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A BHABHA generator for DAΦNE including radiative corrections and ϕ resonance.

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Abstract

A BHABHA generator, developed for PETRA/PEP experiments, has been modified to match DAΦNE characteristics. The elastic channel takes into account both first order radiative corrections and ϕ resonance. The structure is a stand alone program with standard YBOS output bank.

This generator may be helpful for trigger studies, especially when Bhabha events at small angle ($\geq 1^0$) are considered.

1 Introduction

A Bhabha generator was recently included into the GEANFI frame. Such a generator, simultaneously simulating elastic scattering at the Born level, and hard bremsstrahlung diffraction (i.e. scattering with emission of a real photon), is described in a dedicated KLOE note [1]. It can be used for applications, as EmC calibration, in which a good accuracy on the cross sections is not required, but It is inadeguate when the cross sections must be known with a precision of the order of percent: in this case radiative corrections must also be taken into account [2].

In the present note we describe how we have modified for DAΦNE an existing Bhabha generator for positrons (electrons) emitted at a polar angle $\geq 1^0$, including $O(\alpha)$ radiative soft corrections together with hard bremsstrahlung. This new generator has a stand alone structure and does the following tasks:

- Generation of the two processes:

$$e^+(p_+)e^-(p_-) \rightarrow e^+(q_+)e^-(q_-) \quad (1)$$
$$e^+(p_+)e^-(p_-) \rightarrow e^+(q_+)e^-(q_-)\gamma(k) \quad (2)$$

- Calculation either of both elastic (*soft*) cross section with first order radiative corrections in the presence of ϕ resonance, and of inelastic cross section (i.e. when a real photon is emitted). A detailed description of the contributions considered in the elastic cross section [2] is given in Appendix A.1. In Appendix A.2 the formula used for inelastic cross section are shown.

- Storage for each generated event of the outgoing particles (positron, electron,and photon) 4-momenta into the PMBO YBOS Bank (Particle Momentum Object Bank). These 4-momenta refer to the laboratory reference system, where the beam crossing angle (25 mrad) is taken into account.

  The Ybos output file is a standard A.C. data file: the LRID Bank (Logical IDentifier Bank) describes the record type and contains information about the run, while the EVCL Bank (EVent CLassification Bank) stores the event number [3].

- Writing of an output file containing some statistical informations about the run.

2 Structure of the generator

The structure of the generator consists of tree phases:

- Initialization
- Event Generation
- Program Output

The block scheme of the Bhabha generating program is shown in Figure 1.

2.1 Initialization

The initialization phase defines default values, reads data cards, initializes the YBOS package, and evaluates the approximated total elastic and inelastic cross sections in the c.m.s..

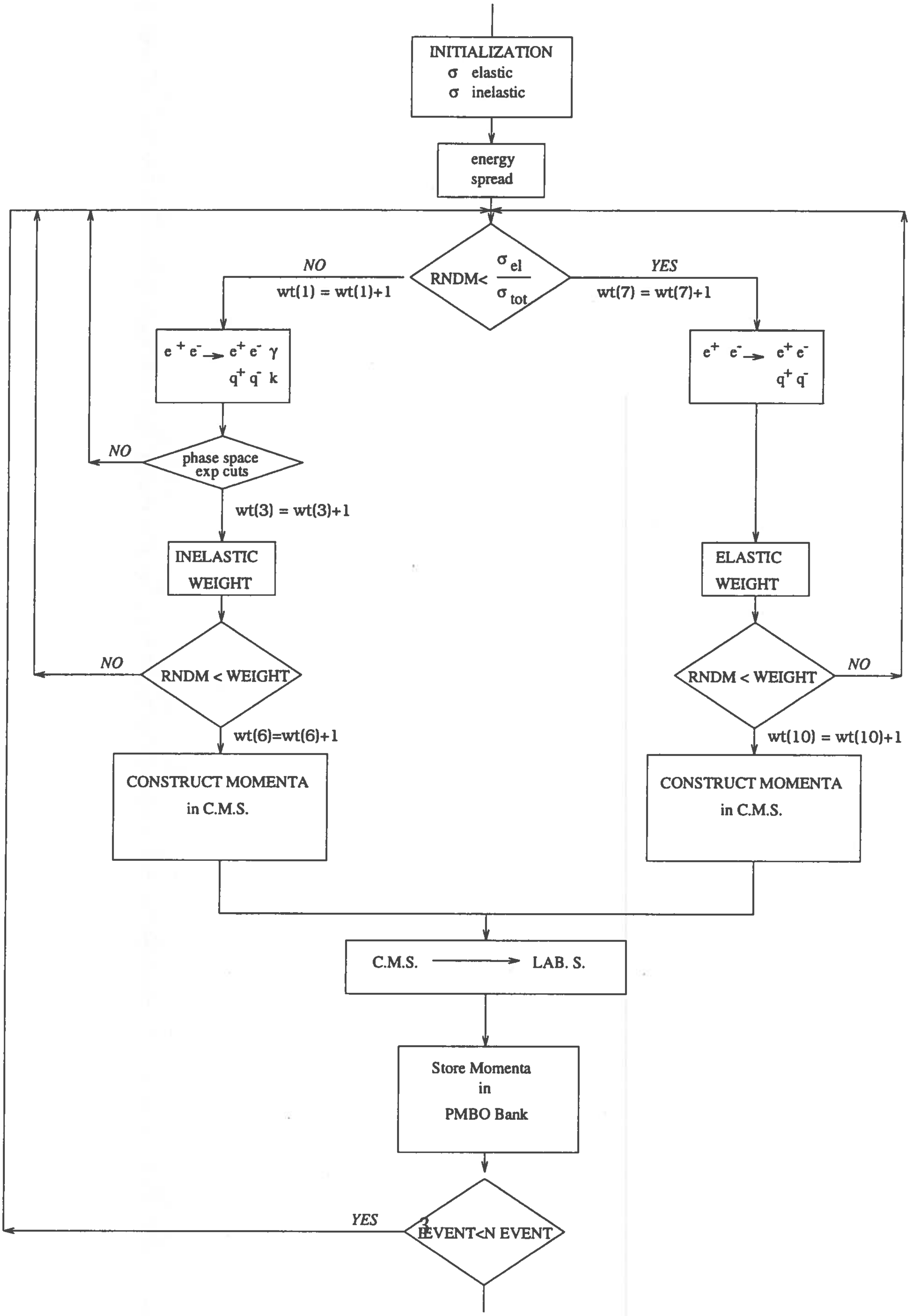

Figure 1: Block scheme of Bhabha generator.

A description of data cards is given in Appendix A.3; the user is expected to specify the following input parameters:

- Number of events to be generated;
- Minimum and maximum positron (electron) polar scattering angles in the Centre of Mass System (c.m.s.);
- Mean value of the beam energy in the c.m.s. ($< E_b >$);
- Run number.

The other parameters, necessary to evaluate the cross sections (see Appendix A.1 and A.2), are:

- Value of the ratio: min. hard bremsstrahlung energy/ Beam energy (XK0) (by def.= 0.01);
- Value of the ratio: max. bremsstrahlung energy/ Beam energy (by def.= 1.);
- Φ mass (1.0194 GeV);
- total Φ width (0.00443 GeV);
- leptonic Φ width ($1.37 \times 10^{-6}\,GeV$).

The approximated elastic cross section σ_{el}^{ap} is obtained by means of a step-by-step integration [2] of the differential cross section described by eq.(17) in Appendix A.1; the accuracy of this calculation is better than 1%.

The approximated inelastic cross section σ_{inel}^{ap} is obtained by integrating the approximated differential cross section $d^5\sigma_a$ described by eq.(27) in Appendix A.2.

The elastic and inelastic cross sections are calculated at the energy fixed by the data card ($< E_b >$).

2.2 Event generation

The event generation is described in detail in [2]: each *trial* event is assigned "elastic" or "inelastic" with a probability proportional to the respective approximated cross section.

For inelastic events the correct values of physical quantities (production angles and momenta) are obtained by accepting the *trial* events falling into the appropriate phase space with a probability proportional to the correspondent weights. The phase space is restricted by cuts on the photon energy ($E_\gamma > k_0$, where k_0 is $XK0 \times < E_b >$) and on positron (electron) polar scattering angle ($\theta_{min} < \theta < \theta_{max}$): the events not satisfying these selection criteria are rejected. Generally more the 90% of the inelastic *trail* events survive these phase space cuts.

These surviving events are finally accepted with a probability proportional to the inelastic weight, defined as:

$$w_{inel} = (\frac{d^5\sigma}{d^5\Gamma})(\frac{d^5\sigma_a}{d^5\Gamma})^{-1} \tag{3}$$

where $d^5\sigma_a$ and $d^5\sigma$ are respectively the approximate and exact differential cross sections, whose expressions are shown in details in Appendix A.2.

In the elastic channel obviously there is no kinematical cut; in this case the *trial* events distribution is directly corrected by the "elastic" weight [2]:

$$w_{el} = \frac{d\sigma_{el}}{d\Omega} / \frac{d\sigma_{el}^{ap}}{d\Omega} \tag{4}$$

with $d\sigma_{el}/d\Omega$ given by (17) and $d\sigma_{el}^{ap}/d\Omega$ defined by:

$$\frac{d\sigma_{el}^{ap}}{d\Omega} = \frac{\sigma_{el}^{ap}}{2\pi(1/(1-cos\theta_{min}) - 1/(1-cos\theta_{max}))}(1-cos\theta)^2, \tag{5}$$

where σ_{el}^{ap} represents the elastic approximated cross section and θ is the positron polar angle. Distributions of weights, for both channels, are shown in Figure 2 for positrons (electrons) emitted with a polar angle $1^0 < \theta < 179^0$.

For each event the beam energy in the c.m.s. is extracted from a gaussian distribution with mean value $< E_b >$, fixed in the data card, and with a standard deviation of 700 keV. The calculation of the cross sections is performed when the program is initialized, at a fixed energy beam ($< E_b >$), without taking into account the spread in beam energy. The error introduced by this approximation is less then 1% because in this range the total cross section is changing slowly with energy.

While the ratio between the number of *trial* events in the elastic channel and *trial* events in the inelastic one is given by $\sigma_{el}^{ap}/\sigma_{inel}^{ap}$, this ratio for the accepted events at the end of this generation procedure is different. This is due to the rejection criterion of the events, as shown in the Block scheme of this generator. The ratio between the number of events accepted in the elastic channel and the number of the events accepted in the inelastic one is given by the ratio of the following *correct* cross sections:

$$\sigma_{el}^{cor} = \sigma_{el}^{ap} \times \frac{\sum_i w_i^{el}}{N_{trial}^{el}} \qquad (6)$$

$$\sigma_{inel}^{cor} = \sigma_{inel}^{ap} \times \frac{\sum_i w_i^{inel}}{N_{trial}^{inel}}, \qquad (7)$$

where w_i^{el} and N_{trial}^{el} (w_i^{inel} and N_{trial}^{inel}) are respectively the weight corresponding to *i-th* event and the total number of *trial* events in the elastic (inelastic) channel.

It's worthwhile to note that, if the weights don't fluctuate too wildly, these *correct* cross sections can be indentified with the exact cross sections, which are expected in the experiment. While this condition is true for the Bhabha elastic cross section, it's only approximated for the hard bremmsstrahlung scattering, as it's shown for example in Figure 2, and the *correct* inelastic cross section σ_{inel}^{cor} can differ from the exact σ_{inel}, with an uncertainity which can't be estimate *a priori* (and could be greater of some per cent).

The results of the generation process are partially summarized in 18 physical quantities, which are incremented event by event during the generation phase. These quantities (wt(i)) are:

wt(1) = no. of *trial* events generated in the inelastic channel;

wt(2) = no. of trials in the inelastic channel to generate $cos\theta$;

wt(3) = no. of *trial* events in the inelastic channel surviving phase space cuts;

wt(4) = sum of "inelastic" weights;

wt(5) = sum of squared "inelastic" weights;

wt(6) = no. of accepted events in the inelastic channel;

wt(7) = no. of *trial* events generated in the elastic channel;

wt(8) = sum of "elastic" weights;

wt(9) = sum of squared "elastic" weights;

wt(10) = no. of accepted events in the elastic channel;

wt(11) = no. of inelastic events with weight < 0;

wt(12) = no. of inelastic events with weight $>$ wmax;

wt(13) = minimum generated weight in the inelastic channel;

wt(14) = maximum generated weight in the inelastic channel;

wt(15) = no. of elastic events with weight <0;

wt(16) = no. of elastic events with weight $>$ wmax;

wt(17) = minimum generated weight in the elastic channel;

wt(18) = maximum generated weight in the elastic channel.

For each accepted event the generator evaluates the outgoing particles 4-momenta in the c.m.s., according to the algorithm described in [2], and then transforms these momenta into the Lab. system, taking into account the beam crossing angle (25 mrad). The Lab. momenta are stored in the PMBO bank.

2.3 Program Output

As last step this generator creates an output file with some statistics, including data cards contents, random numbers seed, wt(i) values, elastic and inelastic (*approximated* and *correct*) cross sections and other informations. An example of this output is shown in A.5.

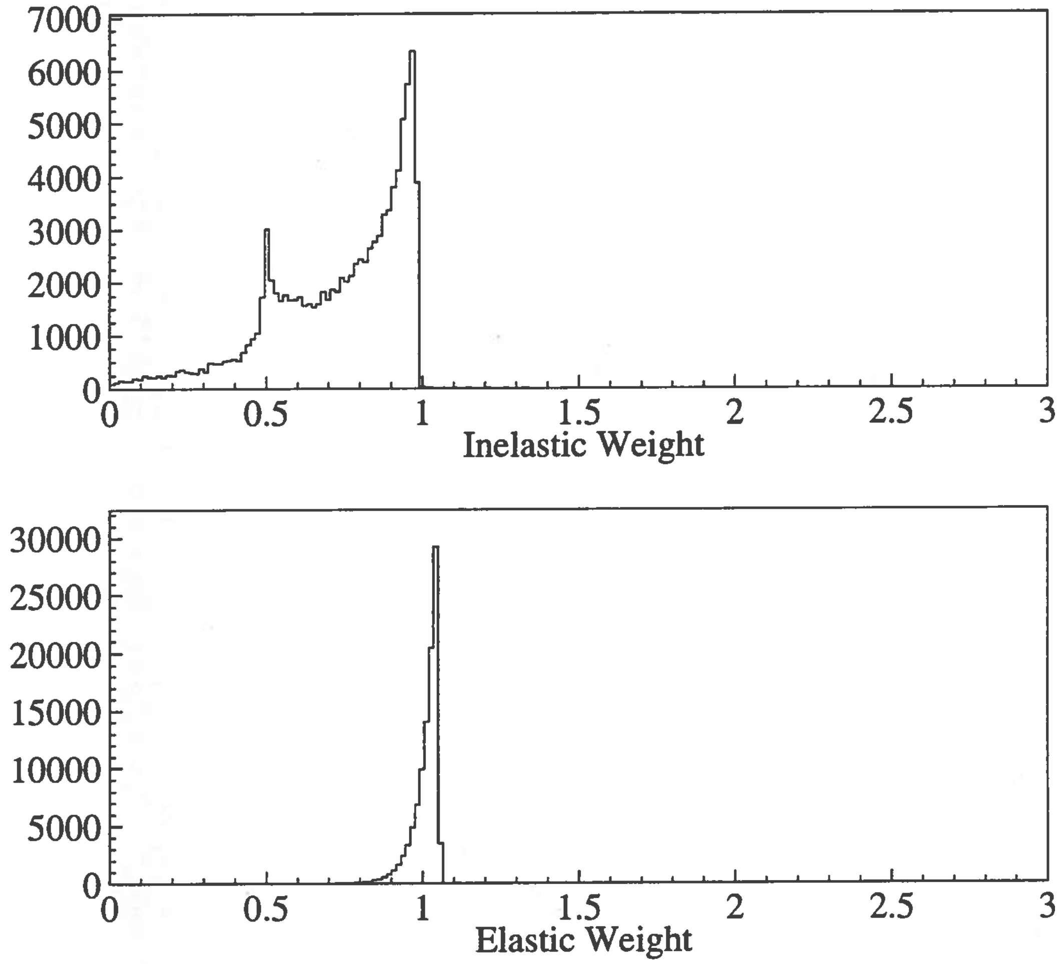

Figure 2: Weights distributions for $1^0 < \theta < 179^0$.

3 Comparison with the GEANFI Bhabha.

With respect to the Bhabha generator in GEANFI [1], this new generator presents some important systematic differences, concerning specifically the event generation and the number of accepted events.

As it is shown in TABLE 1, the approximated cross sections are significantly different for the elastic channels, while they are the same for the inelastic ones. These differences are due to the fact that in the previous generator σ_{el}^{ap} is simply the Born cross section, while in this new generator σ_{el}^{ap} takes into account radiative corrections and ϕ contribution, with a final result to decrease the total cross section (see fig 8).

The accuracy on the evaluation of this $O(\alpha)$ elastic cross section is estimated to be of the order of few percent, by considering:

a) the precision of the numerical integration of differential equation (17) in Appendix A.1, which is better of than 1%;

b) the theoretical accuracy in calculating $O(\alpha)$ radiative corrections (δ_{QED} term in (11)) [2].

The number of the accepted events, for each channel, is also different for the two generator, due to the different structure of the algorithms. While in the previous GEANFI generator, the ratio between the number number of accepted events in the elastic channel and the number in the inelastic one is given by $\sigma_{el}^{ap}/\sigma_{inel}^{ap}$, now this ratio is given by $\sigma_{el}^{cor}/\sigma_{inel}^{cor}$, defined in eqs.(6, 7).

Table 2 shows the values of these *correct* cross sections, for different angular region, and for different minimum hard bremsstrahlung energies (XK0), obtained from 100000 events runs. A comparison between the two tables shows that σ_{el}^{ap} is essentialy the same of σ_{el}^{cor}, while σ_{inel}^{ap} is larger than σ_{el}^{cor}, i.e. the weight in the inelastic channel w_{inel} is often less then one (as it is shown in Figure 2).

Table 3 shows a comparison of cross sections and the respective rates obtained with the two generators in the polar region $3^0 < \theta < 9^0$, assuming a luminosity of $10^{33} cm^{-2}s^{-1}$ and a photon energy threshold of $5.1\, MeV$. (The positron (electron) emitted in this region interact mainly with the quadrupoles, initiating an electromagnetic shower with some energy reaching the calorimeter). It's interesting to note that the difference between the two total rates is $\sim 40\%$!

Selecting the same number of entries in both channels, we have compared some relevant distributions obtained with our generator and with previous one [1]. Both generators have been run using the same selection criteria on the phase space variables: Beam energy= $510\, MeV$; $XK0 = 0.01$; $1^0 < \theta < 179^0$. On Figures 3-7

$e^+(e^-)$ polar angle $9^0 < \theta < 171^0$						
	Previous Generator			New Generator		
$XK0$	σ_{el}^{ap} (μb)	σ_{inel}^{ap} (μb)	σ_{tot}^{ap} (μb)	σ_{el}^{ap} (μb)	σ_{inel}^{ap} (μb)	σ_{tot}^{ap} (μb)
0.002	38.5	25.9	64.4	19.2	25.9	45.1
0.01	38.5	19.2	57.7	25.0	19.2	44.2
0.02	38.5	16.3	54.8	27.5	16.3	43.8

$e^+(e^-)$ polar angle $1^0 < \theta < 179^0$						
	Previous Generator			New Generator		
$XK0$	σ_{el}^{ap} (mb)	σ_{inel}^{ap} (mb)	σ_{tot}^{ap} (mb)	σ_{el}^{ap} (mb)	σ_{inel}^{ap} (mb)	σ_{tot}^{ap} (mb)
0.002	3.28	1.28	4.56	2.32	1.28	3.60
0.01	3.28	0.95	4.23	2.61	0.945	3.555
0.02	3.28	0.80	4.08	2.74	0.80	3.54

Table 1: Comparison between the approximated cross sections for different minimum hard bremsstrahlung energy (XK0): beam energy is fixed to 510 MeV. *Previous Generator* and *New Generator* indicates the results obtained by means of GEANFI Bhabha generator and by this new generator, respectively.

$e^+(e^-)$ polar angle	σ_{el}^{cor} (mb)	σ_{inel}^{cor} (mb)	σ_{tot}^{cor} (mb)	$XK0$
$9^0 < \theta < 171^0$	1.93×10^{-2}	1.95×10^{-2}	3.88×10^{-2}	0.002
	2.51×10^{-2}	1.38×10^{-2}	3.89×10^{-2}	0.01
	2.75×10^{-2}	1.13×10^{-2}	3.88×10^{-2}	0.02
$1^0 < \theta < 179^0$	2.35	0.95	3.30	0.002
	2.63	0.67	3.30	0.01
	2.75	0.56	3.31	0.02

Table 2: *Correct* cross sections for different minimum hard bremsstrahlung energy ($E_b = 510\, MeV$).

$e^+(e^-)$ polar angle $3^0 < \theta < 9^0$							
Previous Generator				New Generator			
σ_{el} (μb)	σ_{inel} (μb)	σ_{tot} (μb)	$Rate_{tot}$ (kHz)	σ_{el} (μb)	σ_{inel} (μb)	σ_{tot} (μb)	$Rate_{tot}$ (kHz)
324	120	544	544	238	81	319	319

Table 3: Comparison between Bhabha elastic and inelastic cross sections obtained by means of GEANFI Bhabha (*Previous* Generator), and by this new one. Photon minimum hard bremsstrahlung energy is 5.1 MeV for both generators and E_b = 510 MeV. It has been assumed a maximum luminosity of $10^{33} cm^{-2} sec^{-1}$.

the solid line is obtained using the new generator, while the shadowed histogram represents previous one.

Figure 3 shows the angular distributions in c.m.s. for emitted positrons in the elastic scattering, with 1^0 polar angle cut. In the new generator the events at large angles are even more suppressed, due to $O(\alpha)$ radiative correction to Born cross section: this effect confirms what we it's expected in the elastic channel, as it is shown in Appendix A.1.

The next figures are plots of some distributions in the laboratory system for the inelastic scattering: as expected the agreement between the data is good. (Detailed notes about these distributions can be found in the previous note [1]).

In conclusion, respect to the previous one, the introduction of radiative corrections in the new generator has a twofold effect: it results in smaller values of the elastic cross section and changes the angular distribution of the scattered positrons (electrons), suppressing even more events at large angles.

Furthermore the expected total cross section for hard bremsstrahlung, calculated with an uncertainty that could be greater of some percent, results to be smaller than one obtained with GEANFI generator, but the angular distributions are essentialy the same.

Finally the ϕ interference gives second order effects, with respect to pure Bhabha scattering, as it will be shown in Appendix A.1.

This Bhabha generator represents a first base to study the response of the KLOE detector to Bhabha events, it is particularly relevant for calibrations and luminosity purposes.

This generator is not yet satisfactory if a precision of (or greater than) 1% is required. In order to obtain more precise calculations a rather different approach is under study.

A Appendix

A.1 Evalutation of Elastic cross section.

The Bhabha elastic scattering is defined to have the same initial and final state $|e^+, e^-\rangle$. We considered for this process the first two non-vanishing terms of QED S-matrix expansion, together with the lowest order diagram with ϕ resonance:

$$\langle e^+, e^- | S | e^+, e^- \rangle \simeq A_0 + A_c + A_\phi \tag{8}$$

where A_0 is the *Feynmann* lowest order amplitude, A_c represents the second order diagrams (loop corrections, vacuum polarization, and so on), and A_ϕ is the diagram of the process $e^+ + e^- \to e^+ + e^-$ via ϕ resonance. (As we will show this term gives small contributions comparison with the others).

The elastic cross section is proportional to the module square of the total amplitude:

$$d\sigma \sim |A_0 + A_c + A_\phi|^2 = |A_0|^2 + |A_c|^2 + |A_\phi|^2 + I(A_0, A_c) + I(A_0, A_\phi) + I(A_c, A_\phi). \tag{9}$$

where I(A_i,A_j) is a real number which take into account interference between the two terms A_i,A_j. (For example I(A_0,A_c) is equal to 2 $Re(A_0 A_c^*)$). We'll now evaluate each term of the expression 9 in the Centre of Mass System (c.m.s.):

- $|A_0|^2$, is the well-know lowest order Bhabha cross section and therefore (in the relativistic limit):

$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{16E_b^2} \times \left(\frac{3 + cos^2\theta}{1 - cos\theta}\right)^2 \tag{10}$$

  where $\alpha \sim \frac{1}{137}$ is the QED fine-structure constant, E_b is the beam energy, $d\Omega$ is the solid angle element and θ is the positron polar scattering angle. This we refer to as the Born cross section.

- The term $|A_c|^2$ is of order α^2 with respect to $|A_0|^2$ being the lowest order. When considering only O(α) corrections to the Born cross section, as in this calculation, this contribution can be neglected.

- The term I(A_0,A_c) derives from interference term of the lowest-order amplitude A_0 with the radiative corrections term A_c, so that is of order α respect

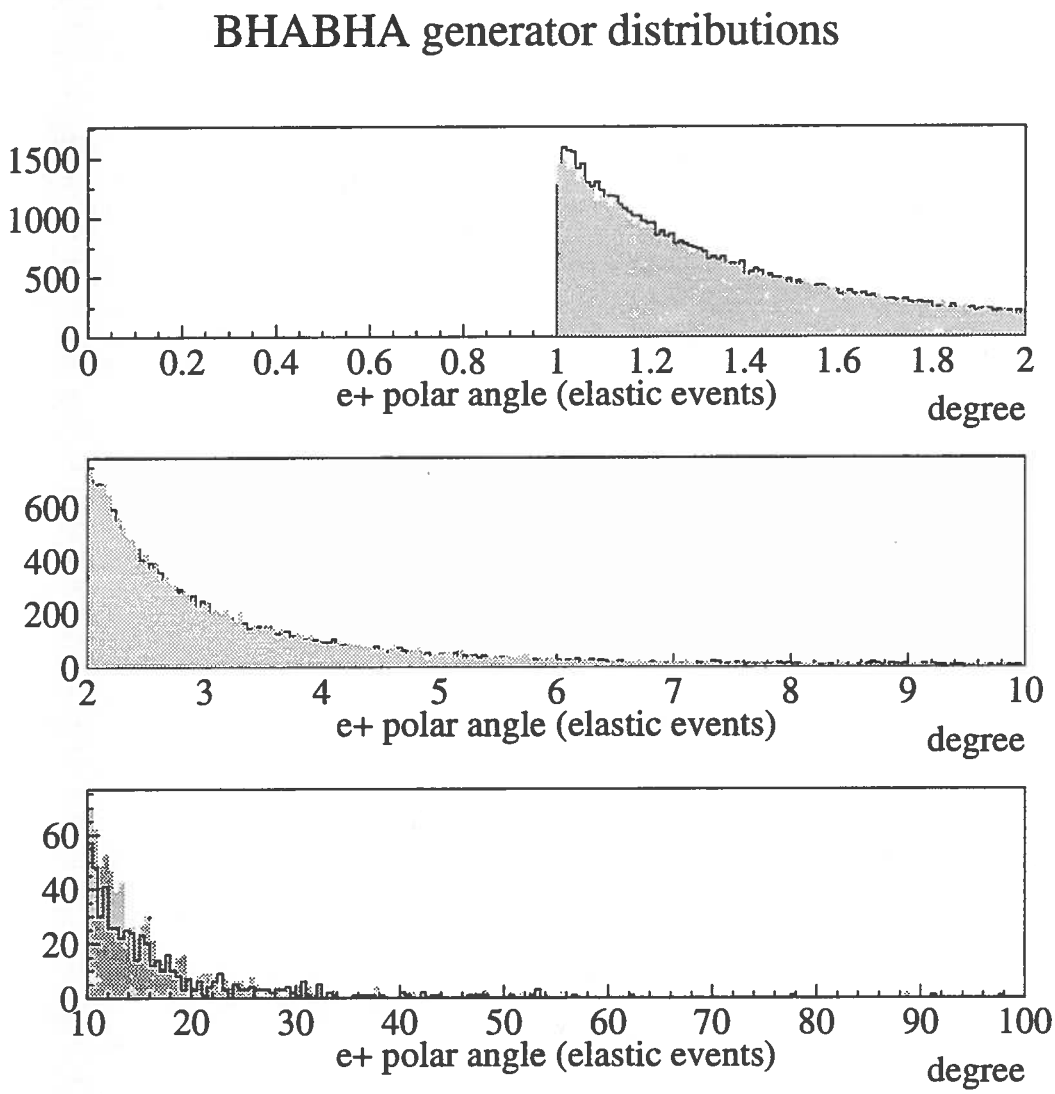

Figure 3: Angular distribution in c.m.s. of emitted positrons in the elastic scattering.

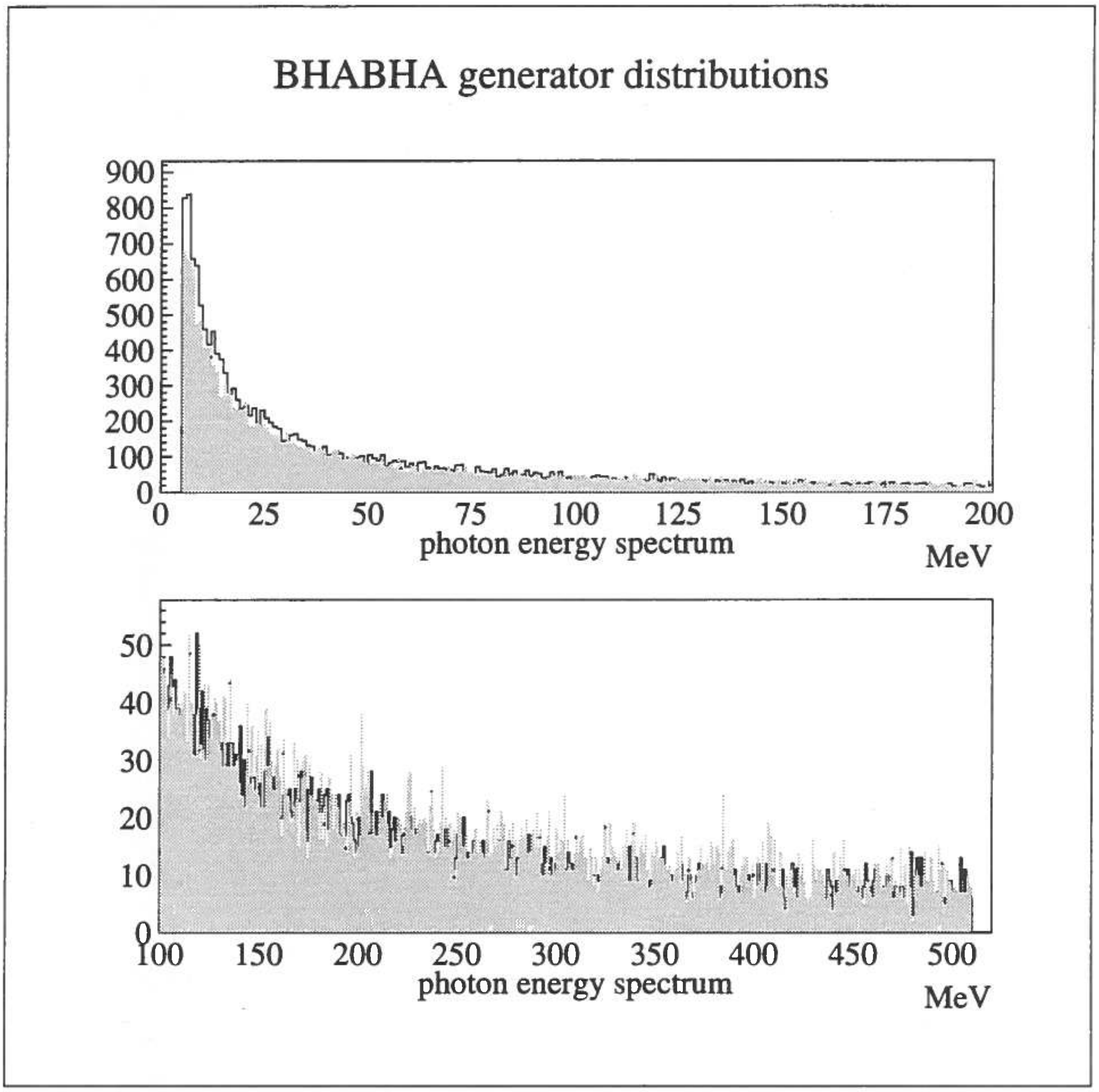

Figure 4: Photon energy spectrum.

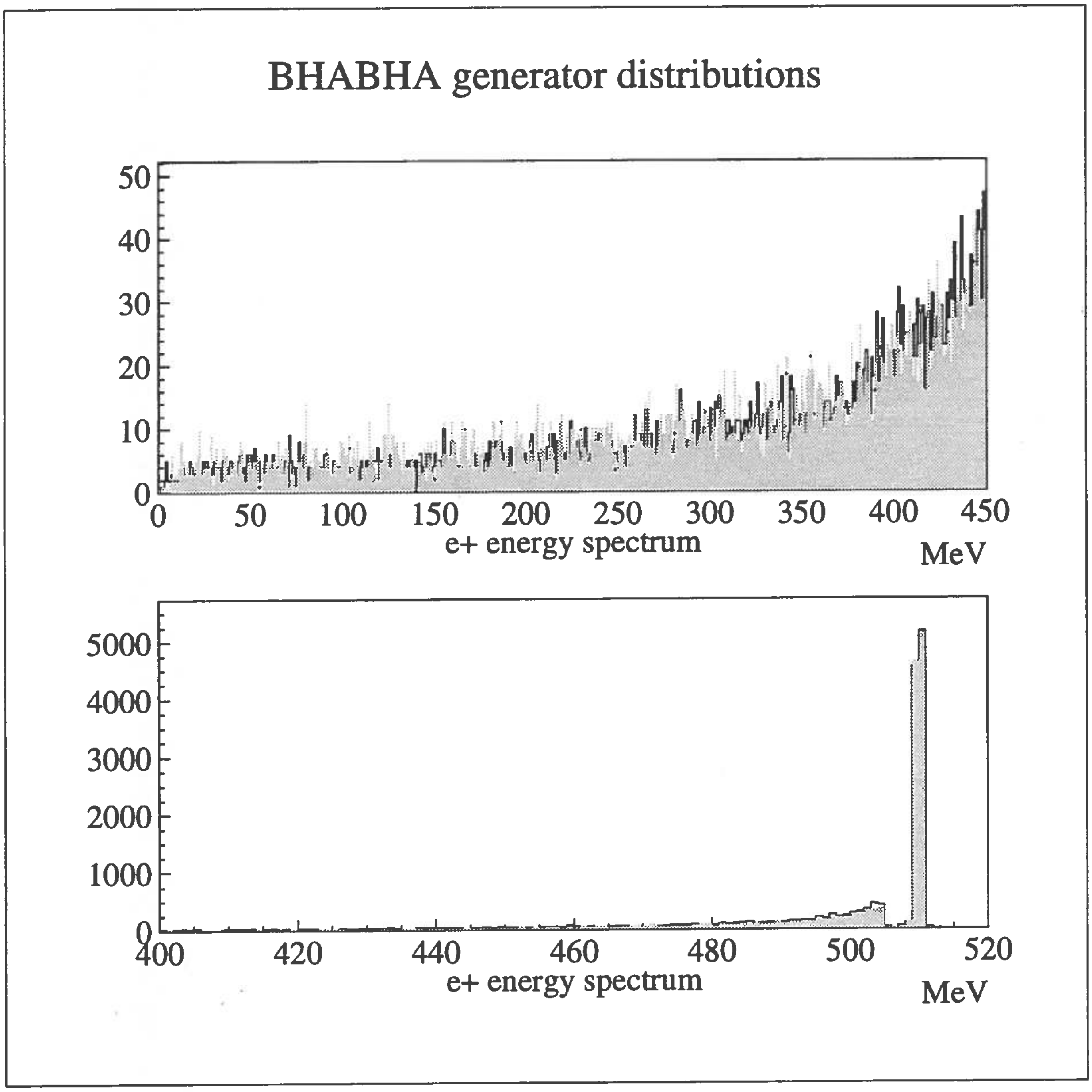

Figure 5: Positron energy spectrum.

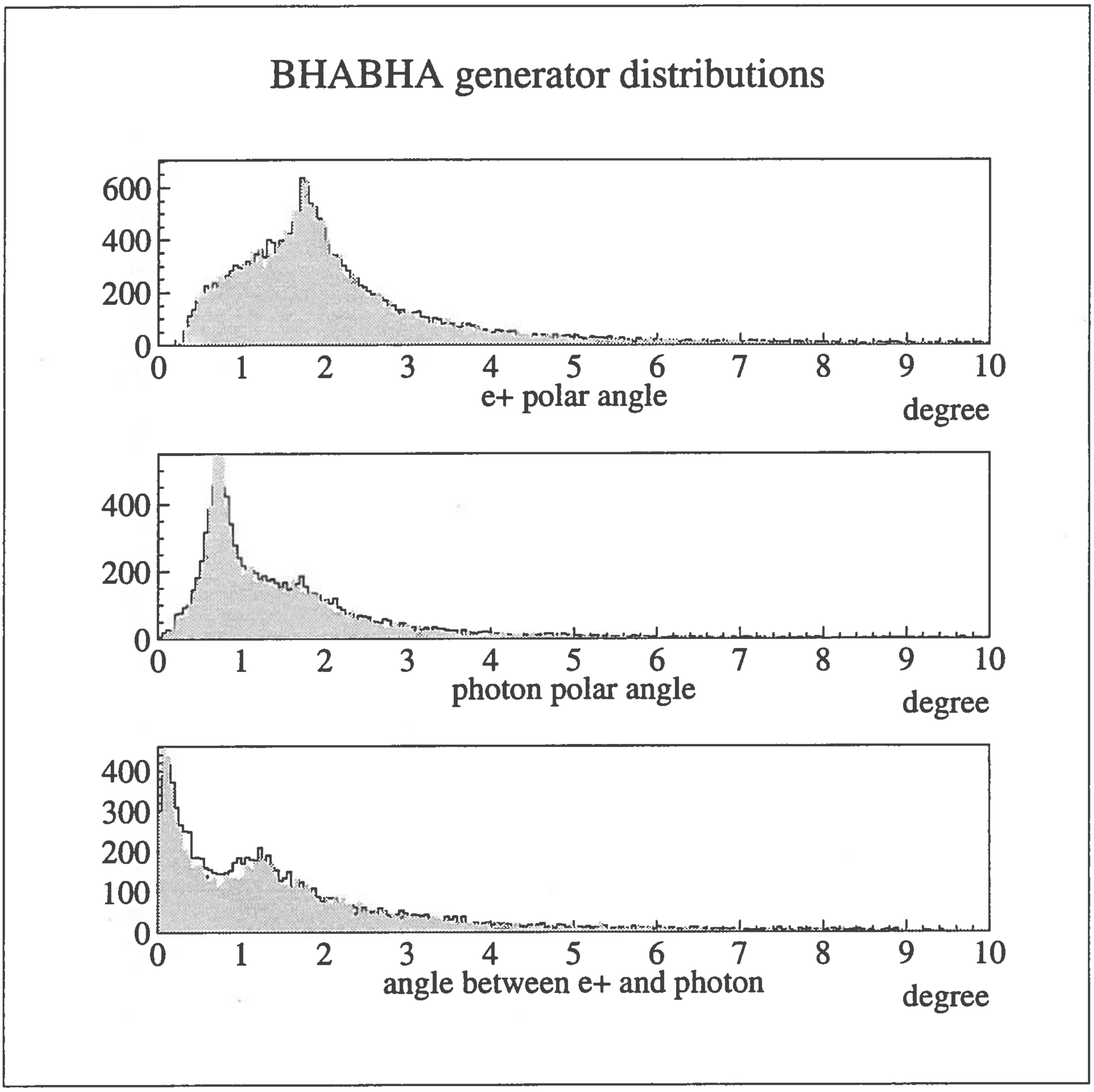

Figure 6: Absolute angles of positron and photon and relative angle between the two. The Lorenz Boost shift from 0^0 the photon peak.

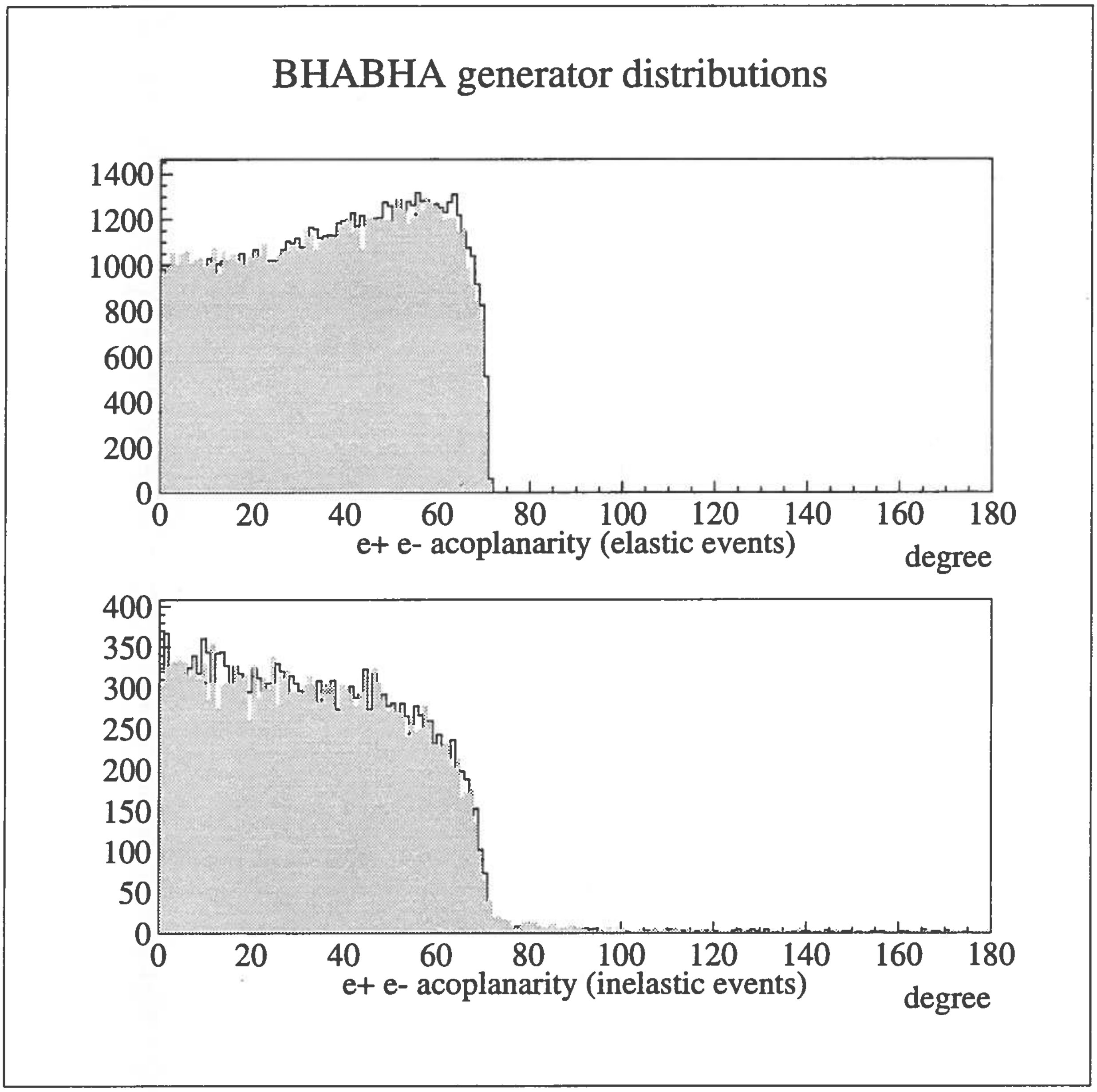

Figure 7: Acoplanarity distributions.

to the lowest order $|A_0|^2$. Because this contribution to the total cross section is infrared divergent, in order to obtain an infrared-finite expression, the contribution from soft bremsstrahlung must be added[1].

Soft bremsstrahlung is the process $e^+ + e^- \to e^+ + e^- + \gamma$, when the emitted photon has such a low-energy, so that it is smaller than the energy threshold of the calorimeter ($E_\gamma < k_0$). The photon is thus not detected and the process is recorded as being elastic. On the contrary, in hard bremsstrahlung, the photon has sufficient energy ($E_\gamma > k_0$) to be detected by the apparatus; in this generator the energy threshold to detect photons k_0 is fixed to 1% of the beam energy, which corresponds to $5.1\,MeV$ for $E_b = 510\,MeV$.

From the above considerations, the differential QED elastic cross section can be conveniently rewritten as:

$$\frac{d\sigma_\gamma}{d\Omega} = \frac{d\sigma_0}{d\Omega}(1 + \delta_{QED}) \tag{11}$$

where δ_{QED} is the α lowest order correction, taking into account both of the radiative[2] and of soft bremsstrahlung contributions. Figure 8 shows the value of δ_{QED} as function of θ polar angle. As can be seen, the effect of this correction is to decrease the total cross section.

- The evalutation of $|A_\phi|^2$ term is simple, considering that for the resonant process the photon propagator varies as:

$$\frac{e^2}{s} \to \frac{e^2}{s}B(s) = \frac{e^2}{s}\left(\frac{\beta s}{s - M_\phi^2 + iM_\phi\Gamma}\right) \tag{12}$$

where $\sqrt{s}$ is the total energy in the c.m.s., M_ϕ is the mass of ϕ ($1019.4\,MeV$), Γ is the total width ($4.43\,MeV$), and β is related to the leptonic width: $\Gamma_{e^+e^-} = \frac{1}{3}\alpha\beta M_\phi = 1.37\,keV$.

The resonant differential cross section in the c.m.s. is:

$$\frac{d\sigma_\phi}{d\Omega} = \frac{\alpha^2}{16E_b^2}(1 + cos^2\theta)|B(s)|^2. \tag{13}$$

[1]This is a particular case of the Bloch-Nordiesck theorem which states that for all processes in QED the infrared divergences cancel exactly to all orders of perturbation theory. In particular the infrared divergences of higher-order radiative corrections exactly cancel those in the inelastic processes with emission of several soft photons.

[2]The leptonic and hadronic contributions to the vacuum polarization are both included. The leptonic contribution is an analytic expression involving the lepton mass; the hadronic contribution is a dispersion integral over the known hadronic cross section. The result used here is a parametrization given by H.Burkhardt et al.: TASSO note 192(1981), Polarization at LEP CERN 88-06 VOL I, H.Burkhardt June 89.

Due to the very small $|B(s)|^2$, the resonant cross section is negligible when compared to the pure Bhabha cross section ($|B(s)|^2$ is at most $\sim 10^{-2}$, for $s = M_\phi^2$):

$$\sigma_{e^+e^-\to\phi\to e^+e^-} = \sigma_{e^+e^-\to\phi} \times \frac{\Gamma_{e^+e^-}}{\Gamma_{tot}} \sim nb, \tag{14}$$

while, for example,

$$\sigma_{e^+e^-\to\phi\to K_SK_L} = \sigma_{e^+e^-\to\phi} \times \frac{\Gamma_{K_SK_L}}{\Gamma_{tot}} \sim 10^3 nb. \tag{15}$$

Nevertheless it's relevant to consider that, if $s \neq M_\phi^2$, the contribution of the ϕ in the process $e^+ + e^- \to e^+ + e^-$ gives a more important effect in the interference term, as it will be shown below.

- The term I(A_0,A_ϕ) represents the interference between the two diagrams of lowest order amplitude of the photon with the ϕ; the interference differential cross section in c.m.s. is:

$$\frac{d\sigma_{\gamma,\phi}}{d\Omega} = \frac{\alpha^2}{8E_b^2}((1+cos^2\theta)ReB(s) - \frac{(1+cos\theta)^2}{(1-cos\theta)}ReB(s)) \tag{16}$$

  where the first term represents the interference with the photon in s-channel and the second is for the photon in t-channel (this term gives a divergent contribution at small angle). At the peak of the ϕ, if the beam energy spread ($\sim 700\,keV$) is taken into account, ReB(s) takes a value of the order of $\sim 10^{-2}$.

- Finally, the term I(A_c,A_ϕ) represents the contribution to the cross section due to the interference between the second order QED diagram and the resonant channel. Again due to the small value of ReB(s), this term ($\sim \alpha ReB(s)$) can be neglected without any conseguence.

Considering all the previously mentioned contributions, the differential elastic cross-section can be conveniently parametrized as:

$$\frac{d\sigma_{el}}{d\Omega} = \frac{d\sigma_\gamma}{d\Omega} + \frac{d\sigma_\phi}{d\Omega} + \frac{d\sigma_{\gamma,\phi}}{d\Omega} = \frac{d\sigma_0}{d\Omega}(1 + \delta_{QED} + \delta_\phi), \tag{17}$$

where δ_ϕ includes all the contributions due to ϕ effects, that are added as a fraction of the pure Born cross section. Figure 9 shows δ_ϕ value as function of θ, for different beam energies. As expected this term is small compared to QED radiative corrections, especially at small angles.

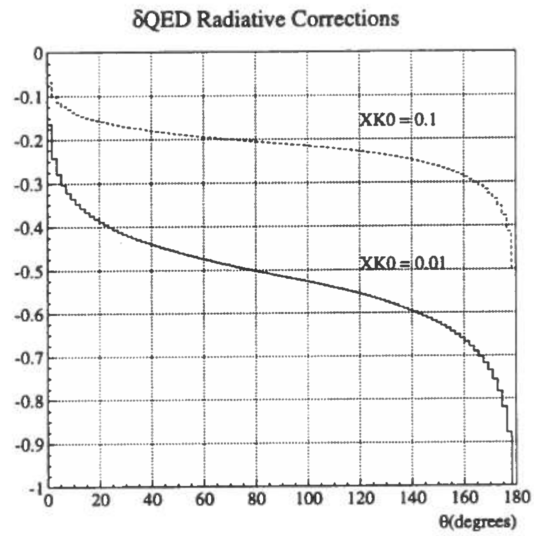

Figure 8: The sum of virtual photon corrections and soft bremsstrahlung contributions as a function of the scattering polar angle for two different value of $XK0 = k_0/E_b$, with $E_b = 510\, MeV$.

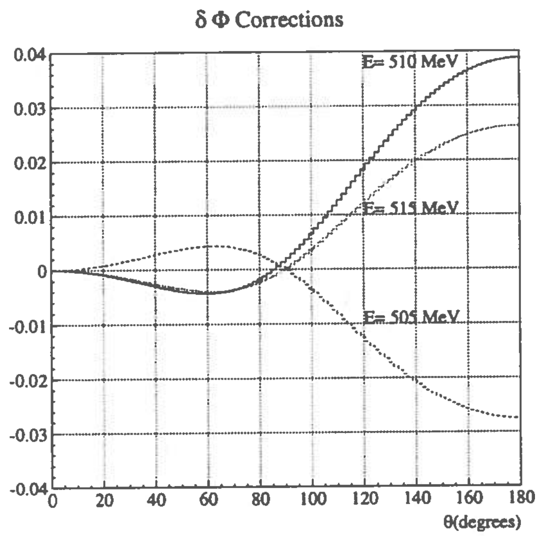

Figure 9: Relative ϕ contribution at different beam energies.

A.2 The Inelastic Process

If in a Bhabha process a real photon is emitted (i.e. if the process is inelastic), the allowed phase space for the final state must be evaluated. The differential cross section in the ultrarelativistic limit is [2]:

$$d^5\sigma = \frac{\alpha^3}{\pi^2 s} A W_{IR} W_m d^5\Gamma, \tag{18}$$

where:

$$A = (ss'(s^2 + s'^2) + tt'(t^2 + t'^2) + uu'(u^2 + u'^2))/(ss'tt'), \tag{19}$$

$$W_{IR} = \frac{s}{x_1 x_2} + \frac{s'}{y_1 y_2} - \frac{t}{x_1 y_1} - \frac{t'}{x_2 y_2} + \frac{u}{x_1 y_2} + \frac{u'}{x_2 y_1}, \tag{20}$$

$$W_m = 1 - \frac{m_e^2(s - s')}{s^2 + s'^2}\left(\frac{s'}{x_1} + \frac{s'}{x_2} + \frac{s}{y_1} + \frac{s}{y_2}\right), \tag{21}$$

$$d^5\Gamma = \delta^4(p_+ + p_- - q_+ - q_- - k)(d^3 q_+/2q_+^0)(d^3 q_-/2q_-^0)(d^3 k/2k^0), \tag{22}$$

and

$$s = (p_+ + p_-)^2, \quad t = (p_+ - q_+)^2, \quad u = (p_+ - q_-)^2, \tag{23}$$

$$s' = (q_+ + q_-)^2, \quad t' = (p_- - q_-)^2, \quad u' = (p_- - q_+)^2, \tag{24}$$

$$x_1 = p_+ k, \quad x_2 = p_- k, \quad y_1 = q_+ k, \quad y_2 = q_- k. \tag{25}$$

W_{IR} describes the infrared contribution for the photon emission, diverging for photons collinear with the incoming or the outgoing beams; W_m takes into account the electron mass terms.

The inelastic weight is:

$$w_{inel} = \left(\frac{d^5\sigma}{d^5\Gamma}\right)\left(\frac{d^5\sigma_a}{d^5\Gamma}\right)^{-1}. \tag{26}$$

where $d^5\sigma_a$ is the following approximate cross section:

$$d^5\sigma_a = \frac{\alpha^3}{\pi^2 E_b^2} \frac{1}{k(1-c)(e+z)(e+c_\gamma)} dk d\Omega_\gamma dc d\phi. \tag{27}$$

(ϕ is the azimuthal angle of q_+ around the beam and

$$k = k^0/E_b, \quad e = (1 + m_e^2/E_b^2)^{0.5} \tag{28}$$

$$c = cos\angle(p_+, q_+), \quad c_\gamma = cos\angle(p_+, k), \quad z = cos\angle(q_+, k). \tag{29}$$

A.3 Data card description

The dimensional units are: centimeters and GeV/c.

```
RNDM 361 108537 0
C* Random seeds

TRIG 10000
C* # of events to generate

TMIN 1.
C* Minimum positron (electron) polar scattering angle (in C.M.S.)

TMAX 179.
C* Maximum positron (electron) polar scattering angle (in C.M.S.)

BEAM 0.510
C* Beam energy mean value in C.M.S.

RUNG 361
C* User run number
```

A.4 PMBO (Particle Momentum Object) Bank

The Bank consists of three blocks, five words each. In each block there are respectively the 4-momenta and the mass of positron, electron and photon.

```
YBOS Bank header:
-----------------
```

```
Bank Name:              PMBO
Bank Number:               1
Bank Data Lenght:         15
Bank Type:            BNKTR4  !  R*4

==============================================

  offset              word type      description

==============================================
1st Block
0                     R*4         Positron Px
1                     R*4         Positron Py
2                     R*4         Positron Pz
3                     R*4         Positron Energy
4                     R*4         Positron mass
----------------------------------------------
2nd Block
5                     R*4         Electron Px
6                     R*4         Electron Py
7                     R*4         Electron Pz
8                     R*4         Electron Energy
9                     R*4         Electron mass
----------------------------------------------
3th Block
10                    R*4         Photon Px
11                    R*4         Photon Py
12                    R*4         Photon Pz
13                    R*4         Photon Energy
14                    R*4         Photon mass
==============================================
```

A.5 The output file: an example

```
Previous value of FOR030 has been superseded
Start event generation          960315      2049
```

```
USER'S DIRECTIVES TO RUN THIS JOB
----------------------------------


***** DATA CARD CONTENT     LIST
***** DATA CARD CONTENT     RNDM 361 108537 0
***** DATA CARD CONTENT     DEBUG -1 1
***** DATA CARD CONTENT     TRIG 10000
***** DATA CARD CONTENT     TMIN 1.
***** DATA CARD CONTENT     TMAX 179.
***** DATA CARD CONTENT     BEAM 0.510
***** DATA CARD CONTENT     RUNG 7
***** DATA CARD CONTENT     JOBID 361  2400002
***** DATA CARD CONTENT     END
Initial Random Number seed values:              361          108537
Following REAL parameters of type  3 have been changed:

    Parameter    1  changed from   5.0000000     to   1.0000000
Following REAL parameters of type  4 have been changed:

    Parameter    1  changed from   175.00000     to   179.00000
--------------------------------------------------------------------------------
INITIALIZATION FOR BHABHA SCATTERING

                        BEAM ENERGY        0.510000 GEV
           MINIMUM SCATTERING ANGLE        1.000000 DEGREES
           MAXIMUM SCATTERING ANGLE      179.000000 DEGREES
 MINIMUM HARD BREMSSTRAHLUNG ENERGY        0.010000
 MAXIMUM HARD BREMSSTRAHLUNG ENERGY        1.000000
          LOWEST ORDER CROSS SECTION    0.328436D+07 NB
 APPROX. CROSS SECTION IN SOFT PART     0.261135D+07 NB
 APPROX. CROSS SECTION IN HARD PART     0.944778D+06 NB
        APPROX. CROSS SECTION TOTAL     0.355612D+07 NB
           APPROX. TOTAL CORRECTION        0.082745
--------------------------------------------------------------------------------
BHABHA SAMPLE STATISTICS
   0.710000D+02   0.157000D+03   0.690000D+02   0.490972D+02   0.382794D+02
  0.280000D+02
   0.137000D+03   0.137886D+03   0.138979D+03   0.720000D+02   0.000000D+00
```

```
 0.000000D+00
  0.721167D-01   0.982566D+00   0.000000D+00   0.000000D+00   0.897424D+00
 0.105098D+01
         LOWEST ORDER CROSS SECTION   0.328436D+07 NB = UNIT
  APPROXIMATED HARD PHOTON XSECTION       0.287660
       CORRECT HARD PHOTON XSECTION       0.198920
                        UNCERTAINTY       0.008429
W.R.P EFFICIENCY IN INTERNAL C LOOP       0.452229
  "      "      OF C/CT RESTRICTION       0.971831
  "      "         FOR HARD WEIGHTS       0.405797
  APPROXIMATED SOFT PHOTON XSECTION       0.795085
       CORRECT SOFT PHOTON XSECTION       0.800224
                        UNCERTAINTY       0.002611
 W.R.P. EFFICIENCY FOR SOFT WEIGHTS       0.525547
   APPROXIMATED TOTAL CROSS SECTION       1.082745
        CORRECT TOTAL CROSS SECTION       0.999144
                        UNCERTAINTY       0.011040

GENERATED WEIGHTS:
         < 0 IN HARD PART      0.000000
      > WMAX IN HARD PART      0.000000
     MINIMUM IN HARD PART      0.072117
     MAXIMUM IN HARD PART      0.982566
         < 0 IN SOFT PART      0.000000
      > WMAX IN SOFT PART      0.000000
     MINIMUM IN SOFT PART      0.897424
     MAXIMUM IN SOFT PART      1.050977
--------------------------------------------------------------------------------
BHABHA SAMPLE STATISTICS
  0.575700D+04   0.130930D+05   0.568000D+04   0.410519D+04   0.325186D+04
 0.207300D+04
  0.159580D+05   0.160718D+05   0.162204D+05   0.802700D+04   0.000000D+00
 0.000000D+00
  0.646022D-03   0.189878D+01   0.000000D+00   0.000000D+00   0.383210D+00
 0.105149D+01
         LOWEST ORDER CROSS SECTION   0.328436D+07 NB = UNIT
  APPROXIMATED HARD PHOTON XSECTION       0.287660
       CORRECT HARD PHOTON XSECTION       0.205124
                        UNCERTAINTY       0.000900
```