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G. Pisent and A.M. Sarius: ELASTIC SCATTERING OF NEUTRONS  
BY  $\text{He}^4$ ,  $\text{C}^{12}$  AND  $\text{O}^{16}$  NUCLEI. PHASE SHIFT ANALYSIS.-

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G. Pisent<sup>(x)</sup> and A.M. Sarius<sup>(o)</sup>: ELASTIC SCATTERING OF NEUTRONS BY He<sup>4</sup>, C<sup>12</sup> AND O<sup>16</sup> NUCLEI. PHASE SHIFT ANALYSIS.

(This work has been partially carried out under Contract Euratom-Cnen)

## 1 - INTRODUCTION.

The elastic scattering of neutrons by the light even-even nuclei He<sup>4</sup>, C<sup>12</sup> and O<sup>16</sup> has been studied at low energies.

As is well known, the angular distribution of nucleons scattered by spinless nuclei may be analysed in terms of one set of phase shifts. The aim of this work is a detailed evaluation of the behaviour of the phase shifts for the reactions in question, in the light of the more recent experiments on angular distributions and polarization. Although the general trend of the phase shifts in n-C<sup>12</sup> and n-O<sup>16</sup> scattering has already been determined<sup>(1)(2)</sup>, recent differential cross section measurements, performed with a very high energy resolution<sup>(3)(4)</sup>, can make possible a detailed analysis of the narrow resonances (see Figs. 1 and 5) and consequently a determination of the level parameters of the compound nuclei from the resonant phase

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shift. The behaviour of the n-He<sup>4</sup> phase shifts has been analysed in a preceding paper<sup>(5)</sup>, and is here compared with the new data of the Wisconsin Group<sup>(6)</sup>.

The motivation for this work is as follows.

First, as is well known, the nuclei which are being considered have a very strong spin-orbit coupling with the bombarding nucleon, resulting in very pronounced polarization effects. A knowledge of the phase shifts allows the nuclei to be employed as analysers in double scattering experiments, provided this knowledge is very precise. In fact, it is well known<sup>(5)</sup> that some non resonant phase shifts, which are not well determined by the differential cross section, produce large contributions to the polarization through interference terms.

Moreover, all possible information about the level splittings would seem to be very useful in shedding light on the still obscure spin-orbit coupling mechanism.

Finally, from experimental phase shifts, the determination of the potential parameters in a simple nuclear model is straightforward. Such calculations are in progress, and will be given in a subsequent paper.

## 2 - PHASE SHIFT ANALYSIS.

The differential cross section for the scattering of neutrons by spin zero nuclei is given by (see for example ref. 7):

$$(1) \quad K^2 \sigma(\theta) = \left| \sum_L (\bar{a}_L / 2i) P_L(\cos\theta) \right|^2 + \left| \sum_L (\bar{b}_L / 2i) P'_L(\cos\theta) \right|^2,$$

where

$$(2a) \quad \bar{a}_L = \left[ (L+1) \cos 2\delta_{2L+1}^L + L \cos 2\delta_{2L-1}^L - (2L+1) \right] + \\ + i \left[ (L+1) \sin 2\delta_{2L+1}^L + L \sin 2\delta_{2L-1}^L \right],$$

$$(2b) \quad \bar{b}_L = \exp(2i \delta_{2L+1}^L) - \exp(2i \delta_{2L-1}^L). \quad (x)$$

These expressions are valid in the energy ranges considered by us, where only the elastic channel is open.

Phase shifts have been calculated from the angular distribution coefficients  $A_N$ , following the method described in (8).

The  $A_N$ , which are defined by

$$(3) \quad k^2 \sigma(\Theta) = \sum_{N=0}^{2L_{\max}} A_N \cos^N \Theta,$$

are obtained by fitting the experimental differential cross section<sup>(+)</sup>. A preliminary fit, with  $N_{\max}=2$ ,  $N_{\max}=3$ , etc., up to  $N_{\max} = 6$  indicated that a D wave approximation ( $N=4$ ) is sufficient at our energies. Table I shows the behaviour of the expression

$$\sum_i \left\{ \chi_i^{(N_{\max})} \right\}^2 = \sum_i \left[ k^2 \sigma(\Theta_i) - \sum_{N=0}^{N_{\max}} A_N \cos^N \Theta_i \right]^2$$

as a function of  $N_{\max}$  for the highest energies considered in the  $n-C^{12}$  and  $n-O^{16}$  scattering.

$$\sum_i \left\{ \chi_i^{(N_{\max})} \right\}^2$$

is there seen to be sufficiently stabilized at  $N_{\max} = 4$ . At very low energies, the analysis has been limited to S and P waves, and the D phase shifts ( $=0$ ) are not reported in Figs. 1 and 5.

Since the method employed gives all the "mathematical" phase shifts, a "physical" discrimination is then required.

In the next paragraphs, the "selected" phase shifts are discussed.

(x) - From now on, the symbol  $\delta_{2J}^L$  is used to represent the phase shifts, and  $\Theta$  indicates the center-of-mass scattering angle.

(+) - All calculations have been set up by means of an IBM 704 computer.

Details about the ambiguities are given in the Appendix.

It is known the difficulty of evaluating the propagation of errors from the cross section to the different phase shifts. The errors in the phase shifts, which strongly depend on energy, can be evaluated by means of the graphical methods described in ref. 8. A consideration of interest for our proposals, is that the errors increase slightly through resonances, where the dependence of the  $\delta_{2J}^L$  on the coefficients  $A_N$  is very critical.

### 3 - RESULTS.

#### a) Oxygen.

Fig. 1 shows experimental phase shifts for  $O^{16}$ . S wave points have been fitted by the effective range formula

$$(4) \quad K \operatorname{ctg} \delta_1^0 = -a_0^{-1} + r_0 K^2/2,$$

(see results in Table II), and resonant phase shifts have been fitted by a one level Breit-Wigner formula. The level parameters of the  $O^{17}$  compound nucleus which were found are given in Table III<sup>(x)</sup>. Curves shown in Fig. 1 have been calculated by means of these parameters. The total width of the level  $\Gamma_{J\pi}$  is given instead of the more meaningful reduced width  $\gamma_{J\pi}^2$ , because the former quantity is an experimental parameter, which is independent of the choice of the interaction radii and also of the boundary conditions used in deriving the dispersion formalism. It is hoped that more detailed information concerning the "physical" character of the interaction will be furnished by an investigation of the potential parameters, which is now in progress.

If the condition of a "smooth" variation with energy is imposed on the phase shift  $\delta_1^1$ , the only acceptable value is seen to be zero throughout the entire energy interval. The phase shift  $\delta_5^2$  is seen to be zero up to a

(x) - A similar analysis on the  $F^{17}$  compound nucleus has been performed by S.R. Salisbury and H.T. Richards<sup>(9)</sup>.

about 1.5 MeV. Assuming as "true" phase shifts the interpolated ones, which are given by the curves of Fig. 1, the differential cross sections have been recalculated and compared with experiment (Fig. 2). For this comparison we have chosen, from the eight angular values where cross sections have been measured, the lowest, the highest and two intermediate ones. The agreement between theory and experiment is equally satisfactory at the other four angles. The polarization, calculated with the same phase shifts, is compared in Fig. 3 with experiments performed by the Wisconsin Group<sup>(10)</sup>. The set of phase shifts is seen to reproduce the polarization and the differential cross section quite well.

The formulas used in calculating the polarization, taking into account S, P and D waves, are (see ref. 5):

$$(5) \quad P(\Theta) = \sin \Theta \sum_{N=0}^3 B_N P_N(\cos \Theta) / K^2 \sigma(\Theta),$$

where

$$(6a) \quad B_0 = 2 \left[ \left( \frac{\delta_1^0}{\delta_3^1} \right) - \left( \frac{\delta_1^0}{\delta_1^1} \right) - \left( \frac{\delta_1^1}{\delta_3^2} \right) + \left( \frac{\delta_1^1}{\delta_5^2} \right) + \right. \\ \left. + 2 \left( \frac{\delta_3^1}{\delta_5^2} \right) - 2 \left( \frac{\delta_3^1}{\delta_3^2} \right) \right],$$

$$(6b) \quad B_1 = 6 \left[ \left( \frac{\delta_1^0}{\delta_5^2} \right) - \left( \frac{\delta_1^0}{\delta_3^2} \right) + \left( \frac{\delta_1^1}{\delta_3^1} \right) + 2 \left( \frac{\delta_3^2}{\delta_5^2} \right) \right],$$

$$(6c) \quad B_2 = 2 \left[ \left( \frac{\delta_3^1}{\delta_5^2} \right) + 5 \left( \frac{\delta_1^1}{\delta_5^2} \right) - 6 \left( \frac{\delta_3^1}{\delta_3^2} \right) \right],$$

$$(6d) \quad B_3 = 18 \left( \frac{\delta_3^2}{\delta_5^2} \right),$$

and

$$(7) \quad (x/y) = \sin x \sin y \sin(x-y).$$

In fig. 4 the polarization is calculated for  $\Theta = 45^\circ, 90^\circ, 135^\circ$ .

b) Carbon.

In Fig. 5 the experimental phase shifts deduced from the Argonne differential cross section measurements are shown. Unfortunately, the polarization calculated from the experimental phase shifts between zero and 2 MeV disagrees with the experimental polarization obtained by the same Argonne Group<sup>(11)</sup>. The disagreement is considerably greater than that which would be expected on the basis of the experimental errors involved<sup>(x)</sup>.

In the absence of other rules for testing the errors on the different phase shifts, an obvious criterion is that of considering the dispersion of the experimental phase shifts, provided one assumes that no systematic error is present. From this viewpoint, Fig. 5 clearly indicates that the S wave phase shift is the best defined. From this starting point, all phase shifts but the  $\sigma_1^0$  have been corrected by a process of successive approximations from experimental values, in order to obtain the best possible agreement with differential cross section and polarization experiments. The general trend which has been obtained, is shown in Fig. 5.

This is clearly only one of the possible procedures. Although the final agreement can be considered satisfactory (see Figs. 6 and 7), the situation is not so clear-cut as in the  $O^{16}$  case, and further experiments would be welcome.

c) Helium.

For the sake of completeness, the behaviour of the n-He<sup>4</sup> phase shifts given in (5) is reproduced in Fig. 8, and is compared with the old measurements<sup>(12)</sup> already considered in ref. (5) and also with the new experimental data<sup>(6)(13)</sup>. The phase shifts are seen to be well determined, except for the "threshold" behaviour of the  $\sigma_1^1$  phase shift.

4 - CONCLUSIONS.

From Fig. 1, 5 and 8, the S wave phase shifts are seen to be repre

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(x) -It has to be noted that the alternative solution, defined as "spurious" in the Appendix, gives rise to no better agreement with polarization.

sented very well by an effective range formula at our energies. For testing the reliability of a "Rigid sphere" approximation for the S waves, one can easily verify that, at very low energies, the simultaneous validity of the effective range and rigid sphere approximation requires that  $a_0 = R$  and  $r_0 = (2/3)R = (2/3)a_0$ .<sup>(x)</sup> So, in our case, where the effective range approximation holds very well, a comparison between  $r_0$  and  $(2/3)a_0$  gives a test of the acceptability of the rigid sphere approach. It can be easily seen from Table II that the rigid sphere formalism, represents an approach which is good for  $\text{He}^4$ , not very good for  $\text{C}^{12}$ , and even worse for  $\text{O}^{16}$ .

Another result to be noted from Table III is that the scattering length  $a_0$  is larger for the light  $\text{C}^{12}$  than for the heavier  $\text{O}^{16}$ . A direct consequence of this is the difficulty of fitting both  $\text{C}^{12}$  and  $\text{O}^{16}$  with a central square well potential of the same depth, and with a radius which increases with the mass number A (for example with the law  $r_0 \propto A^{1/3}$ ).

It does not appear to be possible to account for this apparent discrepancy, even with the use of more realistic potentials, such as that of Saxon and Woods.

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(x) - R is the radius of the rigid sphere.



APPENDIX.

About the resolution of the ambiguities. (x)

The number of mathematical solutions of the phase shifts equation is, in general, 8 in  $L_{\max} = 1$  approximation, and 32 in  $L_{\max} = 2$  approximation.

In practice many of these solutions fall in the complex plane, and are automatically eliminated. Others appear and disappear in limited energy intervals, so that the condition of "smooth" variation with energy is sufficient to eliminate them. Since our analysis starts practically at zero energy, another useful method is to begin with a simple S wave analysis at very low energies. This leads to an unambiguous determination of the S wave behaviour near the P waves threshold. The analysis is then continued, in S and P waves approximation, up to energies at which D waves become important, and so on. Finally, a complete analysis which gives the exact phase shifts is performed. By means of these criteria, the choices made by us were practically unique. An interesting exception is shown in Fig. 9, where a "spurious" solution is shown for  $O^{16}$  and  $C^{12(+)}$ . These non-physical solutions, when compared with the "physical" ones (Figs. 1 and 5 respectively), show the same S phase shift, while the two components of the P and D doublet are inverted. The ambiguity can be considered a Fermi-Yang ambiguity, generalized to angular momenta higher than 1 (see ref. 8). Fig. 9 shows, in both "spurious" solutions, an evident correlation between the resonant wave and the wave with the same L and different  $J^{(o)}$ . This correlation, which exceeds experimental uncertain

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(x) - A general discussion of this argument is given in ref. (8).

(+) - The "non-physical" solutions were, as a matter of interest, given at energies near resonance; but they also exist at lower energies.

(o) - The sudden increase of a non resonant phase shift is easily explained. "Spurious" and "physical" solutions show a resonant wave with  $L=J-1/2$  and  $L=J+1/2$ , respectively. In the first case, the presence of an appreciable non resonant component is clearly needed, in order to reproduce the experimental total cross section.

ties, is sufficient to eliminate these solutions, because they imply a coupling which is forbidden by angular momentum and parity conservation.

It is a pleasure for us to thank Prof. E. Clementel and Prof. C. Villi for many discussions and comments on the work. Thanks are also due to Prof. R.O. Lane, for sending us the detailed tables of his experimental data. The computer programme was written by Dr. F. Fabbri.

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TABLE I

$$\sum_i \xi_i^2 (N_{\max}) \cdot 10^{-3}$$

	E(MeV)	$N_{\max}=2$	$N_{\max}=3$	$N_{\max}=4$	$N_{\max}=5$	$N_{\max}=6$
C <sup>12</sup>	2.129	7.87	2.10	2.10	1.66	1.57
O <sup>16</sup>	1.475	22.25	7.12	5.10	3.91	3.40

TABLE II

S WAVE PARAMETERS

	$a_0$ (fermi)	$r_0$ (fermi)
He <sup>4</sup>	0.243 <sup>+</sup>	0.188 <sup>+</sup>
C <sup>12</sup>	0.610	0.336
O <sup>16</sup>	0.574	0.118

TABLE III

LEVEL PARAMETERS

	$J^\pi$	$E_{\text{res}}$ (KeV)	$\Gamma^{J^\pi}$ (KeV)
He <sup>4</sup>	3/2 <sup>-</sup>	1297 <sup>+</sup>	1442 <sup>+</sup>
C <sup>12</sup>	5/2 <sup>+</sup>	2084	7.90
O <sup>16</sup>	3/2 <sup>-</sup>	442	54.4
	3/2 <sup>+</sup>	1008	106.1
	3/2 <sup>-</sup>	1318	36.4

+ Quantities obtained from ref. (5).

FIGURE CAPTIONS.

- Fig. 1 - Phase shifts  $\delta_{2J}^L$  for the reaction  $O^{16}(n,n)O^{16}$ , derived from differential cross section analysis. The curve fitting  $\delta_1^0$  is obtained from the effective range parameters of Table II. The curves fitting  $\delta_1^1$  and  $\delta_3^2$  are obtained, near the resonances, from the level parameters<sup>3</sup> of Table III.
- Fig. 2 -  $O^{16}(n,n)O^{16}$ . Comparison between calculated cross sections (solid lines) and experimental cross sections (points). The calculated cross sections are obtained from the fitting curves of Fig. 1.
- Fig. 3 -  $O^{16}(n,n)O^{16}$ . The polarization calculated by phase shifts (solid line) is compared with the experimental polarization given in ref. (10) (points).
- Fig. 4 -  $O^{16}(n,n)O^{16}$ . Calculated values of the polarization for center-of-mass angles  $\Theta = 45^\circ$ ,  $\Theta = 90^\circ$  and  $\Theta = 135^\circ$ .
- Fig. 5 - Phase shifts  $\delta_{2J}^L$  for the reaction  $C^{12}(n,n)C^{12}$ , derived from differential cross section analysis. The curves fitting  $\delta_1^0$  and  $\delta_3^2$  (solid lines) are obtained from parameters of Tables II and III.
- Fig. 6 -  $C^{12}(n,n)C^{12}$ . Comparison between calculated cross sections (solid lines), and experimental cross sections (points). The calculated cross sections are obtained from the fitting curves of Fig. 5.
- Fig. 7 -  $C^{12}(n,n)C^{12}$ . The polarization calculated by phase shifts for center-of-mass angles  $\Theta = 48.5^\circ$  (solid line) and  $\Theta = 94.8^\circ$  (dotted line), is compared with the experimental values given in ref. (11) (black and white points).
- Fig. 8 - Phase shifts  $\delta_{2J}^L$  for the reaction  $He^4(n,n)He^4$ . Points shown in Figure refer to measurements due to J.D. Seagrave<sup>(12)</sup>, F. Demanins, G. Pisent, G. Poiani and C. Villi<sup>(13)</sup>, and S.M. Austin, H.H. Barschall and R.E. Shannon<sup>(6)</sup>.
- Fig. 9 -  $O^{16}(n,n)O^{16}$  and  $C^{12}(n,n)C^{12}$  reaction. Example of a "non physical" solution to the phase shifts equations. These "spurious" solutions are discussed in the appendix.

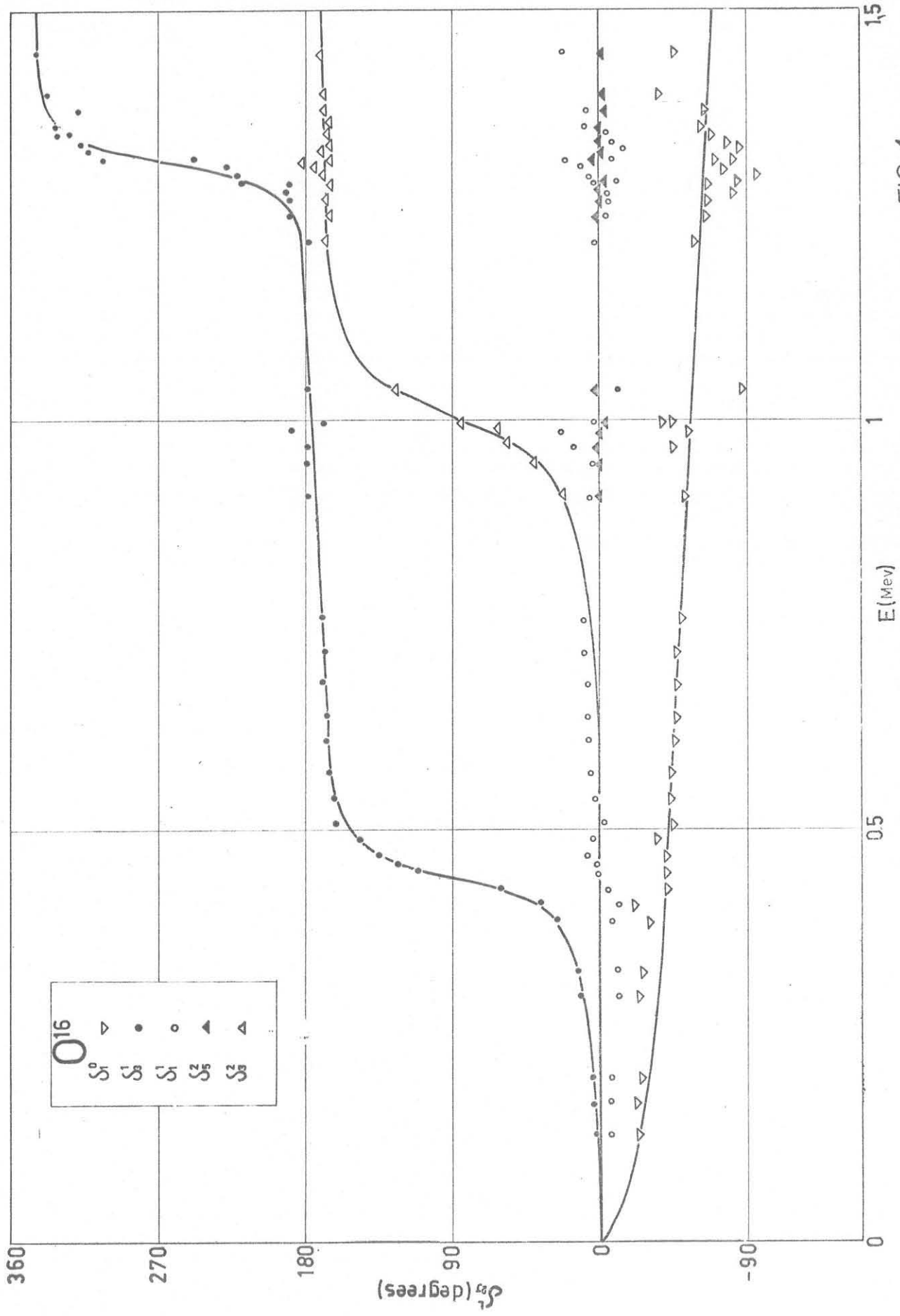


FIG. 1

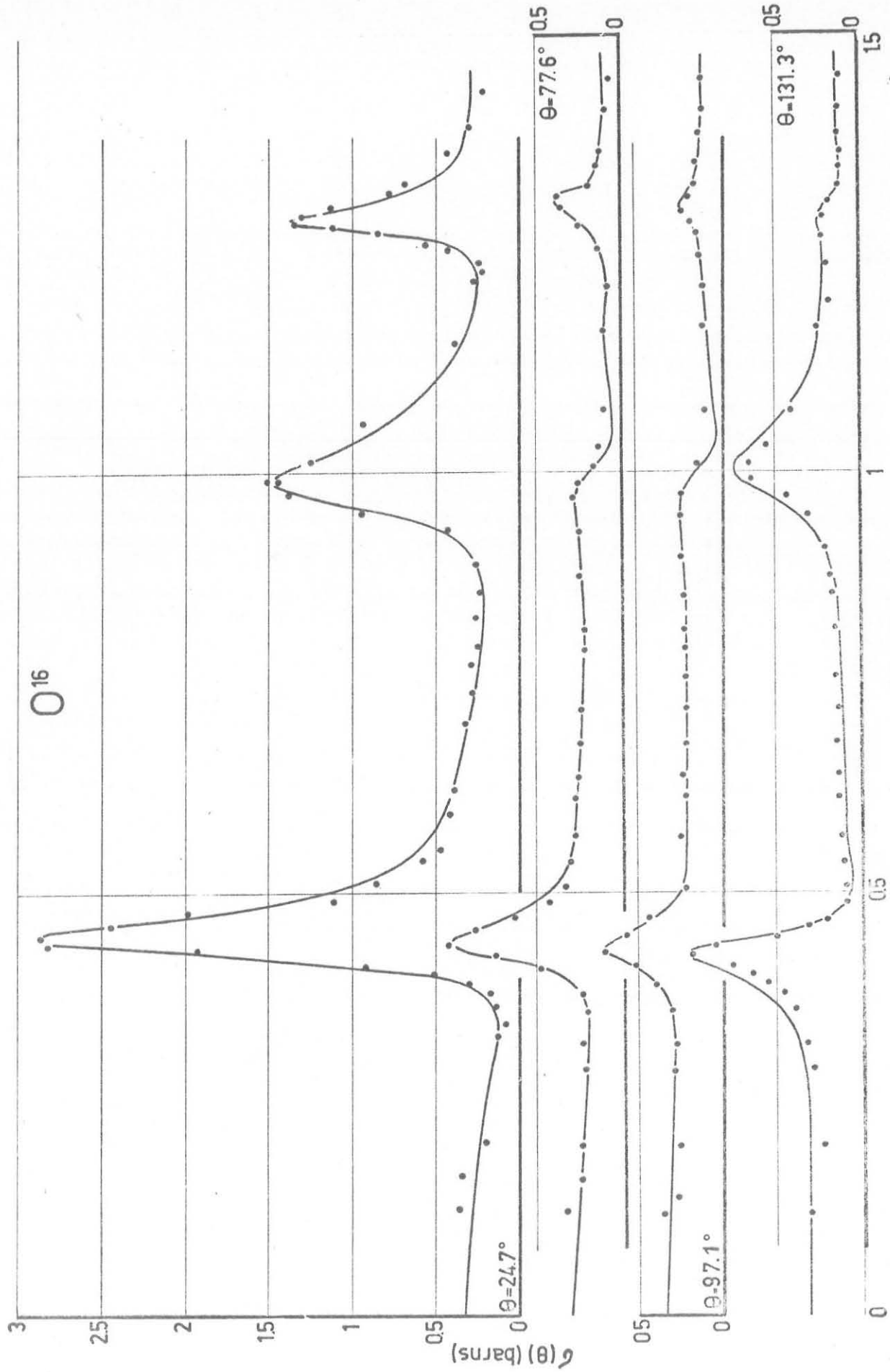


fig. 2

FIG. 2

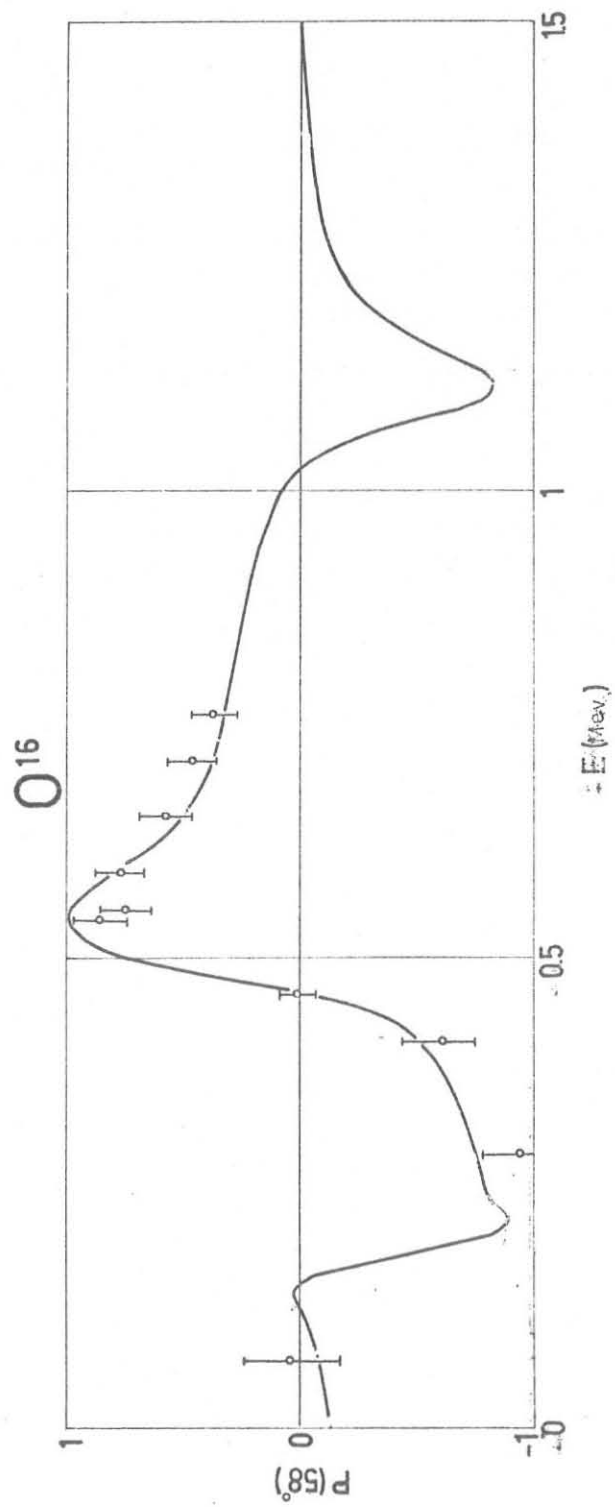


FIG. 3

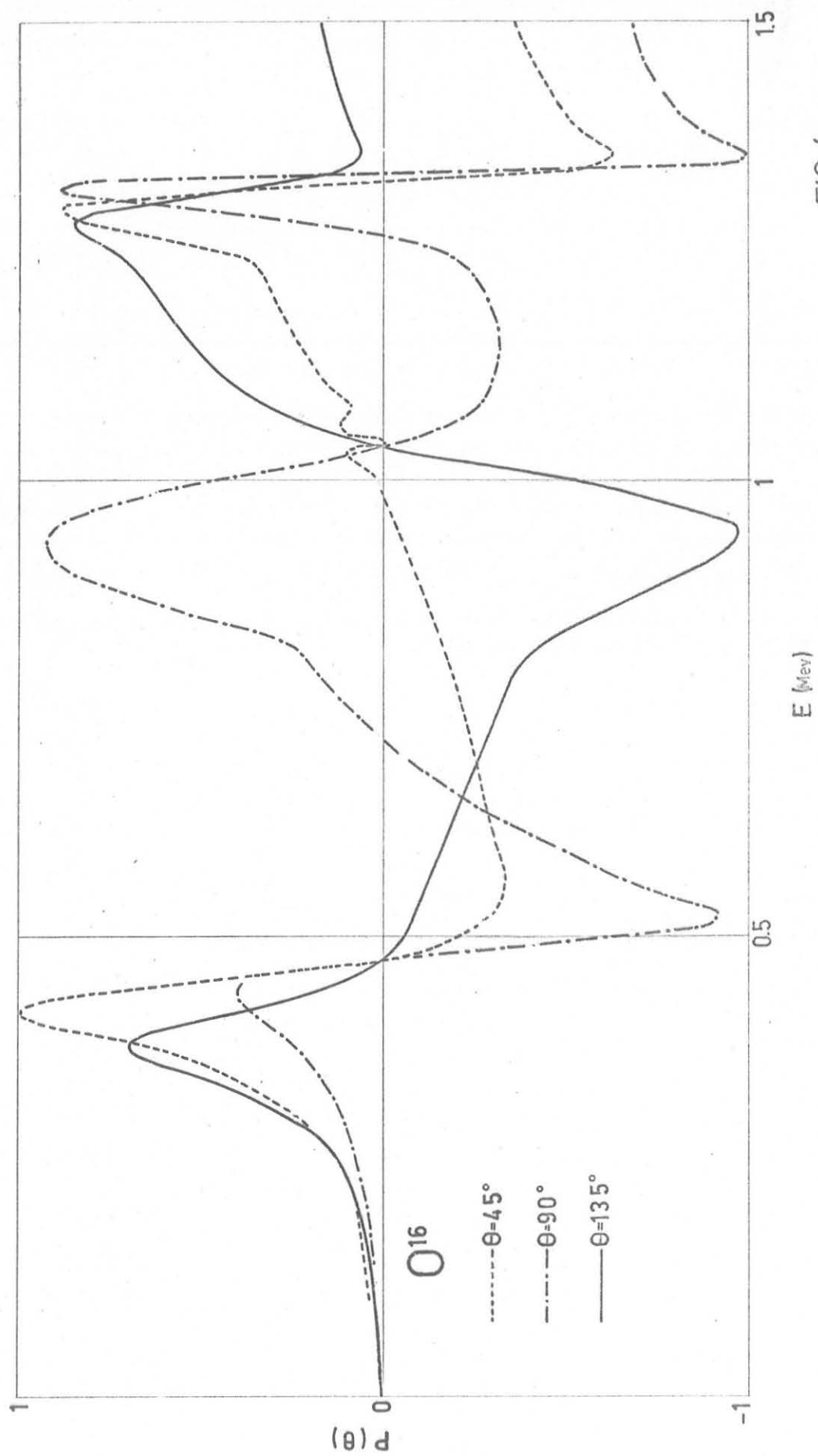
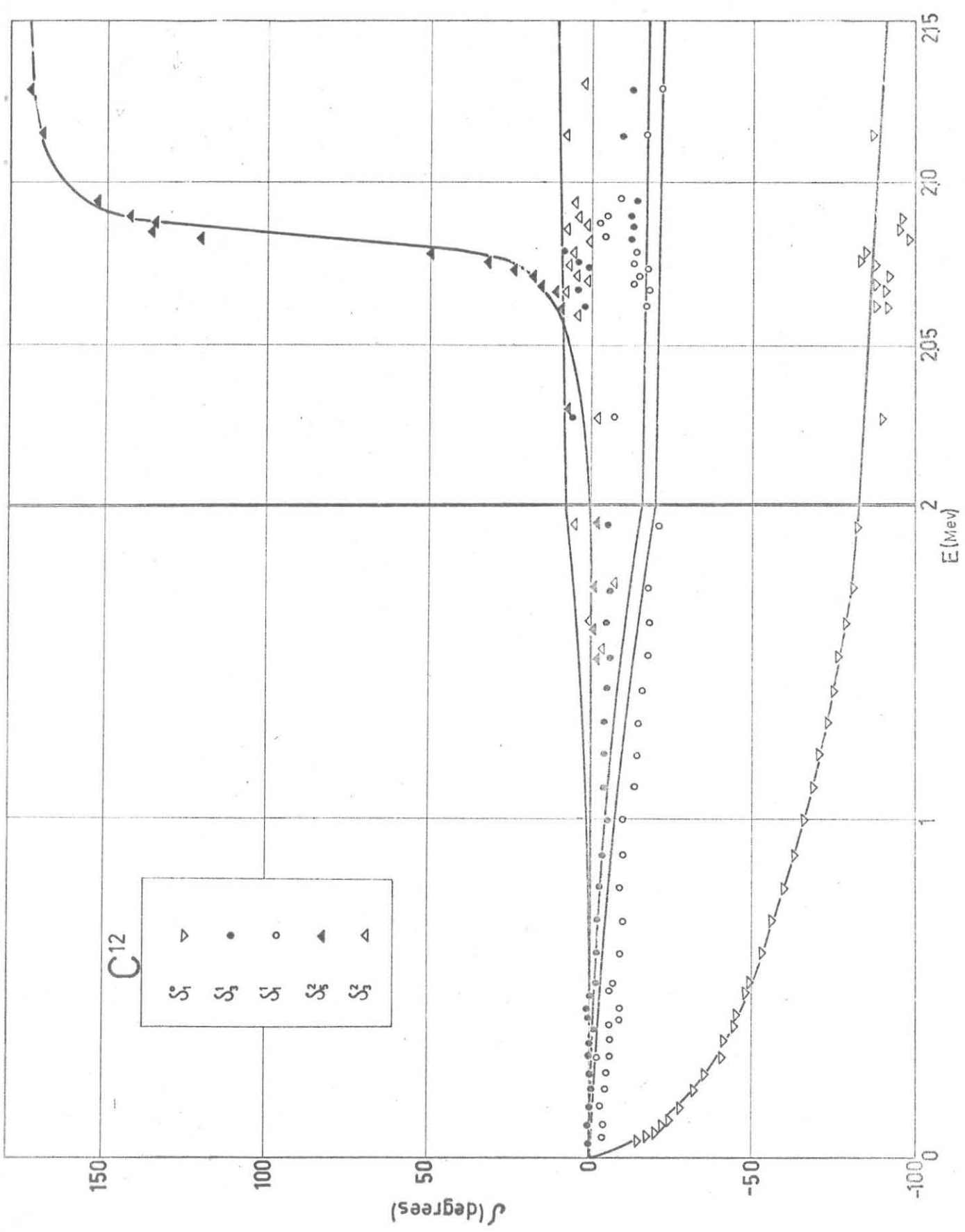


FIG.4



FIG. 5



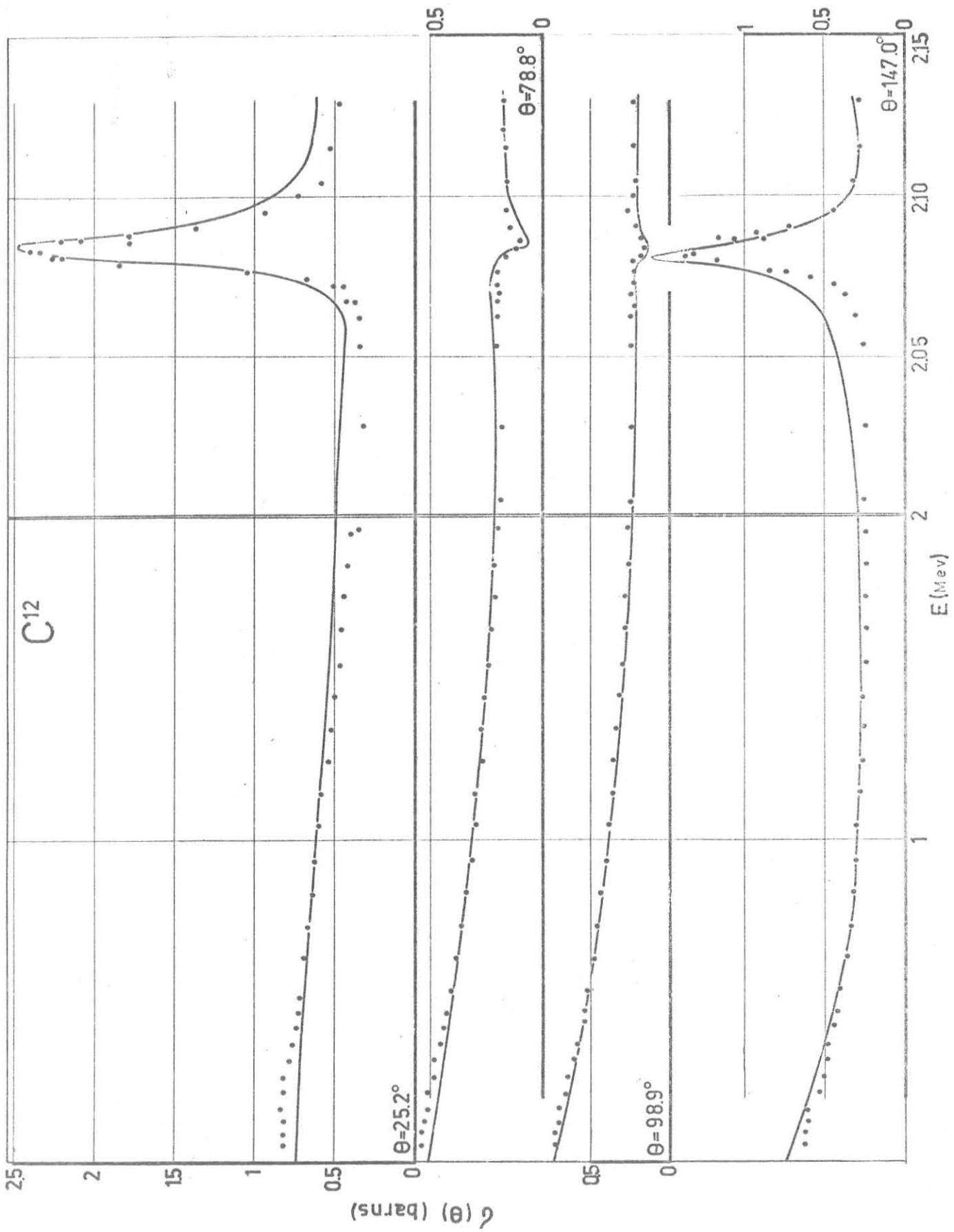


FIG. 6

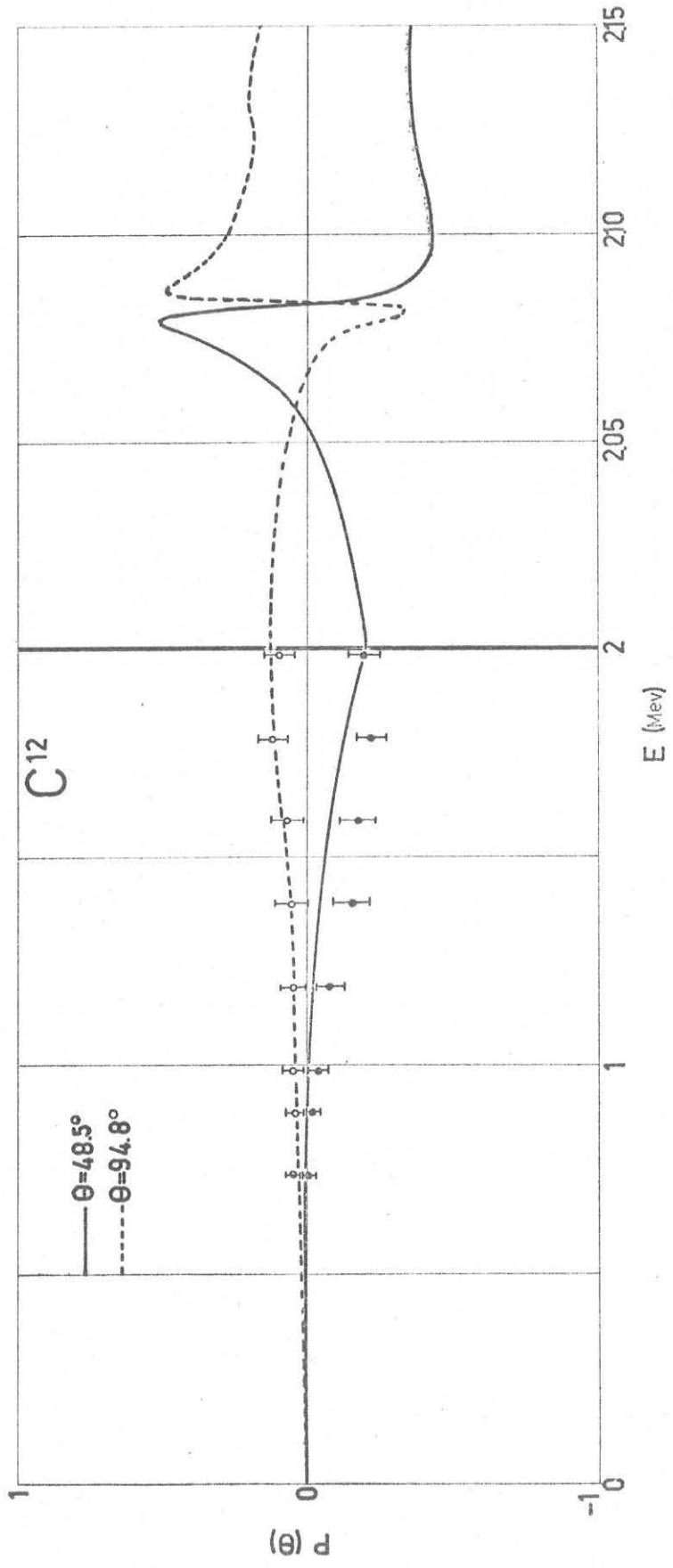


FIG. 7

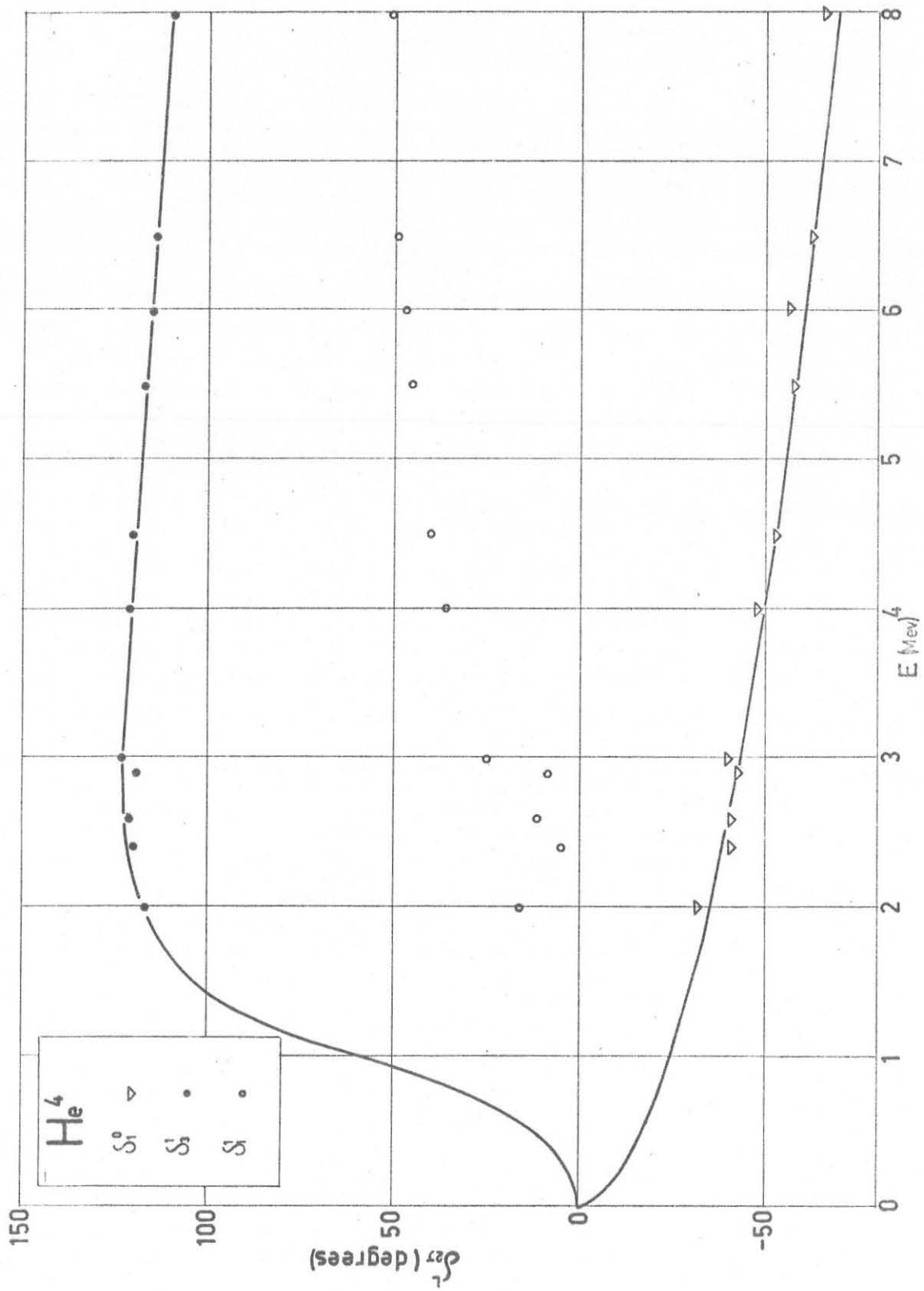


FIG. 8

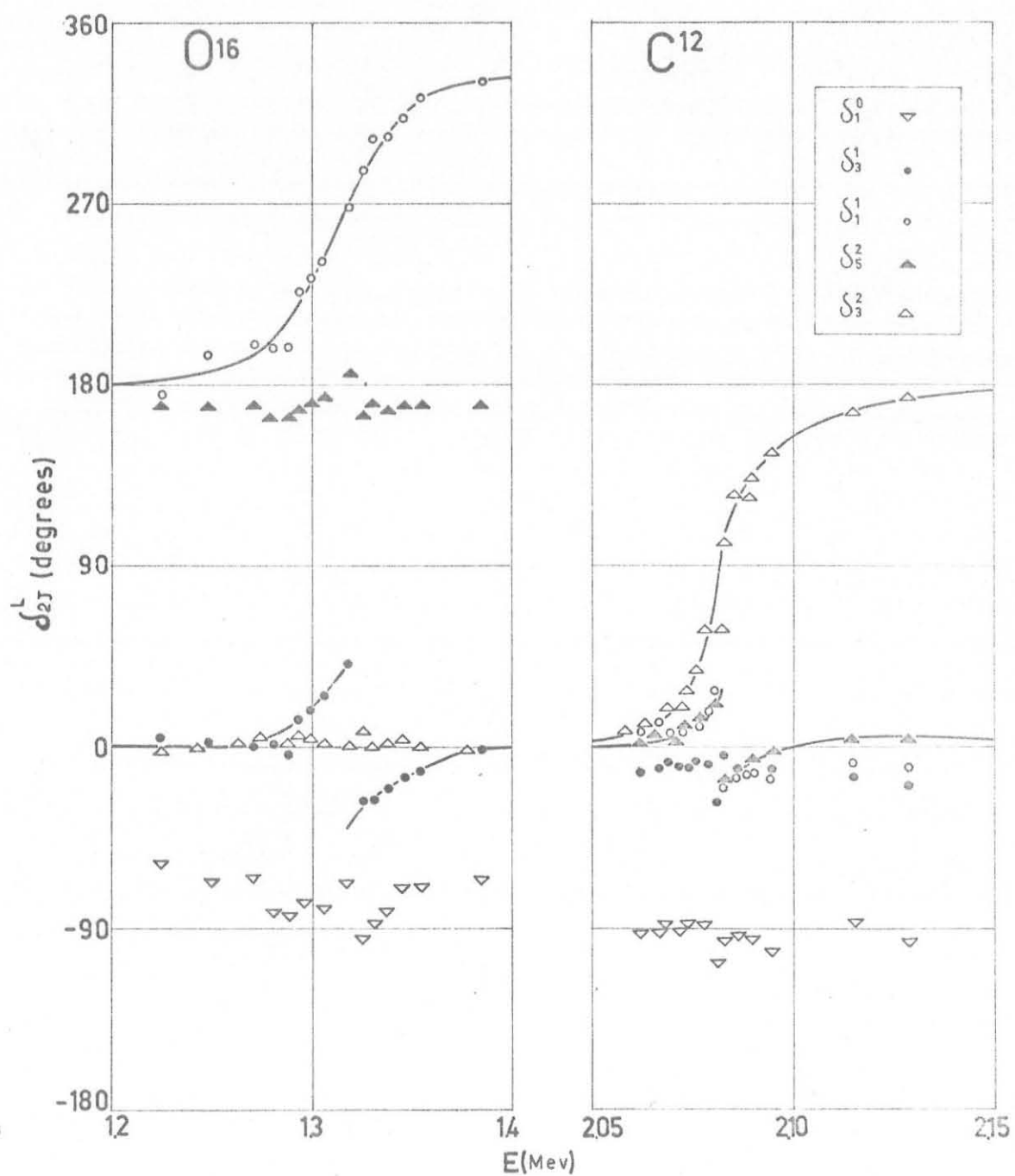


FIG. 9