] . \end{aligned} \tag{5b}$$

The boundary conditions for the solutions are that $u = w = 0$ for $r = 0$, or at the edge of a hard core, and that u, $w \rightarrow 0$ as $r \rightarrow \infty$ for the bound state, while, for the zero-energy α_2-solution, the coefficient of the r^3 term in w is null, i. e. $w_\alpha \propto 1/r^2$ (we return to this later).

2. 1. - The Bound state. -

For any negative value of $\mathcal{E}$ (see eq. (5a)) the eqs. (4) have two linearly independent solutions $u(\mathcal{E})$ and $w(\mathcal{E})$ which are regular at the origin, and two linearly independent solutions which have the decaying asymptotic behaviour required of a bound state. From the linear combination of these an infinity of inward and outward solution can be obtained :

$$\begin{aligned} u_{in}(\mathcal{E}) &= A\, u^{(1)}_{in}(\mathcal{E}) + B\, u^{(2)}_{in}(\mathcal{E}) , \\ w_{in}(\mathcal{E}) &= A\, w^{(1)}_{in}(\mathcal{E}) + B\, w^{(2)}_{in}(\mathcal{E}) , \end{aligned} \tag{6}$$

and

$$u_{out}(\varepsilon) = C\, u^{(1)}_{out}(\varepsilon) + D\, u^{(2)}_{out}(\varepsilon)\,,$$
$$w_{out}(\varepsilon) = C\, w^{(1)}_{out}(\varepsilon) + D\, w^{(2)}_{out}(\varepsilon)\,. \tag{7}$$

Therefore, for a given trial energy ε, two independent solutions are integrated inward from large distances and two independent solutions are integrated outward form the origin, or edge of a hard core, until they meet at some intermediate point $r = R$. At $r = R$, the inward and outward solutions, and their first derivatives, can be made equal with suitable choice of A, B, C and D if and only if the trial energy value ε coincides with an eigenvalue. In other words, an eigenvalue is given as a zero of the determinant

$$\Delta(\varepsilon) = \begin{vmatrix} u^{(1)}_{in}(\varepsilon) & u^{(2)}_{in}(\varepsilon) & u^{(1)}_{out}(\varepsilon) & u^{(2)}_{out}(\varepsilon) \\ w^{(1)}_{in}(\varepsilon) & w^{(2)}_{in}(\varepsilon) & w^{(1)}_{out}(\varepsilon) & w^{(2)}_{out}(\varepsilon) \\ u^{(1)'}_{in}(\varepsilon) & u^{(2)'}_{in}(\varepsilon) & u^{(1)'}_{out}(\varepsilon) & u^{(2)'}_{out}(\varepsilon) \\ w^{(1)'}_{in}(\varepsilon) & w^{(2)'}_{in}(\varepsilon) & w^{(1)'}_{out}(\varepsilon) & w^{(2)'}_{out}(\varepsilon) \end{vmatrix} \tag{8}$$

We wish now to explain why we integrate two inward and two outward solutions rather than limit ourselves to integrating in just one direction. Let us first examine what happens if we integrate outward only. As we proceed toward the asymptotic region the general solution for u and w is of the form

$$u \sim \bar{A}\, e^{-kr} + \bar{B}\, e^{kr}\,,$$
$$w \sim \bar{C}\, e^{-kr}\left(1 + \frac{3}{kr} + \frac{3}{k^2 r^2}\right) + \bar{D}\, e^{kr}\left(1 - \frac{3}{kr} + \frac{3}{k^2 r^2}\right) \tag{9}$$

The boundary conditions impose, however, that $\bar{B}$ and $\bar{D}$ be zero. But rounding off and truncation errors introduce small amounts of the unwanted solution and so sooner or later the outward solution explodes exponentially, in every case, and it is not possible to formulate a criterion which will determine the correct solution unambiguously.

To integrate inward only can be just as inaccurate, since the origin is, in general, a singular point of equs. (4). For example, if the interparticle potentials are constant around the origin, then it can be shown that the general solution for w behaves as $1/r^2$ when $r \rightarrow 0$, while if the potentials are Yukawian then $u \sim 1/r$ and $w \sim 1/r^2$ as $r \rightarrow 0$. It is, therefore, impossible to make u and w vanish at the origin by integrating inward only, since rounding off and truncation errors will always introduce part of the unwanted singular solution. Integrating both outward from the origin, or edge of a hard core, and inward from large distances, with the correct boundary conditions imposed in beginning the integrations, until they meet at some intermediate point, avoids such difficulties, since the unwanted solutions decrease in these directions, respectively.

With regard to the inward solutions, these can be started in the "extreme asymptotic" region, i. e. in the region where the nuclear potentials are zero or completely disregardable, so that the eqs. (4) result uncoupled and have the solution

$$u(r) = N_S\, e^{-kr}\,,$$
$$w(r) = N_D\, e^{-kr}\left(1 + \frac{3}{kr} + \frac{3}{k^2 r^2}\right), \tag{10}$$

where N_S and N_D are constants of integration and $k^2 = -\mathcal{E}$.

The range of integration can, in practice, be halved by the use of a JWKB-type approximation[3] so as to start the inward solutions of eqs. (4) in the "medium asymptotic" region, where the nuclear potentials are small but not negligible. If R_{j-1}, R_j are two successive points of the net which we consider in the numerical integration in this region, with the corresponding values (u_{j-1}, w_{j-1}) and (u_j, w_j) for the solutions of eqs. (4), then[3]

$$\frac{u_{j-1}}{u_j} = \frac{[\bar{u}_o(R_j)]^{1/2}}{[\bar{u}_o(R_{j-1})]^{1/2}} \exp\left[\frac{1}{2}(R_j - R_{j-1})\left\{\bar{u}_o(R_j) + \bar{u}_o(R_{j-1})\right\}\right],$$

and (11)

$$\frac{w_{j-1}}{w_j} = \frac{[\bar{w}_o(R_j)]^{1/2}}{[\bar{w}_o(R_{j-1})]^{1/2}} \exp\left[\frac{1}{2}(R_j - R_{j-1})\left\{\bar{w}_o(R_j) + \bar{w}_o(R_{j-1})\right\}\right],$$

where

$$\bar{u}_o(r) = +\left[-\mathcal{E} + f_1(r) + f_2(r)\,\frac{w(r)}{u(r)}\right]^{1/2},$$
$$\bar{w}_o(r) = +\left[-\mathcal{E} + \frac{6}{r^2} + f_3(r)\,\frac{u(r)}{w(r)}\right]^{1/2}. \tag{12}$$

The right-hand-sides of (11) then involve the ratios w_j/u_j and w_{j-1}/u_{j-1}. It is sufficiently accurate in the "medium asymptotic" region to give an arbitrary value to w_j/u_j and to replace w_{j-1}/u_{j-1} in $\bar{u}_o$ and $\bar{w}_o$ by

$$\frac{w_{j-1}}{u_{j-1}} = \frac{w_j}{u_j}\left(1 + \frac{3}{kR_{j-1}} + \frac{3}{k^2 R_{j-1}^2}\right) \Big/ \left(1 + \frac{3}{kR_j} + \frac{3}{k^2 R_j^2}\right), \tag{13}$$

although in principle one could iterate, i. e. insert the resulting ratios given by (11) in (12) to obtain new values, and so on until convergence is achieved. This iteration procedure has been tested and been found to be unnecessary, giving the same final results.

As has been already observed, $\Delta(\mathcal{E})$ is zero only when $\mathcal{E}$ is an eigenvalue of the system. Integrating two inward and two outward solutions for an arbitrary negative value of $\mathcal{E}$ until some matching-point, $r=R$, is reached, we can choose A, B, C and D so that at $r=R$,

$$\begin{aligned} u_{in}(\mathcal{E}) &= u_{out}(\mathcal{E}) = u_m , \\ w_{in}(\mathcal{E}) &= w_{out}(\mathcal{E}) , \\ w'_{in}(\mathcal{E}) &= w'_{out}(\mathcal{E}) , \end{aligned} \tag{14}$$

where u_m is any constant value which is fixed a priori. It can be shown[3] that if $\mathcal{E}+\delta\mathcal{E}$ is the exact eigenvalue then we can write a corrector formula

$$\delta\mathcal{E} = \frac{u_m\left[u'_{out}(\mathcal{E}) - u'_{in}(\mathcal{E})\right]}{\int_0^R dr\left[u^2_{out}(\mathcal{E}) + w^2_{out}(\mathcal{E})\right] + \int_R^\infty dr\left[u^2_{in}(\mathcal{E}) + w^2_{in}(\mathcal{E})\right]} + 0(\delta\mathcal{E}^2). \tag{15}$$

Alternatively, we can choose A, B, C and D such that, at $r=R$,

$$\begin{aligned} u_{in}(\mathcal{E}) &= u_{out}(\mathcal{E}) , \\ u'_{in}(\mathcal{E}) &= u'_{out}(\mathcal{E}) , \\ w_{in}(\mathcal{E}) &= w_{out}(\mathcal{E}) = w_m , \end{aligned} \tag{16}$$

where w_m is a constant value fixed a priori; and then $\delta\mathcal{E}$ is given by

$$\delta\mathcal{E} = \frac{w_m\left[w'_{out}(\mathcal{E}) - w'_{in}(\mathcal{E})\right]}{\int_0^R dr\left[u^2_{out}(\mathcal{E}) + w^2_{out}(\mathcal{E})\right] + \int_R^\infty dr\left[u^2_{in}(\mathcal{E}) + w^2_{in}(\mathcal{E})\right]} + 0(\delta\mathcal{E}^2). \tag{17}$$

It should be noted that the use of eqs. (14) with the corrector formula (15), or eqs. (16) with the corrector formula (17) should and does yield identical results.

If, alternatively, we assume a given value for the bound state energy $\mathcal{E}$, we can invert the problem, that is to say, it is possible to adjust one of the parameters of the potential so as to reproduce such an energy value. For example, if the variable parameter is a multiplicative factor which is common to the central, tensor and spin-orbit parts of the potential, so that in eqs. (4) we now have the terms λf_1, λf_2, λf_3 in place of f_1, f_2 and f_3 respectively, then the corrector formula for this factor λ is:

$$\delta\lambda = \frac{-u_m\left[u'_{out}(\varepsilon) - u'_{in}(\varepsilon)\right]}{\int_0^R\left[u_{out}^2 f_1 + w_{out}^2 f_3 + 2u_{out}w_{out}f_2\right] + \int_R^\infty dr\left[u_{in}^2 f_1 + w_{in}^2 f_3 + 2u_{in}w_{in}f_2\right]}, \tag{15a}$$

for the case that we satisfy the eqs. (14) at the matching-point $r = R$. This corresponds to eq. (15) for $\delta\varepsilon$. We can also readily deduce the corrector formula for $\delta\lambda$ that may be used in conjuction with eqs. (16):

$$\delta\lambda = \frac{-w_m\left[w'_{out}(\varepsilon) - w'_{in}(\varepsilon)\right]}{\int_0^R dr\left[u_{out}^2 f_1 + w_{out}^2 f_3 + 2u_{out}w_{out}f_2\right] + \int_R^\infty dr\left[u_{in}^2 f_1 + w_{in}^2 f_3 + 2u_{in}w_{in}f_2\right]}. \tag{17a}$$

Finally, if $\gamma^2 = -\varepsilon_d$, where ε_d is the eigenvalue of eqs. (4), then as $r \to \infty$ we have the behaviour for the corresponding solution: $u_d \sim N_S e^{-\gamma r}$ and $w_d \sim N_d e^{-\gamma r}$. The deuteron effective range is defined to be

$$\rho_t = \rho(E_d, E_d) = \frac{1}{\gamma} - \frac{2}{N_S^2 + N_D^2} . \tag{18}$$

2. 2. - The zero-energy solution. -

Imposing the boundary conditions at the origin, or edge of a hard core, it is clear that there are two linearly independent scattering eigensolutions. These are known as the α-solution and β-solution and are mixtures of pure $^3S_1$ and $^3D_1$ states which correspond to pure $^3S_1$ and $^3D_1$ waves respectively in the limit that the tensor force vanishes.

The general solution of eqs. (4), with $\varepsilon = 0$, for asymptotic values of r, i. e. where the potentials vanish, is

$$u \sim gr + h , \qquad w \sim g' r^3 + \frac{h'}{r^2} \tag{19}$$

The α-solution and β-solution have the properties[5] that $g'_\alpha = 0$ and $h_\beta = 0$ respectively, and at zero and low energies the scattering occurs in the α-channel. This may be described in terms of an effective range approximation for such energy values.

Consider the four linearly independent asymptotic solutions (19) of eqs. (4). We can combine these so as to obtain three linearly independent solutions possessing the property that $g' = 0$. Thus, to solve for the α-solution we integrate two linearly independent solutions outward from the origin (or edge of hard core), with the correct zero boundary condition there, and integrate three linearly independent solutions inward from large values of r, until some matching-point $r = R$ is reached, and combine them linearly :

$$u_{out} = A\,u^{(1)}_{out} + B\,u^{(2)}_{out}\,, \qquad u_{in} = C\,u^{(1)}_{in} + D\,u^{(2)}_{in} + E\,u^{(3)}_{in}\,,$$

$$w_{out} = A\,w^{(1)}_{out} + B\,w^{(2)}_{out}\,, \qquad w_{in} = C\,w^{(1)}_{in} + D\,w^{(2)}_{in} + E\,w^{(3)}_{in}\,,$$

so that the solutions and their derivatives are continuous at $r = R$,

$$u_{out}(R) = u_{in}(R)\,, \qquad u'_{out}(R) = u'_{in}(R)\,,$$

$$w_{out}(R) = w_{in}(R)\,, \qquad w'_{out}(R) = w'_{in}(R)\,.$$

There are therefore four equations in four unknowns, the relative mixing of the solutions (the fifth unknown may be given an arbitrary non-zero value), which have clearly a unique solution.

From the resulting solution we can derive the scattering length

$$a_t = -\frac{h_\alpha}{g_\alpha} \tag{20}$$

and the effective range

$$\begin{aligned} r_{0t} &= 2\int_0^\infty dr\left[\left(1-\frac{r}{a_t}\right)^2 - u_\alpha^2 - w_\alpha^2\right] = \\ &= 2\left[r_f\left(1-\frac{r_f}{a_t}+\frac{r_f^2}{3a_t^2}\right) - \int_{r_c}^{r_f} dr\left\{u_\alpha^2 + w_\alpha^2\right\} - \frac{r_f}{3}\left\{w_\alpha(r_f)\right\}^2\right], \end{aligned} \tag{21}$$

where r_c is the hard core radius, or zero if there is no hard core, and r_f is the distance from the origin of any point in the asymptotic region. It should be mentioned that the final term in expression (21) arises from evaluating directly

$$-2\int_{r_f}^\infty dr\, w_\alpha^2\,.$$

The mixed effective range is given by

$$\rho(0, E) = 2\left(\gamma - \frac{1}{a_t}\right)/\gamma^2\,. \tag{22}$$

Finally, the shape parameters P_t and Q_t in the effective range approximation:

$$k\cot\delta_\alpha = -\frac{1}{a_t} + \frac{1}{2}\,r_{0t}k^2 - P_t\,r_{0t}^3k^4 + Q_t\,r_{0t}^5k^6\,, \tag{23}$$

are given by[4] :

$$P_t = \left\{ 3(\gamma - \frac{1}{a_t}) - \gamma^2 (r_{0t} + \frac{1}{2} \rho_t) \right\} / r_{0t}^3 \gamma^4 =$$
$$= \left\{ \frac{3}{2} \ (0, E_d) - r_{0t} - \frac{1}{2} \rho_t \right\} / r_{0t}^3 \gamma^2 \ , \tag{24}$$

and

$$Q_t = \left\{ -2(\gamma - \frac{1}{a_t}) + \frac{1}{2}\gamma^2 (r_{0t} + \rho_t) \right\} / r_{0t}^5 \gamma^6 =$$
$$= \left\{ -\rho(0, E_d) + \frac{1}{2}(r_{0t} + \rho_t) \right\} / r_{0t}^5 \gamma^4 \ . \tag{25}$$

If, on the other hand, we adopt the effective range approximation[(7)] (see also eq. (3. 2) of ref. (8)) :

$$k \cot \delta_d = -\frac{1}{a_t} + \frac{1}{2} r_{0t} k^2 + \frac{C_t}{1 + D_t r_{0t}^2 k^2} \ , \tag{23a}$$

which may well be applicable over a larger range of values of k than eq. (23), then it may be shown that[(4)]

$$C_t = \frac{(\gamma - \frac{1}{a_t} - \frac{1}{2} r_{0t} \gamma^2)^2}{r_{0t}^3 \gamma^4 (\gamma - \frac{1}{a_t} - \frac{1}{2} \rho_t \gamma^2)} = \frac{\left\{ \rho(0, E_d) - r_{0t} \right\}^2}{2 r_{0t}^3 \gamma^2 \left\{ \rho(0, E_d) - \rho_t \right\}} \ , \tag{24a}$$

and

$$D_t = \frac{2(\gamma - \frac{1}{a_t}) - \frac{1}{2}(r_{0t} + \rho_t)\gamma^2}{r_{0t}^2 \gamma^2 (\gamma - \frac{1}{a_t} - \frac{1}{2} \rho_t \gamma^2)} = \frac{2\rho(0, E_d) - r_{0t} - \rho_t}{r_{0t}^2 \gamma^2 \left\{ \rho(0, E_d) - \rho_t \right\}} . \tag{25a}$$

3. PROGRAM DESCRIPTION. -

The program was originally written in FORTRAN II, but at an intermediate stage of the testing it was decided to translate it to FORTRAN IV (due to changes in the computer facilities available to us). We therefore present the FORTRAN IV version of the program, and keeping in mind the fact that FORTRAN II is still being used extensively, we note that the only changes required to obtain a FORTRAN II version are appropriate modifications in the input and output statements and to the names, in call statements, of the subprograms of the system.

The program listing that we present yields double precision operations for all floating-point calculations. To obtain double precision results it is not necessary to perform all these calculations in double precision, but it was felt that the greater simplicity resulting, combined with the very small time difference involved on an IBM 7094 computer, justified our doing so. To change to single precision calculations, all that is then needed is to replace the D-conversion by E-conversion for the floating point constants and in format statements associated with the input and output, and to remove all type statements "Double Precision . . . . " that appear towards the beginning of each subprogram.

The entire program consists of a main routine and two subroutines. The main routine executes the bulk of the calculations and the input and output instructions, subroutine POT calculates the potential functions normalised accordingly, and subroutine SYSTEM performs the solution of a set of simultaneous linear equations of arbitrary order (we need to solve a set of three equations prior to evaluating the corrector formula for the bound state, and a set of four equations to match the zero-energy scattering solutions).

We present the listing of the main program and subroutine POT in Appendix A. For subroutine SYSTEM (A, N, B) any standard subprogram that solves the set of N simultaneous linear equations, which we write in matrix form

$$AX = B, \qquad (26)$$

and which, before returning execution to the calling program, replaces the array B by the solution X of the system, may be used. The subprogram used by us was based on a program MATINV existing in the SHARE catalogue available to users of IBM machines. For compatibility with the main program it is necessary that A and B of subroutine SYSTEM be dimensioned as A(4,4) and B(4) respectively.

Subroutine POT yields the central, tensor and spin-orbit potentials at the set of net-points of the integration range combined, and normalised, in a form suitable for direct insertion in the differential equations. The well shapes of the potentials calculated may be

$$\begin{aligned} &\text{Square well} : && V(r) = \begin{cases} V_0, & r < r_0, \\ 0, & r > r_0, \end{cases} \\ &\text{Exponential well} : && V(r) = V_0\, e^{-\mu r}, \\ &\text{Yukawa well} : && V(r) = \frac{V_0\, e^{-\mu r}}{\mu r}, \\ &\text{Gauss well} : && V(r) = V_0\, e^{-\alpha r^2}, \end{aligned} \qquad (27)$$

and there may or may not be a hard core associated with the two-particle interaction. The information necessary to define the potentials is transmitted to the subroutine from the main program, after being specified by the input data.

3. 1. - Main Subprogram. -

The most important part of this program for the causal user consists of the input statements, so that once a set of input data is specified the machine effects the calculations requested by the data. There are three such input statements: they are preceded by the external formula numbers 2, 4 and 6 and are associated respectively with the format statements having external formula numbers 500, 504 and 516.

Since, in general, the nuclear potentials are short ranged and vary strongly near the origin, it is necessary, when solving the differential equations, to use small integration step-lengths in this region; while on the other hand, when we reach a region where the potentials become small, it may be sufficient to use much larger step-lengths to obtain a particular accuracy and, indeed, it would be extremely wasteful of machine-time to use the same step-length over the whole range of integration besides requiring a corresponding increase in the capacity of the memory of the computer. We have therefore allowed for the possibility of using different integration step-lengths, dividing up the total range of integration of the equations into blocks, each of which is subdivided into a certain number of intervals of equal length. Clearly, the actual step-lengths to be used for a particular problem depends essentially on the accuracy desired for the results; nevertheless we feel it worthwhile stating, as an illustration, that, to obtain very accurate results with some potentials, step-lengths of about 0. 001 fm had to be used near the origin, whereas at about 2 fm away from it step-lengths of 0. 2 fm were already quite adequate.

The first input statement:

```
2    READ(5,500) CØRE, H(1), JØUT, JTØT, (N(L),X(L),L=1,JTØT)
```

associates the numbers on the first few data cards with respectively, the hard core radius, the step-length to be used nearest the origin, the number of blocks to be used in the outward integration, the total number of blocks to be used over the entire integration range, followed by the number of step-lengths to be used in the first block (i. e. the one nearest the origin), the step-length scale factor in going from the first to second block, the number of step-lengths to be used in the second block, the step-length scale factor in going from the second to the third integration block, and so on until the number of step-lengths in the final block is read.

The machine then proceeds to calculate the distance from the origin of each of the integration net-points and the appropriate factors that should be associated with them in the numerical integration of simple quadratures, such as is required later to normalise the wave functions. The numerical integration formula used for these quadratures is the three-point SIMPSON rule.

In the second input statement of the program:

```
4    READ (5,504) IV,   VC,   RC,   VT,   RT,   VL,   RL
```

the first quantity is the one which decides whether square wells, exponentials wells, Yukawa wells or Gauss wells are to be used for the central, tensor and spin-orbit potentials of the nuclear force interaction. This is followed successively by the depth and range of each of these three potentials. The depths should in all cases be presented in MeV, positive for a repulsive potential and negative for an attractive potential, whereas the ranges for a square well, exponential well, Yukawa well, Gauss well should be given in fm, fm^{-1}, fm^{-1}, fm^{-2} respectively. It should be noted that if a calculation is to be made with a neutron-proton potential having no spin-orbit term and using Yukawa potentials for the central and tensor parts, then a non-zero value should be assigned to the range of the spin-orbit term.

The third and final input statement of the program:

```
6     READ (5,516) EO, EPS, IC, IR, IPR, M, KCH, MAXIT
```

reads a trial energy for the bound state (this should be negative), and the minimum significance to be accorded to the successive corrections to the energy value in MeV. Thus, if a correction is less than the latter quantity in absolute value (it may be given as zero), then the energy search is terminated. As has been already mentioned, however, the accuracy is limited rather by the step-lengths used in the integration formula. Therefore, if we require a value of the energy accurate to 0.001 MeV for the potentials of Appendix B, say, then we may obtain such an accuracy with single precision calculations dividing the range of integration into step-lengths of 0.01 fm for $0 < r < 0.2$, 0.02 fm for $0.2 < r < 0.8$, 0.04 fm for $0.8 < r < 2.0$ and 0.4 fm for $2.0 < r < 12.0$. Putting the next number (IC) equal to 1, 2 or 3 instructs the machine to calculate the solution to the bound state problem only, the solutions to the bound state and scattering at zero energy, or the zero-energy scattering solution only respectively. The succeeding numbers stipulate the re-entry point at the end of a given calculation; which wave functions (if any) are desired in the output; whether, in solving the bound state problem, we wish to use eqs. (14) and (15) or eqs. (16) and (17) (we have already pointed out that these will give the same final results); whether to use the "extreme asymptotic" solution given by eqs. (10) or the JWKB solution of eqs. (11), (12) and (13) for starting the inward integrations, in the case of the bound state problem; and, finally, the maximum number of iterations to be used in solving the bound state problem.

With regard to the re-entry point, we have allowed for a re-entry at any one of the three input statements. Hence, for example, if we wish to change the potential without altering the net-points over the range of the integration we can make the re-entry at the second input statement. For starting the inward integrations in the case of the bound state problem, clearly the "extreme asymptotic" solution is the more indicated for square-well potentials since these are exactly zero further than a certain distance from the origin. In all other cases, the JWKB solution makes it possible to reduce the total range of integration by about a factor of two as compared with the "extreme asymptotic" solution, in obtaining the same accuracy. For example, to obtain more than five figure accuracy for the energy of a bound state, it is necessary to integrate inward from about 27 fm if we use the "extreme asymptotic" solution, whereas with the JWKB solution somewhere between 12 and 17 fm suffices for this purpose. To conclude our comments on the input statements we should mention that we have found that the solution to the bound state problem generally converges in between three and five interations. All further details regarding the input statements are contained in comments at the beginning of the subprograms, listing of which are given in Appendix A. In Appendix B we give a sample set of input data, while in Appendix C we give a sample set of results obtained using the first set of data listed in Appendix B.

In conclusion, we feel that some comments on the numerical integration of the differential equations are appropriate. In the program, we have used the NUMEROV recurrence relation[6] to integrate inside each of the blocks into which the range of integration is divided, and a generalisation of it[3] to continue the solution from one block of a given step-length size to another one.

If we write eqs. (4) in matrix form

$$d^2Y/dr^2 = V(r)Y\ , \tag{28}$$

then, if h is the step-length in a given block, writing

$$Z = Y - \frac{1}{12} h^2 Y^{(2)}\ , \tag{29}$$

where $Y^{(2)}$ is the second derivative of Y with respect to r, the NUMEROV recurrence relation is

$$Z_{n+1} = 2Z_n - Z_{n-1} + h^2 Y_n^{(2)} = 12Y_n - 10Z_n - Z_{n-1}\ , \tag{30}$$

with an error of approximately $-(h^6/240)Y_n^{(6)}$.

To continue the solution from one block of step-lengths to the next let Y_n, Y_{n+k}, Y_{n-l} be, respectively, the values of Y at r_n, r_n+kh, r_n-lh for arbitrary values of k and l, then

$$l\left[Y_{n+k} - \frac{1}{12}h^2(k^2+kl-l^2)Y_{n+k}^{(2)}\right] = (l+k)\left[Y_n + \frac{1}{12}h^2(l^2+3lk+k^2)Y_n^{(2)}\right] -$$
$$- k\left[Y_{n-l} - \frac{1}{12}h^2(l^2+lk-k^2)Y_{n-l}^{(2)}\right]\ , \tag{31}$$

with an error $0(h^5)$ when $k \neq l$, and which reduces to eq. (30) when $k = l$.

These are the most accurate three-point formulas that may be used to integrate eqs. (29). More accurate integration formulae involving a larger number of points can be derived, such as for example, the predictor-corrector formulae of Milne

$$\begin{aligned} Y_{n+1} &= Y_n + Y_{n-2} - Y_{n-3} + \frac{h^2}{4}(5Y_n^{(2)} + 2Y_{n-1}^{(2)} + 5Y_{n-2}^{(2)}) + 0(h^6)\ , \\ Y_n &= 2Y_{n-1} - Y_{n-2} + \frac{h^2}{12}(Y_n^{(2)} + 10Y_{n-1}^{(2)} + Y_{n-2}^{(2)}) + 0(h^6)\ , \end{aligned} \tag{32}$$

over five points. The use of more accurate formulae would permit one to use larger integration step-lengths and though they involve a larger number of elementary operations there would probably be a net reduction in the calculation time. More important still is the saving in the memory store of the computer which could be extremely important in the inverse problem of searching for a potential, of a given type, to fit the deuteron data and scattering phase shifts over a range of energy values.

REFERENCES. -

(1) - H. Feshbach and J. Schwinger, Phys. Rev. 84, 194 (1951).
(2) - M. H. Kalos and J. M. Blatt, Univ. of Illinois, Digital Computer Laboratory, Internal report n. 50 (1953, unpublished).
(3) - L. Lovitch and S. Rosati, Phys. Rev. 140, B877 (1965).
(4) - L. Lovitch and S. Rosati, preprint.
(5) - J. M. Blatt and L. C. Biedenharn, Phys. Rev. 86, 399 (1952).
(6) - D. R. Hartree, The Calculation of Atomic Structures (J. Wiley and Sons, Inc., New York, 1957).
(7) - H. P. Noyes, Phys. Rev. 130, 2025 (1963).
(8) - L. C. Biedenharn and J. M. Blatt, Phys. Rev. 93, 1387 (1954).

APPENDIX A

PROGRAM LISTING

```
$IBFTC DEUD    NOLIST,DECK
C
C          DIRECT NUMERICAL SOLUTION, WITH MIDDLE-POINT MATCHING, OF
C      THE DEUTERON EQUATIONS WITH TENSOR FORCES GIVING THE ENERGY
C      EIGENVALUE, PERCENTAGE D-STATE AND QUADRUPOLE MOMENT, AND/OR
C      THE SCATTERING LENGTH AND EFFECTIVE RANGE TOGETHER WITH THE
C      ASSOCIATED WAVE FUNCTIONS.
C
:          VARIOUS STEP-LENGTHS CAN BE USED IN INTEGRATING OVER THE
C      TOTAL RANGE
C
C      JTOT = TOTAL NUMBER OF BLOCKS OF DIFFERENT STEP-LENGTHS
C      JOUT = NUMBER OF BLOCKS USED IN INTEGRATING OUTWARD
C      RMAX = RANGE OF INTEGRATION FOR LAST ITERATION IN SEARCHING FOR
C      THE EIGENVALUE, OR WHEN CALCULATING THE SCATTERING LENGTH AND
C      EFFECTIVE RANGE
C      N(L) = NUMBER OF STEP-LENGTHS IN L-TH BLOCK
C      X(L) = STEP-LENGTH SCALE FACTOR IN GOING FROM L-TH TO (L+1)-TH
C      BLOCK
C
C      CORE = HARD CORE RADIUS OF POTENTIAL
C      VC, RC, VT, RT, VL, RL  ARE THE PARAMETERS OF POTENTIAL
C      AND IV DEFINES ITS TYPE - CF. SUBPROGRAM POT FOR FURTHER DETAILS
C
C      E0 = TRIAL ENERGY VALUE FOR EIGENVALUE PROBLEM - NEGATIVE
C      EPS IS ACCURACY REQUIRED FOR EIGENVALUE - THIS CAN BE ZERO
C      IC = 1 FOR SOLVING THE EIGENPROBLEM ONLY
C      IC = 2 FOR SOLVING THE EIGENVALUE PROBLEM AND SCATTERING LENGTH
C      AND EFFECTIVE RANGE
C      IC = 3 FOR EVALUATING THE SCATTERING LENGTH AND EFFECTIVE RANGE
C      IR = 1, 2, 3 GIVES REENTRY AT 2, 4, 6 RESPECTIVELY
C      IPR=1  TO PRINT U1, W1, U2, W2, U, W AT END OF FINAL ITERATION
C      IPR=2  TO PRINT U, W AT END OF FINAL ITERATION
C      IPR=3 TO PRINT U1, W1, U2, W2, U, W DURING EACH ITERATION
C      IPR=4 TO PRINT U, W DURING EACH ITERATION
C      IPR=5 FOR NO SOLUTION PRINT-OUT
C      M =1, 2 FOR MATCHING EXACTLY U, W RESPECTIVELY AND, IN CASE OF
C      EIGENVALUE PROBLEM, THE CORRECTOR FORMULA IS GIVEN IN TERMS OF
C      THE MISMATCH OF W, U RESPECTIVELY
C
C      KCH = 1,2 FOR EXTREME ASYMPTOTIC, JWKB SOLUTION RESPECTIVELY
C      FOR INITIATING THE INWARD SOLUTION
C      RS1, RS2 ARE INITIAL RATIOS OF THE TWO INDEPENDENT D-WAVE/S-WAVE
C      SOLUTIONS AT THE FIRST STEP LENGTH FROM THE HARD CORE RADIUS
C      RF1, RF2 ARE INITIAL RATIOS OF THE TWO INDEPENDENT D-WAVE/S-WAVE
C      SOLUTIONS AT THE FIRST STEP-LENGTH FROM THE END OF INTEGRATION
C      RANGE
:      G1, G2 ARE THE VALUES OF G FOR THE ZERO-ENERGY ASYMPTOTIC
C      ALPHA S-WAVE SOLUTIONS, 1.+ G*R
C
       DIMENSION N(10)
       DIMENSION H(10),A1(4),HV(10),R(300),SIMP(300),U1(300),U2(300),
     1  U(300),W1(300),W2(300),W(300),B(4),A(4,4),X(10),F1(300),
     2  F2(300),F3(300),U2S(300),W2S(300),U3(300),W3(300)
C
```

```
      COMMON  HTM     , RAD2    , RAD3
      DOUBLE PRECISION  Q       , RHOT    , SCL     , ROT     , ASDP
      DOUBLE PRECISION  SDP     , VC      , RC      , VT      , RT
      DOUBLE PRECISION  VL      , RL      , EPS     , C1      , C2
      DOUBLE PRECISION  H       , HV      , R       , SIMP    , U1
      DOUBLE PRECISION  U2      , U       , W1      , W2      , W
      DOUBLE PRECISION  B       , A       , X       , F1      , F2
      DOUBLE PRECISION  F3      , U2S     , W2S     , U3      , W3
      DOUBLE PRECISION  HTM     , RAD2    , DSQRT   , RAD3    , H1
      DOUBLE PRECISION  CORE    , RM      , DE      , EE      , E0
      DOUBLE PRECISION  P       , P1      , P2      , P3      , P4
      DOUBLE PRECISION  ER      , G1      , G2      , G3      , RF1
      DOUBLE PRECISION  RF2     , C       , C1      , C2      , A1
      DOUBLE PRECISION  B1      , DEXP    , XM      , S31     , Z31
      DOUBLE PRECISION  S32     , Z32     , S11     , Z11     , S21
      DOUBLE PRECISION  Z21     , S12     , Z12     , S22     , Z22
      DOUBLE PRECISION  S13     , Z13     , S23     , Z23     , S33
      DOUBLE PRECISION  Z33     , C3      , A2      , B2      , U1OUT
      DOUBLE PRECISION  W1OUT   , U2OUT   , W2OUT   , DFLOAT  , DU1OUT
      DOUBLE PRECISION  DU2OUT  , DW1OUT  , DW2OUT  , U3IN    , W3IN
      DOUBLE PRECISION  DU3IN   , DW3IN   , U1IN    , W1IN    , U2IN
      DOUBLE PRECISION  W2IN    , U1K2    , U2K2    , W1K2    , W2K2
      DOUBLE PRECISION  DU1IN   , DU2IN   , DW1IN   , DW2IN   , RS1
      DOUBLE PRECISION  RS2     , WM      , UM      , F       , PD
      DOUBLE PRECISION  SSP     , AMFR
C
  500 FORMAT(2D10.5,2I5/5(I5,D10.5))
  504 FORMAT(I5,6D12.6)
  508 FORMAT(1H1 35X,43H SOLUTION OF SCHROEDINGER DEUTERON EQUATION ////
     1/ 25H INITIAL PARAMETER VALUES ////18X,10H HARD CORE 7X,20H INITIA
     2L STEP LENGTH 7X,17H NUMBER OF BLOCKS/D29.4,D21.5,I22    ////  44H
     3INWARD INTEGRATION - NUMBER OF BLOCKS JOUT= I3 // 4(31H      N      H
     4         X            )/(4(I5,2D11.4,4X)))
  510 FORMAT(/// 26H MATCHING POINT ABSCISSA =D12.4 ////43H INWARD INTEG
     1RATION - NUMBER OF BLOCKS JIN= I3 // 4(31H      N      H           X
     2         )/(4(I5,2D11.4,4X)))
  512 FORMAT(////32H PARAMETERS SPECIFYING POTENTIAL ////
     1            7X,3H IV 9X,3H VC 12X,3H RC 12X,3H VT 12X,3H RT 12X,
     2    3H VL 12X,3H RL /I10,3X,6D15.6)
  516 FORMAT(2D10.6,6I5)
  520 FORMAT(////35H REMAINING DATA SPECIFYING SOLUTION ////
     1        5H   IC3X,8H M     KCH5X,3H G1 12X,3H G2 12X,3H G3 12X,4H RF1
     2    11X,4H RF2 11X,4H RS1 11X,4H RS2 /   3I5,7D15.6)
  522 FORMAT(1H1)
  524 FORMAT(    5X,31H UNRENORMALISED S-WAVE SOLUTION //6(21H    R(L)
     1U1(L)      ))
  528 FORMAT(1H15X,31H UNRENORMALISED S-WAVE SOLUTION //6(21H    R(L)
     1U2(L)      ))
  530 FORMAT(1H15X,31H UNRENORMALISED S-WAVE SOLUTION //6(21H    R(L)
     1U3(L)      ))
  532 FORMAT(1H15X,31H UNRENORMALISED D-WAVE SOLUTION //6(21H    R(L)
     1W1(L)      ))
  536 FORMAT(1H15X,31H UNRENORMALISED D-WAVE SOLUTION //6(21H    R(L)
     1W2(L)      ))
  538 FORMAT(1H15X,31H UNRENORMALISED D-WAVE SOLUTION //6(21H    R(L)
     1W3(L)      ))
  540 FORMAT(6(1PD 9.2,  D12.5))
  544 FORMAT(////4H IT= I3,  4H E0= 1PD15.7,    4H EE= D15.7,    4H DE=
     1  D15.7,1X,8H B(1/3)= 3D15.7)
  548 FORMAT(1H18X,25H DEUTERON S-WAVE SOLUTION       / 6(21H   R(L)
     1(L)       ))
  552 FORMAT(1H18X,25H DEUTERON D-WAVE SOLUTION       / 6(21H   R(L)
     1(L)       ))
  556 FORMAT(///20X,4H E0= 1PD15.7,3X,3H Q=0PD15.7,3X,4H PD=1PD15.7//)
  560 FORMAT(/// 35X,18H SCATTERING LENGTH 10X,16H EFFECTIVE RANGE /
     1  D48.6,D26.6//)
  562 FORMAT(///20X,18H SCATTERING LENGTH 10X,16H EFFECTIVE RANGE 10X,
     116H SHAPE PARAMETER/1PD36.6,D27.6,0PD26.6//3X,22H MIXED EFFECTIVE
     2RANGE 3X,25H DEUTERON EFFECTIVE RANGE 3X,24H APPROX. SHAPE PARAMET
     3ER 3X, 23H SECOND SHAPE PARAMETER /1PD17.6,D28.6,0PD29.6,D26.6//)
  564 FORMAT(//56H TRIAL ENERGY HAS GONE POSITIVE, CALCULATION INTERRUPT
     1ED)
C
```

```
      EQUIVALENCE (U1,U),(W1,W),(U2,U2S),(W2,W2S)
C
      HTM=0.0241143D0
      RAD2=DSQRT(2.D0)
      RAD3=DSQRT(3.D0)
C
 2    READ  ( 5, 500) CORE,H(1),JOUT,JTOT,(N(L),X(L),L=1,JTOT)
C
      JIN=JTOT-JOUT
      X(JTOT)=1.D0
      X(JOUT)=1.D0
      H1=H
      DO 3 L=1,JTOT
      H(L)=H1
      HV(L)=H1*H1/12.D0
 3    H1=H1*X(L)
C
C     INTEGRATION COEFFICIENTS
C
 17   K=0
      K1=1
      R=CORE+H
      SIMP=4.D0*H/3.D0
 21   DO 65 L=1,JTOT
      H1=H(L)
      K2=N(L)
 25   IF(L-1)41,41,29
 29   IF(L-JOUT-1)37,33,37
 33   NT2=K1
      NT3=K
      NT4=K-1
      RM=R(K)
 37   A=H1/3.D0
      SIMP(K)=SIMP(K)+A
      SIMP(K1)=4.D0*A
      R(K1)=R(K)+H1
 41   DO 45 IB=2,K2
      K=K1
      K1=K+1
      R(K1)=R(K)+H1
 45   SIMP(K1)=H1+H1-SIMP(K)
 49   IF(SIMP(K1)-H1)53,57,57
 53   SIMP(K1)=0.5D0*SIMP(K1)
      GO TO 61
 57   SIMP(K1)=0.375D0*SIMP(K1)
      SIMP(K)=1.25D0*SIMP(K)
 61   K=K1
 65   K1=K1+1
      NT=K
      NT1=NT-1
      H1=1.D0/H(JOUT)
C
 4    READ  (5,504)IV, VC,RC,VT, RT,VL,RL
C
      CALL POT(IV,VC,VT,VL,RC,RT,RL,JTOT,JOUT,N,H,X,HV,R,F1,F2,F3)
      DE=0.D0
C
 6    READ  (5,516)E0,EPS,IC,IR,IPR,M,KCH,MAXIT
       RS1=2.5D0*H
       RS2=5.D0*H
       RF1=7.D-2
       RF2=1.D-2
       G1=-1.D0
       G2=-0.5D0
       G3=0.1D0
C
```

```
      WRITE (6,522)
      WRITE  ( 6,508)CORE,H(1),JTOT,JOUT,(N(L),H(L),X(L), L=1,JOUT)
C
      L1=JOUT+1
C
      WRITE ( 6,510)RM,JIN,(N(L),H(L),X(L),L=L1,JTOT)
      WRITE (6,512)IV, VC, RC, VT, RT, VL, RL
      WRITE (6,520)IC,M,KCH,G1,G2,G3,RF1,RF2,RS1,RS2
      WRITE (6,522)
C
      GO TO(69,73,73),IC
 69   ICC=IC
      GO TO 77
 73   ICC=IC-1
C
 77   GO TO (81,85),ICC
 81   EE=HTM*E0
      GO TO 87
 85   E0=0.D0
      EE=E0
 87   DE=EE+DE
C
 89   DO 373 IT=1,MAXIT
 93   GO TO (97,107),ICC
C
 97   P=DSQRT(-EE)
C
C     P1-P4 ARE CORRECTIONS FOR THE FINAL INTEGRATION COEFFICIENT
C     SIMP(NT) TO TAKE INTO ACCOUNT TAIL OF WAVE FUNCTION
C
      P1=0.5D0/P
      A=1.D0/(P*R(NT))
      B=1.D0/(1.D0+3.D0*A*(1.D0+A))
      P2=P1*B*B*(1.D0+6.D0*A*(1.D0+A*(2.D0+A)))
      P3=P1*B*(1.D0+A*(4.D0+5.D0*A))
      P4=P1*B*B*(1.D0+A*(7.D0+A*(18.5D0+18.D0*A)))
 107  K=0
C
 109  DO 117 L=1,JTOT
      ER=DE*HV(L)
      L1=N(L)-1
      DO 110 IB=1,L1
      K=K+1
      F1(K)=F1(K)-ER
 110  F3(K)=F3(K)-ER
      IF(L-JOUT) 112,112,111
 111  ER=ER*X(L)*X(L)
 112  K=K+1
      F1(K)=F1(K)-ER
 117  F3(K)=F3(K)-ER
C
      A=R(NT)
      B=R(NT1)
C
 121  GO TO (126,125),ICC
C
C     BOUNDARY VALUES OF ZERO ENERGY FUNCTIONS
C
 125  U1(NT)=1.D0+A*G1
      U2(NT)=1.D0+A*G2
      U3(NT)=U1(NT)
      W1(NT)=1.D0/(A*A)
      W2(NT)=W1(NT)
```

```
      W3(NT)=G3*W1(NT)
      U1(NT1)=1.D0+B*G1
      U2(NT1)=1.D0+B*G2
      U3(NT1)=U1(NT1)
      W1(NT1)=1.D0/(B*B)
      W2(NT1)=W1(NT1)
      W3(NT1)=G3*W1(NT1)
      GO TO 131
C
C     BOUNDARY VALUES OF FUNCTIONS
C
 126  U1(NT)=1.D-10
      U2(NT)=1.D-10
      W1(NT)=U1(NT)*RF1
      W2(NT)=U2(NT)*RF2
      C=A/B
      C1=P*A
      C2=P*B
      C=C*C*(C2*(C2+3.D0)+3.D0)/(C1*(C1+3.D0)+3.D0)
      C1=RF1*C
      C2=RF2*C
C
      GO TO (127,128),KCH
 127  A1=DEXP(P*H(JTOT))
      U1(NT1)=U1(NT)*A1
      U2(NT1)=U2(NT)*A1
      W1(NT1)=W1(NT)*A1*C
      W2(NT1)=W2(NT)*A1*C
      GO TO 131
C
C     JWKB APPROXIMATION
C
 128  A1=DSQRT(F1(NT)+F2(NT)*RF1)
      A1(2)=DSQRT(F1(NT)+F2(NT)*RF2)
      A1(3)=DSQRT(F3(NT)+F2(NT)/RF1)
      A1(4)=DSQRT(F3(NT)+F2(NT)/RF2)
      B1=DSQRT(F1(NT1)+F2(NT1)*C1)
      U1(NT1)=U1(NT)*DEXP((A1+B1)* RAD3 )*DSQRT(A1/B1)
C
      B1=DSQRT(F1(NT1)+F2(NT1)*C2)
      U2(NT1)=U2(NT)*DEXP((A1(2)+B1)* RAD3 )*DSQRT(A1(2)/B1)
C
      B1=DSQRT(F3(NT1)+F2(NT1)/C1)
      W1(NT1)=W1(NT)*DEXP((A1(3)+B1)* RAD3 )*DSQRT(A1(3)/B1)
C
      B1=DSQRT(F3(NT1)+F2(NT1)/C2)
      W2(NT1)=W2(NT)*DEXP((A1(4)+B1)* RAD3 )*DSQRT(A1(4)/B1)
C
C     START INWARD INTEGRATION
C
 131  K=NT
      K1=NT1
      L1=JIN
      L2=-1
C
 135  ICL=ICC+L2-1
      DO 187 L=1,L1
 139  IF(L2)143,143,147
```

```
 143   JR=JTOT-L+1
       XM=1.D0/(X(JR)*X(JR))
       IF(ICL) 163,145,163
 145   S31=(1.D0-F1(K)*XM)*U3(K)-F2(K)*XM*W3(K)
       Z31=(1.D0-F3(K)*XM)*W3(K)-F2(K)*XM*U3(K)
       S32=(1.D0-F1(K1))*U3(K1)-F2(K1)*W3(K1)
       Z32=(1.D0-F3(K1))*W3(K1)-F2(K1)*U3(K1)
       GO TO 163
C
 147   JR=L
 151   IF(L-1) 155,155,159
 155   S11=0.D0
       Z11=0.D0
       S21=0.D0
       Z21=0.D0
       GO TO 167
 159   XM=X(L-1)**2
C
C      S AND Z ARE AUXILIARY VARIABLES FOR THE SOLUTIONS
C
 163   S11=(1.D0-F1(K)*XM)*U1(K)-F2(K)*XM*W1(K)
       Z11=(1.D0-F3(K)*XM)*W1(K)-F2(K)*XM*U1(K)
       S21=(1.D0-F1(K)*XM)*U2(K)-F2(K)*XM*W2(K)
       Z21=(1.D0-F3(K)*XM)*W2(K)-F2(K)*XM*U2(K)
C
 167   S12=(1.D0-F1(K1))*U1(K1)-F2(K1)*W1(K1)
       Z12=(1.D0-F3(K1))*W1(K1)-F2(K1)*U1(K1)
       S22=(1.D0-F1(K1))*U2(K1)-F2(K1)*W2(K1)
       Z22=(1.D0-F3(K1))*W2(K1)-F2(K1)*U2(K1)
C
       K=K+L2
       K1=K1+L2
       K2=N(JR)
C
 171   DO 175 IB=2,K2
       S13=10.D0*(U1(K)-S12)+U1(K)+(U1(K)-S11)
       Z13=10.D0*(W1(K)-Z12)+W1(K)+(W1(K)-Z11)
       S23=10.D0*(U2(K)-S22)+U2(K)+(U2(K)-S21)
       Z23=10.D0*(W2(K)-Z22)+W2(K)+(W2(K)-Z21)
C
       A=1.D0/((1.D0-F1(K1))*(1.D0-F3(K1))-F2(K1)*F2(K1))
       U1(K1)=A*(Z13*F2(K1)+S13*(1.D0-F3(K1)))
       W1(K1)=A*(Z13*(1.D0-F1(K1))+S13*F2(K1))
       U2(K1)=A*(Z23*F2(K1)+S23*(1.D0-F3(K1)))
       W2(K1)=A*(Z23*(1.D0-F1(K1))+S23*F2(K1))
C
       IF(ICL) 174,173,174
 173   S33=10.D0*(U3(K)-S32)+U3(K)+(U3(K)-S31)
       Z33=10.D0*(W3(K)-Z32)+W3(K)+(W3(K)-Z31)
       U3(K1)=A*(Z33*F2(K1)+S33*(1.D0-F3(K1)))
       W3(K1)=A*(Z33*(1.D0-F1(K1))+S33*F2(K1))
       S31=S32
       S32=S33
       Z31=Z32
       Z32=Z33
 174   S11=S12
       S12=S13
       Z11=Z12
       Z12=Z13
       S21=S22
```

```
      S22=S23
      Z21=Z22
      Z22=Z23
C
      K=K+L2
 175  K1=K1+L2
C
C     CONTINUING THE SOLUTIONS FROM ONE STEP-LENGTH TO THE NEXT
C
 177  IF(L2)179,179,181
 179  A=X(JR-1)
      GO TO 183
 181  A=1.D0/X(L)
C
 183  B=1.D0+A*(1.D0-A)
      A=1.D0/A
      C3=1.D0+A*(1.D0-A)
      C=1.D0+A*(3.D0+A)
      K2=K-L2
C
      A1=1.D0-B*F1(K1)
      B1=-B*F2(K1)
      A2=B1
      B2=1.D0-B*F3(K1)
      B=1.D0/(A1*B2-B1*A2)
C
      C1=(1.D0+A)*(U1(K)+C*(U1(K)-S12))-A*(U1(K2)-C3*(U1(K2)-S11))
      C2=(1.D0+A)*(W1(K)+C*(W1(K)-Z12))-A*(W1(K2)-C3*(W1(K2)-Z11))
      U1(K1)=(C1*B2-C2*B1)*B
      W1(K1)=(C2*A1-C1*A2)*B
C
      C1=(1.D0+A)*(U2(K)+C*(U2(K)-S22))-A*(U2(K2)-C3*(U2(K2)-S21))
      C2=(1.D0+A)*(W2(K)+C*(W2(K)-Z22))-A*(W2(K2)-C3*(W2(K2)-Z21))
      U2(K1)=(C1*B2-C2*B1)*B
      W2(K1)=(C2*A1-C1*A2)*B
C
      IF(ICL) 187,185,187
 185  C1=(1.D0+A)*(U3(K)+C*(U3(K)-S32))-A*(U3(K2)-C3*(U3(K2)-S31))
      C2=(1.D0+A)*(W3(K)+C*(W3(K)-Z32))-A*(W3(K2)-C3*(W3(K2)-Z31))
      U3(K1)=(C1*B2-C2*B1)*B
      W3(K1)=(C2*A1-C1*A2)*B
 187  CONTINUE
C
      U1OUT=U1(K)
      W1OUT=W1(K)
      U2OUT=U2(K)
      W2OUT=W2(K)
C
      S13=(1.D0-F1(K1))*U1(K1)-F2(K1)*W1(K1)
      S23=(1.D0-F1(K1))*U2(K1)-F2(K1)*W2(K1)
      Z13=(1.D0-F3(K1))*W1(K1)-F2(K1)*U1(K1)
      Z23=(1.D0-F3(K1))*W2(K1)-F2(K1)*U2(K1)
C
      A=-L2
C
      DU1OUT=A*((S11-S13)+0.5D0*(U1(K1)-U1(K2))) *H1
      DU2OUT=A*((S21-S23)+0.5D0*(U2(K1)-U2(K2)))  *H1
      DW1OUT=A*((Z11-Z13)+0.5D0*(W1(K1)- W1(K2))) *H1
      DW2OUT=A*((Z21-Z23)+0.5D0*(W2(K1)-W2(K2))) *H1
      IF(ICL) 189,188,189
 188  U3IN=U3(K)
      W3IN=W3(K)
      S33=(1.D0-F1(K1))*U3(K1)-F2(K1)*W3(K1)
```

```
      Z33=(1.D0-F3(K1))*W3(K1)-F2(K1)*U3(K1)
      DU3IN=((S31-S33)+0.5D0*(U3(K1)-U3(K2)))*H1
      DW3IN=((Z31-Z33)+0.5D0*(W3(K1)-W3(K2)))*H1
C
  189 IF(L2) 193, 193, 197
  193 U1IN=U1OUT
      W1IN=W1OUT
      U2IN=U2OUT
      W2IN=W2OUT
      U1K2 = U1(K2)
      U2K2 = U2(K2)
      W1K2 = W1(K2)
      W2K2 = W2(K2)
      DU1IN=DU1OUT
      DU2IN=DU2OUT
      DW1IN=DW1OUT
      DW2IN=DW2OUT
C
C     START OUTWARD INTEGRATION
C
      U1=1.D-10
      U2=1.D-10
      W1=U1*RS1
      W2=U2*RS2
C
      K=0
      K1=1
      L1=JOUT
      L2=1
      GO TO 135
C
  197 U1(K1) = U1K2
      U2(K1) = U2K2
      W1(K1) = W1K2
      W2(K1) = W2K2
  198 GO TO (199,231),ICC
C
C     OUTWARD U SOLUTION= B(1)*U1+B(2)*U2
C     OUTWARD W SOLUTION= B(1)*W1+B(2)*W2
C      INWARD U SOLUTION = B(3)*U1+U2
C      INWARD W SOLUTION = B(3)*W1+W2
C     MATCHING POINT CONDITIONS ARE
C     B(1)*U1OUT+B(2)*U2OUT = B(3)*U1IN+U2IN
C     B(1)*W1OUT+B(2)*W2OUT = B(3)*W1I5+W2IN
C     B(1)*DU1OUT+B(2)*DU2OUT = B(3)*DU1IN+DU2IN
C     B(1)*DW1OUT+B(2)*DW2OUT = B(3)*DW1IN+DW2IN
C
  199 C=1.D0/U2IN
      B=1.D0
      B(2)=W2IN*C
C
      A(1,1)=U1OUT*C
      A(1,2)=U2OUT*C
      A(1,3)=-U1IN*C
C
```

```
      A(2,1)=W1OUT*C
      A(2,2)=W2OUT*C
      A(2,3)=-W1IN*C
C
 201  GO TO (205,209),M
 205  B(3)=DU2IN*C
      A(3,1)=DU1OUT*C
      A(3,2)=DU2OUT*C
      A(3,3)=-DU1IN*C
      GO TO 213
C
 209  B(3)=DW2IN*C
      A(3,1)=DW1OUT*C
      A(3,2)=DW2OUT*C
      A(3,3)=-DW1IN*C
C
 213  CALL SYSTEM(A,3,B)
C
 221  GO TO (225,229),M
 225  WM=B(3)*W1IN+W2IN
      GO TO 237
 229  UM=B(3)*U1IN+U2IN
 237  GO TO (241,257,242,257,257),IPR
C
C     OUTWARD U SOLUTION (E=0.) = B(1)*U1+B(2)*U2
C     OUTWARD W SOLUTION (E=0.) = B(1)*W1+B(2)*W2
C     INWARD U SOLUTION (E=0.) = B(3)*U1+B(4)*U2+U3
C     INWARD W SOLUTION (E=0.) = B(3)*W1+B(4)*W2+W3
C
C     MATCHING POINT CONDITIONS (E=0.) ARE
C     B(1)*U1OUT+B(2)*U2OUT=B(3)*U1IN+B(4)*U2IN+U3IN
C     B(1)*W1OUT+B(2)*W2OUT=B(3)*W1IN+B(4)*W2IN+W3IN
C     B(1)*DU1OUT+B(2)*DU2OUT=B(3)*DU1IN+B(4)*DU2IN+DU3IN
C     B(1)*DW1OUT+B(2)*DW2OUT=B(3)*DW1IN+B(4)*DW2IN+DW3IN
 231  C=1.D0/U3IN
      B=1.D0
      B(2)=W3IN*C
      B(3)=DU3IN*C
      B(4)=DW3IN*C
C
      A(1,1)=U1OUT*C
      A(1,2)=U2OUT*C
      A(1,3)=-U1IN*C
      A(1,4)=-U2IN*C
C
      A(2,1)=W1OUT*C
      A(2,2)=W2OUT*C
      A(2,3)=-W1IN*C
      A(2,4)=-W2IN*C
C
      A(3,1)=DU1OUT*C
      A(3,2)=DU2OUT*C
      A(3,3)=-DU1IN*C
      A(3,4)=-DU2IN*C
C
```

```
      A(4,1)=DW1OUT*C
      A(4,2)=DW2OUT*C
      A(4,3)=-DW1IN*C
      A(4,4)=-DW2IN*C
C
      CALL SYSTEM(A,4,B)
C
      B1=1.D0/(B(3)+B(4)+1.D0)
      B=B1*B
      B(2)=B1*B(2)
      B(3)=B1*B(3)
      B(4)=B1*B(4)
C
 233  GO TO (244,257,244,257,257),IPR
C
 241  IF(IT-MAXIT) 257,242,242
C
 242  IF(IT-1) 244,244,243
 243  WRITE (6,522)
      II=0
C
 244  DO 1246 JF=1,NT,300
      IF(II) 245,245,1244
 1244 WRITE (6,522)
 245  WRITE (6,524)
      JL=MIN0(JF+49,NT)
      DO 246 J1=JF,JL
      IL=MIN0(J1+250,NT)
 246  WRITE (6,540)(R(L), U1(L), L=J1,IL,50)
 1246 II=II+1
C
      DO 247 JF=1,NT,300
      WRITE (6,528)
      JL=MIN0(JF+49,NT)
      DO 247 J1=JF,JL
      IL=MIN0(J1+250,NT)
 247  WRITE (6,540)(R(L),U2(L), L=J1,IL,50)
C
      GO TO (250,248),ICC
 248  DO 249 JF=NT4,NT,300
      WRITE (6,530)
      JL=MIN0(JF+49,NT)
      DO 249 J1=JF,JL
      IL=MIN0(J1+250,NT)
 249  WRITE (6,540)(R(L), U3(L), L=J1,IL,50)
C
 250  DO 251 JF=1,NT,300
      WRITE (6,532)
      JL=MIN0(JF+49,NT)
      DO 251 J1=JF,JL
      IL=MIN0(J1+250,NT)
 251  WRITE (6,540)(R(L),W1(L), L=J1,IL,50)
C
      DO 252 JF=1,NT,300
      WRITE (6,536)
```

```
      JL=MINO(JF+49,NT)
      DO 252 J1=JF,JL
      IL=MINO(J1+250,NT)
 252  WRITE (6,540)(R(L), W2(L), L=J1,IL,50)
C
      GO TO (257,253),ICC
 253  DO 254 JF=NT4,NT,300
      WRITE (6,538)
      JL=MINO(JF+49,NT)
      DO 254 J1=JF,JL
      IL=MINO(J1+250,NT)
 254  WRITE (6,540)(R(L), W3(L), L=J1,IL,50)
C
C     SOLUTIONS
C
 257  C1=0.D0
      C2=0.D0
 261  DO 265 L=1,NT3
      U(L)=B*U1(L)+B(2)*U2(L)
      W(L)=B*W1(L)+B(2)*W2(L)
      U2S(L)=SIMP(L)*U(L)*U(L)
      W2S(L)=SIMP(L)*W(L)*W(L)
      C1=C1+U2S(L)
 265  C2=C2+W2S(L)
C
      GO TO (269,329),ICC
 269  DO 273 L=NT2,NT1
      U(L)=B(3)*U1(L)+U2(L)
      W(L)=B(3)*W1(L)+W2(L)
      U2S(L)=SIMP(L)*U(L)*U(L)
      W2S(L)=SIMP(L)*W(L)*W(L)
      C1=C1+U2S(L)
 273  C2=C2+W2S(L)
      U(NT)=B(3)*U1(NT)+U2(NT)
      W(NT)=B(3)*W1(NT)+W2(NT)
      U2S(NT)=(SIMP(NT)+P1)*U(NT)*U(NT)
      W2S(NT)=(SIMP(NT)+P2)*W(NT)*W(NT)
      C1=C1+U2S(NT)
      C2=C2+W2S(NT)
      C3=C1+C2
 281  A1=1.D0/C3
C
C     ENERGY CORRECTOR FORMULA
C
 285  GO TO (289,293),M
 289  F=(B*DW1OUT+B(2)*DW2OUT-B(3)*DW1IN-DW2IN)*WM
      GO TO 297
 293  F=(B*DU1OUT+B(2)*DU2OUT-B(3)*DU1IN-DU2IN)*UM
 297  DE=F*A1
C
      WRITE (6,544)IT, E0, EE, DE,B(1),B(2),B(3)
C
      EE=EE+DE
 301  IF(EE) 309,305,305
 305  WRITE (6,564)
```

```
      DE=DE-EE
      GO TO 409
  309 E0=EE/HTM
      K1=IT-MAXIT
C
  313 IF(DABS(DE)-DABS(EPS)*HTM)317,321,321
  317 K1=0
C
C     NORMALIZE
C
  321 A2=DSQRT(A1)
  323 DO 325 L=1,NT
      U(L)=U(L)*A2
  325 W(L)=W(L)*A2
  337 GO TO (341,341,345,345,353),IPR
C
  329 DO 331 L=NT2,NT1
      U(L)=B(3)*U1(L)+B(4)*U2(L)+B1*U3(L)
      W(L)=B(3)*W1(L)+B(4)*W2(L)+B1*W3(L)
      U2S(L)=SIMP(L)*U(L)*U(L)
  331 W2S(L)=SIMP(L)*W(L)*W(L)
      U(NT)=B(3)*U1(NT)+B(4)*U2(NT)+B1*U3(NT)
      W(NT)=B(3)*W1(NT)+B(4)*W2(NT)+B1*W3(NT)
      U2S(NT)=SIMP(NT)*U(NT)*U(NT)
      W2S(NT)=(SIMP(NT)+R(NT)/3.D0)*W(NT)*W(NT)
  333 GO TO (345,345,345,345,353),IPR
C
  341 IF(K1)353,345,345
  345 DO 347 JF=1,NT,300
      WRITE (6,548)
      JL=MIN0(JF+49,NT)
      DO 347 J1=JF,JL
      IL=MIN0(J1+250,NT)
  347 WRITE (6,540)(R(L), U(L), L=J1,IL,50)
C
      DO 349 JF=1,NT,300
      WRITE (6,552)
      JL=MIN0(JF+49,NT)
      DO 349 J1=JF,JL
      IL=MIN0(J1+250,NT)
  349 WRITE (6,540)(R(L),W(L), L=J1,IL,50)
C
  353 GO TO(357,397),ICC
  357 IF(K1)361,377,377
  361 RF2=(B(3)*RF1+RF2)/(B(3)+1.D0)
  373 RS2=(B*RS1+B(2)*RS2)/(B+B(2))
C
C     PERCENTAGE D-STATE AND QUADRUPOLE MOMENT
C
  377 PD=C2*A1
      B2=RAD2+RAD2
      Q=0.D0
  381 DO 385 L=1,NT1
  385 Q=Q+SIMP(L)*W(L)*R(L)*R(L)*(B2*U(L)-W(L))
      Q=0.05D0*(Q+((SIMP(NT)+P3)*B2*U(NT)-(SIMP(NT)+P4)*W(NT))*W(NT)
```

```
     1      *R(NT)*R(NT))
C
      WRITE (6,556)E0,Q,PD
      WRITE (6,522)
C
      DE=DE-EE
      GO TO (409,393),IC
 393  ICC=2
      A=P*R(NT)
      B=1.D0/A
      RHOT=1.D0/P-2.D0*DEXP(-A-A)/(U(NT)*U(NT)+(W(NT)/(1.D0+3.D0*B*(1.D0
     1+B)))**2)
      GO TO 89
C
C     SCATTERING LENGTH AND EFFECTIVE RANGE
C
 397  SCL=-1.D0/(G1*(B1+B(3))+G2*B(4))
      A=R(NT)/SCL
      ROT=R(NT)*(1.D0-A*(1.D0-A/3.D0))
 401  DO 405 L=1,NT
 405  ROT=ROT-U2S(L)-W2S(L)
      ROT=ROT+ROT
C
      IF(IC-2)406,407,406
 406  WRITE (6,560)SCL, ROT
      WRITE (6,522)
      GO TO 408
C
 407  A=DSQRT(-EE)-1.D0/SCL
      B=1.D0/(EE*EE*ROT**3)
      ASDP=(A+0.5D0*EE*ROT)*B
      SDP=(3.D0*A+EE*(ROT+0.5D0*RHOT))*B
      AMFR=(-A-A)/EE
      SSP=(-AMFR+0.5D0*(ROT+RHOT))*B/(ROT*ROT)
      WRITE (6,562)SCL,ROT,SDP,AMFR,RHOT,ASDP,SSP
C
 408  DE=0.D0
 409  GO TO (2,4,6),IR
C
      END
```

```
$IBFTC PODD     NOLIST,DECK
      SUBROUTINE POT(IV,VC,VT,VL,RC,RT,RL,JTOT,JOUT,N,H,X,HV,R,F1,F2,F3)
      DIMENSION N(10),H(10),HV(10),X(10),R(300),F1(300),F2(300),F3(300)
      COMMON  HTM     , RAD2
C
      DOUBLE PRECISION  H       , HV      , X       , R       , F1
      DOUBLE PRECISION  F2      , F3      , CC1     , VC      , HTM
      DOUBLE PRECISION  CT1     , VT      , CL1     , VL      , A
      DOUBLE PRECISION  DEXP    , RC      , RT      , RL      , H1
      DOUBLE PRECISION  B       , B6      , ECH     , ETH     , ELH
      DOUBLE PRECISION  EC1     , ET1     , EL1     , CC      , CT
      DOUBLE PRECISION  CL      , RAD2    , DFLOAT
      DFLOAT(I)=I
C
C     IV=1,2,3,4 FOR SQUARE, EXPONENTIAL, YUKAWA AND GAUSS WELL
C       POTENTIALS RESPECTIVELY
C
      GO TO (1,5,5,9),IV
 1    CC1=VC*HTM
      CT1=(VT+VT)*HTM
      CL1=3.D0*VL*HTM
C
```

```
      M1=1
      M2=1
      M3=1
      GO TO 13
C
 5    A=R-H
      CC1=VC*DEXP(-RC*A)*HTM
      CT1=(VT+VT)*DEXP(-RT*A)*HTM
      CL1=3.D0*VL*DEXP(-RL*A)*HTM
C
      IF(IV-3)13,11,11
 11   CC1=CC1/RC
      CT1=CT1/RT
      CL1=CL1/RL
      GO TO 13
C
 9    A=R-H
      CC1=VC*DEXP(-RC*A*A)*HTM
      CT1=(VT+VT)*DEXP(-RT*A*A)*HTM
      CL1=3.D0*VL*DEXP(-RL*A*A)*HTM
C
 13   K=0
 17   DO 109 L=1,JTOT
      NL=N(L)
      H1=H(L)
      B=HV(L)
      B6=6.D0*B
C
 21   GO TO (33,25,25,29),IV
 25   ECH=DEXP(-RC*H1)
      ETH=DEXP(-RT*H1)
      ELH=DEXP(-RL*H1)
      GO TO 33
C
 29   ECH=DEXP(-RC*(A+A)*H1)
      ETH=DEXP(-RT*(A+A)*H1)
      ELH=DEXP(-RL*(A+A)*H1)
      EC1=DEXP(-RC*H1*H1)
      ET1=DEXP(-RT*H1*H1)
      EL1=DEXP(-RL*H1*H1)
C
 33   DO 97 IB=1,NL
      K=K+1
C
 37   GO TO (41,77,89,81),IV
 41   GO TO (45,53),M1
 45   IF(R(K)-RC)53,49,49
 49   M1=2
      CC1=0.D0
C
 53   GO TO (57,65),M2
 57   IF(R(K)-RT)65,61,61
 61   M2=2
      CT1=0.D0
C
 65   GO TO(69,85),M3
 69   IF(R(K)-RL)85,73,73
 73   M3=2
      CL1=0.D0
      GO TO 85
C
 77   CC1=CC1*ECH
      CT1=CT1*ETH
      CL1=CL1*ELH
      GO TO 85
C
```

```
 81     CC1=CC1*ECH*EC1
        CT1=CT1*ETH*ET1
        CL1=CL1*ELH*EL1
        ECH=ECH*EC1*EC1
        ETH=ETH*ET1*ET1
        ELH=ELH*EL1*EL1
C
 85     CC=CC1*B
        CT=CT1*B
        CL=CL1*B
        GO TO 93
C
 89     CC1=CC1*ECH
        CT1=CT1*ETH
        CL1=CL1*ELH
        A=B/R(K)
        CC=CC1*A
        CT=CT1*A
        CL=CL1*A
C
 93     F1(K)=CC
        F2(K)=CT*RAD2
  97    F3(K)=CC-CT-CL + B6/(R(K)*R(K))
 C
  101   GO TO (109,109,109,105),IV
  105   A=A+H1*DFLOAT(NL)
  109   CONTINUE
 C
        NL=0
        DO 113 L=1,JOUT
  113   NL=NL+N(L)
        M1=JOUT+1
        M2=JTOT-1
        DO 117 L=M1,M2
        NL=NL+N(L)
        B=X(L)*X(L)
        F1(NL)=B*F1(NL)
        F2(NL)=B*F2(NL)
  117   F3(NL)=B*F3(NL)
        RETURN
        END
```

APPENDIX B

LISTING OF A SAMPLE SET OF DATA

```
$DATA
    0.+0     0.001+0     4    6
100     4.+0   20      5.+0   30      2.5+0   20      1.+0    6       4.+0
60      4.+0
3    -39.53+0    0.84324+0   -33.0431+0    0.65229+0        0.+0         1.+0
-2.0+0       0.+0     2    1    2    2    2    7

   0.4+0     0.01+0     3    5
20      2.+0   30      2.+0   25      1.+0    5     10.+0   25      1.+0
3    -100.7+0        1.23+0       -257.+0       1.203+0       -5.+3        3.7+0
-2.0+0       1.-3     2    2    2    2    2    7

3    -100.7+0        1.23+0       -257.+0       1.203+0        0.+0         1.+0
-2.0+0       1.-4     2    1    2    2    2    7

   0.5+0     0.01+0     3    5
20      2.+0   30      2.+0   25      1.+0    5     10.+0   25      1.+0
3   -6780.2+0    3.0026+0    -49.522+0     7.5064-1         0.+0         1.+0
-2.0+0       1.-3     2    2    2    2    2    5
```

SOLUTION OF SCHROEDINGER DEUTERON EQUATION

INITIAL PARAMETER VALUES

HARD CORE	INITIAL STEP LENGTH	NUMBER OF BLOCKS
0.0000D-38	0.10000D-02	6

INWARD INTEGRATION - NUMBER OF BLOCKS JOUT= 4

N	H	X	N	H	X	N	H	X	N	H	X
100	0.1000D-02	0.4000D 01	20	0.4000D-02	0.5000D 01	30	0.2000D-01	0.2500D 01	20	0.5000D-01	0.1000D 01

MATCHING POINT ABSCISSA = 0.1780D 01

INWARD INTEGRATION - NUMBER OF BLOCKS JIN= 2

N	H	X	N	H	X	N	H	X	N	H	X
6	0.5000D-01	0.4000D 01	60	0.20000 00	0.1000D 01						

PARAMETERS SPECIFYING POTENTIAL

IV	VC	RC	VT	RT	VL	RL
3	-0.395300D 02	0.843240D 00	-0.330431D 02	0.652290D 00	0.000000D-38	0.100000D 01

REMAINING DATA SPECIFYING SOLUTION

IC	M	KCH	G1	G2	G3	RF1	RF2	RS1	RS2
2	2	2	-0.100000D 01	-0.500000D 00	0.100000D 00	0.700000D-01	0.100000D-01	0.250000D-01	0.500000D-01

```
IT=  1 EO= -2.0000000D 00 EE= -4.8228600D-02 DE= -4.8082717D-03  B(1/3)=  9.7654369D-01 -4.7954606D-01  2.0412434D 01

IT=  2 EO= -2.1993950D 00 EE= -5.3036872D-02 DE= -9.6792359D-05  B(1/3)=  1.1368639D-10  7.8319742D-02  1.9650173D 00

IT=  3 EO= -2.2034089D 00 EE= -5.3133664D-02 DE= -3.5067806D-08  B(1/3)=  8.3269286D-13  2.7533173D-02  3.9703612D-02

IT=  4 EO= -2.2034104D 00 EE= -5.3133699D-02 DE=  6.8272428D-15  B(1/3)=  2.8353975D-16  2.6482157D-02  1.4377529D-05

IT=  5 EO= -2.2034104D 00 EE= -5.3133699D-02 DE= -1.0421716D-14  B(1/3)=  3.9287652D-18  2.6481777D-02  1.9197958D-12

IT=  6 EO= -2.2034104D 00 EE= -5.3133699D-02 DE=  2.6319323D-14  B(1/3)=  1.5638308D-19  2.6481777D-02 -3.1032615D-12

IT=  7 EO= -2.2034104D 00 EE= -5.3133699D-02 DE= -1.6460529D-15  B(1/3)= -5.0890973D-19  2.6481777D-02 -4.6750432D-12
```

DEUTERON S-WAVE SOLUTION

R(L)	U(L)	R(L)	U(L)	R(L)	U(L)	R(L)	U(L)	R(L)	U(L)	R(L)	U(L)
1.CCD-03	9.01318D-04	5.10D-02	4.46522D-02	1.04D-01	8.81616D-02	8.30D-01	4.31152D-01	7.08D 00	1.70746D-01		
2.000D-03	1.80170D-03	5.20D-02	4.55005D-02	1.08D-01	9.13252D-02	8.80D-01	4.41796D-01	7.28D 00	1.63075D-01		
3.000D-03	2.70106D-03	5.30D-02	4.63478D-02	1.12D-01	9.44718D-02	9.30D-01	4.51297D-01	7.48D C0	1.55744D-01		
4.000D-03	3.59940D-03	5.40D-02	4.71941D-02	1.16D-01	9.76017D-02	9.80D-01	4.59731D-01	7.68D 00	1.48740D-01		
5.000D-03	4.49671D-03	5.50D-02	4.80393D-02	1.20D-01	1.00715D-01	1.03D 00	4.67172D-01	7.88D 00	1.42049D-01		
6.000D-03	5.39300D-03	5.60D-02	4.88834D-02	1.24D-01	1.03811D-01	1.08D 00	4.73690D-01	8.08D 00	1.35658D-01		
7.000D-03	6.28826D-03	5.70D-02	4.97265D-02	1.28D-01	1.06891D-01	1.13D 00	4.79350D-01	8.28D 00	1.29552D-01		
8.000D-03	7.18249D-03	5.80D-02	5.05685D-02	1.32D-01	1.09954D-01	1.18D 00	4.84213D-01	8.48D 00	1.23721D-01		
9.000D-03	8.07570D-03	5.90D-02	5.14095D-02	1.36D-01	1.13000D-01	1.23D 00	4.88335D-01	8.68D 00	1.18151D-01		
1.000D-02	8.96788D-03	6.00D-02	5.22495D-02	1.40D-01	1.16029D-01	1.28D 00	4.91771D-01	8.88D 00	1.12831D-01		
1.100D-02	9.85903D-03	6.10D-02	5.30883D-02	1.44D-01	1.19042D-01	1.33D 00	4.94570D-01	9.08D 00	1.07750D-01		
1.200D-02	1.07491D-02	6.20D-02	5.39261D-02	1.48D-01	1.22038D-01	1.38D 00	4.96780D-01	9.28D 00	1.02898D-01		
1.300D-02	1.16382D-02	6.30D-02	5.47629D-02	1.52D-01	1.25018D-01	1.43D 00	4.98445D-01	9.48D 00	9.82634D-02		
1.400D-02	1.25263D-02	6.40D-02	5.55986D-02	1.56D-01	1.27981D-01	1.48D 00	4.99604D-01	9.68D 00	9.38376D-02		
1.500D-C2	1.34133D-02	6.50D-02	5.64333D-02	1.60D-01	1.30927D-01	1.53D 0C	5.00297D-01	9.88D 00	8.96109D-02		
1.600D-02	1.42993D-02	6.60D-02	5.72669D-02	1.64D-01	1.33857D-01	1.58D 00	5.00558D-01	1.01D 01	8.55744D-02		
1.700D-02	1.51842D-C2	6.70D-02	5.80994D-02	1.68D-01	1.36770D-01	1.63D 00	5.00422D-01	1.03D 01	8.17195D-02		
1.800D-02	1.60681D-C2	6.80D-02	5.89309D-02	1.72D-01	1.39667D-01	1.68D 00	4.99919D-01	1.05D 01	7.80383D-02		
1.900D-02	1.69510D-02	6.90D-02	5.97613D-02	1.76D-01	1.42548D-01	1.73D C0	4.99077D-01	1.07D 01	7.45227D-02		
2.000D-02	1.78329D-02	7.00D-02	6.05907D-02	1.80D-01	1.45412D-01	1.78D 00	4.97925D-01	1.09D 01	7.11655D-02		
2.100D-02	1.87137D-02	7.10D-02	6.14190D-02	2.00D-01	1.59489D-01	1.83D 00	4.96486D-01	1.11D 01	6.79594D-02		
2.200D-02	1.95934D-02	7.20D-02	6.22463D-02	2.20D-01	1.73163D-01	1.88D 00	4.94784D-01	1.13D 01	6.48977D-02		
2.300D-02	2.04722D-02	7.30D-02	6.30725D-02	2.40D-01	1.86439D-01	1.93D 00	4.92841D-01	1.15D 01	6.19739D-02		
2.400D-02	2.13498D-02	7.40D-02	6.38977D-02	2.60D-01	1.99324D-01	1.98D 00	4.90676D-01	1.17D 01	5.91818D-02		
2.500D-02	2.22265D-02	7.50D-02	6.47218D-02	2.80D-01	2.11822D-01	2.03D 00	4.88309D-01	1.19D 01	5.65155D-02		
2.600D-02	2.31021D-02	7.60D-02	6.55448D-02	3.00D-01	2.23942D-01	2.08D 00	4.85758D-01	1.21D 01	5.39693D-02		
2.700D-02	2.39766D-02	7.70D-02	6.63668D-02	3.20D-01	2.35688D-01	2.28D 00	4.74010D-01	1.23D 01	5.15378D-02		
2.800D-02	2.48502D-02	7.80D-02	6.71877D-02	3.40D-01	2.47070D-01	2.48D 00	4.60415D-01	1.25D 01	4.92158D-02		
2.900D-02	2.57226D-02	7.90D-02	6.80076D-02	3.60D-01	2.58092D-01	2.68D 00	4.45624D-01	1.27D 01	4.69984D-02		
3.000D-02	2.65941D-02	8.00D-02	6.88264D-02	3.80D-01	2.68763D-01	2.88D 00	4.30121D-01	1.29D 01	4.48809D-02		
3.100D-02	2.74645D-02	8.10D-02	6.96442D-02	4.00D-01	2.79089D-01	3.08D 00	4.14263D-01	1.31D 01	4.28588D-02		
3.200D-02	2.83338D-02	8.20D-02	7.04609D-02	4.20D-01	2.89078D-01	3.28D 00	3.98315D-01	1.33D 01	4.09278D-02		
3.300D-02	2.92021D-02	8.30D-02	7.12766D-02	4.40D-01	2.98738D-01	3.48D 00	3.82468D-01	1.35D 01	3.90838D-02		
3.400D-02	3.00693D-02	8.40D-02	7.20912D-02	4.60D-01	3.08074D-01	3.68D 00	3.66861D-01	1.37D 01	3.73228D-02		
3.500D-02	3.09355D-02	8.50D-02	7.29047D-02	4.80D-01	3.17095D-01	3.88D 00	3.51592D-01	1.39D 01	3.56412D-02		
3.600D-02	3.18007D-02	8.60D-02	7.37172D-02	5.00D-01	3.25808D-01	4.08D 00	3.36731D-01	1.41D 01	3.40354D-02		
3.700D-02	3.26648D-02	8.70D-02	7.45286D-02	5.20D-01	3.34219D-01	4.28D 00	3.22321D-01				
3.800D-02	3.35279D-02	8.80D-02	7.53390D-02	5.40D-01	3.42337D-01	4.48D 00	3.08393D-01				
3.900D-02	3.43899D-C2	8.90D-02	7.61483D-02	5.60D-01	3.50167D-01	4.68D 00	2.94962D-01				
4.000D-02	3.52509D-02	9.00D-02	7.69566D-02	5.80D-01	3.57717D-01	4.88D 00	2.82036D-01				
4.100D-02	3.61108D-02	9.10D-02	7.77638D-02	6.00D-01	3.64995D-01	5.08D 00	2.69612D-01				
4.200D-02	3.69696D-02	9.20D-02	7.85700D-02	6.20D-01	3.72005D-01	5.28D 00	2.57688D-01				
4.300D-02	3.78275D-02	9.30D-02	7.93751D-02	6.40D-01	3.78757D-01	5.48D 00	2.46253D-01				
4.400D-02	3.86842D-02	9.40D-02	8.01791D-02	6.60D-01	3.85255D-01	5.68D 00	2.35295D-01				
4.500D-C2	3.95399D-02	9.50D-02	8.09821D-02	6.80D-01	3.91506D-01	5.88D 00	2.24802D-01				
4.600D-02	4.03946D-02	9.60D-02	8.17841D-02	7.00D-01	3.97517D-01	6.08D 00	2.14759D-01				
4.700D-02	4.12482D-C2	9.70D-02	8.25849D-02	7.20D-01	4.03294D-01	6.28D 00	2.05151D-01				
4.800D-02	4.21008D-02	9.80D-02	8.33848D-02	7.40D-01	4.08843D-01	6.48D 00	1.95961D-01				
4.900D-02	4.29523D-02	9.90D-02	8.41835D-02	7.60D-01	4.14170D-01	6.68D 00	1.87174D-01				
5.000D-02	4.38028D-02	1.000D-01	8.49813D-02	7.80D-01	4.19281D-01	6.88D 00	1.78775D-01				

DEUTERON D-WAVE SOLUTION

R(L)	W(L)	R(L)	W(L)	R(L)	W(L)	R(L)	W(L)	R(L)	W(L)	R(L)	W(L)
1.000D-03	7.91222D-07	5.10D-02	1.62982D-03	1.04D-01	5.78328D-03	8.30D-01	9.15567D-02	7.08D 00	2.02218D-02		
2.000D-03	3.07100D-06	5.20D-02	1.68877D-03	1.08D-01	6.16872D-03	8.80D-01	9.52945D-02	7.28D 00	1.88653D-02		
3.000D-03	6.85980D-06	5.30D-02	1.74860D-03	1.12D-01	6.56250D-03	9.30D-01	9.86752D-02	7.48D 00	1.76094D-02		
4.000D-03	1.21205D-05	5.40D-02	1.80927D-03	1.16D-01	6.96434D-03	9.80D-01	1.01712D-01	7.68D 00	1.64463D-02		
5.000D-03	1.88296D-05	5.50D-02	1.87080D-03	1.20D-01	7.37394D-03	1.03D 00	1.04418D-01	7.88D 00	1.53685D-02		
6.000D-03	2.69656D-05	5.60D-02	1.93318D-03	1.24D-01	7.79101D-03	1.08D 00	1.06810D-01	8.08D 00	1.43692D-02		
7.000D-03	3.65085D-05	5.70D-02	1.99638D-03	1.28D-01	8.21530D-03	1.13D 00	1.08903D-01	8.28D 00	1.34422D-02		
8.000D-03	4.74390D-05	5.80D-02	2.06042D-03	1.32D-01	8.64652D-03	1.18D 00	1.10713D-01	8.48D 00	1.25817D-02		
9.000D-03	5.97388D-05	5.90D-02	2.12527D-03	1.36D-01	9.08444D-03	1.23D 00	1.12256D-01	8.68D 00	1.17826D-02		
1.000D-02	7.33904D-05	6.00D-02	2.19094D-03	1.40D-01	9.52879D-03	1.28D 00	1.13548D-01	8.88D 00	1.10399D-02		
1.100D-02	8.83767D-05	6.10D-02	2.25741D-03	1.44D-01	9.97934D-03	1.33D 00	1.14605D-01	9.08D 00	1.03493D-02		
1.200D-02	1.04681D-04	6.20D-02	2.32468D-03	1.48D-01	1.04358D-02	1.38D 00	1.15441D-01	9.28D 00	9.70678D-03		
1.300D-02	1.22288D-04	6.30D-02	2.39274D-03	1.52D-01	1.08981D-02	1.43D 00	1.16072D-01	9.48D 00	9.10855D-03		
1.400D-02	1.41182D-04	6.40D-02	2.46159D-03	1.56D-01	1.13659D-02	1.48D 00	1.16512D-01	9.68D 00	8.55126D-03		
1.500D-02	1.61347D-04	6.50D-02	2.53121D-03	1.60D-01	1.18389D-02	1.53D 00	1.16773D-01	9.88D 00	8.03178D-03		
1.600D-02	1.82770D-04	6.60D-02	2.60160D-03	1.64D-01	1.23171D-02	1.58D 00	1.16870D-01	1.01D 01	7.54727D-03		
1.700D-02	2.05436D-04	6.70D-02	2.67275D-03	1.68D-01	1.28001D-02	1.63D 00	1.16813D-01	1.03D 01	7.09509D-03		
1.800D-02	2.29330D-04	6.80D-02	2.74465D-03	1.72D-01	1.32878D-02	1.68D 00	1.16616D-01	1.05D 01	6.67287D-03		
1.900D-02	2.54440D-04	6.90D-02	2.81731D-03	1.76D-01	1.37800D-02	1.73D 00	1.16290D-01	1.07D 01	6.27837D-03		
2.000D-02	2.80751D-04	7.00D-02	2.89070D-03	1.80D-01	1.42766D-02	1.78D 00	1.15844D-01	1.09D 01	5.90959D-03		
2.100D-02	3.08251D-04	7.10D-02	2.96483D-03	2.00D-01	1.68176D-02	1.83D 00	1.15288D-01	1.11D 01	5.56465D-03		
2.200D-02	3.36927D-04	7.20D-02	3.03969D-03	2.20D-01	1.94410D-02	1.88D 00	1.14632D-01	1.13D 01	5.24185D-03		
2.300D-02	3.66767D-04	7.30D-02	3.11527D-03	2.40D-01	2.21282D-02	1.93D 00	1.13886D-01	1.15D 01	4.93961D-03		
2.400D-02	3.97757D-04	7.40D-02	3.19157D-03	2.60D-01	2.48630D-02	1.98D 00	1.13056D-01	1.17D 01	4.65647D-03		
2.500D-02	4.29887D-04	7.50D-02	3.26858D-03	2.80D-01	2.76312D-02	2.03D 00	1.12150D-01	1.19D 01	4.39111D-03		
2.600D-02	4.63144D-04	7.60D-02	3.34628D-03	3.00D-01	3.04200D-02	2.08D 00	1.11177D-01	1.21D 01	4.14228D-03		
2.700D-02	4.97516D-04	7.70D-02	3.42469D-03	3.20D-01	3.32183D-02	2.28D 00	1.06732D-01	1.23D 01	3.90886D-03		
2.800D-02	5.32992D-04	7.80D-02	3.50378D-03	3.40D-01	3.60163D-02	2.48D 00	1.01665D-01	1.25D 01	3.68979D-03		
2.900D-02	5.69562D-04	7.90D-02	3.58356D-03	3.60D-01	3.88054D-02	2.68D 00	9.62506D-02	1.27D 01	3.48411D-03		
3.000D-02	6.07212D-04	8.00D-02	3.66402D-03	3.80D-01	4.15778D-02	2.88D 00	9.06935D-02	1.29D 01	3.29091D-03		
3.100D-02	6.45934D-04	8.10D-02	3.74515D-03	4.00D-01	4.43269D-02	3.08D 00	8.51406D-02	1.31D 01	3.10936D-03		
3.200D-02	6.85716D-04	8.20D-02	3.82694D-03	4.20D-01	4.70468D-02	3.28D 00	7.96960D-02	1.33D 01	2.93871D-03		
3.300D-02	7.26547D-04	8.30D-02	3.90939D-03	4.40D-01	4.97322D-02	3.48D 00	7.44315D-02	1.35D 01	2.77824D-03		
3.400D-02	7.68418D-04	8.40D-02	3.99250D-03	4.60D-01	5.23786D-02	3.68D 00	6.93940D-02	1.37D 01	2.62729D-03		
3.500D-02	8.11317D-04	8.50D-02	4.07625D-03	4.80D-01	5.49821D-02	3.88D 00	6.46122D-02	1.39D 01	2.48524D-03		
3.600D-02	8.55236D-04	8.60D-02	4.16065D-03	5.00D-01	5.75393D-02	4.08D 00	6.01011D-02	1.41D 01	2.35154D-03		
3.700D-02	9.00163D-04	8.70D-02	4.24568D-03	5.20D-01	6.00473D-02	4.28D 00	5.58661D-02				
3.800D-02	9.46089D-04	8.80D-02	4.33135D-03	5.40D-01	6.25034D-02	4.48D 00	5.19055D-02				
3.900D-02	9.93005D-04	8.90D-02	4.41763D-03	5.60D-01	6.49056D-02	4.68D 00	4.82126D-02				
4.000D-02	1.04090D-03	9.00D-02	4.50454D-03	5.80D-01	6.72522D-02	4.88D 00	4.47775D-02				
4.100D-02	1.08977D-03	9.10D-02	4.59206D-03	6.00D-01	6.95416D-02	5.08D 00	4.15880D-02				
4.200D-02	1.13959D-03	9.20D-02	4.68020D-03	6.20D-01	7.17726D-02	5.28D 00	3.86307D-02				
4.300D-02	1.19037D-03	9.30D-02	4.76893D-03	6.40D-01	7.39444D-02	5.48D 00	3.58916D-02				
4.400D-02	1.24210D-03	9.40D-02	4.85826D-03	6.60D-01	7.60561D-02	5.68D 00	3.33567D-02				
4.500D-02	1.29475D-03	9.50D-02	4.94818D-03	6.80D-01	7.81074D-02	5.88D 00	3.10119D-02				
4.600D-02	1.34833D-03	9.60D-02	5.03869D-03	7.00D-01	8.00979D-02	6.08D 00	2.88437D-02				
4.700D-02	1.40283D-03	9.70D-02	5.12978D-03	7.20D-01	8.20275D-02	6.28D 00	2.68391D-02				
4.800D-02	1.45824D-03	9.80D-02	5.22145D-03	7.40D-01	8.38960D-02	6.48D 00	2.49859D-02				
4.900D-02	1.51454D-03	9.90D-02	5.31369D-03	7.60D-01	8.57038D-02	6.68D 00	2.32723D-02				
5.000D-02	1.57174D-03	1.00D-01	5.40650D-03	7.80D-01	8.74510D-02	6.88D 00	2.16877D-02				

E0= -2.2034104D 00   Q= 0.2793051D 00   PD= 3.8031646D-02

34

DEUTERON S-WAVE SOLUTION

R(L)	U(L)	R(L)	U(L)	R(L)	U(L)	R(L)	U(L)	R(L)	U(L)	R(L)	U(L)
1.00D-03	1.12168D-03	5.10D-02	5.55677D-02	1.04D-01	1.09705D-01	8.30D-01	5.32576D-01	7.08D 00	-3.16249D-01		
2.00D-03	2.24219D-03	5.20D-02	5.66234D-02	1.08D-01	1.13641D-01	8.80D-01	5.45153D-01	7.28D 00	-3.53466D-01		
3.00D-03	3.36143D-03	5.30D-02	5.76778D-02	1.12D-01	1.17556D-01	9.30D-01	5.56245D-01	7.48D 00	-3.90682D-01		
4.00D-03	4.47940D-03	5.40D-02	5.87309D-02	1.16D-01	1.21449D-01	9.80D-01	5.65949D-01	7.68D 00	-4.27895D-01		
5.00D-03	5.59610D-03	5.50D-02	5.97826D-02	1.20D-01	1.25322D-01	1.03D 00	5.74355D-01	7.88D 00	-4.65105D-01		
6.00D-03	6.71151D-03	5.60D-02	6.08331D-02	1.24D-01	1.29174D-01	1.08D 00	5.81549D-01	8.08D 00	-5.02314D-01		
7.00D-03	7.82565D-03	5.70D-02	6.18822D-02	1.28D-01	1.33004D-01	1.13D 00	5.87610D-01	8.28D 00	-5.39520D-01		
8.00D-03	8.93852D-03	5.80D-02	6.29300D-02	1.32D-01	1.36814D-01	1.18D 00	5.92615D-01	8.48D 00	-5.76724D-01		
9.00D-03	1.00501D-02	5.90D-02	6.39765D-02	1.36D-01	1.40603D-01	1.23D 00	5.96633D-01	8.68D 00	-6.13926D-01		
1.00D-02	1.11604D-02	6.00D-02	6.50217D-02	1.40D-01	1.44371D-01	1.28D 00	5.99732D-01	8.88D 00	-6.51127D-01		
1.10D-02	1.22694D-02	6.10D-02	6.60655D-02	1.44D-01	1.48118D-01	1.33D 00	6.01974D-01	9.08D 00	-6.88326D-01		
1.20D-02	1.33772D-02	6.20D-02	6.71081D-02	1.48D-01	1.51844D-01	1.38D 00	6.03416D-01	9.28D 00	-7.25524D-01		
1.30D-02	1.44836D-02	6.30D-02	6.81493D-02	1.52D-01	1.55550D-01	1.43D 00	6.04113D-01	9.48D 00	-7.62721D-01		
1.40D-02	1.55888D-02	6.40D-02	6.91892D-02	1.56D-01	1.59235D-01	1.48D 00	6.04115D-01	9.68D 00	-7.99917D-01		
1.50D-02	1.66926D-02	6.50D-02	7.02278D-02	1.60D-01	1.62899D-01	1.53D 00	6.03471D-01	9.88D 00	-8.37111D-01		
1.60D-02	1.77952D-02	6.60D-02	7.12651D-02	1.64D-01	1.66542D-01	1.58D 00	6.02224D-01	1.01D 01	-8.74305D-01		
1.70D-02	1.88965D-02	6.70D-02	7.23011D-02	1.68D-01	1.70165D-01	1.63D 00	6.00415D-01	1.03D 01	-9.11499D-01		
1.80D-02	1.99965D-02	6.80D-02	7.33357D-02	1.72D-01	1.73767D-01	1.68D 00	5.98084D-01	1.05D 01	-9.48691D-01		
1.90D-02	2.10953D-02	6.90D-02	7.43690D-02	1.76D-01	1.77349D-01	1.73D 00	5.95266D-01	1.07D 01	-9.85883D-01		
2.00D-02	2.21927D-02	7.00D-02	7.54010D-02	1.80D-01	1.80910D-01	1.78D 00	5.91995D-01	1.09D 01	-1.02308D 00		
2.10D-02	2.32888D-02	7.10D-02	7.64317D-02	2.00D-01	1.98409D-01	1.83D 00	5.88302D-01	1.11D 01	-1.06027D 00		
2.20D-02	2.43837D-02	7.20D-02	7.74611D-02	2.20D-01	2.15403D-01	1.88D 00	5.84215D-01	1.13D 01	-1.09746D 00		
2.30D-02	2.54772D-02	7.30D-02	7.84892D-02	2.40D-01	2.31897D-01	1.93D 00	5.79763D-01	1.15D 01	-1.13465D 00		
2.40D-02	2.65695D-02	7.40D-02	7.95159D-02	2.60D-01	2.47399D-01	1.98D 00	5.74970D-01	1.17D 01	-1.17184D 00		
2.50D-02	2.76604D-02	7.50D-02	8.05413D-02	2.80D-01	2.63417D-01	2.03D 00	5.69861D-01	1.19D 01	-1.20903D 00		
2.60D-02	2.87501D-02	7.60D-02	8.15654D-02	3.00D-01	2.78457D-01	2.08D 00	5.64456D-01	1.21D 01	-1.24622D 00		
2.70D-02	2.98384D-02	7.70D-02	8.25882D-02	3.20D-01	2.93027D-01	2.28D 00	5.40276D-01	1.23D 01	-1.28341D 00		
2.80D-02	3.09255D-02	7.80D-02	8.36097D-02	3.40D-01	3.07137D-01	2.48D 00	5.12778D-01	1.25D 01	-1.32060D 00		
2.90D-02	3.20112D-02	7.90D-02	8.46298D-02	3.60D-01	3.20795D-01	2.68D 00	4.82794D-01	1.27D 01	-1.35779D 00		
3.00D-02	3.30957D-02	8.00D-02	8.56487D-02	3.80D-01	3.34009D-01	2.88D 00	4.50946D-01	1.29D 01	-1.39498D 00		
3.10D-02	3.41789D-02	8.10D-02	8.66662D-02	4.00D-01	3.46788D-01	3.08D 00	4.17705D-01	1.31D 01	-1.43217D 00		
3.20D-02	3.52607D-02	8.20D-02	8.76824D-02	4.20D-01	3.59140D-01	3.28D 00	3.83420D-01	1.33D 01	-1.46936D 00		
3.30D-02	3.63413D-02	8.30D-02	8.86973D-02	4.40D-01	3.71075D-01	3.48D 00	3.48357D-01	1.35D 01	-1.50654D 00		
3.40D-02	3.74205D-02	8.40D-02	8.97108D-02	4.60D-01	3.82601D-01	3.68D 00	3.12713D-01	1.37D 01	-1.54373D 00		
3.50D-02	3.84985D-02	8.50D-02	9.07231D-02	4.80D-01	3.93727D-01	3.88D 00	2.76639D-01	1.39D 01	-1.58092D 00		
3.60D-02	3.95751D-02	8.60D-02	9.17340D-02	5.00D-01	4.04462D-01	4.08D 00	2.40244D-01	1.41D 01	-1.61811D 00		
3.70D-02	4.06504D-02	8.70D-02	9.27436D-02	5.20D-01	4.14815D-01	4.28D 00	2.03613D-01				
3.80D-02	4.17245D-02	8.80D-02	9.37519D-02	5.40D-01	4.24794D-01	4.48D 00	1.66809D-01				
3.90D-02	4.27972D-02	8.90D-02	9.47588D-02	5.60D-01	4.34407D-01	4.68D 00	1.29878D-01				
4.00D-02	4.38686D-02	9.00D-02	9.57645D-02	5.80D-01	4.43664D-01	4.88D 00	9.28552D-02				
4.10D-02	4.49387D-02	9.10D-02	9.67688D-02	6.00D-01	4.52573D-01	5.08D 00	5.57664D-02				
4.20D-02	4.60075D-02	9.20D-02	9.77718D-02	6.20D-01	4.61142D-01	5.28D 00	1.86309D-02				
4.30D-02	4.70750D-02	9.30D-02	9.87735D-02	6.40D-01	4.69379D-01	5.48D 00	-1.85371D-02				
4.40D-02	4.81412D-02	9.40D-02	9.97739D-02	6.60D-01	4.77293D-01	5.68D 00	-5.57272D-02				
4.50D-02	4.92060D-02	9.50D-02	1.00773D-01	6.80D-01	4.84891D-01	5.88D 00	-9.29319D-02				
4.60D-02	5.02696D-02	9.60D-02	1.01771D-01	7.00D-01	4.92180D-01	6.08D 00	-1.30146D-01				
4.70D-02	5.13319D-02	9.70D-02	1.02767D-01	7.20D-01	4.99170D-01	6.28D 00	-1.67365D-01				
4.80D-02	5.23928D-02	9.80D-02	1.03762D-01	7.40D-01	5.05867D-01	6.48D 00	-2.04586D-01				
4.90D-02	5.34524D-02	9.90D-02	1.04756D-01	7.60D-01	5.12278D-01	6.68D 00	-2.41808D-01				
5.00D-02	5.45107D-02	1.00D-01	1.05749D-01	7.80D-01	5.18412D-01	6.88D 00	-2.79029D-01				

DEUTERON D-WAVE SOLUTION

R(L)	W(L)	R(L)	W(L)	R(L)	W(L)	R(L)	W(L)	R(L)	W(L)	R(L)	W(L)
1.00D-03	9.84664D-07	5.10D-02	2.02811D-03	1.04D-01	7.19556D-03	8.30D-01	1.13166D-01	7.08D 00	1.61222D-02		
2.00D-03	3.82181D-06	5.20D-02	2.10146D-03	1.08D-01	7.67503D-03	8.80D-01	1.17691D-01	7.28D 00	1.48023D-02		
3.00D-03	8.53690D-06	5.30D-02	2.17590D-03	1.12D-01	8.16487D-03	9.30D-01	1.21763D-01	7.48D 00	1.36092D-02		
4.00D-03	1.50837D-05	5.40D-02	2.25140D-03	1.16D-01	8.66471D-03	9.80D-01	1.25396D-01	7.68D 00	1.25307D-02		
5.00D-03	2.34330D-05	5.50D-02	2.32796D-03	1.20D-01	9.17419D-03	1.03D 00	1.28610D-01	7.88D 00	1.15555D-02		
6.00D-03	3.35581D-05	5.60D-02	2.40557D-03	1.24D-01	9.69296D-03	1.08D 00	1.31424D-01	8.08D 00	1.06737D-02		
7.00D-03	4.54339D-05	5.70D-02	2.48422D-03	1.28D-01	1.02207D-02	1.13D 00	1.33857D-01	8.28D 00	9.87587D-03		
8.00D-03	5.90365D-05	5.80D-02	2.56389D-03	1.32D-01	1.07570D-02	1.18D 00	1.35931D-01	8.48D 00	9.15383D-03		
9.00D-03	7.43432D-05	5.90D-02	2.64459D-03	1.36D-01	1.13017D-02	1.23D 00	1.37666D-01	8.68D 00	8.50000D-03		
1.00D-02	9.13321D-05	6.00D-02	2.72629D-03	1.40D-01	1.18543D-02	1.28D 00	1.39081D-01	8.88D 00	7.90758D-03		
1.10D-02	1.09982D-04	6.10D-02	2.80900D-03	1.44D-01	1.24146D-02	1.33D 00	1.40198D-01	9.08D 00	7.37045D-03		
1.20D-02	1.30272D-04	6.20D-02	2.89270D-03	1.48D-01	1.29824D-02	1.38D 00	1.41034D-01	9.28D 00	6.83306D-03		
1.30D-02	1.52183D-04	6.30D-02	2.97739D-03	1.52D-01	1.35572D-02	1.43D 00	1.41610D-01	9.48D 00	6.44045D-03		
1.40D-02	1.75695D-04	6.40D-02	3.06304D-03	1.56D-01	1.41389D-02	1.48D 00	1.41943D-01	9.68D 00	6.03814D-03		
1.50D-02	2.00790D-04	6.50D-02	3.14967D-03	1.60D-01	1.47271D-02	1.53D 00	1.42050D-01	9.88D 00	5.67211D-03		
1.60D-02	2.27450D-04	6.60D-02	3.23724D-03	1.64D-01	1.53217D-02	1.58D 00	1.41948D-01	1.01D 01	5.33874D-03		
1.70D-02	2.55656D-04	6.70D-02	3.32577D-03	1.68D-01	1.59223D-02	1.63D 00	1.41653D-01	1.03D 01	5.03480D-03		
1.80D-02	2.85390D-04	6.80D-02	3.41524D-03	1.72D-01	1.65287D-02	1.68D 00	1.41181D-01	1.05D 01	4.75737D-03		
1.90D-02	3.16638D-04	6.90D-02	3.50563D-03	1.76D-01	1.71407D-02	1.73D 00	1.40545D-01	1.07D 01	4.50384D-03		
2.00D-02	3.49380D-04	7.00D-02	3.59695D-03	1.80D-01	1.77580D-02	1.78D 00	1.39758D-01	1.09D 01	4.27188D-03		
2.10D-02	3.83602D-04	7.10D-02	3.68919D-03	2.00D-01	2.09169D-02	1.83D 00	1.38835D-01	1.11D 01	4.05939D-03		
2.20D-02	4.19287D-04	7.20D-02	3.78233D-03	2.20D-01	2.41774D-02	1.88D 00	1.37786D-01	1.13D 01	3.86448D-03		
2.30D-02	4.56420D-04	7.30D-02	3.87636D-03	2.40D-01	2.75166D-02	1.93D 00	1.36623D-01	1.15D 01	3.68546D-03		
2.40D-02	4.94985D-04	7.40D-02	3.97129D-03	2.60D-01	3.09140D-02	1.98D 00	1.35357D-01	1.17D 01	3.52083D-03		
2.50D-02	5.34968D-04	7.50D-02	4.06710D-03	2.80D-01	3.43519D-02	2.03D 00	1.33998D-01	1.19D 01	3.36923D-03		
2.60D-02	5.76353D-04	7.60D-02	4.16378D-03	3.00D-01	3.78145D-02	2.08D 00	1.32555D-01	1.21D 01	3.22945D-03		
2.70D-02	6.19126D-04	7.70D-02	4.26133D-03	3.20D-01	4.12879D-02	2.28D 00	1.26113D-01	1.23D 01	3.10039D-03		
2.80D-02	6.63273D-04	7.80D-02	4.35973D-03	3.40D-01	4.47597D-02	2.48D 00	1.18938D-01	1.25D 01	2.98108D-03		
2.90D-02	7.08779D-04	7.90D-02	4.45899D-03	3.60D-01	4.82191D-02	2.68D 00	1.11390D-01	1.27D 01	2.87063D-03		
3.00D-02	7.55632D-04	8.00D-02	4.55909D-03	3.80D-01	5.16566D-02	2.88D 00	1.03735D-01	1.29D 01	2.76827D-03		
3.10D-02	8.03816D-04	8.10D-02	4.66002D-03	4.00D-01	5.50636D-02	3.08D 00	9.61639D-02	1.31D 01	2.67329D-03		
3.20D-02	8.53320D-04	8.20D-02	4.76178D-03	4.20D-01	5.84330D-02	3.28D 00	8.88126D-02	1.33D 01	2.58505D-03		
3.30D-02	9.04130D-04	8.30D-02	4.86436D-03	4.40D-01	6.17580D-02	3.48D 00	8.17725D-02	1.35D 01	2.50297D-03		
3.40D-02	9.56233D-04	8.40D-02	4.96775D-03	4.60D-01	6.50331D-02	3.68D 00	7.51024D-02	1.37D 01	2.42656D-03		
3.50D-02	1.00962D-03	8.50D-02	5.07195D-03	4.80D-01	6.82533D-02	3.88D 00	6.88362D-02	1.39D 01	2.35534D-03		
3.60D-02	1.06427D-03	8.60D-02	5.17695D-03	5.00D-01	7.14144D-02	4.08D 00	6.29896D-02	1.41D 01	2.28890D-03		
3.70D-02	1.12017D-03	8.70D-02	5.28274D-03	5.20D-01	7.45126D-02	4.28D 00	5.75650D-02				
3.80D-02	1.17732D-03	8.80D-02	5.38931D-03	5.40D-01	7.75449D-02	4.48D 00	5.25549D-02				
3.90D-02	1.23570D-03	8.90D-02	5.49666D-03	5.60D-01	8.05084D-02	4.68D 00	4.79456D-02				
4.00D-02	1.29530D-03	9.00D-02	5.60478D-03	5.80D-01	8.34011D-02	4.88D 00	4.37187D-02				
4.10D-02	1.35611D-03	9.10D-02	5.71367D-03	6.00D-01	8.62210D-02	5.08D 00	3.98528D-02				
4.20D-02	1.41811D-03	9.20D-02	5.82331D-03	6.20D-01	8.89667D-02	5.28D 00	3.63254D-02				
4.30D-02	1.48129D-03	9.30D-02	5.93370D-03	6.40D-01	9.16371D-02	5.48D 00	3.31129D-02				
4.40D-02	1.54565D-03	9.40D-02	6.04483D-03	6.60D-01	9.42311D-02	5.68D 00	3.01921D-02				
4.50D-02	1.61118D-03	9.50D-02	6.15670D-03	6.80D-01	9.67482D-02	5.88D 00	2.75401D-02				
4.60D-02	1.67785D-03	9.60D-02	6.26929D-03	7.00D-01	9.91880D-02	6.08D 00	2.51349D-02				
4.70D-02	1.74566D-03	9.70D-02	6.38261D-03	7.20D-01	1.01550D-01	6.28D 00	2.29556D-02				
4.80D-02	1.81460D-03	9.80D-02	6.49665D-03	7.40D-01	1.03835D-01	6.48D 00	2.09825D-02				
4.90D-02	1.88467D-03	9.90D-02	6.61140D-03	7.60D-01	1.06043D-01	6.68D 00	1.91971D-02				
5.00D-02	1.95584D-03	1.000D-01	6.72685D-03	7.80D-01	1.08173D-01	6.88D 00	1.75823D-02				

SCATTERING LENGTH	EFFECTIVE RANGE	SHAPE PARAMETER
5.377921D 00	1.641093D 00	0.595947D-01

MIXED EFFECTIVE RANGE	DEUTERON EFFECTIVE RANGE	APPROX. SHAPE PARAMETER	SECOND SHAPE PARAMETER
1.677354D 00	1.721885D 00	0.772040D-01	0.123057D 00