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A THEORETICAL REVIEW OF THE $B \rightarrow D^{(*)} \bar{D}^{(*)}$ DECAY CHANNEL

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A theoretical review of the $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decay channel

Guido Michelon¹

Abstract

We review the present theoretical comprehension of the $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decay channel, with particular attention to the strong and electroweak penguin diagrams contribution to the amplitude and to the strong phase problem. We further present the complete angular analysis for the decay $B \rightarrow D^* \bar{D}^*$ and suggest some potentially useful isospin relation.

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1 Introduction

The $\bar{B} \rightarrow D^{(*)+}D^{(*)-}$ decay, governed by the quark subprocess $b \rightarrow c\bar{c}d$, is expected to give an important contribution to the determination of the unitarity angle β [1]. Now, while the D^+D^- state is an eigenstate of CP with positive parity, the vector channel $D^{*+}D^{*-}$ is an admixture of CP-even and CP-odd states, due to the fact that the decay occurs through three waves (s,p,d) with different CP-parity. However, as already well known, angular analysis seems to provide an efficient tool for extracting contributions of different CP-parity, leading to a potentially clean measure of the angle β (see [2, 3]).

An important question related to such a measure is the role of the hadronic uncertainties. The main potential sources of these uncertainties are the fact that two amplitudes depending on different *weak* phases can contribute to the decay process, and the presence of final state interaction (FSI) *strong* phases.

Before addressing the specific problem of our interest, we briefly review the general formalism about mixing and CP violation in B meson physics. We consider the decay of the B in a CP-even eigenstate f_{CP} :

$$B \rightarrow f_{CP} .$$

The amplitudes for the decay in such a state are defined in the following way:

$$\begin{aligned} A &= \langle f_{CP} | \mathcal{H}^{\Delta B=1} | B^0 \rangle \\ \bar{A} &= \langle f_{CP} | \mathcal{H}^{\Delta B=1} | \bar{B}^0 \rangle \end{aligned} \quad (1)$$

and the time dependence of the state due to the $B^0 - \bar{B}^0$ mixing is given by:

$$|B^0(t)\rangle = e^{-imt - \frac{\Gamma t}{2}} \left[\cos\left(\frac{\Delta mt}{2}\right) |B^0\rangle - i\frac{q}{p} \sin\left(\frac{\Delta mt}{2}\right) |\bar{B}^0\rangle \right] \quad (2)$$

$$|\bar{B}^0(t)\rangle = e^{-imt - \frac{\Gamma t}{2}} \left[\cos\left(\frac{\Delta mt}{2}\right) |\bar{B}^0\rangle - i\frac{p}{q} \sin\left(\frac{\Delta mt}{2}\right) |B^0\rangle \right] \quad (3)$$

where q/p is the mixing parameter of the $B^0 - \bar{B}^0$ system:

$$\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} = e^{-2i\beta} . \quad (4)$$

From these equations it is possible to obtain the time dependent expression of the rates:

$$\begin{aligned} \Gamma(B^0(t) \rightarrow f_{CP}) &\propto |A|^2 e^{-\Gamma t} \left[\frac{1 + |\lambda|^2}{2} \right. \\ &\quad \left. + \frac{1 - |\lambda|^2}{2} \cos(\Delta mt) - \text{Im}\lambda \sin(\Delta mt) \right] \end{aligned} \quad (5)$$

$$\begin{aligned} \Gamma(\bar{B}^0(t) \rightarrow f_{CP}) &\propto |A|^2 e^{-\Gamma t} \left[\frac{1 + |\lambda|^2}{2} \right. \\ &\quad \left. - \frac{1 - |\lambda|^2}{2} \cos(\Delta mt) + \text{Im}\lambda \sin(\Delta mt) \right] \end{aligned} \quad (6)$$

where we have introduced:

$$\lambda = \frac{q}{p} \frac{\bar{A}}{A}. \quad (7)$$

We then define the time dependent asymmetry, which is the experimentally measured quantity:

$$a_{CP}(t) = \frac{\Gamma(B^0(t) \rightarrow f_{CP}) - \Gamma(\bar{B}^0(t) \rightarrow f_{CP})}{\Gamma(B^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}^0(t) \rightarrow f_{CP})} \quad (8)$$

and substituing the eqs.(5) and (6) we obtain:

$$a_{CP}(t) = \frac{(1 - |\lambda|^2) \cos(\Delta mt) - 2Im\lambda \sin(\Delta mt)}{1 + |\lambda|^2}. \quad (9)$$

The term depending on $\cos(\Delta mt)$ represents the *direct* CP-violation effect, which requires $A \neq \bar{A}$ or, since $|q/p| = 1$ with very good approximation for the B system, $|\lambda| \neq 1$, while the $\sin(\Delta mt)$ term represents the *indirect* CP-violation effect due to the $B^0 - \bar{B}^0$ mixing. If there is only one amplitude contributing to the decay process, or if there are several amplitudes but all with the *same* weak phase, it clearly results $A = \bar{A}$ so that:

$$|\lambda| = 1. \quad (10)$$

Using this assumption, we can simplify the time dependent and time integrated expression (9) of the asymmetry obtaining:

$$\begin{aligned} a_{CP}(t) &= -Im\lambda \sin(\Delta mt) \\ a_{CP} &= -\frac{x}{1+x^2} Im\lambda ; \end{aligned} \quad (11)$$

this corresponds to CP-violation only through the mixing. The parameter $Im\lambda$ is directly related to the CKM matrix elements, and, for the decay of our interest $B \rightarrow D\bar{D}$ it results:

$$Im\lambda = -\sin 2\beta. \quad (12)$$

Until now we have treated the B meson decay to a CP eigenstate; if, however, the final state of the decay has not a definite behaviour under CP, the situation obviously is not so simple[2]. In the particular case of $B \rightarrow D^{*+}D^{*-}$, the final particles have spin 1 and consequently the final state is a superposition of different waves, with different CP parity; we can therefore write the rates of the conjugated B and \bar{B} decays in the following way:

$$\begin{aligned} \Gamma(B^0(t) \rightarrow D^{*+}D^{*-}) &= \Gamma_+(1+a) + \Gamma_-(1-a) \\ \Gamma(\bar{B}^0(t) \rightarrow D^{*+}D^{*-}) &= \Gamma_+(1-a) + \Gamma_-(1+a) \end{aligned} \quad (13)$$

where Γ_+ and Γ_- are respectively the CP-even and CP-odd decay widths and a is the asymmetry parameter defined in (9) for a pure CP-even final state. Now,

constructing the rate asymmetry, which is the experimentally tested quantity, we find:

$$A(t) = \frac{\Gamma(B^0(t) \rightarrow D^{*+}D^{*-}) - \Gamma(\bar{B}^0(t) \rightarrow D^{*+}D^{*-})}{\Gamma(B^0(t) \rightarrow D^{*+}D^{*-}) + \Gamma(\bar{B}^0(t) \rightarrow D^{*+}D^{*-})} = Ka(t) \quad (14)$$

where

$$K = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}. \quad (15)$$

As we can see, the asymmetry results to be *diluted* by an unknown parameter K which depends on the CP composition of the final state. As we will briefly show in section 3, the angular analysis of the decay provides a tool for measuring the dilution factor K , so the measurement of the unitarity angle β from this channel is expected to have a good accuracy[3].

Now we pass to the theoretical description of the amplitude; the diagrams describing $\bar{B} \rightarrow D^{(*)+}D^{(*)-}$ decay are depicted in Figure 1: they are the *tree* diagram (\mathcal{T}), the *strong* (\mathcal{ST}) and *electroweak* (\mathcal{EW}) *penguin* (\mathcal{P}) diagrams and the *exchange* (\mathcal{E}) diagram. Neglecting the exchange process, which is expected to be suppressed¹, the two main contributions to the amplitude come from tree and penguin diagrams. The effective hamiltonian up to one loop order in electroweak interactions (and to all orders in strong interactions) for the decay $b \rightarrow c\bar{c}d$ can be written in the following way[5]:

$$\mathcal{H}_{eff}^{\Delta B=1} = \frac{4G_F}{\sqrt{2}} \left[V_{cb}V_{cd}^*(c_1O_1 + c_2O_2) - V_{tb}V_{td}^* \sum_{i=3}^{10} c_iO_i \right] \quad (16)$$

where the operators O_i and the Wilson coefficients $c_i(\mu)$ (at $\mu = m_b \simeq 5 \text{ GeV}$) are[6, 7]:

$$\begin{aligned} O_1 &= \bar{d}\gamma_\mu Lc\bar{c}\gamma^\mu Lb & c_1 &= 1.1502 \\ O_2 &= \bar{d}_\alpha\gamma_\mu Lc_\beta\bar{c}_\beta\gamma^\mu Lb_\alpha & c_2 &= -0.3125 \\ O_{3(5)} &= \bar{d}\gamma_\mu Lb\bar{c}\gamma^\mu L(R)c & c_{3(5)} &= 0.0174(0.0104) \\ O_{4(6)} &= \bar{d}_\alpha\gamma_\mu Lb_\beta\bar{c}_\beta\gamma^\mu L(R)c_\alpha & c_{4(6)} &= -0.0373(-0.0459) \\ O_{7(9)} &= \frac{3}{2}\bar{d}\gamma_\mu Lbe_c\bar{c}\gamma^\mu R(L)c & c_{7(9)} &= 1.050 \times 10^{-5}(-0.0101) \\ O_{8(10)} &= \frac{3}{2}\bar{d}_\alpha\gamma_\mu Lb_\beta e_c\bar{c}_\beta\gamma^\mu R(L)c_\alpha & c_{8(10)} &= 3.839 \times 10^{-4}(1.959 \times 10^{-3}) \end{aligned} \quad (17)$$

where $L(R) = (1 \mp \gamma_5)/2$, α and β are color indices.

O_1 and O_2 represent tree level processes, while O_{3-6} are the so called strong penguin (\mathcal{STP}) operators, due to one gluon exchange, and O_{7-10} describe the electroweak penguin (\mathcal{EWP}) processes, mediated by γ and Z^0 exchange.

¹One would expect a suppression factor of order $f_B/m_B \sim \mathcal{O}(\lambda^2)$ in the amplitude[4]; in any case this channel depends on the same weak phase as the tree diagram, so it does not represent a polluting contribution. See the discussion in section 4.

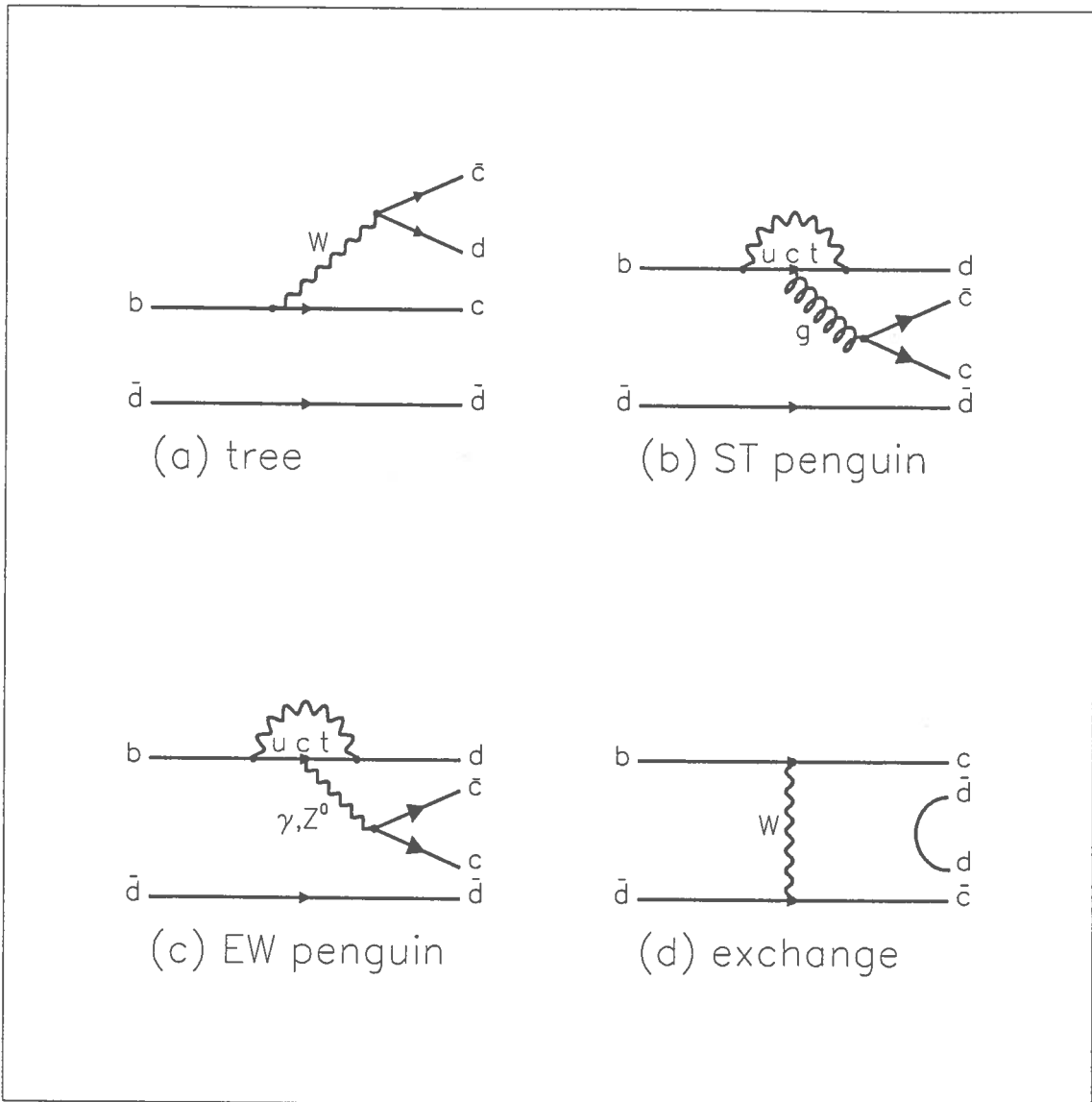


Figure 1: The four diagrams responsible for $b \rightarrow c \bar{c} d$ process: (a) tree (\mathcal{T}), (b) strong penguin (\mathcal{STP}), (c) electroweak penguin (\mathcal{EWP}), (d) exchange (\mathcal{E}).

The Wilson coefficients c_i were calculated at the next to leading order corrections in QCD by the authors of ref. [6, 7], using $\alpha_s(m_Z) = 0.118$, $\alpha_{em}(m_Z) = 1/128$, $m_t = 176 \text{ GeV}$.

As we see from eq.(16), the tree amplitude is proportional to CKM elements $V_{cb}V_{cd}^*$, while the penguin contribution from the internal loop with a t quark depends on $V_{tb}V_{td}^*$, so that we can parametrize the decay amplitude in the following way:

$$\begin{aligned} \bar{A}(\bar{B} \rightarrow D^{(*)+} D^{(*)-}) &= V_{cb}V_{cd}^* \mathcal{T} - V_{tb}V_{td}^* \mathcal{P} \\ &= V_{cb}V_{cd}^* (\mathcal{T} + \mathcal{P}) + V_{ub}V_{ud}^* \mathcal{P} , \end{aligned} \quad (18)$$

where use has been made of the CKM matrix unitarity relation

$$V_{cb}V_{cd}^* + V_{tb}V_{td}^* + V_{ub}V_{ud}^* = 0 .$$

Since the CKM coefficients of each term are of comparable magnitude, we cannot neglect the contribution of the penguin diagram in comparison with that of the tree diagram, albeit the coefficients of the penguin operators are much smaller than those of the tree operators (see eq.(17)). On the contrary this happens in the case of the decay $\bar{B} \rightarrow J/\Psi K_s$, where there is a suppression factor of the order $|V_{ub}V_{us}^*|/|V_{cb}V_{cs}^*| \sim \lambda^2$ in the Wolfenstein parametrization. Therefore, together with the $V_{cb}V_{cd}^*$ term, which is related through the mixing angle q/p to the unitarity angle β , there is also a polluting contribution from the $V_{ub}V_{ud}^*$ term (or the $V_{tb}V_{td}^*$ term, which is the same).

Concerning the strong FSI effects and restricting to the scalar decay $\bar{B}^0 \rightarrow D^+ D^-$, we can account for them by a relative strong phase δ between the tree and penguin amplitudes²: this, too, leads to an uncertainty on the extracted value of β . These combined effects can be parametrized in the decay amplitude in the following way:

$$\bar{A}(\bar{B} \rightarrow D^+ D^-) = V_{cb}V_{cd}^* \mathcal{T} (1 - \mathcal{R} e^{i\beta} e^{i\delta}) \quad (19)$$

where for convenience we have defined:

$$\mathcal{R} = - \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| \frac{\mathcal{P}}{\mathcal{T}} , \quad \delta = \delta_{\mathcal{P}} - \delta_{\mathcal{T}}$$

with $\delta_{\mathcal{P}}$ and $\delta_{\mathcal{T}}$ the strong phases respectively of the penguin and tree amplitudes; β is the unitarity angle defined as (see also eq.(4)):

$$\beta = \text{Arg} \left[- \frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right] \quad (20)$$

²This is valid also for the decay $\bar{B}^0 \rightarrow D^* D$ where there is only one possible polarization state, namely longitudinal polarization. $D^* D$, however, is obviously not a CP eigenstate. The decay to two vector mesons has three amplitudes corresponding to the three different polarization states of the final particles so the situation is much more complex. See the discussion in the next sections.

and the amplitudes \mathcal{T} and \mathcal{P} are real. Using this expression, we find that the asymmetry parameter $Im\lambda_{\mathcal{P}}$ in the presence of penguin contribution takes the following form:

$$Im\lambda_{\mathcal{P}} = -\sin(2\beta) + \mathcal{R} \frac{-2\sin\beta\cos(2\beta-\delta) + \mathcal{R}\sin(2\beta)}{1 - 2\mathcal{R}\cos(\beta-\delta) + \mathcal{R}^2}. \quad (21)$$

Obviously for $\mathcal{R} = 0$, i.e. neglecting the penguin processes, we obtain the ‘clean’ expression (only tree diagram) already given, see eq.(12).

We can obtain an analogous expression for the time dependent asymmetry eq.(8), which is what we actually measure, in terms of \mathcal{R} , δ and β :

$$a_{\mathcal{R}}(t) = a_0 \sin(\Delta mt) + b_0 \cos(\Delta mt) \quad (22)$$

where we have introduced

$$a_0 = \sin(2\beta) + \mathcal{R} \frac{2\sin\beta\cos(2\beta)\cos\delta - \mathcal{R}^2\sin(2\beta)}{1 - 2\mathcal{R}\cos\beta\cos\delta + \mathcal{R}^2} \quad (23)$$

$$b_0 = -\mathcal{R} \frac{2\sin\beta\sin\delta}{1 - 2\mathcal{R}\cos\beta\cos\delta + \mathcal{R}^2}$$

and the time integrated expression reads:

$$a_{\mathcal{R}} = \frac{xa_0 + b_0}{(1+x^2)} \quad (24)$$

where $x = \Delta m/\Gamma = 0.73 \pm 0.05$ [8]. These expressions have to be confronted with the $\mathcal{R} = 0$ case, namely eqs.(11). In this contest, we stress that, due to the contribution of the penguin diagram, the simplified assumption in which $|\lambda| = 1$ is no more true. In other words, this means that the asymmetry is not correlated in the usual simple way to $Im\lambda$; in particular, the time dependent expression will now contain a term depending on $\cos(\Delta mt)$.

Clearly, to extract β from the measurement of $a_{\mathcal{R}}$ in this channel, we must somehow evaluate \mathcal{R} and δ . We note here, however, that \mathcal{R} is not properly a constant, since it contains a hidden dependence on the unitarity angles α and β through the the CKM elements $|V_{td}V_{tb}^*|/|V_{cd}V_{cb}^*|$: this makes evident the complexity of the case. For the purpose of obtaining a numerical estimate, we will make the following somewhat arbitrary approximation:

$$\left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| \sim 1$$

which is the central value of the range permitted by the present experimental knowledge of the CKM elements[8] and corresponds to the choice $\beta = \pi - 2\alpha$. Therefore, the numerical values of \mathcal{R} given in the following must be considered only as order of magnitude estimates.

In the next section, we will review the actual theoretical estimates (if there are any) of \mathcal{R} and δ , making use of factorization of the hadronic amplitudes and of the approach [9] to the calculation of perturbative strong phases. In section 3, we will pass to the complete angular distribution for the decay $B \rightarrow D^* \bar{D}^*$, and in section 4 we will see how angular and isospin analysis can help in the evaluation and perhaps the measurement of these parameters. Finally, we will present some conclusive remarks in section 5.

2 Theoretical Predictions

In order to obtain numerical results for the penguin/tree ratio \mathcal{R} we will make use of the hypothesis of factorization of the hadronic amplitudes (which seems to be on a rather firm basis for color allowed decays like that of interest). We can obtain explicit expression for the amplitudes from the following definitions of the relevant matrix elements[10]:

$$\begin{aligned}
\langle D(p) | A_\mu | 0 \rangle &= -i f_D p_\mu \\
\langle D^*(p; \epsilon) | V_\mu | 0 \rangle &= m_{D^*} f_{D^*} \epsilon_\mu^* \\
\langle D(p) | V_\mu | B(P) \rangle &= (P + p)_\mu F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu (F_0(q^2) - F_1(q^2)) \\
\langle D^*(p; \epsilon) | A_\mu | B(P) \rangle &= i(m_B + m_{D^*}) A_1(q^2) \left(\epsilon_\mu^* - \frac{\epsilon^* \cdot q}{q^2} q_\mu \right) - \\
&\quad -i A_2(q^2) \frac{\epsilon^* \cdot q}{m_B + m_{D^*}} \left((p + P)_\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q_\mu \right) + 2i m_{D^*} A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q_\mu \\
\langle D^*(p; \epsilon) | V_\mu | B(P) \rangle &= \frac{2V(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\rho\sigma} P^\nu p^\rho \epsilon^{*\sigma}
\end{aligned} \tag{25}$$

where in all cases $q = (P - p)$. Using these definitions we easily find the following expressions for the decay amplitudes:

1. scalar-scalar (SS):

$$A(\bar{B}^0 \rightarrow D^+ D^-) = \frac{G_F}{\sqrt{2}} [V_{cb} V_{cd}^* a_1 - V_{tb} V_{td}^* (a_4 + 2(a_6 + a_8)D + a_{10})] F_{SS} \tag{26}$$

2. scalar-vector (SV):

$$A(\bar{B}^0 \rightarrow D^+ D^{*-}) = \frac{G_F}{\sqrt{2}} [V_{cb} V_{cd}^* a_1 - V_{tb} V_{td}^* (a_4 + a_{10})] F_{SV} \tag{27}$$

3. vector-scalar (VS):

$$A(\bar{B}^0 \rightarrow D^{*+} D^-) = \frac{G_F}{\sqrt{2}} [V_{cb} V_{cd}^* a_1 - V_{tb} V_{td}^* (a_4 - 2(a_6 + a_8)D' + a_{10})] A_{VS} \tag{28}$$

4. vector-vector (VV):

$$A(\bar{B}^0 \rightarrow D^{*+}D^{*-}) = \frac{G_F}{\sqrt{2}} [V_{cb}V_{cd}^*a_1 - V_{tb}V_{td}^*(a_4 + a_{10})] A_{VV} \quad (29)$$

where N is the number of colors,

$$a_{2i-1} = c_{2i-1} + \frac{c_{2i}}{N} \quad a_{2i} = c_{2i} + \frac{c_{2i-1}}{N} ,$$

and

$$\begin{aligned} D &= \frac{m_D^2}{(m_b - m_c)(m_c + m_d)} \quad , \quad D' = \frac{m_D^2}{(m_b + m_c)(m_c + m_d)} \\ F_{SS} &= i(m_B^2 - m_D^2)f_D F_0^{BD}(m_D^2) \\ F_{SV} &= \lambda^{1/2}(m_B^2, m_{D^*}^2, m_D^2)f_{D^*} F_1^{BD}(m_{D^*}^2) \\ A_{VS} &= -\lambda^{1/2}(m_B^2, m_{D^*}^2, m_D^2)f_D A_0^{BD^*}(m_D^2) \\ A_{VV} &= m_{D^*} f_{D^*} \left(2 \frac{\epsilon_{\mu\nu\rho\sigma} P^\nu p^\rho \eta^{*\sigma} \epsilon^{*\mu}}{m_B + m_{D^*}} V^{BD^*}(m_{D^*}^2) \right) \\ &\quad - i m_{D^*} f_{D^*} \left[(m_B + m_{D^*})(\epsilon^* \cdot \eta^*) A_1^{BD^*}(m_{D^*}^2) - 2 \frac{(\epsilon^* \cdot q)(\eta^* \cdot p)}{m_B + m_{D^*}} A_2^{BD^*}(m_{D^*}^2) \right] . \end{aligned} \quad (30)$$

In the VV case the form factors $A_1^{BD^*}$, $A_2^{BD^*}$ and V^{BD^*} contribute respectively to s , d waves (CP-even) and p wave (CP-odd). In the cases SV and VS there is obviously only one possible state of polarization for the D^* particle, i.e. the longitudinal polarization.

Using now the above expressions, we can obtain predictions for the amplitudes, in particular for the ratios \mathcal{R} and

$$\mathcal{R}_{\mathcal{EWP}} = \left| \frac{\mathcal{EWP}}{STP} \right| .$$

The results are, for the SS and VS cases:

$$\mathcal{R}_{\mathcal{EWP}} \simeq \begin{cases} 0.01 & \text{for } N = 3 \\ 0.02 & \text{for } N = \infty \end{cases} \quad \mathcal{R} \simeq \begin{cases} 0.09 & \text{for } N = 3 \\ 0.09 & \text{for } N = \infty \end{cases} \quad (31)$$

and for the other cases:

$$\mathcal{R}_{\mathcal{EWP}} \simeq \begin{cases} 0.04 & \text{for } N = 3 \\ 0.05 & \text{for } N = \infty \end{cases} \quad \mathcal{R} \simeq \begin{cases} 0.03 & \text{for } N = 3 \\ 0.03 & \text{for } N = \infty \end{cases} \quad (32)$$

It results that the \mathcal{EWP} contribution is always small, so that we can safely neglect it at this level; the penguin contribution to the overall decay amplitude is important in the SS case, where roughly it can reach the 10% level. On the contrary, reflecting the vector character of (at least) one of the final particles such a contribution is

suppressed over the tree by a factor of ~ 30 in the other cases. (This agrees with the results of references [11, 12]).

We must here remember that these results are model-dependent, they were obtained using factorization. However, within this approach they are independent of form factors models and, as it can be seen, the results are not strongly dependent on the value chosen for N , in particular for the case $N = \infty$ in which factorization is supposed to be exact. So we can retain the value obtained with good confidence, at least at the order of magnitude estimates level.

Now we turn our attention to the strong FSI phases, which can be of two kinds: *soft* FSI phases, which are due to genuine non perturbative long-distance effects such as rescattering of the final particles and the contribution of inelastic channels to the final state[13], and *hard* phases, determined by short-distance physics and calculable by means of the perturbative theory[9].

The latter are given by the absorptive parts of the loop integrals of penguin diagrams, namely from the internal loop propagation of *on shell* $q - \bar{q}$ quarks, which generates an imaginary part of the respective Wilson coefficient. In this contest, we must include in the effective hamiltonian (16) responsible for the quark level process $b \rightarrow c\bar{c}d$ the contributions of the penguins processes with internal c and u quarks; this can be done by rewriting the hamiltonian in the more general form[14]:

$$\mathcal{H}_{eff}^{\Delta B=1} = \sum_i \left[V_{cb}V_{cd}^*c_i^c + V_{ub}V_{ud}^*c_i^u + V_{tb}V_{td}^*c_i^t \right] O_i \quad (33)$$

where O_i are the operators already defined in eq.(17) and c_i are Wilson coefficients; c_i^c contain the contributions of tree diagram and penguin diagram with internal c -quark, while c_i^u and c_i^t contain internal u -quark and t -quark contributions. Clearly, due to the values of the masses, only u and c quarks can access on shell states in the internal loop, so that no absorptive part is generated for the *top*-quark penguin and c_i^t is surely real. However, the statement of CPT invariance requires that also c_i^c is real, because a $c - \bar{c}$ pair is created in the final state[15, 16]. Naively we could say that the rule for CPT conservation requires that, at the considered perturbative order, all *flavour-diagonal* contributions are real, i.e. that no absorptive part is generated for those terms in which the internal quarks pair coincides with the final one. Finally one obtains that, in this case, only c_i^u contributes with an imaginary part.

By introducing this contribution in the calculation of the amplitude, it is possible to compute the perturbative strong phase between tree and penguin diagrams at $O(\alpha_s^2)$, obtaining[11]:

$$\delta \simeq 12^\circ, \quad (34)$$

and we can assume this value as an estimate of the true relative strong FSI phase between tree and penguin amplitudes.

Before continuing, we note that including the contributions of the c and u penguins in the effective hamiltonian of the process affects the results just obtained for

the penguin/tree ratio \mathcal{R} . It results that, at the $\mathcal{O}(\alpha_s)$ order, the penguin terms could change up to 24%[17], so that the values previously cited for \mathcal{R} retain an uncertainty of this order of magnitude.

On the other hand, the problem of evaluating soft phases effects is usually approached by isospin analysis: the possible values of the isospin in the final state $D\bar{D}$ are $I = 0$ and $I = 1$ and correspondingly there are two amplitudes A_0 and A_1 . However, both tree and penguin processes contribute to A_0 and A_1 so that in this case we cannot separate their contributions by means of isospin analysis (see the discussion in section 4). Moreover, since D^+ and D^- do not belong to a same isospin multiplet, a conventional isospin analysis with the use of the charged B decays is not possible (contrary to the case of $B \rightarrow \pi\pi$). Also, we cannot expect that these phases should be small as in $B \rightarrow \pi\pi$, because the final particles are heavy so that they are emitted with slow velocity.

In addition, in the case of $\bar{B}^0 \rightarrow D^{*+}D^{*-}$ there could be relative strong phases between the different amplitudes[18] corresponding to the polarization states so that the number of unknown parameters is potentially greater. However, by means of the angular analysis which we will present in the next section it is possible to fit the values of the strong relative phases between the amplitudes. An analogous analysis was recently performed with success for the $B \rightarrow J/\Psi K^*$ decay[19].

Now, if we assume that the relative strong phase between penguin and tree amplitudes is the same for the three polarization amplitudes involved in the vectorial decay (VV), as it should be on the basis of factorization and perturbative approach, since the three partial waves have a common overall factor depending on the Wilson coefficients (see eq.(29)), also in this case the main source of uncertainty on the extracted value of β remains just the strong phase δ initially introduced in eq.(19).

Under the previous assumptions, we can finally use the results obtained to give an estimate of the error on the measured asymmetry due to the presence of penguin and strong phase contributions. Substituting the numerical values in eq.(24) we obtain (SS decay):

$$a_{\mathcal{R}} = \begin{cases} -0.482 & \text{for } \beta = 45^\circ \\ -0.087 & \text{for } \beta = 5^\circ \end{cases} . \quad (35)$$

These results have to be confronted with the corresponding values of eq. (11):

$$a_{CP} = \begin{cases} -0.476 & \text{for } \beta = 45^\circ \\ -0.083 & \text{for } \beta = 5^\circ \end{cases} . \quad (36)$$

As we can see, the percentual variations due to the polluting contributions of penguins and strong phases is relatively small ($\mathcal{O}(0.05)$) if the strong phase has the considered (small) value. We have studied the dependence of the asymmetry function $a_{\mathcal{R}}(\mathcal{R}, \delta; \beta)$ over a complete range of values for δ ($0 - \pi$) and with \mathcal{R} varying between 0 and 10%. The results are plotted in figure 2 and refer to fixed values of the angle β , respectively $\beta = 5^\circ$ (a), $\beta = 25^\circ$ (b), $\beta = 45^\circ$ (c), where cases (a) and (c) represent the minimum and maximum permitted values of β [20].

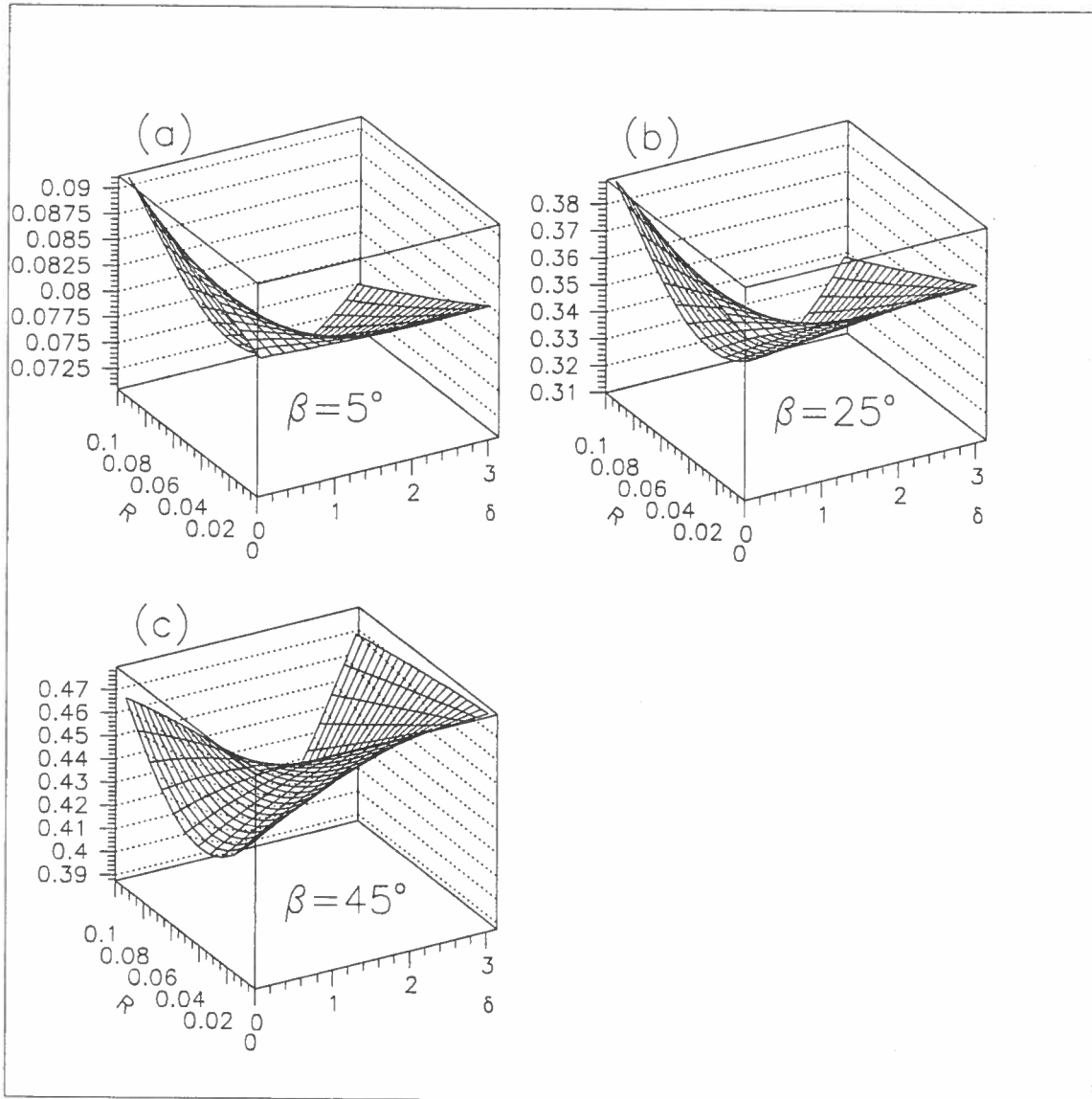


Figure 2: The asymmetry function in dependence on δ and \mathcal{R} for different values of β : $\beta = 5^\circ$ (a), $\beta = 25^\circ$ (b) and $\beta = 45^\circ$ (c).

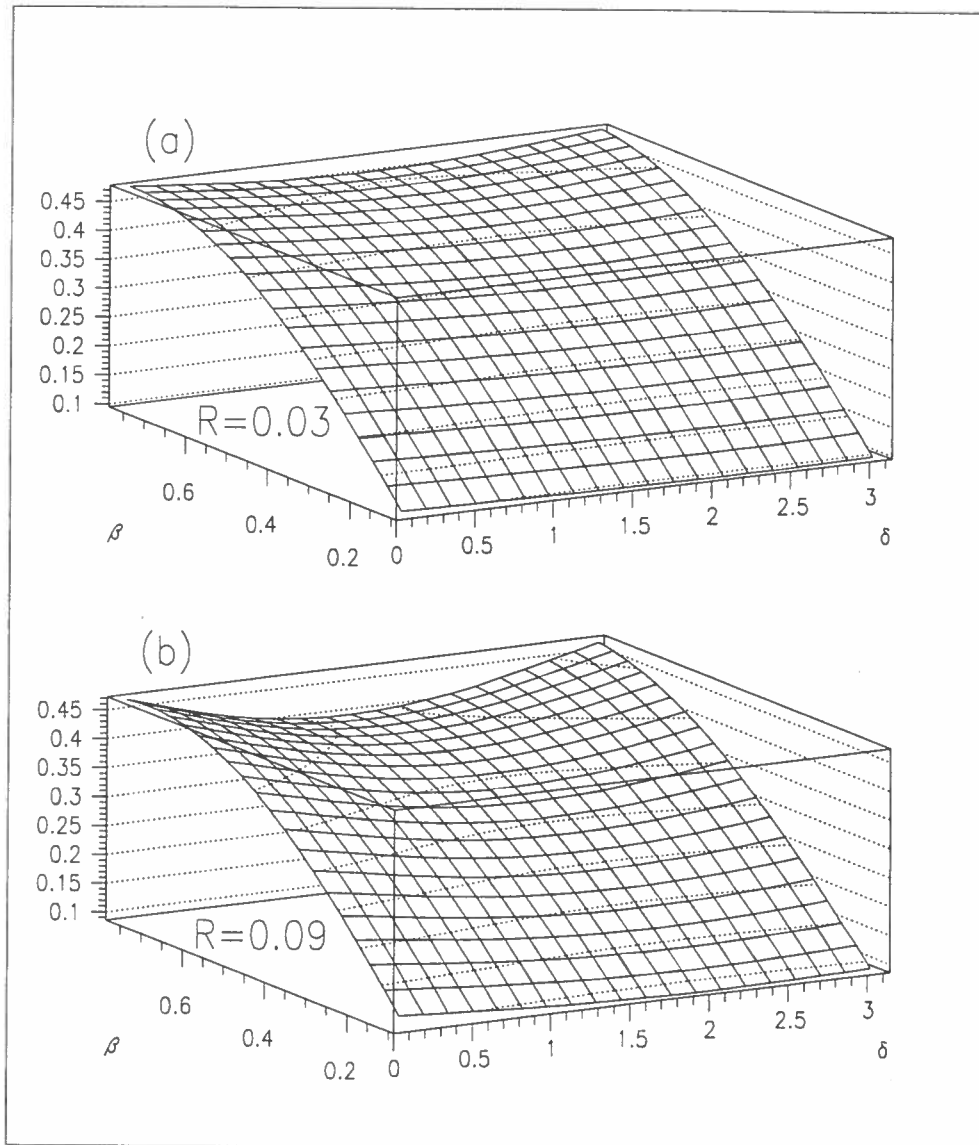


Figure 3: The asymmetry function in dependence on δ and β for fixed values of \mathcal{R} : $\mathcal{R} = 0.03$ (a) and $\mathcal{R} = 0.09$ (b).

From this study, we can see that the effect on the asymmetry from the presence of the strong phase δ varies almost linearly with \mathcal{R} . To be more precise, the following approximated relation holds:

$$\Delta a_{\mathcal{R}}[0 - \pi]_{\delta} \sim 2\mathcal{R} . \quad (37)$$

This means that if the penguin contribution is relevant (as for $B \rightarrow D^+D^-$) the uncertainty due to the strong phase is significant too, while, on the contrary, if the penguin amplitude is suppressed, the influence of δ will be correspondingly smaller. Moreover, this result holds over the complete variability range of β , as can be seen from figure 3, where the asymmetry function is plotted depending on (β, δ) for fixed values of the penguin/tree ratio: $\mathcal{R} = 0.03$ (a), $\mathcal{R} = 0.09$ (b). In particular, we note that, for fixed value of the parameter \mathcal{R} , the biggest discrepancy from the ‘clean’ value is reached at $\delta = \pi/2$.

The implications of this result are important: even if the parameter \mathcal{R} is known, the uncertainty on the asymmetry (and thus on β) due only to the contribution of the strong phase δ is of order $O(\mathcal{R})$, and this is true both for the scalar and the vectorial decay, if, as one could expect, the relative penguin/tree phase is the same for all the polarization amplitudes of the VV decay. This means that, in the case of $B \rightarrow DD$, we predict an uncertainty $\Delta a_{\delta} \sim 20\%$ (when \mathcal{R} is known); on the contrary, for $B \rightarrow D^*D^*$ we expect an uncertainty of the 6% level (with $\mathcal{R} = 3\%$). In this sense, the vectorial channel should be more reliable for the measurement of the unitarity angle than the scalar one. Moreover, as we will see in the next section, we can use the simplifying assumption in which we neglect the penguin contribution: the accuracy of this approximation is $O(0.06)$.

3 The decay $B \rightarrow D^{*+}D^{*-}$

We now turn our attention to the study of the vectorial channel $B \rightarrow D^{*+}D^{*-}$; as already said, since the D^* mesons are spin 1 particles, the decay goes through three waves (s, p, d). By means of the usual symmetry considerations, it is possible to parametrize the decay amplitude in the following way:

$$\begin{aligned} \langle D^{*+}(p, \epsilon), D^{*-}(k, \mu) | T | B(P) \rangle = & \quad (38) \\ (\epsilon^* \cdot \mu^*) f_s + \frac{(\epsilon^* \cdot k)(p \cdot \mu^*)}{m_{D^*}^2} f_d + i\epsilon_{\mu\nu\rho\sigma} \frac{\epsilon^{*\mu} \mu^{*\nu} k^\rho P^\sigma}{m_{D^*}^2} f_p \end{aligned}$$

where we conventionally chose the B meson rest frame, (p, ϵ) and (k, μ) are respectively the D^{*+} and D^{*-} four-momentum and polarization vector, and f_i ($i = s, p, d$) are the amplitudes corresponding to the three waves. Following from the conservation of angular momentum, there are only three possible polarization states for $D^{*+}D^{*-}$, namely the longitudinal polarization state ($D_L^{*+}(0)D_L^{*-}(0)$) and two transverse polarization states ($D_T^{*+}(\pm)D_T^{*-}(\pm)$). We then define the helicity amplitudes

corresponding to the polarization states as follows:

$$\begin{aligned} H_{\pm 1} &= \langle D_T^{*\pm}(\pm) D_T^{*-}(\pm) | T | B^0 \rangle \\ H_0 &= \langle D_L^{*+}(0) D_L^{*-}(0) | T | B^0 \rangle ; \end{aligned} \quad (39)$$

now using eq.(38) and substituting the explicit expressions of the polarization vectors we easily find:

$$\begin{aligned} H_0 &= \frac{1}{m_{D^*}^2} \left[-\frac{m_B^2 - 2m_{D^*}^2}{2} f_s - \frac{m_B^2(m_B^2 - 4m_{D^*}^2)}{4m_{D^*}^2} f_d \right] \\ H_1 &= f_s + \frac{m_B}{2m_{D^*}^2} (m_B^2 - 4m_{D^*}^2)^{1/2} f_p \\ H_{-1} &= f_s - \frac{m_B}{2m_{D^*}^2} (m_B^2 - 4m_{D^*}^2)^{1/2} f_p . \end{aligned} \quad (40)$$

Recalling the expression of the amplitude eq.(29), obtained using factorization and the Bauer-Stech-Wirbel model[10], it is possible to write the explicit expressions for the three waves amplitude:

$$\begin{aligned} f_s &= \frac{G_F}{\sqrt{2}} V(a) f_{D^*} m_{D^*}^2 \left(\frac{m_B + m_{D^*}}{m_{D^*}} \right) A_1(m_{D^*}^2) \\ f_d &= -2 \frac{G_F}{\sqrt{2}} V(a) f_{D^*} m_{D^*}^2 \left(\frac{m_{D^*}}{m_B + m_{D^*}} \right) A_2(m_{D^*}^2) \\ f_p &= 2 \frac{G_F}{\sqrt{2}} V(a) f_{D^*} m_{D^*}^2 \left(\frac{m_{D^*}}{m_B + m_{D^*}} \right) V(m_{D^*}^2) \end{aligned} \quad (41)$$

where we have introduced:

$$V(a) = V_{cb} V_{cd}^* a_1 - V_{tb} V_{td}^* (a_4 + a_{10}) .$$

Remembering that the three waves have definite CP-parity (s and d are CP-even and p is CP-odd) we can construct the amplitudes with definite behaviour under CP; we define:

$$H_{\pm 1} = \frac{1}{\sqrt{2}} (A_{\parallel} \pm A_{\perp}), \quad H_0 = A_0 . \quad (42)$$

where A_0 and A_{\parallel} are CP-even and A_{\perp} is CP-odd. Now, making use of the Heavy Quark Effective Theory[21] it is possible to obtain numerical results for the relative rates corresponding to these definite CP contributions; one finds that[22]:

$$\begin{aligned} \frac{\Gamma_0(B \rightarrow D^{*+} D^{*-})}{\Gamma(B \rightarrow D^{*+} D^{*-})} &\simeq 54\% \\ \frac{\Gamma_{\parallel}(B \rightarrow D^{*+} D^{*-})}{\Gamma(B \rightarrow D^{*+} D^{*-})} &\simeq 40\% \\ \frac{\Gamma_{\perp}(B \rightarrow D^{*+} D^{*-})}{\Gamma(B \rightarrow D^{*+} D^{*-})} &\simeq 6\% . \end{aligned} \quad (43)$$

The model so predicts a final state which is almost a pure CP-even eigenstate:

$$D^{*+}D^{*-} \Rightarrow \begin{cases} 94\% & \text{CP-even} \\ 6\% & \text{CP-odd} \end{cases} . \quad (44)$$

This numerical result is expected to be quite stable since it does not depend on the particular parametrization chosen for the form-factor of the Isgur-Wise model and, moreover, it can be essentially rederived in the framework of the Bauer-Stech-Wirbel model (in this case retaining a form-factor dependence). From this value, we obtain a prediction for the dilution factor introduced in eq.(15); the result is:

$$K = 0.88 . \quad (45)$$

Having defined the amplitudes involved and reviewed the theoretical prediction, we pass to the complete angular analysis of the decay. The full angular distribution of the rate in the VV case is specified by three angles; in the *helicity* basis (see fig.4a) they are defined to be the polar angle of π^+ in the D^{*+} rest frame (ϑ_1), the polar angle of π^- in the D^{*-} rest frame (ϑ_2) and the angle between the D^{*+} and the D^{*-} decay planes (ϕ). In this frame we have:

$$\begin{aligned} \frac{d^3\Gamma(B^0 \rightarrow D^{*+}D^{*-})}{d \cos \vartheta_1 d \cos \vartheta_2 d\phi} &= \frac{k}{4m_B^2} \frac{9}{8 \cdot (2\pi)^2} \left[\frac{1}{2} \sin^2 \vartheta_1 \sin^2 \vartheta_2 (|H_1|^2 + |H_{-1}|^2) \right. \\ &+ \sin^2 \vartheta_1 \sin^2 \vartheta_2 (\text{Re}(H_1 H_{-1}^*) \cos(2\phi) \\ &- \text{Im}(H_1 H_{-1}^*) \sin(2\phi)) + 2|H_0|^2 \cos^2 \vartheta_1 \cos^2 \vartheta_2 \\ &+ \frac{1}{2} \sin(2\vartheta_1) \sin(2\vartheta_2) (\text{Re}(H_0 H_{-1}^* + H_1 H_0^*) \cos \phi \\ &\left. - \text{Im}(H_0 H_{-1}^* + H_1 H_0^*) \sin \phi \right] , \end{aligned} \quad (46)$$

where the amplitudes were defined in eq.(39) and k is the final momentum of the D^* mesons:

$$k = \frac{\sqrt{m_B^2 - 4m_{D^*}^2}}{2} . \quad (47)$$

Another basis particularly useful in studying the composition in CP-eigenstates of the final state is *transversity* basis: in this system (see fig. 4b) the angles are the polar angle of the π^+ in the D^{*+} rest frame (ϑ_1), the polar angle between the normal to the D^{*+} decay plane and the π^- flight line (ϑ_{tr}) and the relative azimuthal angle (ϕ_{tr}). With these definitions we obtain the following angular distribution:

$$\begin{aligned} \frac{d^3\Gamma(B \rightarrow D^*D^*)}{d \cos \vartheta_1 d \cos \vartheta_{tr} d\phi_{tr}} &= \frac{k}{4m_B^2} \frac{9}{8 \cdot (2\pi)^2} \left[|A_{\parallel}|^2 \sin^2 \vartheta_1 \sin^2 \vartheta_{tr} \sin^2 \phi_{tr} \right. \\ &+ |A_{\perp}|^2 \sin^2 \vartheta_1 \cos^2 \vartheta_{tr} \\ &+ 2|A_0|^2 \cos^2 \vartheta_1 \sin^2 \vartheta_{tr} \cos^2 \phi_{tr} \\ &\left. + \text{Im}(A_{\perp} A_{\parallel}^*) \sin^2 \vartheta_1 \sin(2\vartheta_{tr}) \sin \phi_{tr} \right] \end{aligned} \quad (48)$$

$$\begin{aligned}
& + \frac{1}{\sqrt{2}} \text{Re}(A_0 A_{\parallel}^*) \sin(2\vartheta_1) \sin^2 \vartheta_{tr} \sin(2\phi_{tr}) \\
& + \frac{1}{\sqrt{2}} \text{Im}(A_0^* A_{\perp}) \sin(2\vartheta_1) \sin(2\vartheta_{tr}) \cos \phi_{tr} .
\end{aligned}$$

where the amplitudes with definite CP were introduced in eq.(42). The following relations hold for the angles in the two basis:

$$\begin{aligned}
\cos \vartheta_2 &= -\sin \vartheta_{tr} \cos \phi_{tr} \\
\sin \vartheta_2 \sin \phi &= \cos \vartheta_{tr} \\
\sin \vartheta_2 \cos \phi &= -\sin \vartheta_{tr} \sin \phi_{tr} .
\end{aligned} \tag{49}$$

This basis has the important property that the dependence on the so called *transversity angle* ϑ_{tr} distinguishes between different CP contributions. Infact, integrating over $d \cos \vartheta_1$ and $d\phi_{tr}$ we obtain:

$$\frac{d\Gamma(B^0 \rightarrow D^* D^*)}{d \cos \vartheta_{tr}} \propto \Gamma_+ (1+a) \frac{3}{4} \sin^2 \vartheta_{tr} + \Gamma_- (1-a) \frac{3}{2} \cos^2 \vartheta_{tr} \tag{50}$$

where a is just the asymmetry parameter proportional to $\sin(2\beta)$ and the definite CP widths and amplitudes were defined in eq.(13) and eq.(42):

$$\begin{aligned}
\Gamma_+ &= |A_{\parallel}|^2 + |A_0|^2 \\
\Gamma_- &= |A_{\perp}|^2 .
\end{aligned} \tag{51}$$

As we have seen in the previous section, the penguin contribution to the rate in the factorization framework is rather small ($\mathcal{R} \simeq 3\%$). So, with a great advantage in simplicity, we can work with a relatively good accuracy in the approximation in which we neglect the penguin contribution to the amplitude and consider the tree amplitude as completely dominating; in this case it is possible to parametrize the time dependent transversity amplitudes as follows[18]:

$$\begin{aligned}
A_{\parallel}(t) &= \mathcal{M}_{\parallel} e^{i\alpha_{\parallel}} e^{-imt - \Gamma t/2} \left[e^{i\beta} \cos\left(\frac{\Delta mt}{2}\right) + i e^{-i\beta} \sin\left(\frac{\Delta mt}{2}\right) \right] \\
A_{\perp}(t) &= \mathcal{M}_{\perp} e^{i\alpha_{\perp}} e^{-imt - \Gamma t/2} \left[-e^{i\beta} \cos\left(\frac{\Delta mt}{2}\right) + i e^{-i\beta} \sin\left(\frac{\Delta mt}{2}\right) \right] \\
A_0(t) &= \mathcal{M}_0 e^{i\alpha_0} e^{-imt - \Gamma t/2} \left[e^{i\beta} \cos\left(\frac{\Delta mt}{2}\right) + i e^{-i\beta} \sin\left(\frac{\Delta mt}{2}\right) \right]
\end{aligned} \tag{52}$$

where \mathcal{M}_i are the absolute values of the amplitudes introduced in eq.(42) and α_i are the strong phases ($i = 0, \parallel, \perp$). From these it results:

$$\begin{aligned}
\Gamma_+(t) &= |A_{\parallel}(t)|^2 + |A_0(t)|^2 = (\mathcal{M}_{\parallel}^2 + \mathcal{M}_0^2) e^{-\Gamma t} [1 + \sin(2\beta) \sin(\Delta mt)] \\
\Gamma_-(t) &= |A_{\perp}(t)|^2 = \mathcal{M}_{\perp}^2 e^{-\Gamma t} [1 - \sin(2\beta) \sin(\Delta mt)] .
\end{aligned} \tag{53}$$

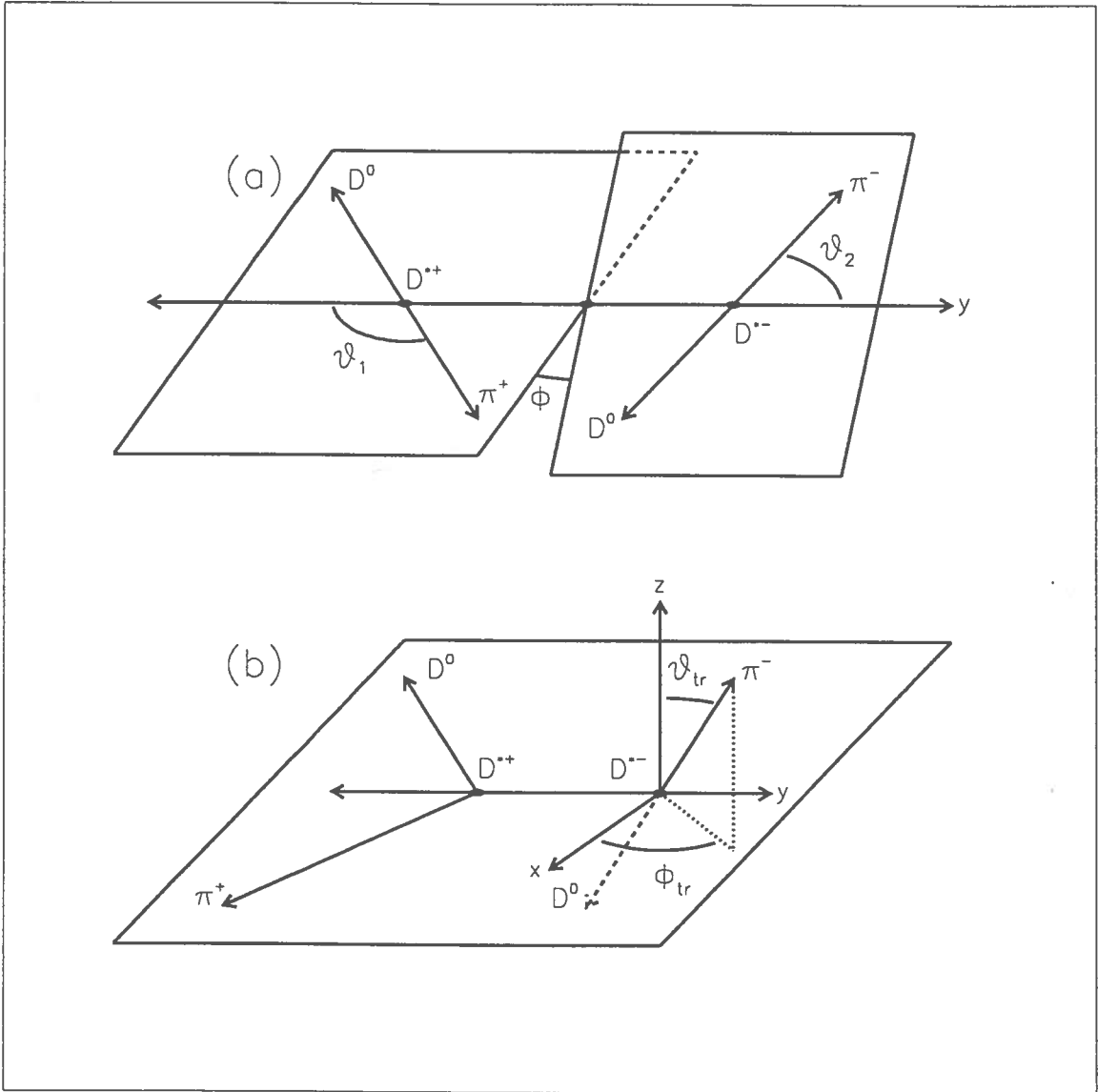


Figure 4: (a) Helicity Frame, (b) Transversity Frame.

We can now substitute these parametrizations in eqs. (46) and (48) to obtain the complete expressions of the rate: explicit formulas are given in the Appendix.

We recall that, in the factorization hypothesis, all the amplitudes are assumed to be real, so that it is not possible to obtain any information about the phases: we can now see how the complete angular analysis permits to fit the values of the three amplitudes and of the relative strong phases between the amplitudes $\alpha_{\parallel} - \alpha_0$ and α_{\perp} (where we have conventionally chosen $\alpha_{\perp} = 0$). This point is particularly important, because the knowledge of these phases could allow to shed further light on the strong phases effects in this channel and so, possibly, to reduce the uncertainty which we must retain at the moment. In particular it is possible to construct conveniently chosen rate asymmetries which directly depend on strong phases. As examples, we present the following two expressions in the helicity frame (see appendix):

- Up-Down asymmetry:

$$\int_{-1}^1 d \cos \vartheta_1 \int_{-1}^1 d \cos \vartheta_2 \int_0^{\infty} dt A^{UD} = \frac{\tau_B}{\pi^2} \left(4\mathcal{M}_{\perp}^2 - \frac{\sqrt{2}}{2} \frac{\mathcal{M}_0 \mathcal{M}_{\perp}}{1+x^2} \sin \alpha_0 \right) \quad (54)$$

where we have defined

$$A^{UD} = \left[\int_0^{\pi} d\phi - \int_{\pi}^{2\pi} d\phi \right] \frac{d^3\Gamma(B^0 \rightarrow D^{*+} D^{*-})}{d \cos \vartheta_1 d \cos \vartheta_2 d\phi} ;$$

- Left-Right asymmetry:

$$\int_{-1}^1 d \cos \vartheta_1 \int_{-1}^1 d \cos \vartheta_2 \int_0^{\infty} dt A^{LR} = -\frac{4\tau_B}{\pi\sqrt{2}} \frac{x \sin(2\beta) \mathcal{M}_0 \mathcal{M}_{\parallel} \sin(\alpha_0 - \alpha_{\parallel})}{1+x^2} \quad (55)$$

where

$$A^{LR} = \left[\int_{\pi/2}^{3\pi/2} d\phi - \int_{-\pi/2}^{\pi/2} d\phi \right] \frac{d^3\Gamma(B^0 \rightarrow D^{*+} D^{*-})}{d \cos \vartheta_1 d \cos \vartheta_2 d\phi} .$$

Obviously analogous expressions could be found in the transversity frame.

4 Isospin analysis

We have seen in the previous section how angular analysis permits the determination of the different polarization amplitudes and of the relative strong phases. Another useful tool to obtain information on the considered channel is represented in principle by isospin analysis: even if, as already said, a conventional analysis as that performed for the $B \rightarrow \pi\pi$ decay is not obviously applicable in this case, we can anyway make some interesting comments.

We can express the decay amplitudes for $B \rightarrow D\bar{D}$ in terms of isospin amplitudes and FSI phases; one finds:

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^+D^-) &= \frac{1}{2} (A_0 e^{i\delta_0} - A_1 e^{i\delta_1}) \\ A(\bar{B}^0 \rightarrow D^0\bar{D}^0) &= \frac{1}{2} (A_0 e^{i\delta_0} + A_1 e^{i\delta_1}) . \end{aligned} \quad (56)$$

In the factorization model the amplitude for the exchange process vanishes:

$$A_{fact}(\bar{B}^0 \rightarrow D^0\bar{D}^0) = 0 ;$$

however in this framework the amplitudes are assumed to be real: this means that the following relation holds

$$A_0 = -A_1 . \quad (57)$$

Obviously, taking into account the presence of FSI phases can modify this prediction: infact, if the phase $\delta_I = \delta_0 - \delta_1$ takes the value $\delta_I = \pi$ one would clearly obtain a quite dramatic result (from eq.(56)):

$$A(\bar{B}^0 \rightarrow D^+D^-) = 0 .$$

It is therefore interesting to obtain phase-independent quantities which could be experimentally tested; from eq.(56) one finds the following combination:

$$B(\bar{B}^0 \rightarrow D^+D^-) + B(\bar{B}^0 \rightarrow D^0\bar{D}^0) \simeq 4.7 \times 10^{-4} \quad (58)$$

where the numerical value is obtained using factorization.

A theoretical evaluation of the FSI phase can come from inelastic effects: the decay $\bar{B}^0 \rightarrow D\bar{D}$, infact, could pass through a two-step process such as $\bar{B}^0 \rightarrow D^*\bar{D}^* \rightarrow D\bar{D}$ with the generation of *inelastic* FSI phases[13]. In this framework also the absolute values of the amplitudes are affected and one obtains a branching ratio for the exchange process $B(\bar{B}^0 \rightarrow D^0\bar{D}^0)$ which is suppressed by a factor $10^{-1} - 10^{-2}$ with respect to $B(\bar{B}^0 \rightarrow D^+D^-)$, which is probably a more realistic estimate. Obviously this result is strongly dependent on the particular choice for the parameters which describe the inelastic scattering process.

According to simple isospin symmetry and neglecting the exchange process, the following equality holds:

$$A(\bar{B}^0 \rightarrow D^{(*)+}D^{(*)-}) = A(B^- \rightarrow D^{(*)0}D^{(*)-}) \quad (59)$$

where in the vectorial case the relation is intended to hold separately for the three polarization amplitudes. So, even if it is not possible to obtain simple relations between isospin strong phases, because neither D^+D^- nor $D^0\bar{D}^0$ belong to the same isospin multiplet, we can use isospin correlated charged decay in order to estimate

amplitude absolute values, in particular to evaluate the parameter \mathcal{R} previously defined. Moreover, since charged B do not mix with each other, this measure should presumably be more clean than that directly obtained from B^0 decays. This discussion is valid for the scalar as for the vectorial decay, since, obviously, the angular dependence of the neutral and charged B decays to two D vector mesons is the same. As an example we compare the expressions of the rate of the B^0 and B^+ decays in the simple case of transversity analysis (see eq.(50)): we account for the presence of penguin and strong phases using the parametrizations (19) and (52) of the amplitudes. We find:

$$\begin{aligned}
\frac{d\Gamma(B^0 \rightarrow D^{*+} D^{*-})}{d \cos \vartheta_{tr}} &\propto (\mathcal{M}_{\parallel}^2 + \mathcal{M}_0^2)[1 + \mathcal{R}^2] \\
&+ \sin(2\beta) \sin(\Delta mt) \\
&- 2\mathcal{R} \cos \delta (\cos \beta + \sin \beta \sin(\Delta mt)) \\
&- 2\mathcal{R} \sin \delta \sin \beta \cos(\Delta mt) \Big] \frac{3}{4} \sin^2 \vartheta_{tr} \\
&+ \mathcal{M}_{\perp}^2 [1 + \mathcal{R}^2] \\
&- \sin(2\beta) \sin(\Delta mt) \\
&- 2\mathcal{R} \cos \delta (\cos \beta - \sin \beta \sin(\Delta mt)) \\
&- 2\mathcal{R} \sin \delta \sin \beta \cos(\Delta mt) \Big] \frac{3}{2} \cos^2 \vartheta_{tr}
\end{aligned} \tag{60}$$

where we have supposed that the relative penguin/tree strong phase δ is the same for the three amplitudes (see previous discussion), and correspondingly:

$$\begin{aligned}
\frac{d\Gamma(B^+ \rightarrow D^{*+} \bar{D}^{*0})}{d \cos \vartheta_{tr}} &\propto (\mathcal{M}_{\parallel}^2 + \mathcal{M}_0^2)[1 + \mathcal{R}^2] \\
&- 2\mathcal{R} \cos \delta_+ \cos \beta \\
&- 2\mathcal{R} \sin \delta_+ \sin \beta \Big] \frac{3}{4} \sin^2 \vartheta_{tr} \\
&+ \mathcal{M}_{\perp}^2 [1 + \mathcal{R}^2 - 2\mathcal{R} \cos \delta_+ \cos \beta \\
&- 2\mathcal{R} \sin \delta_+ \sin \beta \Big] \frac{3}{2} \cos^2 \vartheta_{tr}
\end{aligned} \tag{61}$$

with δ_+ the relative penguin/tree strong phase of this decay. From these expressions it should be possible to fit separately the values of \mathcal{R} and \mathcal{M}_i ($i = \parallel, \perp, 0$) from the charged decays, even if no direct relation exists between the strong phases of the two decays.

Actually, the complete relation between charged and neutral B decays has the form[4]:

$$A(\bar{B}^- \rightarrow D^{(*)0} D^{(*)-}) = A(\bar{B}^0 \rightarrow D^{(*)+} D^{(*)-}) + A(\bar{B}^0 \rightarrow D^{(*)0} D^{(*)0}) . \tag{62}$$

Since from the previous discussion we know that the left hand side of eq.(62) is phase-independent, the measurement of $\bar{B}^0 \rightarrow D^- D^0$ could furnish the value of the isospin

amplitude A_0 , which, in the factorization framework, is the same as A_1 (see eq.(57)). Thus, the separate measurement of $\bar{B}^0 \rightarrow D^- D^+$ and $\bar{B}^0 \rightarrow D^0 \bar{D}^0$ would give the value of the relative FSI isospin phase δ_I . From these considerations we see that the measurement of the decay $\bar{B}^0 \rightarrow D^0 \bar{D}^0$ can have a significant relevance for detecting the FSI effects and could potentially allow to extract the value of the relative phase δ_I between the two different isospin amplitudes. We stress, however, that, since both tree and penguin processes contribute to $I = 0$ and $I = 1$ amplitudes, this phase is not directly related to the relative penguin/tree phase δ previously introduced in eq.(19) and responsible for the uncertainty on the CP asymmetry. However, as results from a recent analysis[23], a careful study of the isospin amplitudes of the $B \rightarrow D\bar{D}$ together with the information extracted from the $B \rightarrow J/\Psi K_s$ channel could lead, at least in principle, to the determination of the penguin-induced phase contribution.

Introducing for completeness the $B \rightarrow DD^*$ channel, which obviously is not a CP eigenstate we note that, since only one wave contributes to the decay, the angular analysis is not useful in this case. However, one can construct the linear combination with definite CP-parities:

$$|\pm\rangle = \frac{1}{\sqrt{2}} (|D^{*+} D^- \rangle \pm |D^{*-} D^+ \rangle) . \quad (63)$$

It has been demonstrated[24] that, in the heavy quark limit (HQL)[21], the process $B \rightarrow |+\rangle$ occurs at the tree level, while $B \rightarrow |-\rangle$ can occur only through the exchange diagram. As a result, the state $D^{*+} D^- + D^{*-} D^+$ is predicted to be a CP-eigenstate with an error that should be as small as 1%. Clearly, this result is model-dependent and can be somewhat affected by deviations from the HQL. Moreover, the measurement of the rate of the $\bar{B}^0 \rightarrow D^0 \bar{D}^0$ decay can provide useful tests about the significance of the exchange contribution. However this channel, having only one polarization amplitude, avoids the problem of the "multiplication" of strong phases which occurs in the decay to two vector mesons, and, moreover, has little contribution from the penguin diagram since a vector meson is present in the final state ($\mathcal{R} \simeq 0.03$). Therefore, $B \rightarrow DD^*$ decays offer interesting opportunities for the measurement of the unitarity angle β coming from $D^* D^*$ and DD decays.

5 Conclusion

In this note, we have presented a theoretical analysis of the $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decay modes, with particular attention to the strong penguin and strong phases pollution to the CP violating asymmetry. We have found that, since the penguin/tree rate \mathcal{R} is expected to be important in the scalar case the uncertainty on the asymmetry coming from the presence of the strong phase δ can be substantial, reaching the 20% level.

On the contrary, the penguin contribution to vectorial decays is a factor of 3 smaller and consequently even the strong phase effects are expected to be less important. This allows, as a first approximation (roughly at the 6% level), to neglect these contributions, performing a much more simplified analysis. In any case, the study of isospin correlated charged B decays and in particular of the decay $\bar{B}^0 \rightarrow D^0 \bar{D}^0$ can provide important informations on the absolute values of the amplitudes involved and moreover could give the values of the isospin amplitudes and relative phases. At least in principle, this could allow the extraction of some information on the relative strong phase δ between penguin and tree amplitudes, which at the moment is probably the main source of uncertainty on the CP-violating asymmetry.

Finally, the 'mixed' vectorial decay $B \rightarrow DD^*$, though not being a CP eigenstate, is expected to be a source of interesting experimental information and could provide comparative tests on the measurement of the unitarity angle β from the other channels.

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Appendix

These are the complete angular distributions (we have chosen $\alpha_{\perp} = 0$).

Helicity Frame

$$\begin{aligned}
& \frac{d^3\Gamma(B^0(\bar{B}^0) \rightarrow D^{*+}D^{*-})}{d \cos \vartheta_1 d \cos \vartheta_2 d\phi} \\
= & \frac{k}{4m_B^2} \frac{9}{8 \cdot (2\pi)^2} e^{-\Gamma t} \left\{ \frac{1}{2} \sin^2 \vartheta_1 \sin^2 \vartheta_2 [(\mathcal{M}_{\parallel}^2 + \mathcal{M}_{\perp}^2) - (+) \right. \\
- (+) & (\mathcal{M}_{\perp}^2 - \mathcal{M}_{\parallel}^2) \sin(2\beta) \sin(\Delta mt)] + \\
+ & \frac{1}{2} \sin^2 \vartheta_1 \sin^2 \vartheta_2 [(\mathcal{M}_{\parallel}^2 - \mathcal{M}_{\perp}^2) + (-) \\
+ (-) & (\mathcal{M}_{\parallel}^2 + \mathcal{M}_{\perp}^2) \sin(2\beta) \sin(\Delta mt)] \cos(2\phi) - \\
- & 2\mathcal{M}_{\parallel}\mathcal{M}_{\perp} (\sin \alpha_{\parallel} \cos(\Delta mt) + (-) \\
+ (-) & \cos \alpha_{\parallel} \cos(2\beta) \sin(\Delta mt)] \sin(2\phi) + \\
+ & 2 \cos^2 \vartheta_1 \cos^2 \vartheta_2 [\mathcal{M}_0^2 + (-)\mathcal{M}_0^2 \sin(2\beta) \sin(\Delta mt)] + \\
+ & \frac{1}{2} \sin(2\vartheta_1) \sin(2\vartheta_2) [(\mathcal{M}_{\parallel}\mathcal{M}_0 \cos(\alpha_{\parallel} - \alpha_0) + (-)
\end{aligned}$$

$$\begin{aligned}
& +(-) \mathcal{M}_{\parallel} \mathcal{M}_0 \cos(\alpha_{\parallel} - \alpha_0) \sin(2\beta) \sin(\Delta mt) \cos \phi - \\
& - (\mathcal{M}_{\perp} \mathcal{M}_0 \sin \alpha_0 \cos(\Delta mt) + (-) \\
& +(-) \mathcal{M}_{\perp} \mathcal{M}_0 \cos \alpha_0 \cos(2\beta \sin(\Delta mt)) \sin \phi] \}
\end{aligned}$$

Transversity frame

$$\begin{aligned}
& \frac{d^3 \Gamma(B^0(\bar{B}^0) \rightarrow D^{*+} D^{*-})}{d \cos \vartheta_1 d \cos \vartheta_{tr} d \phi_{tr}} \\
= & \frac{k}{4m_B^2} \frac{9}{8 \cdot (2\pi)^2} e^{-\Gamma t} \{ \mathcal{M}_{\parallel}^2 (1 + (-) \\
+(-) & \sin(2\beta) \sin(\Delta mt) \sin^2 \vartheta_1 \sin^2 \vartheta_{tr} \sin^2 \phi_{tr} + \\
& + \mathcal{M}_{\perp}^2 (1 - (+) \sin(2\beta) \sin(\Delta mt) \sin^2 \vartheta_1 \cos^2 \vartheta_{tr} + \\
& + 2\mathcal{M}_0^2 (1 + (-) \sin(2\beta) \sin(\Delta mt) \cos^2 \vartheta_1 \sin^2 \vartheta_{tr} \cos^2 \phi_{tr} \\
& + \mathcal{M}_{\perp} \mathcal{M}_{\parallel} \sin^2 \vartheta_1 \sin(2\vartheta_{tr}) \sin \phi_{tr} [\sin \alpha_{\parallel} \cos(\Delta mt) + (-) \\
+(-) & \cos(2\beta) \cos \alpha_{\parallel} \sin(\Delta mt)] + \\
& + \frac{1}{\sqrt{2}} \mathcal{M}_0 \mathcal{M}_{\parallel} \sin(2\vartheta_1) \sin^2 \vartheta_{tr} \sin(2\phi_{tr}) (1 + (-) \\
+(-) & \sin(2\beta) \sin(\Delta mt) \cos(\alpha_{\parallel} - \alpha_0) + \\
& + \frac{1}{\sqrt{2}} \mathcal{M}_0 \mathcal{M}_{\perp} \sin(2\vartheta_1) \sin(2\vartheta_{tr}) \cos \phi_{tr} [\sin \alpha_0 \cos(\Delta mt) + (-) \\
+(-) & \cos(2\beta) \cos \alpha_0 \sin(\Delta mt)] \} .
\end{aligned}$$

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