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1. INTRODUCTION -

It is well known that the cross section of nucleons elastically scattered by light nuclei shows a structure of well resolved resonances at the low energies. In some cases the average behaviour of the interaction and some broad resonances have been well accounted for using a single-particle potential model. An example is given by the interaction of neutrons with the even-even light nuclei ^4He , ^{12}C , ^{16}O . In these cases it has been possible to reproduce the behaviour of the non-resonant phase shifts, and the following single-particle resonances:

- The $3/2^+$ level in the n- ^{16}O process (neutron energy in the laboratory frame $E=1$ MeV; total width of the level $\Gamma = 106$ KeV)(1).
- The $3/2^-$ level in the n- ^4He process ($E=1.3$ MeV; $\Gamma = 1440$ KeV)(2).
- The $3/2^+$ level in the n- ^{12}C process ($E=3.67$ MeV; $\Gamma = 1200$ KeV).

In spite of these results, the model is unsatisfactory, because many narrow resonances are found by experience, which can not be reproduced by this simple mechanism. We investigate here the possibility of explaining these narrow resonances by means of the coupled channel theory⁽³⁾, which takes into account the internal degrees of freedom of the target nucleus. Until now this method has been successfully applied to the

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high energy nucleon-nucleus interaction^{(4)...}(7). In this paper the method is applied to the n-¹²C scattering process in the elastic region, taking into account the virtual excitation of the first target level, which is supposed to be collective.

The n-¹²C process seems to be an ideal case, in order to check the possibilities of the model. The reasons are as follows:

- The experimental knowledge of the interaction is complete and detailed: many measurements of integral and differential cross section, and polarization, are available in the literature^(8, 9). From these experimental data, a set of phase shifts has been unambiguously extracted⁽⁸⁾⁽¹⁰⁾. We shall refer throughout to the phase shifts behaviour vs energy, which summarize coherently all the experimental information available.
- The collective nature of the ¹²C low energy spectrum has been already emphasized in the literature⁽¹¹⁾.

The analysis has been carried out in three steps, employing the following interaction potentials: (1) a spherical square well potential; (2) a deformed square well potential; (3) a deformed potential with a Woods-Saxon radial form. This approach gives a complete phenomenological survey of the interaction, and leads to a step by step formulation of the inverse problem (determination of the potential parameter from experimental data).

The physical reliability of the model here employed is limited mainly by the following approximations:

- Only the first 2⁺ excited state is supposed to be effective in the target nucleus.
- For the low energy states of the target nucleus, harmonic oscillator wave functions are assumed, whereas the realistic states would be more complicated, as shown in references^(11, 12).

In spite of these limitations, the model seems to work quite well, as demonstrated "a posteriori" by the good results obtained.

2. SINGLE-PARTICLE MODEL. SPHERICAL SQUARE-WELL POTENTIAL

A single-particle potential taking into account the spin-orbit coupling, reads:

$$(1) \quad V_0(r) = -U_0 f(r) + U_{SO} (\vec{l} \cdot \vec{s}) (\hbar/M_{\pi} c)^2 \cdot (1/r) (df/dr),$$

where U_0 and U_{s0} are positive parameters for attractive forces. For a square well shape, the radial dependence is given by

$$(2a) \quad f(r)=1 \quad \text{for } r \leq R_0$$

$$(2b) \quad f(r)=0 \quad \text{for } r > R_0$$

$$(2c) \quad df/dr = -\delta(r-R_0).$$

By inserting eq. (1) into the radial Schrödinger equation

$$(3) \quad (T_\ell + V(r) - E_{CM})u_{j\ell}(r) = 0,$$

one obtains the connection between the phase shift and the unknown parameters of the potential:

$$(4) \quad P_\ell(\rho_0) \cot(\delta_{j\ell} + \phi_\ell(\rho_0)) = x_{j\ell}.$$

The expressions introduced in eqs (3) and (4) are defined as follows (for details see ref. (13))

$$(5a) \quad T_\ell = (\hbar^2/2M) \left(-d^2/dr^2 + \frac{\ell(\ell+1)}{r^2} \right) = \text{kinetic energy of relative motion,}$$

$$(5b) \quad x_{j\ell} = \mu_0 (F_{\ell-1}(\mu_0)/F_\ell(\mu_0)) - \ell - S_\ell^{(+)}(\rho_0) + \chi_{j\ell},$$

$$(5c) \quad \chi_{j\ell} = -U_{s0} (2M/\hbar^2) (\hbar/M_\pi c)^2 (\vec{\ell} \cdot \vec{s}),$$

$$(5d) \quad \rho_0/R_0 = \sqrt{(2M/\hbar^2)E_{CM}} = \text{external momentum,}$$

$$(5e) \quad \mu_0/R_0 = \sqrt{(2M/\hbar^2)(E_{CM} + U_0)} = \text{internal momentum,}$$

$$(5f) \quad \phi_\ell(\rho_0) = \text{arctg}(F_\ell(\rho_0)/G_\ell(\rho_0)) = \text{hard-sphere scattering phase shift}$$

$$(5g) \quad P_\ell(\rho_0) = \rho_0 (F_\ell^2(\rho_0) + G_\ell^2(\rho_0))^{-1} = \text{penetrability,}$$

$$(5h) \quad S_\ell^{(+)}(\rho_0) = P_\ell(\rho_0) (F_\ell(\rho_0)F_{\ell-1}(\rho_0) + G_\ell(\rho_0)G_{\ell-1}(\rho_0)) - \ell = \\ = \text{shift factor}$$

4.

with M_π = pion mass, M = reduced mass of the nucleon-nucleus system, F_ℓ and G_ℓ , regular and irregular solutions of the Schrodinger equation for positive-energy channels.

We assume that the depth of the central and spin-orbit part of the potential, are slow varying linear functions of E (neutron energy in the laboratory frame):

$$(6a) \quad U_0 = U_0^{(0)} + U_0^{(1)} E,$$

$$(6b) \quad U_{SO} = U_{SO}^{(0)} + U_{SO}^{(1)} E.$$

Neglecting the two narrow $3/2^+$ and $5/2^+$ resonances, the parameters $U_0^{(0)}$ and $U_0^{(1)}$ have been evaluated for a particular value of the square well radius R_0 from the S - wave behaviour and from the existence of the $1/2^+$ bound state at -4.14 MeV, respectively.

The parameters $U_{SO}^{(0)}$ and $U_{SO}^{(1)}$ are calculated from the energy and the width of the 3.67 MeV, $3/2^+$ level.

The nuclear radius is assumed to be defined by the well known relation $R_0 = r_0 A^{1/3}$, A being the target nucleus mass number.

The relation between the potential parameters and the energy E^r and total width Γ of a resonance are obtained by inserting a Breit - Wigner one level formula in eq. (4):

$$(7a) \quad -P_\ell(E^r) \operatorname{tg} \phi_\ell(E^r) = x_{j\ell}(E^r)$$

$$(7b) \quad \Gamma_{j\ell} = -2P_\ell(E^r) \left[\frac{dx_{j\ell}}{dE} \right]_{E=E^r}$$

It can be easily seen that ϕ_ℓ for $\ell > 1$ can be neglected in calculations, in the energy range here considered. In this approximation eq. (7a) becomes

$$(8) \quad x_{j\ell}(E^r) = 0$$

The parameters so obtained are listed in Table II, 1), for two different values of the parameter r_0 , while the calculated phase shifts are shown in fig. 1. In Table I, 1) the calculated parameters of the level are compared with experience. The agreement is in this case complete.

The choice of the radius is not critical. No appreciable diffe

rence exists between the phase shifts calculated by parameters of Table II, 1a) and II, 1b). The very well known approximate relation $R_0^2 U_0 \approx \text{constant}$, is seen to be satisfied.

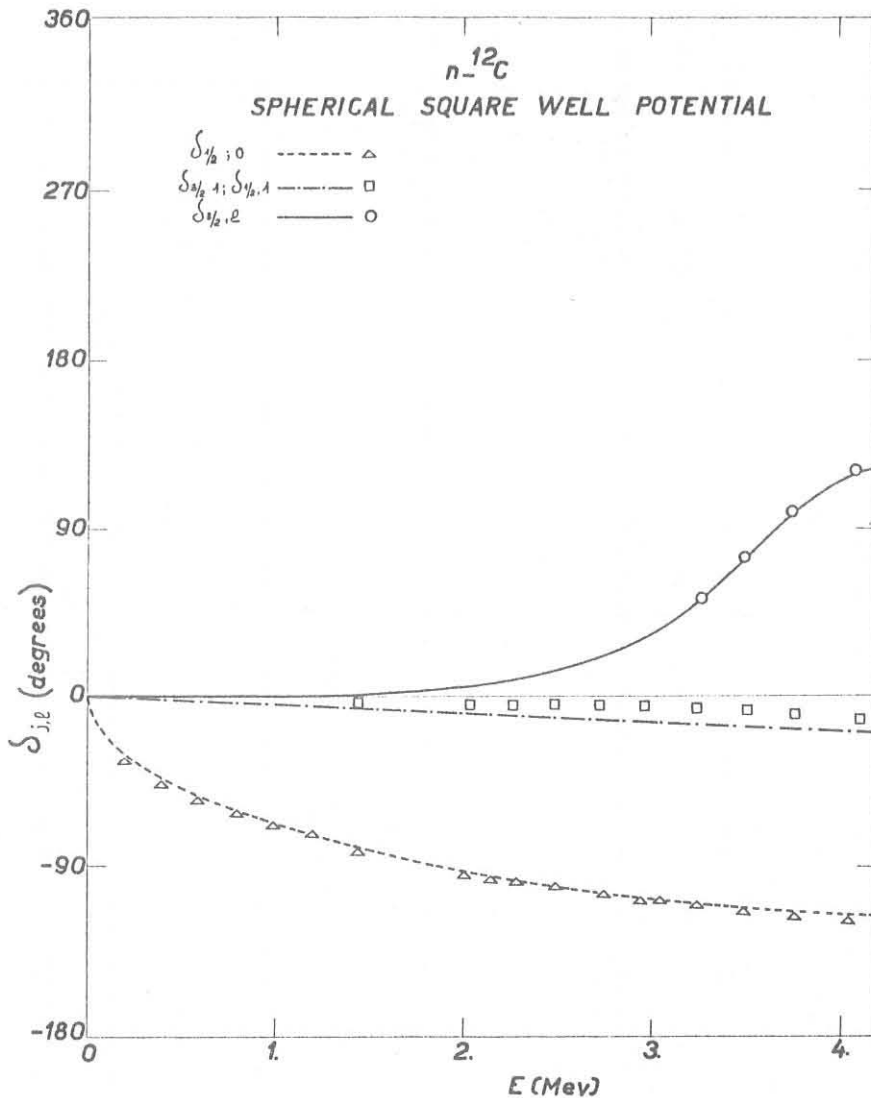


FIG. 1 - Phase shifts calculated by parameters of table II, 1), compared with experiments. Experimental points are taken from refs (8) and (10). The $5/2^+$ level and the first $3/2^+$ level have been dropped. The phase shifts of the P doublet, both theoretical and experimental, are indistinguishable in the figure.

We may conclude that the square well spherical potential is able to reproduce all the features of the interaction, if one neglects the two narrow levels. By changing the radial shape of the potential, one can only modify energy and width of the level already obtained, without reproducing new levels.

It can be pointed out finally that the values obtained for the $U_0^{(1)}$ and $U_{SO}^{(1)}$ parameters, are very small. In the next paragraph these parameters are dropped, and U_0 and U_{SO} are taken constant in energy.

TABLE I
Level Parameters

J^π	Experimental values		Calculated values							
	E^r (MeV)	Γ (KeV)	1) Square well central potential		2) Square well deformed potential				3) Woods-Saxon deformed potential	
			a) $r_0 = 1.25$ f and b) $r_0 = 1.5$ f		a) $r_0 = 1.3$ f		b) $r_0 = 1.4$ f		$r_0 = 1.5$ f	
			E^r (MeV)	Γ (KeV)	E^r (MeV)	Γ (KeV)	E^r (MeV)	Γ (KeV)	E^r (MeV)	Γ (KeV)
$5/2^+$	2.08	7.9			2.14	0.7	2.03	3	3.09	1
$3/2^+$	2.95	90			2.95	39	2.97	114	2.90	212
$3/2^+$	3.67	1200	3.67	1200	3.69	130	3.65	178	3.70	1395
$5/2^+$	--				3.05	0.01	3.37	11	--	--

TABLE II
Potential parameters

	1) Square-well central potential		2) Square-well deformed potential		3) Woods-Saxon deformed potential
	(a)	(b)	(a)	(b)	
r_0 (f)	1.25	1.50	1.30	1.40	1.50
U_0 (MeV)	65.58-0.818E (MeV)	46.91-0.458E (MeV)	61.91	53.25	44.60
U_{so} (MeV)	11.89-0.140E (MeV)	17.06+0.223E (MeV)	18.30	18.50	10.00
β	-	-	0.041	0.065	0.080
a (f)	-	-	-	-	0.5

3. CHANNEL COUPLING MODEL -

Let us consider now the coupling between the 0^+ ground state and the first 2^+ collective state of ^{12}C (energy of the level $\epsilon_2 = 4.43$ MeV).

For details on the model see references(5, 6, 7).

In this case the hamiltonian, for the neutron-nucleus interaction in the centre-of-mass system, reads:

$$(9) \quad H = T_p(r) + H_t(\xi) + V(\vec{r}, \xi)$$

where $\vec{r}(\equiv r, \theta, \phi)$ is the coordinate of the relative motion; ξ stands for the angular coordinates of the target; $H_t(\xi)$ is the target hamiltonian. The interaction potential $V(r, \xi)$, for weak deformations, can be developed as follows:

$$(10) \quad V(\vec{r}, \xi) = V_0(r) - V_2(r) \beta R_0 \sqrt{4\pi/5} \cdot \sum_q (-1)^q Y_2^{-q}(\theta, \phi) T_2^q(\xi).$$

$V_0(r)$ is defined by eq. (1), while

$$(11) \quad V_2(r) = -U_0(df/dr) .$$

β is the deformation parameter and T_2^q is a II rank spherical tensor. The definition of β and T_2^q requires the specification of the rotational or vibrational character of the model.

In the rotational case the surface of the nucleus is assumed to be permanently deformed, and β is defined by $R(\theta, \phi) = R_0 (1 + \beta Y_2^0(\theta, \phi))$.

In the vibrational case the nuclear surface exhibits small instantaneous deformations expressed by the collective variables α_q : $R(\theta, \phi) = R_0 [1 + \sum_q \alpha_q Y_2^q(\theta, \phi)]$. In this case β is a dynamical deformation parameter defined by the reaction:

$$\beta^2 = (0 | \sum_{\mu} | \alpha_{\mu} |^2 | 0),$$

Let us expand now the total wave function $\psi^{JM}(\vec{r}, \xi)$ as follows:

$$(12) \quad \psi^{JM}(\vec{r}, \xi) = (1/r) \sum_{Ij\ell} u_{Ij\ell}^J(r) \phi_{Ij\ell}^{JM}(\theta, \phi, \xi),$$

where

$$(13) \quad \phi_{Ij\ell}^{JM}(\theta, \phi, \xi) = \sum_{m_I, m_\ell, m_S} C(jm \ell m_I; j I J M) \cdot C(\ell m_\ell m_S; \ell S j m) \psi_I^{m_I}(\xi) \{i^\ell Y_\ell^{m_\ell}(\theta, \phi)\} \chi_S^{m_S}$$

8.

is obtained by coupling the neutron spin \vec{s} with the angular momentum of relative motion \vec{l} , and the resultant \vec{j} with the target spin \vec{I} .

Then, introducing the excitation energies of the target levels, ϵ_I , which are supposed to be known

$$(14) \quad (H_t(\xi) - \epsilon_I) \psi_I^{m_I} = 0,$$

one obtains, from the hamiltonian (9), the following system of coupled differential equations:

$$(15) \quad \sum_{I'j\ell} (T_\ell + \epsilon_I - E_{CM}) \delta_{I'I} \delta_{j'j} \delta_{\ell'\ell} + \langle \phi_{I'j'\ell'}^{JM} | V | \phi_{Ij\ell}^{JM} \rangle u_{Ij\ell}^J = 0.$$

The calculation of the matrix elements $\langle \phi_{I'j'\ell'}^{JM} | V | \phi_{Ij\ell}^{JM} \rangle$ in eq. (15) requires the specification of the II rank tensor $T_2^q(\xi)$ and of the target wave function $\psi_I^{m_I}(\xi)$. Therefore the result, given below, depends on the vibrational or rotational nature of the target level:

$$(16) \quad \langle \phi_{I'j'\ell'}^J | V | \phi_{Ij\ell}^J \rangle = V_0(r) - V_2(r) \beta R_0 \omega_{I'j'\ell';Ij\ell}$$

$\omega_{I'j'\ell';Ij\ell}$ is given, for the rotational and the vibrational case respectively, by

$$(17a) \quad \omega_{I'j'\ell';Ij\ell} (\text{rot.}) = \frac{1}{\sqrt{4\pi}} (-1)^{j+j'-I+J+s} \frac{(-1)^{\ell-\ell'}}{i}$$

$$\left\{ (2\ell+1)(2\ell'+1)(2j+1)(2j'+1)(2I'+1) \right\}^{\frac{1}{2}}$$

$$C(I'020; I'2I0) C(\ell'0\ell 0; \ell'\ell 20)$$

$$W(j'I'jI; J2) W(\ell j \ell' j'; s2),$$

$$(17b) \quad \omega_{I'j'\ell';Ij\ell} (\text{vibr.}) = \omega_{I'j'\ell';Ij\ell} (\text{rot.}) (1 - \delta_{I'I}).$$

The $C(j_1 m_1 j_2 m_2; j_1 j_2 j m)$ and $W(abcd; ef)$ of eq. (17a) are the Clebsch Gordan coefficients and Racah coefficients⁽¹⁴⁾.

The phase shifts $\delta_{j\ell}$ are then calculated from the asymptotic behaviour of the elastic channel radial function

$$(18) \quad u_{0j\ell} \sim I_{\ell}^{+}(\rho_0) - e^{2i\delta_{j\ell}} O_{\ell}^{+}(\rho_0),$$

where

$$(19a) \quad I_{\ell}^{+} = G_{\ell} - iF_{\ell}$$

and

$$(19b) \quad O_{\ell}^{+} = G_{\ell} + iF_{\ell}$$

are the ingoing and outgoing waves.⁽¹³⁾

3. 1. - Deformed square well potential. - As far as we consider the coupling between the ground state ($I=0$) and the first collective level ($I=2$) of an even-even target nucleus, the number of coupled channels to be considered, taking into account the total momentum and parity conservation, is $N \leq 6$, including the elastic channel. Let us classify these channels by an index $\lambda = (Ij\ell)$ ($\lambda=1$ standing for the elastic channel). Then by solving the system (17) under the assumption that the 2^{+} level is virtually excited, and that the radial dependence of the potential is a square well eq.

(2), the following expression for the $\delta_{j\ell}$ phase shift is obtained:

$$(20) \quad P_{\ell}(\rho_0) \cot(\delta_{j\ell} + \phi_{\ell}(\rho_0)) = D^{j\ell} / D_{11}^{j\ell},$$

where $D^{j\ell}$ is a symmetric determinant of order N

$$(21) \quad D^{j\ell} = \begin{vmatrix} x_1 + \alpha_{11} & \alpha_{12} & \alpha_{13} & \dots & \alpha_{1N} \\ \alpha_{12} & x_2 + \alpha_{22} & \alpha_{23} & \dots & \alpha_{2N} \\ \alpha_{13} & \alpha_{23} & x_3 + \alpha_{33} & \dots & \alpha_{3N} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{1N} & \alpha_{2N} & \alpha_{3N} & \dots & x_N + \alpha_{NN} \end{vmatrix}$$

10.

and $D^{j\ell}$ is the determinant obtained by dropping the first row and the first column in $D^{j\ell}$,

The function $x_1 = x_{0j\ell}$ is defined by eq. (5b), while for the inelastic channels ($\lambda > 1$), the following expression must be used:

$$(22) \quad x_\lambda (\lambda > 1) = x_{2j\ell} = \mu_2 (F_{\ell-1}(\mu_2) / F_\ell(\mu_2))^{-\ell - S_\ell^{(-)}(\rho_2)} + \chi_{j\ell},$$

where

$$(23a) \quad \rho_2 / R_0 = \sqrt{(2M/\hbar^2)(\epsilon_2 - E_{CM})} = \text{external momentum for coupled channels}$$

$$(23b) \quad \mu_2 / R_0 = \sqrt{(2M/\hbar^2)(E_{CM} + U_0 - \epsilon_2)} = \text{internal momentum for coupled channels,}$$

and $S_e^{(-)}$ is the shift factor for negative energy channels.

The coefficients $\alpha_{\lambda'\lambda}$ are given by the evaluation of the angular part of the matrix element $\langle \phi_{I'j'\ell'}^J | V | \phi_{Ij\ell}^J \rangle$,

They are related to the $\omega_{\lambda'\lambda}$, defined in eqs (17); as follows:

$$(24) \quad \alpha_{\lambda'\lambda} \equiv \alpha_{I'j'\ell'; Ij\ell} = \beta U_0 R_0^2 (2M/\hbar^2) \omega_{I'j'\ell'; Ij\ell}.$$

When β equals zero, eq. (20) reduces to eq. (4) as expected.

In the vibrational case all matrix elements between inelastic channels vanish (see eq. 17b), and the phase shift equation (20) assumes the very simple form

$$(25) \quad P_\ell(\rho_0) \cot(\delta_{j\ell} + \phi_\ell(\rho_0)) = x_1 - \beta^2 \sum_{\lambda=2}^N (\alpha_{1\lambda}')^2 / x_\lambda,$$

where

$$(26) \quad \alpha_{\lambda'\lambda}' = \alpha_{\lambda'\lambda} / \beta.$$

It is interesting to note that, as expected, the phase shift (25) depends on the square of the coupling constant β , while in the rotational

case (see eq. 20) an explicit dependence on the sign of β , (i. e. on the prolate or oblate form of the ellipsoidal target nucleus) is observed.

Let us extend now the calculations of §2, under the assumption that the 2^+ excited state of ^{12}C is a vibrational level.

The energy and width of a resonance, in the approximation $\phi_\ell \ll 1$, are immediately calculated from eq. (25):

$$(27a) \quad x_1(E^r) = \beta^2 \sum_{\lambda=2}^N (\alpha'_{1\lambda})^2 / x_\lambda(E^r),$$

$$(27b) \quad \Gamma = -2P_\ell(E^r) \left\{ \left[\frac{dx_1}{dE} \right]_{E=E^r} + \right. \\ \left. + \beta^2 \sum_{\lambda=2}^N (\alpha'_{1\lambda})^2 \left[\frac{dx_\lambda}{dE} \right]_{E=E^r} / x_\lambda^2(E^r) \right\}^{-1}.$$

For small deformations, a first approach to calculations can be given by investigating the behaviour of eqs (25) and (27) in the limit $\beta \rightarrow 0$. It can be easily recognized that, for β sufficiently small, the spectrum is composed by single-particle resonances at the energies satisfying eq. (8), whose widths are given by eq. (7b), and by "collective" resonances with $\Gamma \rightarrow 0$, whenever the equation

$$(28) \quad x_\lambda(E) = 0$$

is satisfied. In other words, as β approaches to zero, the "collective" resonances become infinitely narrow, and tend to definite energy positions, which depend on the parameters of the central potential. This very peculiar situation is a consequence of the "non perturbative" nature of the method. Since every inelastic channel ($I=2, j, \ell$) is coupled in general with more than one entrance channel ($I=0, j, \ell$), we have some degeneracies, which are listed in Table III. As β increases, the levels enlarge and the degeneracies are removed.

We start our calculations from the parameters of the spherical potential (Table II, 1), assuming a very small deformation. In this way the spectra of Table IV are obtained. It can be immediately recognized that the model is able to generate, at the energies considered:

- one single-particle $3/2^+$ level,
- one "collective" $3/2^+$ level,
- two "collective" $5/2^+$ levels,
- other "collective" levels with $j > 5/2$.

At the low energies considered by us, the levels with very high

TABLE III

Structure of the "collective" spectrum foreseen by the square well deformed potential in the limit $\beta \rightarrow 0$

$I, j, \ell^{(x)}$	$J^{\pi}(x)$
2, 1/2, 0	$3/2^+, 5/2^+$
2, 1/2, 1	$3/2^-, 5/2^-$
2, 3/2, 1	$1/2^-, 3/2^-, 5/2^-, 7/2^-$
2, 3/2, 2	$1/2^+, 3/2^+, 5/2^+, 7/2^+$
2, 5/2, 2	$1, 2^+, 3/2^+, 5/2^+, 7/2^+, 9/2^+$
2, 5/2, 3	$1/2^-, 3/2^-, 5/2^-, 7/2^-, 9/2^-$
2, 7/2, 3	$3/2^-, 5/2^-, 7/2^-, 9/2^-, 11/2^-$
2, 7/2, 4	$3/2^+, 5/2^+, 7/2^+, 9/2^+, 11/2^+$
2, 9/2, 4	$5/2^+, 7/2^+, 9/2^+, 11/2^+, 13/2^+$

(x) - The quantum numbers of the first column refer to the channel function $x_{Ij\ell}$. At the energies where $x_{Ij\ell} = 0$, resonances of quantum numbers J^{π} (listed in the second column) are observed.

TABLE IV

Levels calculated by the potential parameters of Table II, 1a and II, 1b in the limit $\beta \rightarrow 0$

$I, j, \ell^{(x)}$	$J^{\pi}(x)$	a) $r_0 = 1.25 f$		b) $r_0 = 1.50 f$	
		$E^{\Gamma}(\text{MeV})$	$\Gamma(\text{KeV})$	$E^{\Gamma}(\text{MeV})$	$\Gamma(\text{KeV})$
0, 3/2, 2	$3/2^+$	3.67	1200	3.67	1200
2, 1/2, 0	$3/2^+, 5/2^+$	5.40 ^(x)	0	4.16	0
2, 9/2, 4	$5/2^+, 7/2^+, 9/2^+, 11/2^+, 13/2^+$	-	-	4.32	0

(x) - See footnote on Table III

(x) - This value has been obtained by extrapolation

angular momentum ($l=4, 6$) can be disregarded. Then the problem is only to increase β and vary slightly the phenomenological parameters, in order to reach the best agreement with experimental energies and widths of the resonances. But, since two possible $5/2^+$ levels are foreseen by the model, while one only $5/2^+$ resonance has been till now experimentally found, the problem arises to identify the $5/2^+$ experimental resonance at $E=2.08$ MeV: (i) either with the level $5/2^+$ (2,1/2,0), (ii) or with the level $5/2^+$ (2,9/2,4) of Table IV(x).

Let us consider in detail these two cases:

- (i) As β increases, the degeneracy of the states $5/2^+$ (2,1/2,0) and $3/2^+$ (2,1/2,0) is removed, but the two levels cannot be separated in energy as much the experience indicates. The calculations show that, under the hypothesis (i) the three levels cannot be reproduced simultaneously at the observed energies;
- (ii) Under the second hypothesis the positions of the three levels can be reproduced exactly, if one assumes r_0 ranging between ~ 1.3 and ~ 1.4 f. Table II,2) gives the potential parameters which reproduce correctly the resonance energies, and Table I,2) shows ^{the} widths calculated from these parameters. The best agreement with experience is given by the value $r_0=1.4$ f. The phase shifts calculated by means of these parameters (Table II,2b) are shown in fig. 2. An obvious extension of the model is then the introduction of a deformed potential with a more realistic diffuse radial shape. In the next paragraph, the influence of the diffuseness on the "collective" levels will be widely investigated, with particular reference to the behaviour of the $5/2^+$ (2,1/2,0) and $5/2^+$ (2,9/2,4) resonances.

3.2. - Deformed potential with a diffuse radial shape.

The usual Woods-Saxon form

$$(29) \quad f(r) = \{1 + \exp((r-R_0)/a)\}^{-1},$$

is introduced into the radial part of the interaction potential (10), through eqs (1) and (11).

The coupled equations (15) are then integrated numerically, by means of the 7094 IBM Computer, and the phase shifts are obtained from eq. (18).

(x) - Now and in the following, when necessary, the levels will be indicated by the symbol $J^{\pi} (I, j, \ell)$, with explicit reference to the quantum numbers of the channel which generates that level in the limit $\beta \rightarrow 0$.

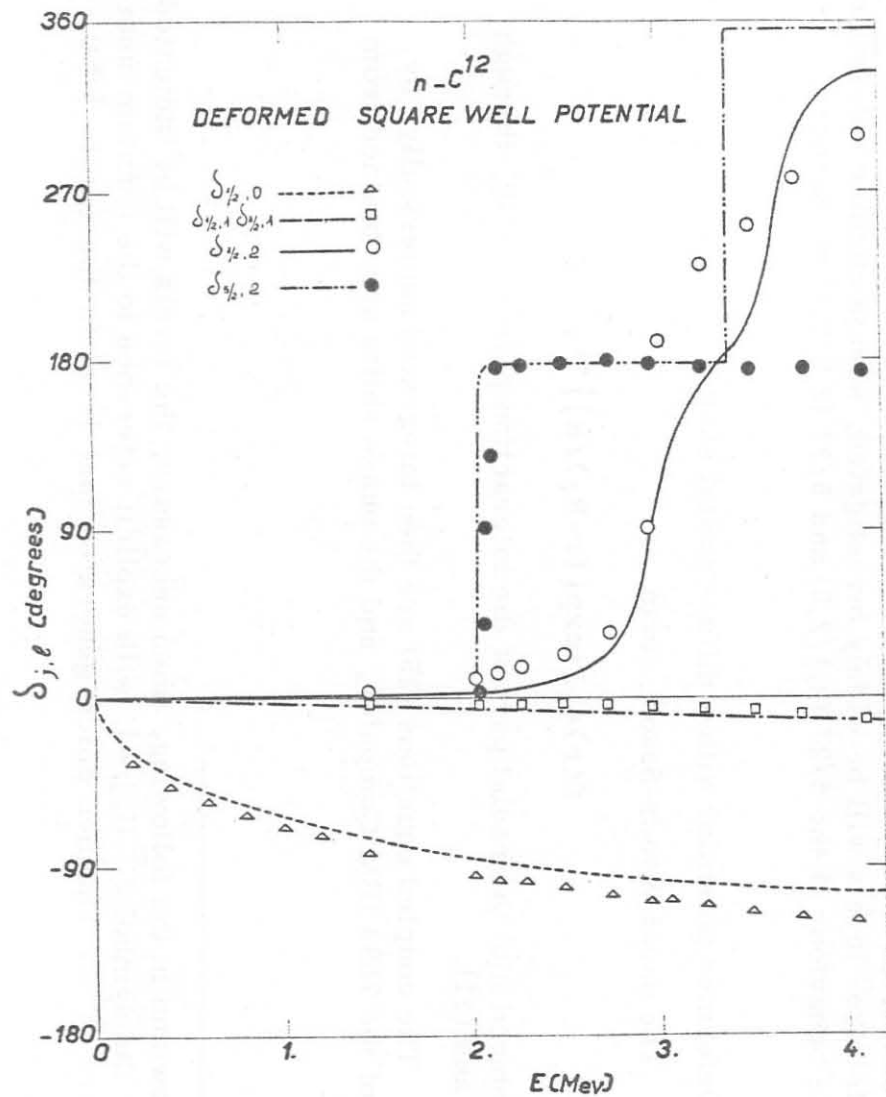


FIG. 2 - Phase shifts calculated by parameters of Table II, 2b), compared with experiments.

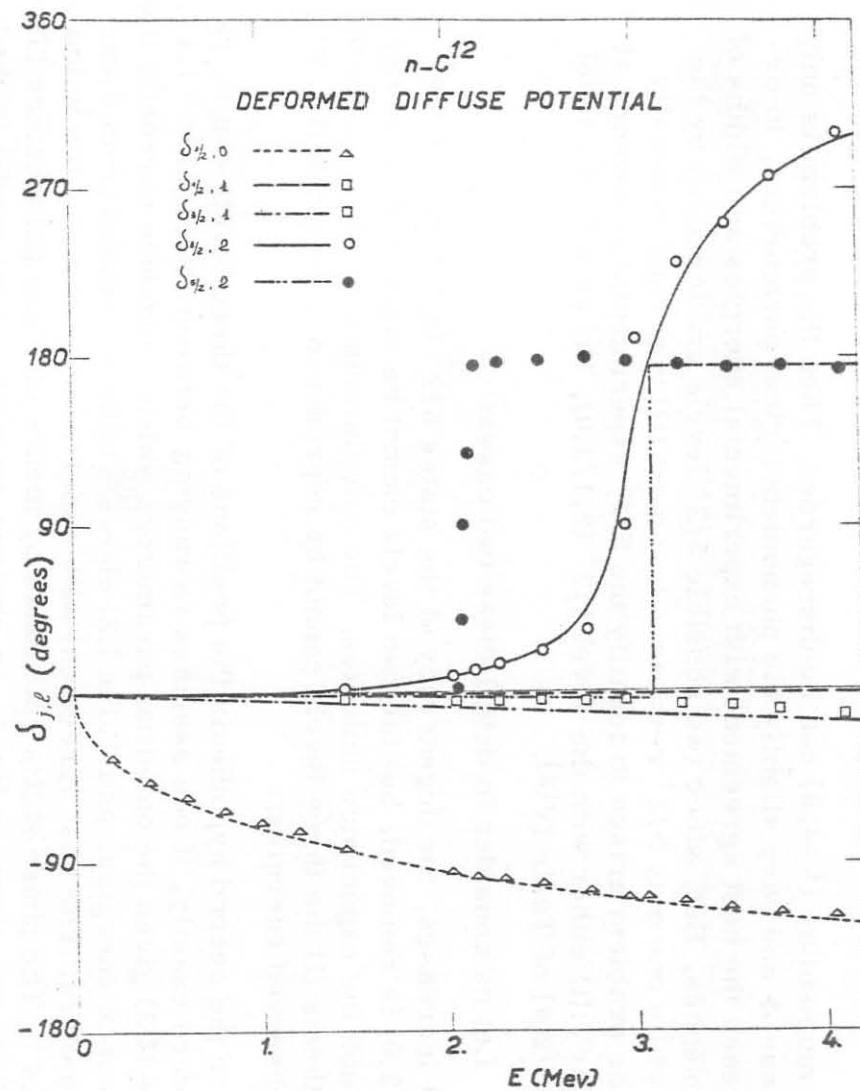


FIG. 3 - Phase shifts calculated by parameters of Table II, 3), compared with experiments.

Since the square well can be considered as a Woods-Saxon well in the limit $a \rightarrow 0$, we have calculated first the effect of a very small diffuseness on the levels, starting from the parameters of Table II, 2b. The results are shown in Table V. The levels $3/2^+$ (0,3/2,2), $3/2^+$ (2,1/2,0) and $5/2^+$ (2,1/2,0) remain practically unchanged, while the level $5/2^+$ (2,9/2,4) is dropped by the introduction of a very small diffuseness ($a=10^{-2}f$). This can be explained since this level, being generated by a coupled channel with $\ell=4$, is highly influenced by small surface effects^(x). Owing to the impossibility of improving the data of Table II, 2b) by the introduction of a diffuseness $a \neq 0$, we have undertaken an independent parameter analysis of the experimental data following the procedure here outlined:

Assuming $r_0 = 1.5 f$ and $a = 0.65 f^{(+)}$, the parameters U_0 , U_{S0} and β have

TABLE V

Effect of a small diffuseness a on the spectra calculated by the square well deformed potential (the parameters are those of Table II, 2b).

a(f)	$J^{\pi} (I, j, \ell)$							
	$5/2^+ (2, 9/2, 4)$		$3/2^+ (2, 1/2, 0)$		$5/2^+ (2, 1/2, 0)$		$3/2^+ (0, 3/2, 2)$	
	E^r (MeV)	Γ (KeV)	E^r (MeV)	Γ (KeV)	E^r (MeV)	Γ (KeV)	E^r (MeV)	Γ (KeV)
0	2.03	3	2.97	114	3.37	11	3.65	178
0.01	-	-	3	382	3.5	-(x)	3.67	407
0.015	-	-	3.05	423	3.6	-(x)	3.75	445
0.02	-	-	3.07	460	3.35	-(x)	3.80	319

(x) - These very small widths have not been evaluated because too long computation times were necessary

(x) - To obtain deeper information on the mechanism of the model, we have calculated the spectra obtained by the parameters of Table II, 2b) (square well deformed potential), assuming the 2^+ excited state of the target to be a rotational level. The results are that the levels $3/2^+$ (0,3/2,2), $3/2^+$ (2,1/2,0) and $5/2^+$ (2,1/2,0) remain practically unchanged in position and width (so that the rotational or vibrational nature of the target could not be decided on the ground of these results only), while the $5/2^+$ (2,9/2,4) level is shifted in energy by an amount of 1.5 MeV, toward the high ($\beta < 0$) or the low ($\beta > 0$) energies. This is a further indication of the high instability of that level.

(+) - This is the value assumed for high energy scattering calculations⁽⁶⁾.

been varied, until the double resonant $3/2^+$ phase shift has been well reproduced. The values $U_0 = 43$ MeV, $U_{SO} = 10$ MeV, $\beta = 0.08$ have been so obtained. Differences of 5% in U_0 , and of 20% in U_{SO} and β were sufficient to change completely the phase shifts behaviour.

Then, by slight variations of all parameters, with the exception of r_0 , fit of both the $1/2^+$ and $3/2^+$ phase shifts behaviour was obtained. The potential parameters are listed in Table II,3). The corresponding phase shifts and level parameters are reported in fig. 3 and Table I,3) respectively^(x).

Since the energy position of the $5/2^+$ level is not correct, we have analyzed the dependence of this level on the parameters r_0 , U_0 , U_{SO} , β and a , around the values of Table II,3). The conclusion was that in no way the energy position of this level can be reproduced. This means that even the perturbation introduced by the diffuseness is insufficient to remove completely the degeneracy of the levels $3/2^+$ (2,1/2,0) and $5/2^+$ (2,1/2,0), which was found in the limit $\beta \rightarrow 0$ and $a \rightarrow 0$.

We are able now to draw the final conclusions on the two $5/2^+$ "collective" levels:

The assumption (ii) seems to give the best agreement with experience in the square well approximation ($a \rightarrow 0$), but the $5/2^+$ (2,9/2,4) level is too sensitive to small variations of the potential parameters.

In particular it is completely cancelled out by the introduction of a very small diffuseness. Furthermore, under the assumption (ii) the second level $5/2^+$ (2,1/2,0) is still present between ~ 3 and ~ 3.5 MeV.

The assumption (i) gives rise to a much more stable resonance, although the level $5/2^+$ (2,1/2,0) could not be placed exactly at the experimental energy. This discrepancy could be ascribed to the limitations of the model, which considers the excitation of only one collective state in the ^{12}C nucleus.

As a final remark, it can be pointed out that the values found here for the Woods-Saxon potential parameters (r_0 , U_0 , U_{SO} , a) are in good agreement with values reported in literature, as obtained from analyses of bound single particle states⁽¹⁵⁾ and scattering reactions⁽⁶⁾.

4. SUMMARY AND CONCLUSION -

The n- ^{12}C elastic scattering process has been analyzed employing a single-particle model and a channel coupling model. The spherical poten-

(x) - The uncertainties in the parameters of Table II,3) are: $\Delta U_0/U_0 \sim 0.01$;
 $\Delta U_{SO}/U_{SO} \sim \Delta a/a \sim 0.05$; $\Delta \beta/\beta \sim 0.1$.

tial is able to reproduce the behaviour of the non-resonant phase shifts, and the broad single-particle resonances. The introduction of a deformed potential gives rise to new possibilities of excitation of the $n\text{-}^{12}\text{C}$ system, because the internal degrees of freedom of the target nucleus are taken into account. The complete experimental spectrum is so reproduced, and a rather satisfactory agreement with the experimental phase shifts is obtained, in spite of the simplifying assumption introduced.

Investigations on the behaviour of the model when the deformation β approaches to zero give useful information about the mechanism of generation of the "collective" resonances.

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