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P. G. Bizzeti and P. R. Maurenzig: ANALYSIS OF CROSS SECTION FLUCTUATIONS IN ISOSPIN FORBIDDEN REACTIONS. -

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P. G. Bizzeti[+] and P. R. Maurenzig: ANALYSIS OF CROSS SECTION FLUCTUATIONS IN ISOSPIN FORBIDDEN REACTIONS[x]. -

1. INTRODUCTION. -

The statistical model of nuclear reactions has been proved to be very useful to interpret average values of the reaction cross section, as well as their fluctuating behaviour as a function of the energy[1, 2]. In the last few years, cross section fluctuations have been extensively investigated, both experimentally[o] and theoretically[2-9].

The aim of this paper is to extend the theory of cross section fluctuations to the "partially forbidden" reactions; a typical example of this kind of reactions, though possibly not the only one, is supplied by transitions which do not conserve isospin. In a previous note[10] it was shown that an anomalous behaviour of cross section fluctuations may be expected in this case, with respect to the normal Ericson's theory[2]. In the following section 2 we give now a more complete report, while in section 3 the calculation is extended to take into account the influence of possible "coherent" effects in the reaction mechanism. Finally in section 4 some possible experiments are proposed and discussed.

Before going into details, a preliminary discussion of the mechanism of forbidden reactions is perhaps worthwhile. The simple, time dependent procedure by Morinaga[11] and Wilkinson[12] may be used for this purpose.

Suppose the initial channel has pure isospin T_1 and a compound system is formed that may (in principle) decay into a channel C_2 of pure isospin $T_2 \neq T_1$. If the interaction Hamiltonian were strictly charge independent, the reaction $C_1 \rightarrow C_2$ would be of course forbidden.

Obviously, the Hamiltonian contains a charge dependent term H^c owing to Coulomb forces. This in turn will induce transitions between states of

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(o) - For a revue of the argument, see ref. (8).

different isospin, during the time of interaction. If H^c is small, this transition rate can be evaluated by first order perturbation.

With some symplifying assumptions (see Appendix 1), we obtain for the transition probability during time δt

$$(1.1) \qquad P(T_1, T_2)\, \delta t = \frac{2\pi \langle |(T_1|H^c|T_2)|^2 \rangle \delta t}{\hbar D_2}$$

where D_2 is the mean spacing of compound levels of isospin T_2 (and given J^{π}) and the angular brackets indicate the average value of the squared element of H^c between two states of different isospin(x).

The interaction time is $\hbar/\Gamma_1$, Γ_1 being the mean level width (or "coherence width") for isospin T_1 states. The average cross section for a forbidden reaction is therefore reduced by approximately a factor of

$$|\alpha|^2 = \frac{\hbar P(T_1, T_2)}{\Gamma_1} \approx \frac{2\pi \langle |(T_1|H^c|T_2)|^2 \rangle}{D_2} \frac{1}{\Gamma_1}$$

with respect to the prediction of a purely statistical theory. Clearly, the factor $|\alpha|^2$ may depend on quantum numbers J and π, so that the angular distribution may be different from that given by Hauser and Feshbach[13]. The energy dependence of $|\alpha|^2$ has been discussed by Wilkinson[12].

Since $P(T_1, T_2)$ is finite, the isotopic spin cannot change abruptly: the "forbidden" part of the state vector needs some time to grow up. As a consequence, the time behaviour of the forbidden transition cannot be the same as for allowed ones.

The outgoing flux u(t) following an instantaneous excitation of the compound system by channel C_1 is shown schematically in fig. 1, for both allo-

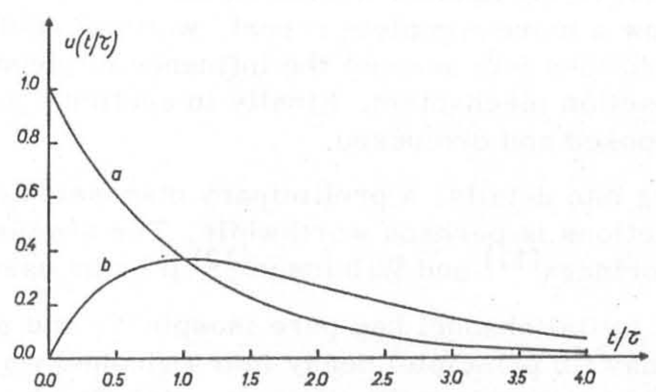

FIG. 1 - Time response fundtion u(t) for allowed channels (curve a) and for a forbidden channel (curve b, for the case $\Gamma_1 = \Gamma_2$, unnormalized).

(x) - Here and in the following, the angular brackets $\langle \ \rangle$ are reserved to average values(i.e. "expectation values" of statistical quantities), and round brackets will be used for state vectors $|\ )$ and matrix elements $(\ |\ |\ )$.

wed and forbidden channels. The decay is (approximately) exponential in the allowed channel. It is the combination of two exponential functions, with $U(0) = 0$, for the forbidden one.

Now, it has been shown by Brink[5] that the time response $u(t)$ is just the Fourier transform of the function $C(\varepsilon)$, occurring in the theory of cross section fluctuations. This function[3] is defined by

$$\langle f^{*}(E)\, f(E+\varepsilon) \rangle = \langle \sigma \rangle\, C(\varepsilon)\,.$$

$f(E)$ being the transition amplitude at energy E and $\langle \sigma \rangle$ the average cross section; and the square modulus of $C(\varepsilon)$ is proportional to the cross section correlation function

$$\langle \left[\sigma(E) - \langle \sigma \rangle \right] \left[\sigma(E+\varepsilon) - \langle \sigma \rangle \right] \rangle$$

For allowed (statistical) reaction $u(t) = \exp(-t\,\Gamma/\hbar)$ and $C(\varepsilon) = (1 - i\,\varepsilon/\Gamma)^{-1}$. In the case of forbidden transitions, $u(t)$ is quite different; therefore a different $C(\varepsilon)$ is expected. The width of $u(t)$ being larger in this case with respect to the allowed one (fig. 1), we may expect a smaller width of $|C(\varepsilon)|^2$. More detailed predictions could be made. However, in view of their indirect character we shall not insist on this approach further.

2. DETAILED CALCULATION WITH THE R-MATRIX THEORY. -

2. 1. - Formal introduction of isobaric spin states. -

A good revue of the R-matrix formalism is given in a classic paper by Lane and Thomas[14]. We shall adopt the same notation used in that paper whenever possible, and refer to it for any undefined symbol. We will need, however, some more symbols, which we now define:

a) Total Hamiltonian. - We put $H = H^{o} + H^{c}$, where H^{o} is the charge independent part, and H^{c} is a (small) perturbation term.

b) Eigenstates for the "internal" region. - With proper boundary conditions, the complete orthonormal set of eigenstates $|X_n)$ of H may be defined as in Lane and Thomas:

$$H|X_n) = E_n|X_n)\,.$$

In addition, we define the complete orthogonal set of eigenstates $|X^{o}_{nT'})$ of H^{o} and T^2, with the same boundary conditions:

$$H^{o}|X^{o}_{nT'}) = E^{o}_{n}|X^{o}_{nT'}) \quad ; \qquad T^2|X^{o}_{nT'}) = T'(T'+1)|X^{o}_{nT'})\,.$$

c) Channel region. - Channel wave functions are defined as in Lane and Thomas paper[14]. Throughout this paper, the symbols c, c', ..., are intended to specify the channel state completely (i.e. $c = \alpha J l s \ldots$).

d) Surface region. - Surface functions φ_c are defined as in Lane and Thomas, so that reduced width amplitudes (r. w. a.) are

(2.1) $$\gamma_{nc} = \int_S \varphi_c^* X_n \, dS \; .$$

We also define the amplitudes

(2.2) $$\gamma_{nc}^o = \int_S \varphi_c^* X_{nT'}^o \, dS \; .$$

Clearly, γ_{nc}^o is zero if the component of channel state c with isospin T' vanishes. The width amplitudes γ_{nc} are real[14], when internal and channel wave functions behave in the same way under time reversal. The same argument also applies to γ_{nc}^o. In addition, matrix elements $(X_n^o | H^c | X_m^o)$ are real and symmetric in this case[x].

2.2. - Eigenvalue expansion of the scattering matrix. -

The collision matrix U is given in terms of R-matrix by the relations

(2.3) $$U = \Omega W \Omega$$
$$W_{cc'} = \delta_{cc'} + 2iP_c^{1/2}\left[(1 - L^oR)^{-1} R\right]_{cc'} P_{c'}^{1/2}$$

(for open channels only).

Here Ω represents the hard sphere scattering matrix, and $P = \mathrm{Im}L^o$ is the penetrability matrix. The matrices Ω and $L^o = S+iP$ are diagonal in the channel representation, and their elements are slowly varying functions of the energy. From now on, we assume that their change throughout the energy region of interest be small, and treat the matrix elements of Ω and L^o as strictly constant in our energy range. At high excitation energy this approximation appears to be usually in order, in the spirit of the statistical approach.

Now the energy dependent factor $(1 - L^oR)^{-1}R$ in W may be expanded in terms of r.w.a., namely

(2.4) $$\left[(1 - L^oR)^{-1} R\right]_{cc'} = \sum_{nm} \gamma_{nc} A_{nm} \gamma_{mc'} = \tilde{\gamma}_c A \gamma_{c'} \; .$$

Here γ_c is a one column matrix in the level space, and

$$A = \left[e - \xi - E\right]^{-1}$$

(x) - In fact, H^c is invariant under space rotation and reflections, so that $(X_n^o | H^c | X_m^o) \neq 0$ only if $J_n = J_m$, $M_n = M_m$ and $\pi_n = \pi_m$. In addition H^c is also invariant under the time reversal τ, namely

$$(X_{nJM}^o | H^c | X_{mJM}^o) = (X_{nJM}^o | \tau H^c \tau | X_{mJM}^o) =$$
$$= (-1)^{2J-2M} (X_{mJ-M}^o | H^c | X_{nJ-M}^o) = (X_{mJM}^o | H^c | X_{nJM}^o)$$

We have so proved that $H_{mn}^c = H_{nm}^c$ and therefore, H^c being Hermitian, it is also real in this representation.

where e is the (diagonal) Hamiltonian matrix in the representation of basis $|X_m\rangle$, E is the actual energy(of the system)times the unit matrix, and $\xi_{nm} = \sum_c \gamma_{nc} L_c^o \gamma_{mc}$. The invariant matrix product $\tilde{\gamma}_c A \gamma_{c'}$ can be evaluated in the more convenient representation with basis $|X^o_{nT}\rangle$, in which H^o reduces to its diagonal form e^o. We get

$$\begin{aligned} &\left[(1 - L^oR)^{-1}R\right]_{cc'} = \tilde{\gamma}^o_c A^o \gamma^o_{c'} \\ (2.5)\qquad &A^o = \left[e^o + H^c - \xi^o - E\right]^{-1} = \left[\mathcal{H} - E\right]^{-1} \end{aligned}$$

with $\mathcal{H} = e^o + H^c - \xi^o \; ; \quad \xi^o_{nm} = \sum_c \gamma^o_{nc} L^o_c \gamma^o_{nc}$

We may find a complex orthogonal transformation operating in the sub-space with isospin T, which diagonalizes the submatrix $(T|\mathcal{H}|T)$ of $\mathcal{H}$ connecting states of equal isospin T

$$a^{(T)}(T|\mathcal{H}|T)\,\tilde{a}^{(T)} = \mathcal{E}^{(T)} \qquad \text{(diagonal)}$$

$$\tilde{a}^{(T)}a^{(T)} = a^{(T)}\tilde{a}^{(T)} = 1$$

and define the transformation a as the direct product of transformation $a^{(T)}$ for all possible values of T.

When the transformation a is applied to every factor of the invariant matrix product $\tilde{\gamma}^o_c A^o \gamma^o_{c'}$, eq. (2.5) becomes

$$(2.6)\qquad \left[(1 - L^oR)^{-1}R\right]_{cc'} = \sum_{nm} \gamma'_{nc} A'_{nm} \gamma'_{mc}$$

where

$$\gamma' = a\gamma^o$$

$$\mathcal{H}' = a\mathcal{H}\tilde{a}$$

and A' is the inverse matrix of $\mathcal{H}'$-E.

Non-diagonal elements of $\mathcal{H}'$ different from zero are now confined in the submatrices $(T|\mathcal{H}'|T')$ $(T \neq T')$ and are assumed to be small if the isobaric spin has to be approximately conserved. As a consequence, A' may be expanded around its diagonal part, giving

$$(2.7)\qquad \left[(1 - L^oR)^{-1}R\right]_{cc'} = \sum_n \frac{\gamma'_{nc}\gamma'_{nc'}}{\mathcal{E}_n - E} - \sum_{n\neq m} \frac{\gamma'_{nc}\mathcal{H}'_{nm}\gamma'_{mc'}}{(\mathcal{E}_n - E)(\mathcal{E}_m - E)} + \cdots$$

If the second and higher order terms may be neglected with respect to the first one, eq. (2.7) is equivalent to the complex eigenvalues expansion by Kapur and Peierls. Isospin selection rule also follows, since the product $\gamma_{nc}\gamma_{nc'}$, is zero when channels c and c' have definite and different isospin. "Forbidden" reactions may actually arise, though usually with a reduced rate, owing to the second terms in the eq. (2.7).

2.3. - Consequences for statistical reactions. -

Till now, no statistical assumptions have been introduced. We specialize the calculation to the extreme case of many channels and overlapping levels, namely we assume:

1) Levels with a definite spin, parity and isospin are nearly equally spaced, with mean energy distance $D_T^{J\pi}$.
2) Their widths are approximately constant, with mean value $\Gamma_T^{J\pi}$.
3) Mean values of both D and Γ are energy independent throughout the range of interest, which is assumed to be large compared to Γ.
4) For all values of J, π, T of practical interest, $D \ll \Gamma$.

The above assumptions are very reasonable at high excitation energy, at least if "intermediate structure" effects do not play an important role in the mechanism of the reaction. Some more detailed hypoteses have now to be introduced on the statistical distribution of amplitudes γ'_{nc} and matrix elements $\mathcal{H}'_{nm}$. The simplest correspond to the extreme random phase assumption, namely:

5) Quantities γ'_{nc}, $\mathcal{H}'_{nm}$ may be considered as random variable and with zero mean;
6) They are not correlated one to another[(x)].

Obviously the last point is the most questionable among the six. We will drop it out in the next section, and substitute it with a less restrictive condition.

Let us accept, for the moment, the absolute validity of the six points above. The eq. (2.7) gives now a very useful, strongly convergent expansion for the collision matrix, since the ratio of the root mean square value of the second term to that of the first is of the order of $\sqrt{2\pi D/\Gamma}$ or less, and higher order terms decrease as the corresponding power of this expression. Only the first non vanishing term has therefore to be taken into account. In the case of the allowed reactions we obtain:

$$W_{cc'} = 2i \sum_n \frac{P_c^{1/2}\, \gamma'_{nc}\, P_{c'}^{1/2}\, \gamma'_{nc'}}{\mathcal{E}_n - E} \qquad c \neq c'$$

which is the usual starting point for Ericson's theory of fluctuations, without direct interactions. For instance, direct effects are automatically excluded as long as correlation between γ'_{nc}, $\gamma'_{nc'}$ are neglected (see also part 3). We quote here just the result of the statistical theory for the particular case of single initial and single final channel[(2, 3)], spinless particles

(x) - Apart from the obvious correlation among channels belonging to the same isospin multiplet (Only minor changes in the formalism are sufficient to take this effect into account).

and s waves

$$\langle f_c^{x}(E)\, f_c(E+\mathcal{E})\rangle = \langle \sigma^c\rangle \frac{1}{1 - i\frac{\mathcal{E}}{\Gamma}}$$

(2.8)

$$\langle \sigma\rangle = \langle \sigma^c\rangle = \pi \lambda\!\!\!^{-2} \frac{2\pi}{D} \frac{(2P_c\langle|\gamma'_{nc}|^2\rangle)(2P_{c'}\langle|\gamma'_{nc'}|^2\rangle)}{\Gamma}$$

(see ref.(2) for the angular momentum effects).

In the case of forbidden transitions, the first term of eq.(2.7) is zero and we have

(2.9)
$$W_{cc'} = -2i \sum_n^{(T)} \sum_m^{(T')} \frac{P_c^{1/2}\, \gamma'_{nc}\, \mathcal{H}'_{nm}\, \gamma'_{mc'}\, P_{c'}^{1/2}}{(\mathcal{E}_n - E)(\mathcal{E}_m - E)} \qquad c \neq c'$$

From eq.(2.9) we get, under the same assumptions (see Appendix 2)

$$\langle f(E)\, f(E+\mathcal{E})\rangle = 0$$

(2.10)
$$\langle f^{x}(E)\, f(E+\mathcal{E})\rangle = \langle \sigma^c\rangle \frac{1}{(1-\frac{i\mathcal{E}}{\Gamma_T})(1-\frac{i\mathcal{E}}{\Gamma_{T'}})}$$

$$\langle \sigma\rangle = \langle \sigma^c\rangle = \pi \lambda\!\!\!^{-2} \frac{(2\pi)^2}{D_T D_{T'}} \frac{(2\langle|\gamma'_{nc}|^2\rangle P_c)\langle|\mathcal{H}'_{nm}|^2\rangle(2\langle|\gamma'_{mc'}|^2\rangle P_{c'})}{\Gamma_T\, \Gamma_{T'}}$$

If the random variables γ'_{nc}, $\mathcal{H}'_{nm}$ are not correlated, values of the transition amplitudes for different energy are distributed according to an approximately normal distribution, whose correlation functions are given by eq.(2.10). As a consequence, the expectation value for cross-section correlation function

$$F(\mathcal{E}) = \langle \sigma(E)\, \sigma(E+\mathcal{E})\rangle - \langle\sigma\rangle^2$$

for single initial and final channel reaction(x), is given by

(2.11)
$$F(\mathcal{E}) = \langle\sigma\rangle \frac{1}{\left[1+(\frac{\mathcal{E}}{\Gamma_T})^2\right]\left[1+(\frac{\mathcal{E}}{\Gamma_{T'}})^2\right]}$$

Alternatively eq.(2.11) can be derived directly from eq.(2.9) without explicit use of eq.(2.10) and the result can be taken as a proof of the gaussian nature of the statistical distribution for f(E), at least to its fourth moment.

2.4. - The width at half maximum of the correlation curve. -

In particular the expected half width Γ_{ex} of $F(\mathcal{E})$ can be obtained from eq.(2.11). It comes out smaller than the smallest of the two widths Γ_T and

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(x) - The usual damping factor 1/N must be introduced when more than one channel contribute to the reaction(2).

$\Gamma_{T'}$, approaching this value when they are very different in magnitude. For the case $\Gamma_T = \Gamma_{T'}$ the expected value of Γ'_{ex} becomes $\Gamma'_{ex} = 0.64\ \Gamma_T$. The ratios Γ_{ex}/Γ_T and $\Gamma'_{ex}/(1/2)(\Gamma_T + \Gamma_{T'})$, evaluated from eq. (2.11) for pure statistical reactions, are shown in fig. 2 as a function of $\Gamma_{T'}/\Gamma_T$.

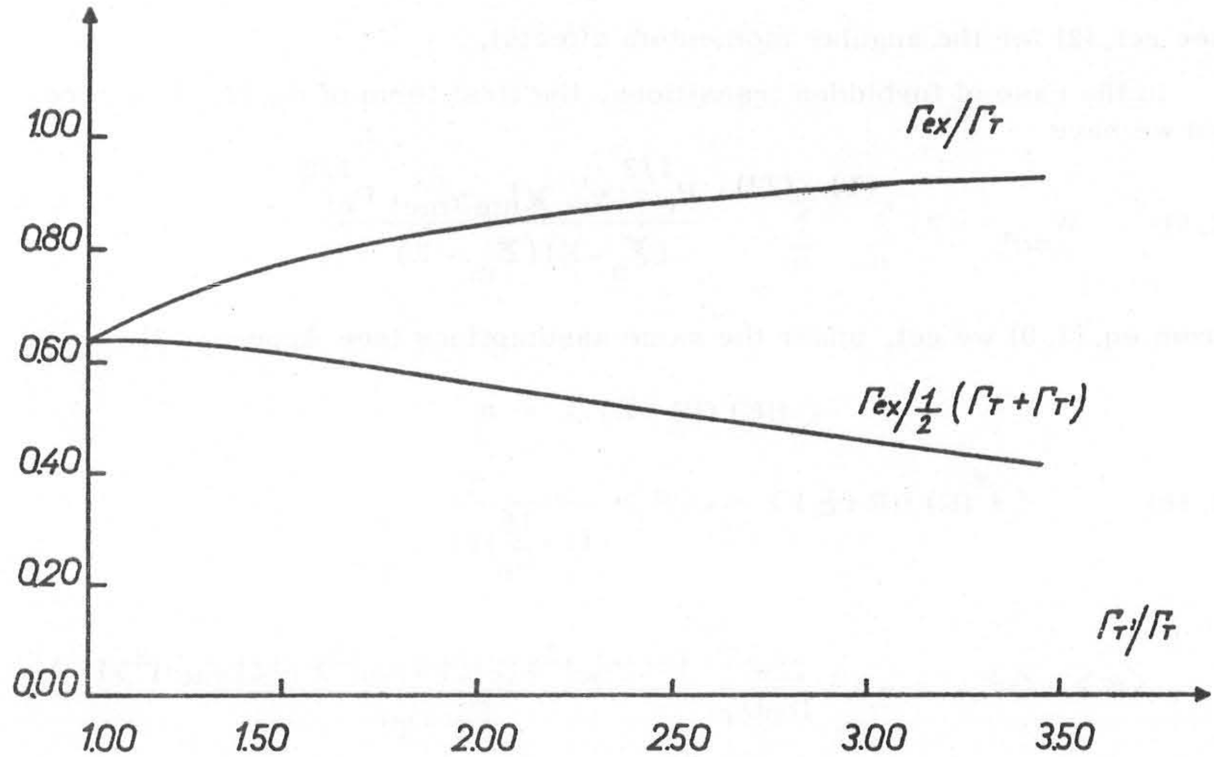

FIG. 2 - Ratios Γ_{ex}/Γ_T and $\Gamma_{ex}/\frac{1}{2}(\Gamma_T + \Gamma_{T'})$ evaluated from eq. (2.11) for pure statistical reactions, as a function of $\Gamma_{T'}/\Gamma_T$.

At least for $\Gamma_{T'}/\Gamma_T \sim 2$, the reduction in the w.h.m. appears to be large enough to be detected by a proper experiment(x).

3. INCLUSION OF DIRECT EFFECTS. -

3.1. - Possible failure of the random phase assumption. -

Let us discuss, in some more detail, assumption n° 6 in section 2.3, namely, that γ'_{nc} are uncorrelated random variables with zero mean. As we shall see the first implication of this statement is that direct effects are excluded in this way, not only for the forbidden but also for the allowed reactions. In the R - matrix formalism, in fact, direct reactions are (somewhat artificially) accounted for as the result of coherent contributions from many distant states(+).

To take direct effects into account, we must therefore assume long range correlation among the r.w.a., i.e. $\langle \gamma'_{nc}\gamma'_{nc'} \rangle \neq 0$, in contrast with previous

(x) - The statistical relative error[15] on the experimental value of Γ is of the order of $\sqrt{(\pi\,\Gamma/\Delta E)}$, so that an energy interval $\Delta E >> \Gamma$ is needed to obtain a sufficient statistical accuracy.

(+) - See section XI, 6 of ref. 14.

assumption 6. Clearly, this may happen only for allowed reactions, since $\gamma'_{nc}\gamma'_{nc'} = 0$ for the forbidden ones. But if we drop the random phase assumption for the width amplitudes γ'_c, it is hard to maintain it for the elements of the matrix $\mathcal{H}'$. We are instead forced to admit that non diagonal elements of $\mathcal{H}'$ are possibly correlated(x) to the r. w. a., and to substitute the condition 6 by a more reliable one.

Quite reasonably, the random variables γ'_{nc}, $\mathcal{H}'_{nm}$ may still be assumed to have zero mean. Moreover, they are actually correlated, but probably not too much. We shall assume that the correlation coefficients of two random variables is small, of the order of $\sqrt{D/\Gamma}$ compared to the product of the root mean square values of the two variables. That is

$$(3.1)\qquad \frac{|\langle \gamma'_{nc}\gamma'_{nc'}\rangle|^2}{\langle|\gamma'_{nc}|^2\rangle\langle|\gamma'_{nc'}|^2\rangle} \lesssim \frac{D}{\Gamma}$$

$$(3.2)\qquad \frac{|\langle \gamma'_{nc}\mathcal{H}'_{nm}\rangle|^2}{\langle|\gamma'_{nc}|^2\rangle\langle|\mathcal{H}'_{nm}|^2\rangle} \lesssim \frac{D}{\Gamma}$$

$$(3.3)\qquad \frac{|\langle \gamma'_{nc}\mathcal{H}'_{nm}\gamma'_{mc'}\rangle|^2}{\langle|\gamma'_{nc}|^2\rangle\langle|\mathcal{H}'_{nm}|^2\rangle\langle|\gamma'_{mc'}|^2\rangle} \lesssim \frac{D_1}{\Gamma_1}\frac{D_2}{\Gamma_2}$$

and so on. Under this condition, the series expansion (2.7) of the scattering matrix can still be used. We shall see (in the following section 3.2) that condition (3.1) is compatible with a direct cross section having the same order of magnitude of the statistical one. It is therefore reasonable to expect that assumptions as (3.2) and (3.3) also hold, when the statistical process accounts for a substantial part of the reaction.

3.2. - Average cross section. -

We have now to evaluate the average cross section, starting from eq. (2.7) of part 2, namely

$$\left[(1-L^oR)^{-1}R\right]_{cc'} = \sum_n \frac{\gamma'_{nc}\gamma'_{nc'}}{\mathcal{E}_n-E} - \sum_{n\neq m}\frac{\gamma'_{nc}\mathcal{H}'_{nm}\gamma'_{mc'}}{(\mathcal{E}_m-E)(\mathcal{E}_n-E)} + \dots$$

(x) - In fact, from (2.5) and following equation we obtain

$$\mathcal{H}' = a(e^o+H^c)\tilde{a} - \xi' \qquad \text{with} \qquad \xi'_{mn} = \sum_{c''}\gamma'_{mc''}L^o_{c''}\gamma'_{nc''}\,.$$

Since state $|m)$ and $|n)$ have different isospin T and T', only channels c'' having both, isospin components T and T' can contribute to ξ'_{mn}. Therefore $\langle\xi'_{mn}\gamma'_{mc}\rangle = 0$, as long as the r. w. a. are assumed uncorrelated, but it is no longer so if the channel c is coupled by a direct reaction to some channel c'' having no pure isospin. See also ref. (16) for possible correlation between r. w. a. and the Coulomb term H^C.

For instance, the average square modulus of the first term is

$$\left\langle \left| \sum_n \frac{\gamma'_{nc}\gamma'_{nc'}}{\mathcal{E}_n - E} \right|^2 \right\rangle = \left\langle \sum_n \frac{\gamma'_{nc}\gamma'_{nc'}}{\mathcal{E}_n - E} \sum_{m \neq n} \frac{\gamma'_{mc}\gamma'_{mc'}}{\mathcal{E}_m - E} \right\rangle +$$

(3.4)

$$+ \left\langle \sum_n \left| \frac{\gamma'_{nc}\gamma'_{nc'}}{\mathcal{E}_n - E} \right|^2 \right\rangle \approx \left| \left\langle \sum_n \frac{\gamma'_{nc}\gamma'_{nc'}}{\mathcal{E}_n - E} \right\rangle \right|^2 + \left\langle \sum_n \left| \frac{\gamma'_{nc}\gamma'_{nc'}}{\mathcal{E}_n - E} \right|^2 \right\rangle$$

but for terms of the order of D/Γ (see ref. 2).

The second term at the right hand side of eq. (3.4) was already found in section 2.3 and corresponds therefore to the statistical process. The resulting average cross section is (neglecting angular momentum effects)

$$(3.5) \qquad \langle \sigma^c \rangle = \pi \lambda\!\!\!^{-2} \frac{2\pi}{D} \frac{(2\langle |\gamma'_{nc}|^2 \rangle P_c)(2\langle |\gamma'_{nc'}|^2 \rangle P_{c'})}{\Gamma}$$

The other term, i.e. the square modulus of the expectation value, is interpreted as a "direct" contribution. The corresponding cross section may be estimated, as for the order of magnitude only(x), to about

$$(3.6) \qquad \sigma^d = \pi \lambda\!\!\!^{-2} \, 4P_c P_{c'} \left| \frac{\pi \langle \gamma'_{nc}\gamma'_{nc'} \rangle}{D} \right|^2$$

In this case, the ratio of the direct to the statistical cross section is

$$\frac{\sigma^d}{\sigma^c} \approx \frac{\pi \Gamma}{2D} \frac{|\langle \gamma_{nc}\gamma_{nc'} \rangle|^2}{\langle |\gamma_{nc}|^2 \rangle \langle |\gamma_{nc'}|^2 \rangle}$$

i.e. of the order of one if the condition (3.1) holds. Then this condition is compatible with a direct cross section as large as the statistical one.

The square modulus of the second term in the series espansion (2.7) comes out small (of the order of $\langle |\mathcal{H}'_{nm}|^2 \rangle / D_2 \Gamma_2$ compared to the first

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(x) - In fact,

$$\left\langle \sum_n \frac{\gamma_{nc}\gamma_{nc'}}{\mathcal{E}_n - E} \right\rangle \approx \langle \gamma_{nc}\gamma_{nc'} \rangle \int \frac{dE'/D}{E' - (1/2)i\Gamma} \approx \frac{\pi}{D} \langle \gamma_{nc}\gamma_{nc'} \rangle$$

This estimation however is not very satisfactory, since an important contribution to the integral comes from rather distant states, and the correlation coefficient $\langle \gamma_{nc}\gamma_{nc'} \rangle$ can in principle depend on the energy E_n.

one(x)), and can therefore be neglected unless the first term is zero.

When this happens, that is for a forbidden reaction (c $\longrightarrow$ c') the cross section is

$$(3.7) \qquad \sigma = \pi \lambda\!\!\!^{-2}\, 4P_c P_{c'} \left| \sum_n^{(T)} \sum_m^{(T')} \frac{\gamma'_{nc}\, \mathcal{H}_{nm}\, \gamma'_{mc'}}{(\mathcal{E}_n - E)(\mathcal{E}_m - E)} \right|^2$$

and for its average value we obtain (as for eq. (3.4))

$$\langle \sigma \rangle = \pi \lambda\!\!\!^{-2}\, 4P_c P_{c'} \Bigg\{ \left| \left\langle \sum_n^{(T)} \sum_m^{(T')} \frac{\gamma'_{nc}\, \mathcal{H}'_{nm}\, \gamma'_{mc'}}{(\mathcal{E}_n - E)(\mathcal{E}_m - E)} \right\rangle \right|^2 +$$

$$+ \sum_n^{(T)} \frac{\langle |\gamma'_{nc}|^2 \rangle}{|\mathcal{E}_n - E|^2} \left| \sum_m^{(T')} \frac{\langle \mathcal{H}'_{nm}\, \gamma'_{mc'} \rangle}{\mathcal{E}_m - E} \right|^2 + \left| \sum_n^{(T)} \frac{\langle \gamma'_{nc}\, \mathcal{H}'_{nm} \rangle}{\mathcal{E}_n - E} \right|^2 \cdot$$

$$(3.8) \qquad \cdot \sum_m^{(T')} \frac{\langle |\gamma'_{mc'}|^2 \rangle}{|\mathcal{E}_m - E|^2} + \sum_n^{(T)} \sum_m^{(T')} \frac{\langle |\gamma'_{nc}|^2 \rangle \langle |\mathcal{H}'_{nm}|^2 \rangle \langle |\gamma'_{mc'}|^2 \rangle}{|\mathcal{E}_m - E|^2\, |\mathcal{E}_n - E|^2} =$$

$$= \sigma^{dd} + \langle \sigma^{cd} \rangle + \langle \sigma^{dc} \rangle + \langle \sigma^{cc} \rangle .$$

If the conditions (3.1) ÷ (3.3) hold, the four terms in eq. (3.8) can reach the same order of magnitude, smaller by a factor $(|\mathcal{H}'_{mn}|^2)/(D_2 \Gamma_2)$ with respect to $\langle \sigma \rangle$ of the allowed reactions. We can try to interpret the four terms in eq. (3.7) as resulting from four different kind of reaction process.

The term σ^{dd} is the modulus square of the average transition amplitude. We can consider it as a contribution from direct process, just as we did for the similar term in eq. (3.4).

The structure of σ^{dc} and of its symmetric term σ^{cd} is not so simple. For instance, σ^{dc} contains a statistical "sum over resonances" of isospin T', but the coupling to c is coherent, through the non vanishing average of $\gamma'_{nc} \mathcal{H}'_{nm}$. In this sense, the mechanism of isospin violation acts like an isospin impurity in the channel c, and will combine coherently with it, if this impurity actually exists.

Finally, the term σ^{cc}, already obtained in section 2, is clearly the result of a purely statistical process.

(x) - Incidentally we observe that the second term in eq. (2.7) vanishes when channel c and c' have pure and equal isospin, $T_c = T_{c'}$. Interference between first and second term can also appear, but the contribution to the cross section is small (as $\sqrt{\frac{\langle |\mathcal{H}'_{nm}|^2 \rangle}{\Gamma D}}$).

3.3. - Cross section fluctuations. -

Now the average correlation function $F(\varepsilon)$ of the cross section can be calculated, with the same procedure used for the average value of the cross section. The cross section correlation function for allowed reactions, in the presence of direct effects, is given (e.g.) in ref. (2). For single initial and single final channel, we have

$$F(\varepsilon) = \langle\sigma(E)\sigma(E+\varepsilon)\rangle - \langle\sigma\rangle^2 = \langle\sigma^c\rangle(2\sigma^d + \langle\sigma^c\rangle)\frac{1}{1+(\frac{\varepsilon}{\Gamma})^2}$$

In the case of forbidden $c \rightarrow c'$ reaction, we obtain from eq. (3.7)

$$\langle\sigma(E)\sigma(E+\varepsilon)\rangle = (\pi\lambda\!\!\!^{-2}\, 4\, P_c P_{c'})^2 \cdot$$

$$\cdot\left\langle\left|\sum_n^{(T)}\sum_m^{(T')}\frac{\gamma'_{nc}\mathcal{H}'_{nm}\gamma'_{mc'}}{(\mathcal{E}_n-E)(\mathcal{E}_m-E)}\right|^2\left|\sum_l^{(T)}\sum_k^{(T')}\frac{\gamma'_{nc}\mathcal{H}'_{nm}\gamma'_{mc'}}{(\mathcal{E}_l-E-\varepsilon)(\mathcal{E}_k-E-\varepsilon)}\right|^2\right\rangle \qquad (3.9)$$

The average value can be obtained by standard, though tedious algebra, as shown in Appendix 3. The assumption (3.1) to (3.3) can be used now, to discord higher order terms in D/Γ. The final result is (for single initial and single final channel)

$$\langle\sigma(E)\sigma(E+\varepsilon)\rangle - \langle\sigma\rangle^2 = \langle\sigma^{cc}\rangle\Big\{\langle\sigma^{cc}\rangle + 2\langle\sigma^{dc}\rangle + 2\langle\sigma^{cd}\rangle +$$

$$+ 4\sigma^{dd}\Big\}\frac{1}{\left[1+(\frac{\varepsilon}{\Gamma_T})^2\right]\left[1+(\frac{\varepsilon}{\Gamma_{T'}})^2\right]} + \langle\sigma^{dc}\rangle\Big\{\langle\sigma^{dc}\rangle + \qquad (3.10)$$

$$+ 2\sigma^{dd}\Big\}\frac{1}{1+(\frac{\varepsilon}{\Gamma_{T'}})^2} + \langle\sigma^{cd}\rangle\Big\{\langle\sigma^{cd}\rangle + 2\sigma^{dd}\Big\}\frac{1}{1+(\frac{\varepsilon}{\Gamma_T})^2}$$

3.4. - The expected width of the correlation curve. -

Now we discuss how the conclusions of section 2 are modified by the appearence of direct contributions in the eq. (3.8) and (3.10). Firstly, we observe that the (presumably small) direct term σ^{dd} does not alter the shape of the correlation curve, just as it happens in the usual fluctuation theory. The shape of the curve changes, however, if terms $\langle\sigma^{cd}\rangle$ or $\langle\sigma^{dc}\rangle$ substantially contribute to the total cross section. For comparison, the values of Γ_{ex}/Γ_T, for $\Gamma_T = \Gamma_{T'}$ and $\langle\sigma^{cd}\rangle = 0$ are shown in table I as a function of the cross sections σ^{dd}, $\langle\sigma^{cc}\rangle$ and $\langle\sigma^{dc}\rangle$. The same comparison, but with $\langle\sigma^{dc}\rangle = \langle\sigma^{cd}\rangle$, has been done in table II. The change in Γ_{ex}/Γ_T is not large, when $\langle\sigma^{cd}\rangle$ and $\langle\sigma^{dd}\rangle$ are smaller than $\langle\sigma^{cc}\rangle$, ranging from 0.64 in the purely statistical case to 0.75 in the (rather improbable) case $\sigma^{cd} = \sigma^{dc} = \sigma^{dd} = \sigma^{cc}$. It is however clear from eq. (3.10) that Γ_{ex} should approach Γ_T or $\Gamma'_{T'}$ when either $\langle\sigma^{cd}\rangle$ or $\langle\sigma^{dc}\rangle$ dominates over the other cross sections.

TABLE I - Values of Γ_{ex}/Γ_T as a function of $\sigma^{dd}/\langle\sigma^{cc}\rangle$ and $\langle\sigma^{dc}\rangle/\langle\sigma^{cc}\rangle$, for $\Gamma_T = \Gamma_T'$ and $\langle\sigma^{cd}\rangle = 0$.

$\langle\sigma^{dc}\rangle/\langle\sigma^{cc}\rangle$ \ $\sigma^{dd}/\langle\sigma^{cc}\rangle$	0	0. 5	1	2
0	0. 64	0. 64	0. 64	0. 64
0. 5	0. 67	0. 68	0. 68	0. 69
1	0. 71	0. 72	0. 72	0. 72
2	0. 77	0. 77	0. 78	0. 78

TABLE II - Values of Γ_{ex}/Γ_T as a function of $\sigma^{dd}/\langle\sigma^{cc}\rangle$ and $\langle\sigma^{dc}\rangle/\langle\sigma^{cc}\rangle$, for $\Gamma_T = \Gamma_T'$ and $\langle\sigma^{cd}\rangle = \langle\sigma^{dc}\rangle$.

$\langle\sigma^{dc}\rangle/\langle\sigma^{cc}\rangle$ \ $\sigma^{dd}/\langle\sigma^{cc}\rangle$	0	0. 5	1	2
0	0. 64	0. 64	0. 64	0. 64
0. 5	0. 68	0. 70	0. 71	0. 72
1	0. 72	0. 74	0. 75	0. 76
2	0. 77	0. 79	0. 80	0. 82

4. CONCLUSION. -

We now discuss the results of the calculation, in connection with possible experiments.

From the comparison of experimental correlation function for allowed and forbidden reactions, we could obtain either (a) a general check of the theory or (b) informations on the amount of "coherent" contribution to the isospin mixing process. For this purpose, experiments are most significant in the following two cases:

a) $\Gamma_T \approx \Gamma_{T'}$ (within a factor 2). The w. h. m. Γ_{ex} of the experimental correlation curve for the forbidden reaction will be smaller than both Γ_T and $\Gamma_{T'}$, if the isospin mixing is mainly statistical. The shape of the correlation curve is not significantly different from the usual Lorentzian shape. Small amounts of coherent isospin mixing and/or isospin impurity in channels c and c' do not alter significantly the theoretical estimates, and are therefore hard to be detected. Measurements in the situation $\Gamma_T \approx \Gamma_{T'}$ are therefore suitable mainly as a global test of the theory.

b) $\Gamma_T \gg \Gamma_{T'}$: in this case the expected Γ_{ex} is very near to $\Gamma_{T'}$ but another structure with width Γ_T also exists, when the contribution from the coherent mixing process in channel c is large enough. This intermediate structure can be made apparent by the procedure proposed by Pappalardo[17].

For a test of the theory, the most favourable case is that of (d, α) or (α, d) reactions on light, selfconjugate nuclei, from Mg^{24} to Ca^{40}. All final nuclei from these reactions show a typical triplet of low lying levels, including the ground state. One of them, is a 0^+, T=1 level, i. e. the isospin analog of the g. s. of neighbour nuclei. Since the first 0^+, T=0 state will appear only at rather high excitation, the low lying 0^+ state is expected to have very pure isospin. Unfortunately, this state is very closed to the others in the triplet, so that it may be difficult to distinguish in the energy spectrum the small "forbidden" peak from the tail of the large "allowed" one. A broad range magnetic spectrograph, or possibly an advanced solid-state counter technique, can in principle allow this experiment to be done at least in the easiest cases, $Si^{28}(d,\alpha)Al^{26}$ and $Ca^{40}(d,\alpha)K^{38}$. Additional troubles may take place in the measurement of $\Gamma_{T=1}$ since $\Gamma_{T=0}$ can be obtained from the analysis of allowed (d, α) reactions, but $\Gamma_{T=1}$ can not. We are therefore forced to use the (p, α) reaction to the T=1 state, i. e. an initial channel having no pure isospin. The forbidden contribution from the T=0 part of the proton channel will therefore interfere with that from T=1. As a consequence, the correlation function f(E) will contain not only a Lorentzian term with $\Gamma_{T=1}$, but also another with $\Gamma_{T=0}$ (though the relative importance of the second term is $\approx 2|\alpha|^2$). If this fact is not carefully taken into account, an erroneus $\Gamma_{T=1}$ is likely to come out, particularly if $\Gamma_{T=0} \ll \Gamma_{T=1}$.

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APPENDIX 1

We want to discuss the time evolution of a highly excited nuclear system, that at time 0 has pure isospin T_1, following the procedure firstly proposed by Morinaga[11]. Suppose the total Hamiltonian is $H = H^o + H^c$, with H^o charge independent and H^c small. We may define eigenstates $|X^o_{nT})$ of H^o and T^2 in a closed region of nuclear dimensions, with proper boundary conditions. For $t = 0$, the state vector $|\psi(t))$ of the system is described by a proper superposition of states $|X^o_{nT})$. For symplicity sake, we assume $|\psi(0))$ actually coincide with a particular $|X^o_{pT_1})$ and neglect decay. This is indeed a very crude assumption, since isospin mixing may occur through the coupling to the continuum, if some of the channels have not pure isospin.

The transition amplitude from state $|X^o_{pT_1})$ to a state $|X^o_{nT_2})$ in the time interval $0 \div t$ is, to the first order,

$$A_{p,n}(t) = -\frac{1}{i\hbar}\int_0^t (nT_2|H^c|pT_1) \exp\frac{i(E^o_p - E^o_n)t'}{\hbar}\, dt' =$$

$$= (nT_2|H^c|pT_1)\frac{\exp\{i(E^o_p - E^o_n)t/\hbar\} - 1}{E^o_p - E^o_n} .$$

Now we assume that the matrix elements $(nT_2|H^c|pT_1)$ (for the given p and all n) are random uncorrelated variables with zero mean and their variance is independent from $E^o_p - E^o_n$ within an energy interval of the order of $\hbar/t$.

Under these assumptions, the total transition probability to states of isospin T_2 is

$$P_{T_1T_2}(t) = \sum_n |a_{pn}(t)|^2 =$$

$$= \sum_n |(nT_2|H^c|pT_1)|^2 \left[\frac{\sin(E^o_p - E^o_n)t/2\hbar}{(E^o_p - E^o_n)/2}\right]^2$$

Substituting the squared matrix elements by their expectation value $\langle|(T_2|H^c|T_1)|^2\rangle$ and the sum on n by an integral, we get

$$\langle P_{T_1T_2}(t)\rangle = 2\pi \frac{\langle|(T_2|H^c|T_1)|^2\rangle}{D_2}\frac{t}{\hbar}.$$

APPENDIX 2

For the forbidden transitions (without coherent effects) the transition amplitude is

$$f = A\sum_n^{(T)}\sum_m^{(T')}\frac{\gamma'_{nc}\mathcal{H}'_{nm}\gamma'_{mc'}}{(\mathcal{E}_n-E)(\mathcal{E}_m-E)}$$

where A is a kinematic constant. We have therefore

$$\langle f^*(E)\,f(E+\varepsilon)\rangle = |A|^2\sum_{nn'}^{(T)}\sum_{mm'}^{(T')}\frac{\langle\gamma'_{nc}\gamma'^*_{n'c'}\mathcal{H}'_{nm}\mathcal{H}'^*_{n'm'}\gamma'_{mc'}\gamma'^*_{m'c'}\rangle}{(\mathcal{E}^*_{n'}-E)(\mathcal{E}^*_{m'}-E)(\mathcal{E}_{n'}-E-\varepsilon)(\mathcal{E}_m-E-\varepsilon)}$$

But the average value at the right hand side is zero unless $n=n'$, $m=m'$; therefore

$$\langle f^*(E)\,f(E+\varepsilon)\rangle = |A|^2\sum_n^{(T)}\frac{1}{(\mathcal{E}^*_n-E)(\mathcal{E}_n-E-\varepsilon)}\,.$$

$$\cdot\sum_m^{(T')}\frac{1}{(\mathcal{E}^*_m-E)(\mathcal{E}_m-E-\varepsilon)}\langle|\gamma'_{nc}|^2\rangle\langle|\mathcal{H}'_{nm}|^2\rangle\langle|\gamma'_{mc'}|^2\rangle$$

If the level spacing is small compared to the level width, we may treat each sum as an integral and extend the integration on the real axis from $-\infty$ to $+\infty$.

$$\sum_n^{(T)}\frac{1}{(\mathcal{E}^*_n-E)(\mathcal{E}_n-E-\varepsilon)} = \sum_n^{(T)}\frac{1}{(E_n+\frac{1}{2}i\Gamma_T-E)(E_n-\frac{1}{2}i\Gamma_T-E-\varepsilon)} =$$

$$= \int\frac{dE'/D_T}{(E'+\frac{1}{2}i\Gamma_T)(E'-\varepsilon-\frac{1}{2}i\Gamma_T)} = \frac{2\pi i}{D_T}\frac{1}{\varepsilon+i\Gamma_T} = \frac{2\pi}{\Gamma_T D_T}\frac{1}{1-i\varepsilon/\Gamma_T}$$

Finally

$$\langle f^{*}(E)\, f(E+\mathcal{E})\rangle = |A|^{2} \langle |\gamma'_{nc}|^{2}\rangle \langle |\mathcal{H}'_{nm}|^{2}\rangle \langle |\gamma'_{mc'}|^{2}\rangle \cdot$$

$$\cdot \frac{2\pi}{D_T \Gamma_T} \frac{2\pi}{D_{T'} \Gamma_{T'}} \frac{1}{(1 - \frac{i\mathcal{E}}{\Gamma_T})(1 - \frac{i\mathcal{E}}{\Gamma_{T'}})}$$

The same procedure shows that $\langle f(E)\, f(E+\mathcal{E})\rangle = 0$, since in this case both poles are on the same side of the real axis.

APPENDIX 3

We want to evaluate the average value of the cross section correlati on function, in the case of forbidden reaction. From eq. (3. 9) we get

$$\langle \sigma(E)\, \sigma(E+\mathcal{E})\rangle = |A|^{4} \sum_{nn'll'}^{(T)} \sum_{mm'kk'}^{(T')} \frac{\gamma'_{nc}\, \gamma'^{*}_{n'c'}\, \mathcal{H}'_{nm}\, \mathcal{H}'^{*}_{n'm'}\, \gamma'_{mc}\, \gamma'^{*}_{m'c'}}{(\mathcal{E}_n - E)(\mathcal{E}^{*}_{n'} - E)(\mathcal{E}_m - E)(\mathcal{E}^{*}_{m'} - E)} \cdot$$

(A. 3. 1.)

$$\cdot \frac{\gamma'_{lc}\, \gamma'^{*}_{l'c'}\, \mathcal{H}'_{lk}\, \mathcal{H}'^{*}_{l'k'}\, \gamma'_{kc}\, \gamma'^{*}_{k'c'}}{(\mathcal{E}_l - E - \mathcal{E})(\mathcal{E}^{*}_{l'} - E - \mathcal{E})(\mathcal{E}_k - E - \mathcal{E})(\mathcal{E}^{*}_{k'} - E - \mathcal{E})}$$

For the moment we only consider these terms in the sum for which none of the indices n, m, n', m' coincides with one among k, l, k', l'.

The two fractions at the right hand side of eq. (A. 3. 1) are statistically independent in this case, and the result is therefore

$$\left\{ |A|^{2} \left\langle \sum_{nn'}^{(T)} \sum_{mm'}^{(T')} \frac{\gamma'_{nc}\, \gamma'^{*}_{n'c}\, \mathcal{H}'_{nm}\, \mathcal{H}'^{*}_{n'm'}\, \gamma'_{mc'}\, \gamma'^{*}_{m'c'}}{(\mathcal{E}_n - E)(\mathcal{E}^{*}_{n'} - E)(\mathcal{E}_m - E)(\mathcal{E}^{*}_{m'} - E)} \right\rangle \right\}^{2} = \langle \sigma \rangle^{2}$$

Additional terms of the same order of magnitude come from the cases we neglected till now, that is

1) n = l' or n' = l
   or m = k' or m' = k ;
2) two of the above equalities hold simultaneously;
3) three of themhold;
4) all four hold at the time.

The terms with $n=1$, or $m=k$, ... have zero mean (within the limits of D/Γ).

Case 1 - For $n=1'$ and $(m,n',m') \neq (1,k,k')$ we get

$$|A|^2 < \sum_n \frac{\gamma'_{nc}\,\gamma'^{*}_{nc}}{(\mathcal{E}_n - E)(\mathcal{E}^*_n - E - \varepsilon)} \sum_m \frac{\mathcal{H}'_{nm}\,\gamma'_{mc'}}{(\mathcal{E}_m - E)} \sum_{k'} \frac{\mathcal{H}'^{*}_{nk'}\,\gamma'^{*}_{k'c'}}{(\mathcal{E}^*_{k'} - E - \mathcal{E})} \,.$$

$$\cdot \sum_{n'm'} \frac{\gamma'^{*}_{n'c}\,\mathcal{H}'^{*}_{n'm'}\,\gamma'^{*}_{m'c'}}{(\mathcal{E}^*_{n'} - E)(\mathcal{E}^*_{m'} - E)} \sum_{1k} \frac{\gamma'_{1c}\,\mathcal{H}'_{1k}\,\gamma'_{kc'}}{(\mathcal{E}_1 - E - \mathcal{E})(\mathcal{E}_k - E - \mathcal{E})}$$

Apart from higher order terms in D/Γ , the average of the product is equal to the product of the average values of each sum. Moreover, the average values of the 3^{rd} and 4^{th} sums are equal to the complex conjugate of the 2^{nd} and 5^{th} respectively. Taking into account the results of appendix 2, we obtain

$$\sum_n \frac{\langle |\gamma'_{nc}|^2 \rangle}{(\mathcal{E}_n - E)(\mathcal{E}^*_n - E - \mathcal{E})} \left| \langle \sum_m \frac{\mathcal{H}'_{nm}\,\gamma'_{mc'}}{(\mathcal{E}_m - E)} \rangle \right|^2 \left| \langle \sum_{n'm'} \frac{\gamma'_{n'c}\,\mathcal{H}'_{n'm'}\,\gamma'_{m'c'}}{(\mathcal{E}_{n'} - E)(\mathcal{E}_{m'} - E)} \rangle \right|^2 =$$

$$= \langle \sigma^{cd} \rangle \sigma^{dd}\, C_1^* \qquad \text{with } C_1(\mathcal{E}) = \left(1 - i\frac{\mathcal{E}}{\Gamma_1}\right)^{-1}$$

If $n'=1$ instead of $n=1'$, we obtain the complex conjugate of this expression. But

$$C_1 + C_1^* = \left(1 + i\frac{\mathcal{E}}{\Gamma_1}\right)^{-1} + \left(1 - i\frac{\mathcal{E}}{\Gamma_1}\right)^{-1} = 2\left[1 + (\mathcal{E}/\Gamma_1)^2\right]^{-1} = 2|C_1|^2$$

The contribution of case 1 coming from the two terms $(m=k', k=m')$ add two similar terms, but with $\langle \sigma^{dc} \rangle$, Γ_2 instead of $\langle \sigma^{cd} \rangle$, Γ_1, is therefore

$$2\,\sigma^{dd}\left(\langle \sigma^{cd} \rangle |C_1|^2 + \langle \sigma^{dc} \rangle |C_2|^2\right).$$

Case 2a - For $m=1'$ and $n'=1$; $(m,m') \neq (k,k')$ we get

$$|A|^2 < \sum_n \frac{\gamma'_{nc}\,\gamma'^{*}_{nc}}{(\mathcal{E}_n - E)(\mathcal{E}^*_n - E - \mathcal{E})} \sum_1 \frac{\gamma'_{1c}\,\gamma'^{*}_{1c}}{(\mathcal{E}^*_1 - E)(\mathcal{E}_1 - E - \mathcal{E})} \sum_m \frac{\mathcal{H}'_{nm}\,\gamma'_{mc'}}{(\mathcal{E}_m - E)} \,.$$

$$\sum_{m'} \frac{\mathcal{H}'^{*}_{1m'}\,\gamma'^{*}_{m'c'}}{(\mathcal{E}^*_{m'} - E)} \sum_k \frac{\mathcal{H}'_{1k}\,\gamma'_{kc'}}{(\mathcal{E}_k - E - \mathcal{E})} \sum_{k'} \frac{\mathcal{H}'_{nk'}\,\gamma'_{k'c'}}{(\mathcal{E}_{k'} - E - \mathcal{E})} \rangle =$$

$$= |A|^2 \left| \sum_n \frac{\langle |\gamma'_{nc}|^2 \rangle}{(\mathcal{E}_n - E)(\mathcal{E}^*_n - E - \varepsilon)} \right|^2 \left| \langle \sum_m \frac{\mathcal{H}'_{nm}\, \gamma'_{mc'}}{(\mathcal{E}_m - E)} \rangle \right|^4 = \langle \sigma^{cd} \rangle^2 |C_1|^2$$

A term with $\langle \sigma^{dc} \rangle$, Γ_2 instead of $\langle \sigma^{cd} \rangle$, Γ_1 comes from $m = k'$, $m' = k$.

Case 2b - For $n = 1'$, $m = k'$: $(n', m') \neq (1, k)$

$$\langle \sum_{nm} \frac{|\gamma'_{nc}|^2 |\mathcal{H}'_{nm}|^2 |\gamma'_{mc'}|^2}{(\mathcal{E}_n - E)(\mathcal{E}^*_n - E - \varepsilon)(\mathcal{E}_m - E)(\mathcal{E}^*_m - E - \varepsilon)} \sum_{n'm'} \frac{\gamma'^*_{n'c}\, \mathcal{H}'^*_{n'm'}\, \gamma'^*_{m'c'}}{(\mathcal{E}^*_{n'} - E)(\mathcal{E}^*_{m'} - E)}$$

$$\sum_{1k} \frac{\gamma'_{1c}\, \mathcal{H}'_{1k}\, \gamma'_{kc}}{(\mathcal{E}_1 - E - \varepsilon)(\mathcal{E}_k - E - \varepsilon)} \rangle = \langle \sigma^{cc} \rangle\, \sigma^{dd}\, C^*_1\, C^*_2$$

Taking also into account the three possible choices $(n = 1', m' = k)$; $(n' = 1, m = k')$ and $(n' = 1, m' = k)$ we obtain for the case 2^b

$$\langle \sigma^{cc} \rangle\, \sigma^{dd} (C^*_1 C^*_2 + C^*_1 C_2 + C_1 C^*_2 + C_1 C_2) =$$

$$= \langle \sigma^{cc} \rangle\, \sigma^{dd} (C_1 + C^*_1)(C_2 + C^*_2) = 4 \langle \sigma^{cc} \rangle\, \sigma^{dd} |C_1 C_2|^2$$

Case 3 - For $n = 1'$, $n' = 1$, $m = k'$ and $m' \neq k$

$$\langle \sum_{nm} \frac{|\gamma'_{nc}|^2 |\mathcal{H}'_{nm}|^2 |\gamma'_{mc'}|^2}{(\mathcal{E}_n - E)(\mathcal{E}^*_n - E - \varepsilon)(\mathcal{E}_m - E)(\mathcal{E}^*_m - E - \varepsilon)} \sum_1 \frac{|\gamma'_{1c}|^2}{(\mathcal{E}^*_1 - E)(\mathcal{E}_1 - E - \varepsilon)}$$

$$\sum_k \frac{\mathcal{H}'_{1k}\, \gamma'_{kc'}}{(\mathcal{E}_k - E - \varepsilon)} \sum_{k'} \frac{\mathcal{H}'^*_{1'k'}\, \gamma'^*_{k'c'}}{(\mathcal{E}^*_{k'} - E - \varepsilon)} \rangle = \langle \sigma^{cc} \rangle\, C^*_1\, C^*_2 \langle \sigma^{cd} \rangle\, C_1$$

Taking into account the terms $(m \neq k')$, $(n \neq 1')$ and $(n' \neq 1)$ we finally obtain

$$\langle \sigma^{cc} \rangle \left[\langle \sigma^{cd} \rangle |C_1|^2 (C^*_2 + C_2) + \langle \sigma^{dc} \rangle |C_2|^2 (C^*_1 + C_1) \right] =$$

$$= 2 \langle \sigma^{cc} \rangle \left[\langle \sigma^{cd} \rangle + \langle \sigma^{dc} \rangle \right] |C_1 C_2|^2 .$$

Case 4 - For $n = 1'$, $n' = 1$, $m = k'$, $m' = k$ we obtain

$$\left\langle \sum_{nm} \frac{|\gamma'_{nc}|^2 \, |\mathcal{H}'_{nm}|^2 \, |\gamma'_{mc'}|^2}{(\mathcal{E}_n - E)(\mathcal{E}^*_n - E - \varepsilon)(\mathcal{E}_m - E)(\mathcal{E}^*_m - E - \varepsilon)} \right. \cdot$$

$$\sum_{lk} \frac{|\gamma'_{lc}|^2 \, |\mathcal{H}'_{lk}|^2 \, |\gamma'_{kc'}|^2}{(\mathcal{E}^*_l - E)(\mathcal{E}_l - E - \varepsilon)(\mathcal{E}^*_k - E)(\mathcal{E}_k - E - \varepsilon)} = \langle \sigma^{cc} \rangle^2 \, |C_1 C_2|^2$$

Combining the results for cases 1 to 4, we finally obtain (for the single $c \longrightarrow c'$ transition)

$$\langle \sigma(E)\, \sigma(E+\varepsilon) \rangle - \langle \sigma^2 \rangle = \langle \sigma^{cc} \rangle \Big[\langle \sigma^{cc} \rangle + 2 \langle \sigma^{cd} \rangle + 2 \langle \sigma^{dc} \rangle +$$

$$+ 4 \quad \sigma^{dd} \Big] \, |C_1 C_2|^2 + \langle \sigma^{cd} \rangle \Big[\langle \sigma^{cd} \rangle + 2\, \sigma^{dd} \Big] \, |C_1|^2 +$$

$$+ \langle \sigma^{dc} \rangle \Big[\langle \sigma^{dc} \rangle + 2\, \sigma^{dd} \Big] \, |C_2|^2 .$$

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