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MEASUREMENT OF m_W AT $\sqrt{s} = 172$ GeV WITH A DIRECT RECONSTRUCTION METHOD USING THE DELPHI DETECTOR AT LEP

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1 Introduction

This note reports on the analysis for the determination of m_W with the DELPHI data at $\sqrt{s} = 172$ GeV and is organized as follows: after describing the event selection, the constrained fitting technique is illustrated, with some emphasis on the mass error treatment; the fitting procedure using differential distributions obtained with the WPHACT [1] generator is then presented, and the results in the semileptonic and the fully hadronic channel are discussed, together with the systematic errors. Finally a combined measurement of m_W is given, followed by the conclusions.

2 Event selection

The track selection was performed as follows:

- charged particles were selected if they fulfilled the following criteria:
  - polar angle with respect to the beam direction between 10° and 170°;
  - momentum greater than 0.4 GeV and its relative error less than 1;
  - track length greater than 15 cm;
  - transverse and longitudinal (with respect to the beam direction) impact parameter less than 4 cm.
- for neutral particles the energy of the shower was required to be greater than 0.5 GeV.

The efficiency of the selection cuts on the WW signal and the background contamination were obtained using PYTHIA [2] Monte Carlo samples interfaced with the JETSET hadronization package [2] and the full detector simulation given by DELSIM [3].

2.1 Semileptonic channel

To select topologies where one W decays hadronically and the other in one lepton and one neutrino, events with a visible energy above $0.4\sqrt{s}$, at least six charged tracks and three jets reconstructed by the clustering algorythm LUCLUS[1] [4] were considered. The event was required to have at least 10 GeV of missing momentum not pointing to the beam pipe region and no isolated photon with energy above 30 GeV. The lepton was identified as an isolated charged track with momentum $p > 5$ GeV. The isolation was assessed with respect to the closest charged track with energy above 1 GeV and the isolation angle θ_{iso} was required to be at least 10°. If more than one lepton candidate was found, the one with highest $p * \theta_{iso}$ was chosen.

At the present state of the analysis no useful mass information can be extracted from the τ channel, therefore only $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$ final states were studied:

[1] $d_{join} = 6.5$ GeV was used in this analysis.

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MEASUREMENT OF m_W AT $\sqrt{s} = 172$ GeV WITH A DIRECT RECONSTRUCTION METHOD USING THE DELPHI DETECTOR AT LEP

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Abstract

In this note a technique for the determination of the W mass using the direct reconstruction method is discussed. The selection criteria of WW events, the constrained fitting method for the reconstruction of the resonance and its effects on the error on the mass are described. The technique is based on an unbinned maximum likelihood fit to the reconstructed mass distributions of the $d\sigma/dm$ function obtained with the WPHACT generator and is applied to both fully hadronic and mixed hadronic-leptonic final states. The performance of the method and the results obtained on the DELPHI data at $\sqrt{s} = 172$ GeV are presented.

- the muons were identified requiring hits in the muon chambers or energy deposits in the hadron calorimeter consistent with a minimum ionizing particle and no electromagnetic deposits above 5 GeV;

- the electrons were identified as charged tracks with associated electromagnetic energy greater than 5 GeV and hadronic energy below 5 GeV.

The semileptonic channel suffers from very little background contamination, coming from QCD (especially $Z \to b\bar{b}$ events) and four fermion neutral currents processes. The following quality cuts were applied to further increase the purity:

- in the $q\bar{q}\mu\nu_\mu$ channel the angle between the muon and the missing momentum was required to be greater than 50°; if the muon momentum was lower than 25 GeV the missing momentum was required to be at least 15° away from the beam direction, and if lower than 15 GeV the additional cut $\theta_{iso}(\mu) > 30°$ was applied;

- similar cuts were used in the $q\bar{q}e\nu_e$ channel. Moreover, in the forward region only electron candidates with energy in the interval 20-60 GeV were selected and the event aplanarity was required to be greater than 20°, to reject topologies with a converted ISR photon and wrong association of neutral clusters to charged tracks;

- to further reduce the four fermion neutral currents background, in both channels events were rejected if two isolated leptons were found with same flavour, opposite charge and sum of the two momenta greater than 50 GeV.

With these cuts, the efficiency is 0.60 ± 0.01 in the electron channel and 0.83 ± 0.01 in the muon channel, with an expected overall purity of 0.92 ± 0.01.

2.2 Hadronic channel

Doubly resonant processes with both W's decaying hadronically were selected by looking for events with a visible energy above 90 GeV and with at least four jets reconstructed by LUCLUS with more than three particles each. Radiative QCD background was removed requiring the effective centre-of-mass energy, calculated using the SPRIME package [5], above 100 GeV and rejecting events with an isolated photon with energy above 40 GeV. The selection efficiency, estimated on WW samples with $m_W = 80.35$ GeV, was found to be 0.83 ± 0.01 with an expected purity of 0.60 ± 0.03. The left background is mainly due to QCD non radiative events with a small contribution from ZZ resonant processes.

3 Mass reconstruction technique

When an event passed the described selection criteria, a kinematical constrained fit was applied to improve the mass resolution. The constrained fitting method is used when a set of parameters to be determined must obey at the same time to some physical law. The technique of the Lagrange multipliers is generally used to fit constrained parameters. Indicating with $\mathcal{L}(\vec{x};\vec{\theta})$ the usual likelihood function and with $\vec{g}(\vec{\theta}) = \vec{0}$ the set of constraints applied on the parameters $\vec{\theta}$, one has to maximize:

$$F(\vec{x};\vec{\theta}) = \log \mathcal{L}(\vec{x};\vec{\theta}) \ + \ \vec{\lambda}^{\top} \cdot \vec{g}(\vec{\theta}) \tag{1}$$

where $\vec{\lambda}$ are the Lagrange multipliers. The maximization of F with respect to $\vec{\theta}$ and $\vec{\lambda}$ represents the usual parameter fitting and, at the same time, the imposition of the constraints [6].

In the problem of the W mass reconstruction in four fermions final state the parameters and the chosen constraints are, respectively:

$$\vec{\theta} = (q_i^{(k)}, E^{(k)}) \ \ (i = 1,3; k = 1,4)$$

$$g_i = \sum_{k=1}^{4} q_i^{(k)} \ \ ; \ \ g_4 = \sqrt{s} - \sum_{k=1}^{4} E^{(k)}$$

$$g_5 = \sqrt{(E^{(i)} + E^{(j)})^2 - (\vec{q^{(i)}} + \vec{q^{(j)}})^2} - \sqrt{(E^{(k)} + E^{(l)})^2 - (\vec{q^{(k)}} + \vec{q^{(l)}})^2}$$

To reconstruct the 4-momenta of the fermions from the W's the LUCLUS clustering algorithm was used, forcing the hadronic candidates to four jets and the semileptonic ones to two jets plus the chosen lepton. The total number of constraints is therefore five in the hadronic channel (5C fit) and two in the semileptonic (2C fit), as three of them are used to determine the neutrino direction. The constrained fit was performed using the PUFITC [7] program, parametrizing the input jet momenta as described in [8].

4 Mass dependence of the errors and their treatment

It can be shown (see for example [9]) that in general a constrained fit changes the errors on the fitted parameters: the new variance may have new terms which represent the additional information coming from the constraints. This terms give rise to a peculiar behaviour when a kinematical limit is introduced, for example when two of the reconstructed masses in the final state are imposed to be equal. A proof of this is given in the appendix under the simplified hypothesis of only two particles in the final state.

Performing a 5C fit on WW hadronic Montecarlo samples at different centre-of-mass energies, a strong correlation between δm and m is observed, as shown in figure 1.

The δm–m correlation is introduced by the kinematical limit and is independent on $\sqrt{s}$. Given that the bulk of the mass distribution corresponds to the central value of the Breit-Wigner m_W, the estimated errors on the mass are on average smaller at low rather than at high centre-of-mass energies. To check with high precision if the error behaviour actually corresponds to the true mass resolution (without Breit-Wigner effects) given by the constrained fit, a simplified Monte Carlo was built generating WW-like events with WPHACT [1] at $\sqrt{s} = 172$ GeV, setting $\Gamma_W = 0$ and turning off ISR. The generated 4-momenta were smeared with a gaussian

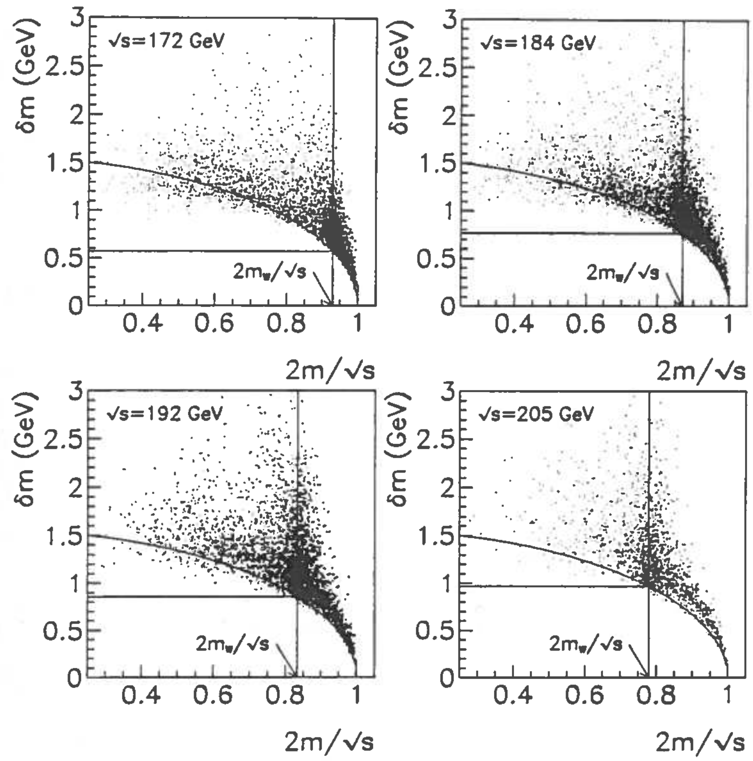

Figure 1: Mass errors versus normalized fitted mass at different centre-of-mass energies, after a five constraints fit.

resolution function in order to reproduce detector effects and then input to PUFITC to perform a 5C fit with the same error parametrization used for the smearing. We want to stress that the physical quantities measured experimentally usually have non gaussian errors, therefore an ideal situation is represented here.

The mean value δm_{fit} of the distribution of the error given by PUFITC can then be compared to the mass resolution δm_{true}, defined as the root-mean-square of the distribution $m_W - m_{rec}$, where m_W is the generated mass and m_{rec} is the fitted mass. In the upper plot of figure 4 this comparison is shown for different values of $x = 2m_W/\sqrt{s}$: both quantities are well approximated, near to the kinematical limit, by a function of the form $const. \times \sqrt{1-x^2}$. Their ratio $\delta m_{true}/\delta m_{fit}$, which can be seen as an estimator of the reliability of the fit errors, is shown in the lower plot: its behaviour is nearly constant apart for high x values, where it steeply increases. This means that the error on the fitted mass seems to strongly underestimate the "true" error when very close to the kinematical limit; in the remaining range of x there is a simple scale factor between the two errors, due to non gaussian effects in the fit.

To compensate for the discrepancy between the "true" mass errors and the ones coming from the 5C fit, one can think of correcting the errors from the fit on an event by event basis. There are several ways to do this:

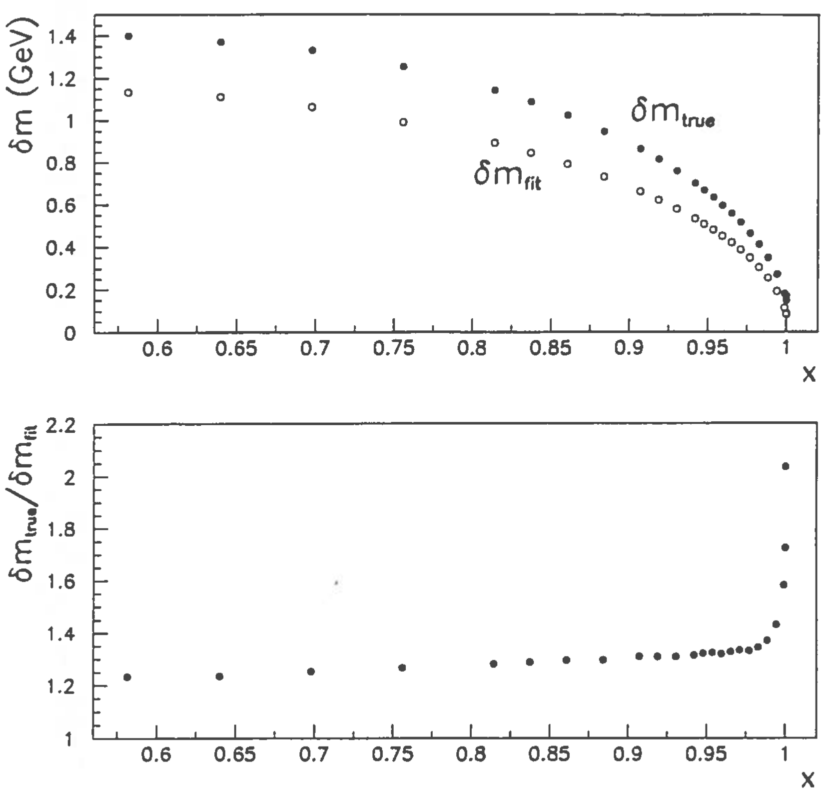

Figure 2: Comparison between the true mass spread δm_{true} and the mass error δm_{fit} given by the fit (upper plot) and their ratio (lower plot) as a function of x.

- summing in quadrature to the error some constant value: this removes the vanishing behaviour of the fitted errors at the kinematical limit but does not fully cancel the dependence of δm on m.
- summing in quadrature to the error some function of m, for example the lower limit of the $\delta m(m)$ distribution. This will move the average error to the value obtained in the limit of large $\sqrt{s}$.
- dividing the error by some function $f(m)$, which in principle should allow to definitely remove the dependence on the fitted mass. One may work out $f(m)$ using empirical parametrizations of the observed behaviour of $\sigma_m'^2$.

In figure 3 the effect of these corrections is shown; the last method is chosen because removes the $\delta m - m$ correlation. This is useful because an asymmetric δm distribution with respect to m_W would lead to a biased measurement of the W mass. The value of the absolute error scale is determined from Montecarlo simulations, tuning to unity the width of the pull function, defined as:

$$Pull_m = \frac{m - m_W}{\delta m} \tag{2}$$

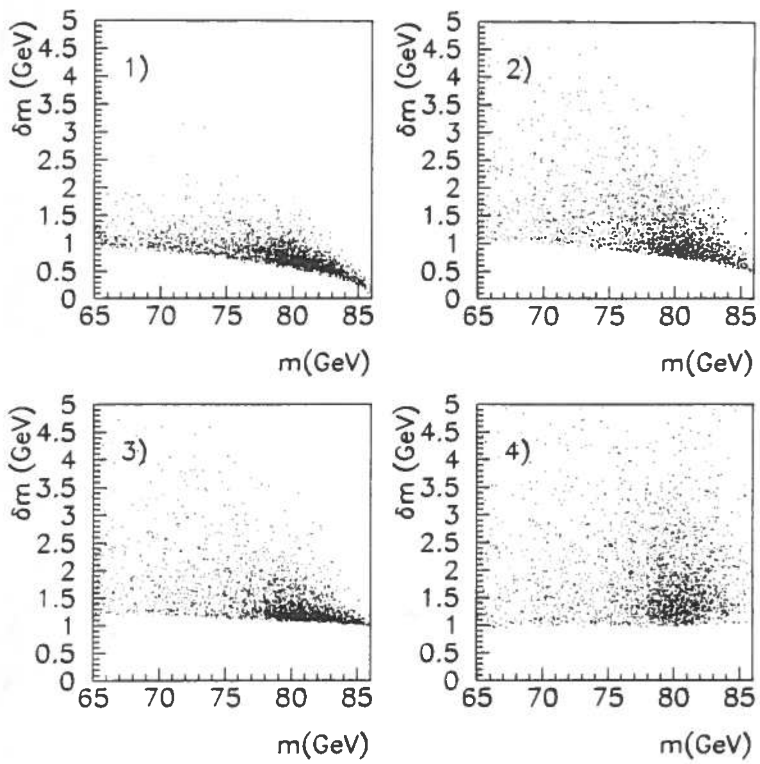

Figure 3: Mass errors versus fitted mass: 1) without correction, 2) adding 0.5 GeV in quadrature, 3) adding the function $\sqrt{4m^2/s}$ GeV in quadrature, 4) dividing by the function $1.5\sqrt{1-4m^2/s}$. Before applying the corrections, the errors were rescaled according to the χ^2 as explained in the text.

where $m, \delta m$ are the experimental measurement of the mass and its error.

After the error rescaling, the mass pull functions are expected to be gaussian with resolution close to unity and independent on any other variable, in particular m and δm. However, the corrections for the kinematical limit effect previously discussed allow only an overall tuning, but do not take into account the local dependence of the pull width on the mass. When in the event there is a lack of information leading to poorly reconstructed 4-momenta, the fitted mass m_{rec} may be not anymore correlated to the true generated mass m_{gen}. The errors coming from the fit appear to be underestimated in this case, resulting in a dependence of the residuals $m_{gen} - m_{rec}$ on the fitted mass m_{rec}, so that the spread of the residuals increases when m_{rec} moves away from the central value of the Breit-Wigner.

The χ^2 from the fit can be used to correct for this effect, multiplying by $\sqrt{\chi^2/NDF}$ the errors of those events for which the χ^2 is greater than the number of degrees of freedom NDF. The result of this rescaling combined with the division by $1.5\sqrt{1-4m^2/s}$ is shown in figure 4, where the pull function per event is plotted versus the fitted mass, for a WW hadronic Monte Carlo sample at $\sqrt{s}$ = 172 GeV.

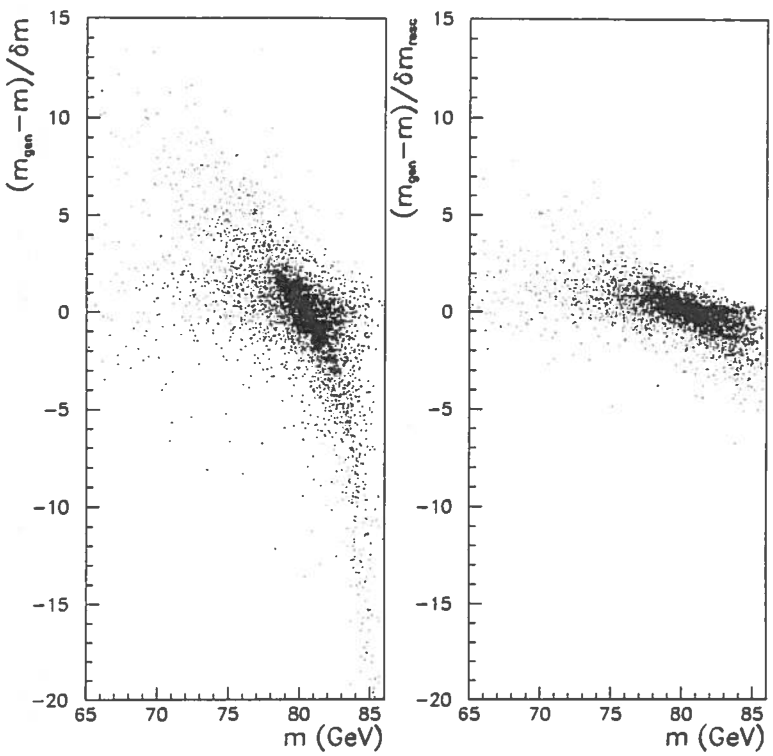

Figure 4: Pull per event as a function of the fitted mass before (left) and after (right) the rescaling described in the text for a WW hadronic Monte Carlo sample. Only the mass corresponding to the correct jets pairing is plotted.

The χ^2 rescaling is effective for badly reconstructed events while the correction based on the mass cures the kinematical limit effect.

5 Fitting procedure

After the constrained fitting procedure of the event, an unbinned maximum likelihood fit was performed on the mass distributions obtained from the data, with a technique similar to the one described in [8]. Only the solutions with mass lying above 65 GeV were used in both channels, as the choice of this window allows to reduce the uncertainty on the shape and the amount of background with a negligible loss of signal (a few permills of the total number of events after the experimental cuts).

5.1 Likelihood function with WPHACT

The parametrization of the WW signal was done numerically instead of analytically, using the CC3 differential cross section $d\sigma/dm$ obtained with the WPHACT [1] gen-

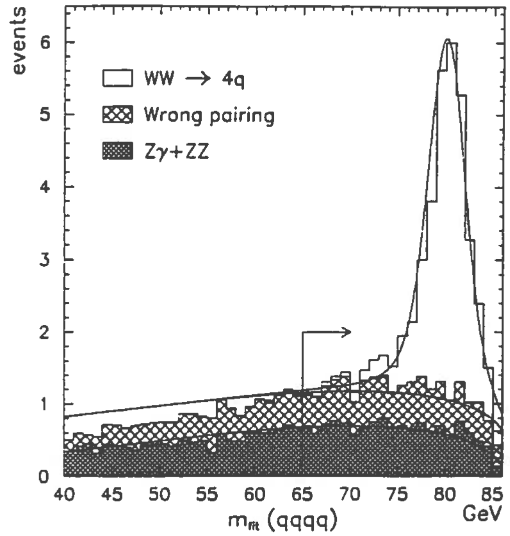

Figure 5: Mass distributon of WW hadronic events plus physical background. The arrow marks the lower limit of the interval in which the fit is performed.

erator, where m is the average of the two generated masses in the event. In this way the correct phase space term and the mass dependence of the matrix elements are automatically taken into account; the method is therefore interesting in the perspective of having greater statistics, when a better knowledge of the signal probability density function is going to be important.

The produced $d\sigma/dm$ distributions include ISR, Coulomb corrections, running W width and naive QCD corrections; the W mass was changed from 75 GeV to 84 GeV in steps of 100 MeV. Every m_W point required approximately 2 hours of CPU on an AlphaStation 600 5/333; the relative uncertainty achieved in the peak of the distribution is lower than 10^{-4}, approaching 10^{-3} in the tails.

The WW signal inside the likelihood is then formed by convoluting the WPHACT distributions with a gaussian function, defined event by event with central value and resolution given respectively by the fitted mass m and its error δm. In both semileptonic and hadronic channels, the physical background is described by empirical parametrizations obtained from the DELPHI simulation. In the hadronic case the likelihood contains an additional term to account for the combinatorial background coming from multiple solutions (due to three different ways of combining the jets 4-momenta to give two equal masses), which was again extracted from the simulation. The likelihood for a WW event can be eventually expressed as:

$$\mathcal{L}(m_W) = \prod_k \eta\{\rho_k[s(m|m_W) \otimes g(m|m_k, \delta m_k)] + (1-\rho_k)c\} + (1-\eta)b_k(m_k) \quad (3)$$

where:

- m_k, δm_k are the measured mass and its error for the k-th mass solution (only one in the semileptonic case);

- η is the signal purity, set to a fixed value in the semileptonic channel and determined event by event in hadronic case (as explained in section 6.2);
- ρ_k is the "correct pairing efficiency" (combination dependent), defined as the fraction of events where the jets have been correctly associated to the parent quarks;
- $s(m|m_W)$ is the WW signal described by the WPHACT $d\sigma/dm$ distributions;
- $g(m|m_k, \delta m_k)$ is a gaussian function which convolves the signal with central value m_k and resolution δm_k;
- c is a constant term describing the combinatorial background;
- $b_k(m_k)$ are polynomial functions describing the physical background.

The functions which describe the signal, the combinatorial and the physical background are normalized to unity inside the chosen mass window. The global likelihood for observing all events is then built as the product of the event likelihoods. Figure 5 shows, for the best χ^2 solution in the hadronic channel, the contributions from the physical background, the combinatorial background and the signal after the selection and the constrained fit, with superimposed the corresponding terms of the probability density function; a constant value $\delta m = 1.5$ GeV has been used for the gaussian convoluted with the signal.

6 Results

6.1 Semileptonic channel

Applying the selection cuts on the 172 GeV data, 29 events are found in total (13 electron candidates and 16 muon candidates), in good agreement with the expected 27.2 WW events (for $m_W = 80.35$ GeV) plus 1 background event; the corresponding mass distribution is shown in figure 6. The likelihood fit on the combined sample gives:

$$m_W = 80.22 \pm 0.58 \text{ GeV} \tag{4}$$

To correct the experimental measurement of m_W for detector and physics effects, the method has been calibrated on the DELPHI full simulation, using samples with different input values of m_W. A linear dependence exists between the reconstructed mass m^W_{fit} and the generated mass m^W_{true}, as shown in figure 7, with a slope compatible with one. Thanks to the error rescaling described previously, the measurement is almost bias free. The correction is obtained from a fit of the points in the plot to the function:

$$m^W_{fit} - 80.35 = a(m^W_{true} - 80.35) + b \tag{5}$$

which gives:

$$a = 0.97 \pm 0.03 \ , \ b = -0.02 \pm 0.03 \text{ GeV} \tag{6}$$

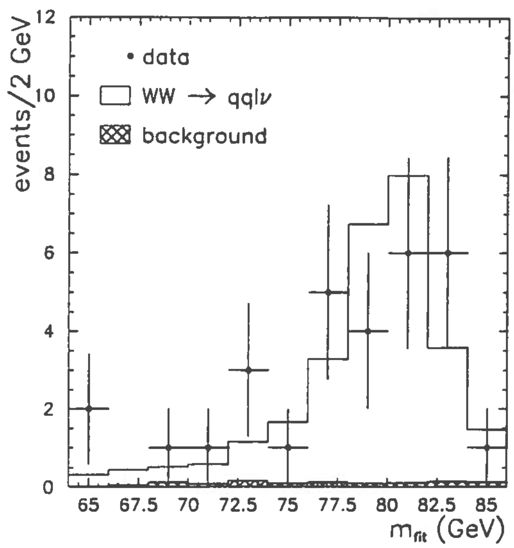

Figure 6: Distribution of the fitted mass for the semileptonic channel. The histogram superimposed to the data is obtained from full simulation of signal (with m_W = 80.35 GeV) plus background and is normalized to the number of expected events.

The value of m_W after the correction becomes:

$$m_W = 80.24 \pm 0.60 \text{ GeV} \tag{7}$$

To compute the expected error on m_W and test its reliability, a high number of samples with the same integrated luminosity as the data has been simulated. The samples were created merging signal and background events according to the expected purity and taking into account poissonian fluctuations.

The error distribution obtained from the samples is plotted in figure 7, toghether with the width of the pull distribution versus the error itself. The flat shape of the latter shows that the method is not sensitive to the particular mass distribution and statistics of the chosen sample; the average value is consistent with one within the statistical precision, as expected.

6.2 Hadronic channel

Given the relatively high amount of physical background in this channel, an event by event purity was defined (as in [8]) to quantify the probability that the event comes from WW processes and carries information on m_W.

To discriminate between signal and background the variable $E_m\theta_m$ was used, where E_m is the energy of the less energetic jet in the event and θ_m is the minimum interjet angle. The variable $E_m\theta_m$ tends to have a lower value for non radiative Z background with gluon emission in the final state, because on average a gluon jet has energy lower than a quark jet and tends to follow the direction of the parent

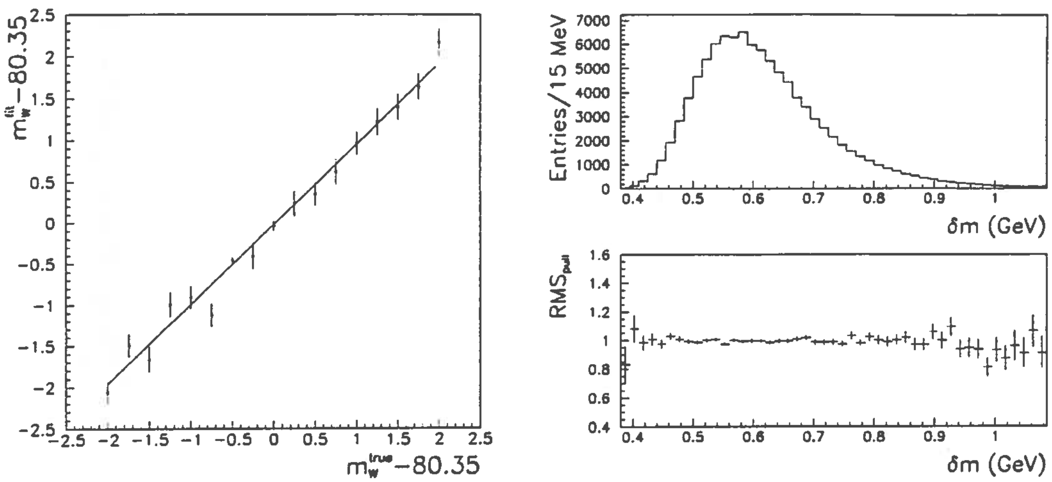

Figure 7: Left: linear fit of the fitted versus the generated mass for the semileptonic channel. Right: error distribution from simulated samples (upper plot) and width of the pull distribution as a function of the error (lower plot).

quark. The agreement on $E_m\theta_m$ between data and Monte Carlo is good, as shown in figure 8.

To further improve the separation, one can use the fact that WW hadronic events are more likely to give a higher number of reconstructed jets N_j because of four quarks in the final state instead of two. The signal and background Monte Carlo samples were therefore divided in two groups with $N_j = 4$ and $N_j > 4$, and the event by event purity as a function of $E_m\theta_m$ was computed for both dividing the corresponding signal distribution by the signal plus background (see figure 8).

A hadronic event which passes all cuts was then forced into four jets and three constrained fits were performed to take into account all possible combinations of four objects to build two invariant masses. The choice of the correct combination is a complicated problem with no solution in case of two (or three) pairings with high invariant mass and similar χ^2 from the fit: taking only the combination with the best χ^2 results in a loss of information [10]. It was therefore decided to use also the other solutions if the difference between their χ^2 and the best one was $\Delta\chi^2 < 5$. The probability density function in the likelihood is then combination dependent in order to take into account different amount and shape of combinatorial and physical background.

The mass distribution obtained in the data for the first combination is shown in figure 9. The number of selected events is 54 (for a total of 46 first combinations and 16 second combinations), in good agreement with the sum of 42.4 expected WW events (for $m_W = 79.85$ GeV) plus 14.5 events of background. Performing the likelihood fit on this sample gives:

$$m_W = 79.78 \pm 0.68 \;\; \text{GeV} \qquad (8)$$

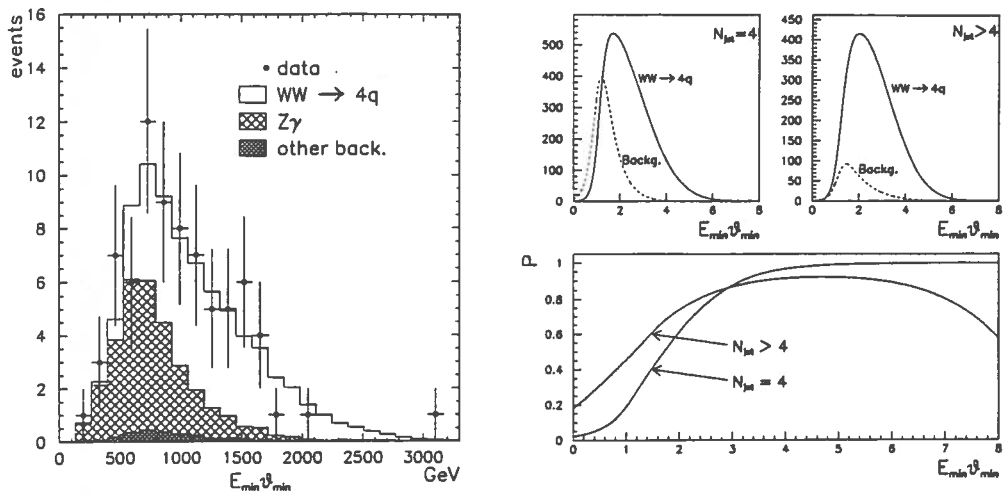

Figure 8: Comparison of expected $E_m\theta_m$ distribution with data (left), fitted behaviour of signal and background $E_m\theta_m$ distribution for events with $N_j = 4$ and $N_j > 4$ (right upper plots) and derived expected purity as a function of $E_m\theta_m$ (right lower plot).

A procedure identical to the one described for the semileptonic channel has been adopted to correct the experimental result. The linear fit between m_W^{fit} and m_W^{true} gives:

$$a = 0.98 \pm 0.02\ ,\ b = 0.15 \pm 0.02\ \text{GeV} \tag{9}$$

where the small bias is due to the approximated shape of the combinatorial and physical background. The linearity plot and the error behaviour for the hadronic channel are shown in figure 10. The corrected value of m_W becomes:

$$m_W = 79.62 \pm 0.69\ \text{GeV} \tag{10}$$

7 Systematic errors

The systematic errors in the determination of m_W are connected to the LEP beam energy measurement, the selection cuts, the jet and lepton energies, the parameters entering in the likelihood and the approximate knowledge of physics effects. They are reported in table 1 and were assessed as follows:

- The systematic uncertainty on the LEP energy is around 30 MeV and moves m_W by the same amount.
- The selection cuts were varied according to their resolution or by 10% and the corresponding variations on m_W were added in quadrature to give the total systematic error.
- From the comparison between data and simulation at $\sqrt{s} = m_Z$, the maximum distortion in the energy scale was found to be 2% for jets, 1% for

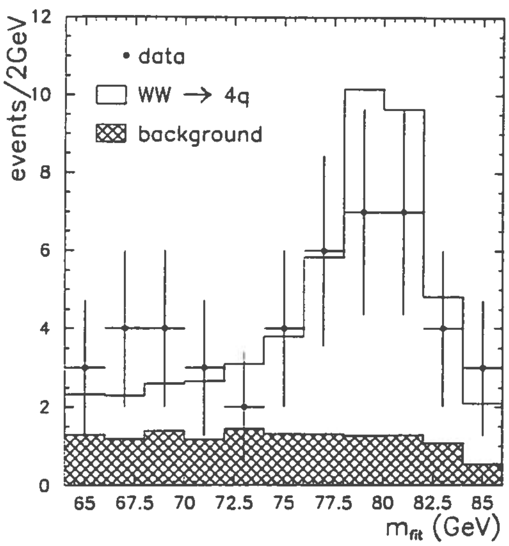

Figure 9: Distribution of the fitted mass for the best χ^2 solution in the hadronic channel. The histogram superimposed to the data is obtained from full simulation of signal (with m_W = 79.85 GeV) plus background and is normalized to the number of expected events.

electrons and 0.5% for muons [8]. Conservatively varying the energies by this amount, the maximum shift on m_W after the whole fitting procedure was taken as systematic uncertainty; half of the effect due to the jet energy scale was assumed to be fully correlated between the semileptonic and the hadronic channel as in [8].

- The physical and combinatorial background were varied according to their statistical error (from Monte Carlo) producing a very small shift on m_W and the maximum effect in the two channels was conservatively used.
- The uncertainty on m_W due to the statistical errors on the linearity parameters has also been taken as systematic error.
- To account for possible systematics coming from the description of the jet resolution inside the kinematical fit the mass errors were rescaled by $\pm 10\%$ and the maximum bias on m_W was taken as systematic error, fully correlated in the two channels. This has to be considered as a very conservative estimate as the pull function was not retuned after changing the errors.
- the systematic effects due to the fragmentation modelling both in the WW signal and in the background are not expected to depend strongly on the details of the analysis, therefore the results shown in [8] are used here.
- The errors coming from initial state uncertaintes on the ISR spectrum and final state uncertaintes on Bose-Einstein and colour reconnection effects were conservatively taken as indicated in [11], [8].

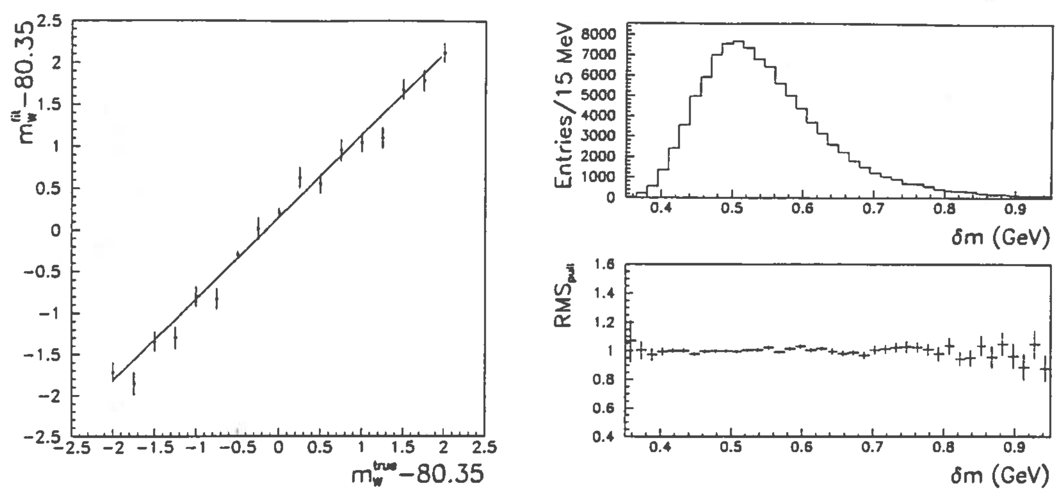

Figure 10: Left: linear fit of the fitted versus the generated mass for the hadronic channel. Right: error distribution from simulated samples (upper plot) and width of the pull distribution as a function of the error.

- Systematics effects coming from four fermion final state interference, not included in the CC3 WPHACT distributions, were studied at the generator level and found to be 38 MeV in the $q\bar{q}e\nu$ channel and negligible in the other channels.

Conclusions

The mass of the W boson was measured using the DELPHI data at $\sqrt{s} = 172$ GeV with the direct reconstruction of the invariant mass of the fermion pairs in both decay channels $q\bar{q}\ell\nu$, $q\bar{q}q\bar{q}$ and performing a maximum likelihood fit on the mass distributions. The probability density function was built using the CC3 differential cross section computed with the WPHACT generator. The combination of the m_W measurements in the semileptonic and hadronic channel, taking into account the correlation between systematic errors, yields:

$$m_W = 79.98 \pm 0.45(\text{stat.}) \pm 0.04(\text{syst.}) \pm 0.05(\text{theo.}) \pm 0.03(\text{LEP}) \;\; \text{GeV/c}^2 \quad (11)$$

where the first error is statistical, the second one is the experimental systematic error and the third one is the theoretical systematic error (see table 1). The last systematic error comes from the LEP beam energy uncertainty.

Acknowledgements

We would like to thank Alessandro Ballestrero for his help and Niels Kjaer for useful discussions and suggestions.

Source	$q\bar{q}\ell\nu$	$q\bar{q}q\bar{q}$	Combined
Selection cuts	20	25	16
Monte Carlo calibration	30	28	20
Jet energy scale	28	28	24
Lepton energy scale	16	-	8
Background fraction	2	5	5
Constrained fit	10	10	10
Signal fragmentation	10	25	13
Background fragmentation	-	10	5
Total experimental	50	55	40
ISR	10	10	10
FSI	-	100	50
4-fermion correction	19	-	10
Total theoretical	21	100	52
LEP	30	30	30

Table 1: Systematic errors (in MeV) in the determination of m_W.

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Appendix A

It can be shown ([9]) that the maximization of F instead of $\mathcal{L}$ (see text) changes the errors on the fitted parameters: if V is the variance matrix of the unconstrained parameters, the new variance matrix after applying the constraints can be written as:

$$\mathsf{V}'(\vec{\theta}) = \mathsf{V} - \mathsf{VB}(\mathsf{B}^T\mathsf{VB})^{-1}\mathsf{B}^T\mathsf{V} \tag{12}$$

where $\mathsf{B} = \partial\vec{g}/\partial\vec{\theta}$. The second term in the right hand side of equation (12) represents the reduction in variance obtained by adding the information of the constraints. This term shows a peculiar behaviour when in presence of a kinematical limit: to show this let us consider a simplified problem in which only two particles (W bosons in our case) are present in the final state and a 5C fit is imposed. With the same syntax used in the text, the constraint functions $\vec{g}$ and the parameters $\vec{\theta} = (\vec{p_1}, E_1, \vec{p_2}, E_2)$ become:

$$g_i = \vec{p}_{1i} + \vec{p}_{2i} \ ; \ g_4 = E_1 + E_2 - \sqrt{s} \ ; \ g_5 = \sqrt{E_1^2 - p_1^2} - \sqrt{E_2^2 - p_2^2} \tag{13}$$

The matrix B is:

$$\mathsf{B}^T = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \frac{-p_{1x}}{m_1} & \frac{-p_{1y}}{m_1} & \frac{-p_{1z}}{m_1} & \frac{E_1}{m_1} & \frac{p_{2x}}{m_2} & \frac{p_{2y}}{m_2} & \frac{p_{2z}}{m_2} & \frac{-E_2}{m_2} \end{pmatrix} \tag{14}$$

Approaching the kinematical limit all the centre-of-mass energy is spent to produce massive particles at rest, so that the 3-momenta in the final state go to zero and all terms of the kind $p_{\alpha i}/m_\alpha$ vanish in the matrix B.
Making now the assumption that V is diagonal, $\mathsf{B}^T\mathsf{VB}$ becomes:

$$\mathsf{B}^T\mathsf{VB} = \begin{pmatrix} \sigma^2_{p_{x1}} + \sigma^2_{p_{x2}} & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{p_{y1}} + \sigma^2_{p_{y2}} & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{p_{z1}} + \sigma^2_{p_{z2}} & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{E_1} + \sigma^2_{E_2} & \sigma^2_{E_1} - \sigma^2_{E_2} \\ 0 & 0 & 0 & \sigma^2_{E_1} - \sigma^2_{E_2} & \sigma^2_{E_1} + \sigma^2_{E_2} \end{pmatrix} \tag{15}$$

and its inversion is straightforward, leading to:

$$\mathsf{V}'(\vec{\theta}) = \mathsf{V}(\vec{\theta}) \left[\mathsf{I} - \begin{pmatrix} \mathsf{K}_1 & 0 & \mathsf{K}_2 & 0 \\ 0 & 1 & 0 & 0 \\ \mathsf{K}_1 & 0 & \mathsf{K}_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right] \tag{16}$$

where I is the identity matrix and K_i are the 3×3 matrices:

$$\mathsf{K}_i = \begin{pmatrix} \frac{\sigma^2_{p_{xi}}}{\sigma^2_{p_{x1}}+\sigma^2_{p_{x2}}} & 0 & 0 \\ 0 & \frac{\sigma^2_{p_{yi}}}{\sigma^2_{p_{y1}}+\sigma^2_{p_{y2}}} & 0 \\ 0 & 0 & \frac{\sigma^2_{p_{zi}}}{\sigma^2_{p_{z1}}+\sigma^2_{p_{z2}}} \end{pmatrix} \tag{17}$$

From equation (16) one can see that $\sigma'^{\,2}_E = \mathsf{V}'_{44} = \mathsf{V}'_{88} = 0$. Given the expression for the variance on the mass:

$$\sigma'^{\,2}_m = \left(\frac{E}{m}\right)^2 \sigma'^{\,2}_E + \left(\frac{p}{m}\right)^2 \sigma'^{\,2}_p + 2\left(\frac{pE}{m^2}\right) \sigma'^{\,2}_{pE} \tag{18}$$

and using the fact that $\sigma'^{\,2}_p$, $\sigma'^{\,2}_{pE}$ are finite, it follows that $\sigma'^{\,2}_m = 0$ at the kinematical limit.