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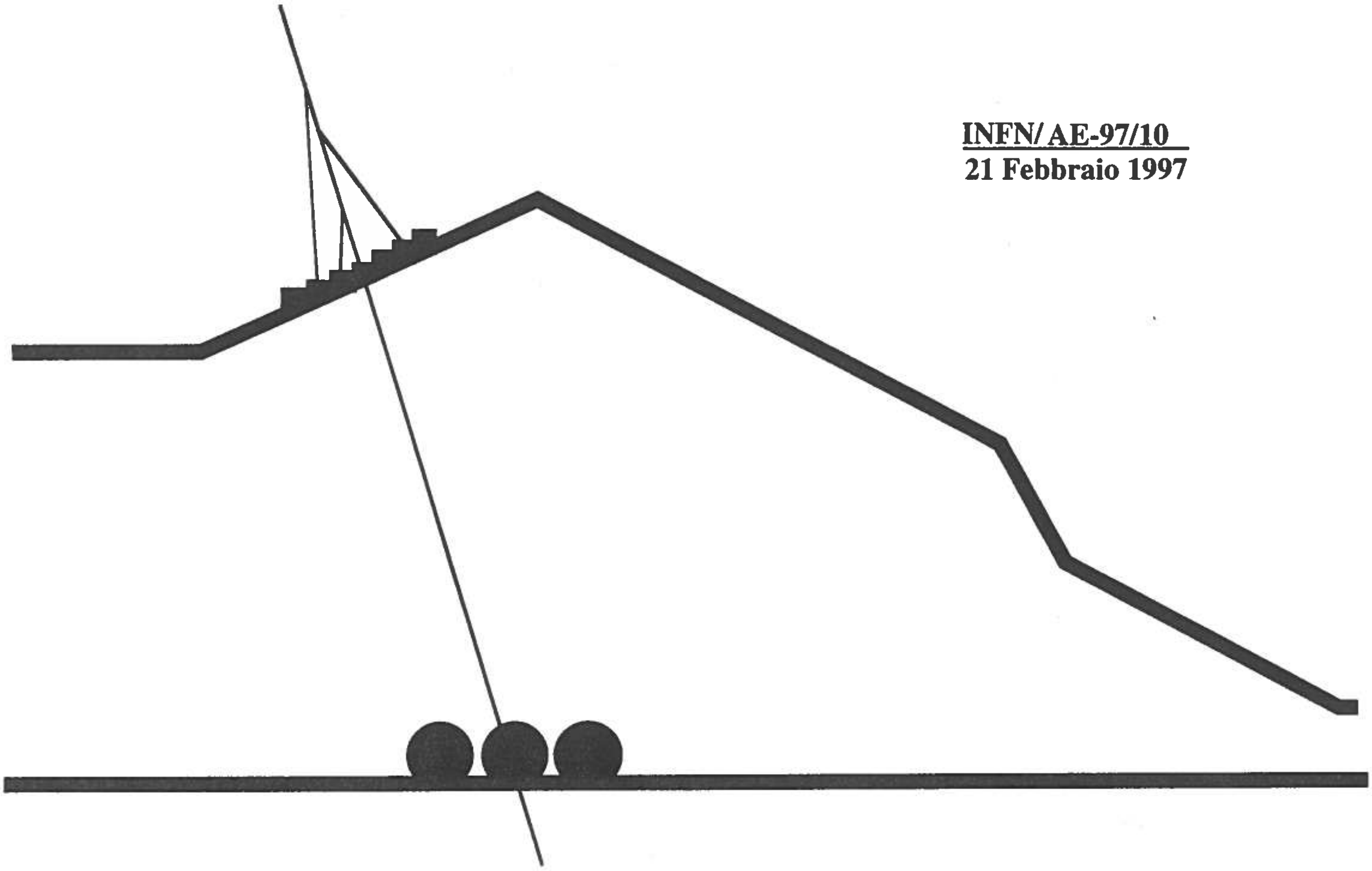
21 Febbraio 1997



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Submitted to Nuclear Instruments & Methods in Physics Research

INFN - Laboratori Nazionali del Gran Sasso

Published by SIS-Pubblicazioni
dei Laboratori Nazionali di Frascati

INFN – Istituto Nazionale di Fisica Nucelare
Laboratori Nazionali del Gran Sasso

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Simulation of Muon Transport at High Energy: Comparison of Few Different Codes

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Abstract

Some comparison among a few transport codes for high energy muons (GEANT, FLUKA and PROPMU), in the framework of the activity of underground experiments for cosmic rays, is performed. A few important differences in the sampling of e.m. interactions and energy loss calculations have been found. The possible consequences on muon physics are discussed.

1 Introduction

A precise treatment of muon transport at high energy is an essential part of many research activities, such as those related to underground [1], underwater or underice experiments[2]. In those experimental frameworks, the calculation of survival probabilities of high energy muons (at energies ranging from GeV to many TeVs), of their deflection and of their residual energy at a given depth, are essential tools to perform analyses concerning both indirect measurements on nuclear cosmic rays and atmospheric neutrinos. Besides the quoted examples, muon calculations are also extremely important for aspects such as radiation shielding at high energy colliders.

Simulation of muon interaction with matter at high energy presents two different order of problems. The first one is of mere theoretical nature; there is now a general agreement in the high energy community to consider the cross sections reviewed in[3] as a sort of standard reference for muon calculations: Petrukhin and Shestakov[4] for bremsstrahlung, Kokoulin and Petrukhin[5] for pair production and Bezrukov and Bugaev[6] for photo-nuclear interaction. However, many uncertainties have been discussed recently[7]. In fact, despite the leptonic nature of muon, the relevance of nuclear form factors and of their uncertainties at TeV energies becomes noticeable as far as screening is concerned even for processes like bremsstrahlung and pair production. The implication of non perturbative hadronic physics are even more fundamental in the case of the inelastic photo-nuclear interaction of muons. Unfortunately, there are not yet experimental data with sufficient accuracy to solve these questions. There are also other aspects which, for their complexity, contribute to the overall uncertainties, such as radiative corrections and higher order Born terms. For a review of this problematics see [7] and also [8].

The second aspect is just of computational technique in Monte Carlo codes, due to some algorithmic complexities and to numerical cancellations which may occur in particular conditions, and mostly at the highest energies. Sometimes, approximations and simplified treatment are applied to speed up computations, but they can introduce systematic effects. In order to evaluate the performance and the possible systematics associated to the calculations of muon interactions, mainly in the framework of the analysis activity of underground experiments at Gran Sasso, we have performed comparison test of few MC codes at TeV energies: GEANT 3.21[9] (for a few comparisons we have tested, in addition, also the 3.15 version), FLUKA[10], the transport code of HEMAS Monte Carlo[11, 12] (adopted by some experiments at Gran Sasso, see for example [13]), and the code PROPMU by Lipari and Stanev[14], specifically designed for underground applications. In part, this work originates by the need to understand the differences between existing calculations of muons survival probability, as those of ref. [15], [14] and [16]. Our aim is limited here to those aspects which mainly affect the energy loss calculation, without considering other processes like the calculation of deflections due to multiple scattering or to other interactions, which are anyway fundamental for three-dimensional calculations. We do not even consider the comparison, when possible, of the different implementations for the production of secondary particles in the radiative interaction of muons, although it should deserve some attention for some particular application, since it can be neglected in the calculation of muon energy loss. In fact, some of the considered codes do not follow secondary products.

In the next section we summarize briefly the relevant ingredients of the proposed codes. In section 3 the organization of the comparative test is described, while the results are reported in section 4, discussing separately, whenever possible, the main different interaction processes. Section 5 offers some conclusive remarks.

2 The selected codes

In the organization of the discussion of the performance in muon transport of the selected codes, we shall first consider the content of GEANT (we have restricted ourselves to versions 3.15 and 3.21), FLUKA, and PROPMU, leaving HEMAS, which was intentionally written in a much more simplified way with respect to standard codes,to a subsequent consideration.

The description of the physical processes concerning muon interactions implemented in GEANT is extensively reported in its manual[9] as far as bremsstrahlung and pair production is concerned, where the relevant cross sections are those contained in the review of Lohmann et. al.[3], (that is, respectively, the cross sections of Petrukhin and Shestakov[4] and of Kokoulin and Petrukhin[5]), but only limited information is given for the photonuclear interaction, for which reference is given to the GHEISHA Monte Carlo[17], which is taken from [18]. As discussed later in more detail, this code contains a rather different, and inaccurate, choice with respect to that of Bezrukov and Bugaev[6], reported in ref. [3]. Both FLUKA[10] and the PROPMU code[14] use the cross sections listed in the quoted review. Both FLUKA and GEANT sample the discrete process above user-defined cuts. Below these cuts the cross sections are integrated to calculate the contribution to the continuous energy loss. As far as the sampling algorithms are concerned, the GEANT manual alerts that for the the discrete simulation of pair production, some simplifications are performed. The original version of PROPMU, used for this work, performs the discrete sampling of bremsstrahlung, pair production and photo-nuclear processes above a determined fractional energy loss in a single collision ($\Delta E/E \sim 0.01$)[1]. The step length in muon transport is variable, and muons are stopped when their energy is less than 1 GeV.

It is important also to compare the performances of the ionization energy losses, since their contribution to the energy loss of TeV muons remains non-negligible. GEANT allows either to consider the average energy loss calculated along the tracking step, or to account for fluctuations either by explicit δ-ray production above a given energy cut or by a computed fluctuation function (gaussian or approximating the Landau shape). The version of FLUKA used for the test reported here can introduce fluctuations just by explicit δ-ray production . The PROPMU code considers only the average ionization losses. In this code, muon transport is performed with variable optimized steps, and the multiple scattering and lateral displacements are sampled from the proper Gaussian distributions. Even if not really relevant for the discussion contained in this paper, in GEANT we selected Moliere scattering. FLUKA allows only the Moliere treatment.

Coming now to HEMAS, we note that the muon transport routines have been also constructed following the philosophy of a simplified transport. The average energy loss of muons in standard rock (Z=11, A=22, ρ=2.65 g/cm^3), has been expressed as

$$-\frac{dE}{dx} = a(E) + b(E)E \tag{1}$$

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[1] A second version of PROPMU allows, optionally, to treat the pair production as a continuous energy loss

where $a(E)$ accounts for the ionization loss[19] and $b(E)$ is the is the sum of the contributions coming from pair production ($b_P(E)$, bremsstrahlung ($b_B(E)$)) and photonuclear interaction ($b_B(E)$). Both $a(E)$ and the different $b(E)$ are expressed by the results of detailed fits to the calculated energy behaviour. In particular, the $b_i(E)$ are given by polynomials in $\log(E_\mu/m_\mu)$. The transport is arranged in fixed tiny steps of standard rock, and the actual path is corrected for multiple scattering in the Gaussian approximation. At each step, the muon energy loss is the sum of the average ionization and pair production loss, plus a term stochastically sampled from a simplified distribution of energy loss due to bremsstrahlung and nuclear interaction. Below 5 Gev only the range-energy relation as obtained by ionization losses is considered. Surviving muons are defined as those having a residual total energy greater than 0.5 GeV. Apparently, a more accurate version of this code, with an improved treatment of the stochastic losses has been used to calculate the survival probabilities of ref.[15].

From the point of view of the choice of basic cross sections, HEMAS quotes the same basic references as GEANT, (except for the photonuclear cross section, where ref[20] is quoted) but the algorithmic implementation of the discrete process appear in HEMAS not only much more simplified with respect to all other quoted codes, but also too approximative in some respect, as we shall discuss later. As correctly stressed in [14], an important requirement for a Montecarlo is to implement all cross-sections with extreme precision, otherwise, after thousands of samplings (as needed for large depths) minor deviations from the true expressions add up to significative factors. Due to the relevance given to the HEMAS Monte Carlo for its performance for underground muon analysis, some effort in the evaluation of its features in muon transport is mandatory.

3 Simulation results

Comparison between different codes has been performed propagating 50000 muons with E_μ=3 TeV, since this is roughly the mean energy in atmosphere of muons detected underground at Gran Sasso. Muons are propagated through a thickness of 3500 hg/cm^2 of standard rock.

In the following we discuss the results mostly in terms of residual energy and survival probability, stressing the contribute coming from the treatment of radiative process.

3.1 Residual energy distribution and survival probability

Fig. 1 shows muon residual energy distribution for the muon transport codes considered. FLUKA and PROPMU codes are substantially in agreement. PROPMU neglects fluctuations in the ionization energy loss. The agreement between FLUKA and PROPMU is impressive if these fluctuations are neglected in FLUKA too (Fig. 2). HEMAS and GEANT residual muon energy distributions are clearly in disagreement with FLUKA and PROPMU ones. Table 1 shows muon survival probabilities and muon mean residual energy for the four codes.

The survival probability calculated with the 3.21 version is now very close to that predicted by PROPMU and FLUKA. That was not true for the previous 3.15 version. The survival probability from HEMAS is significantly lower. The differences are so relevant that a detailed examination of the code was carried on. We found that HEMAS makes

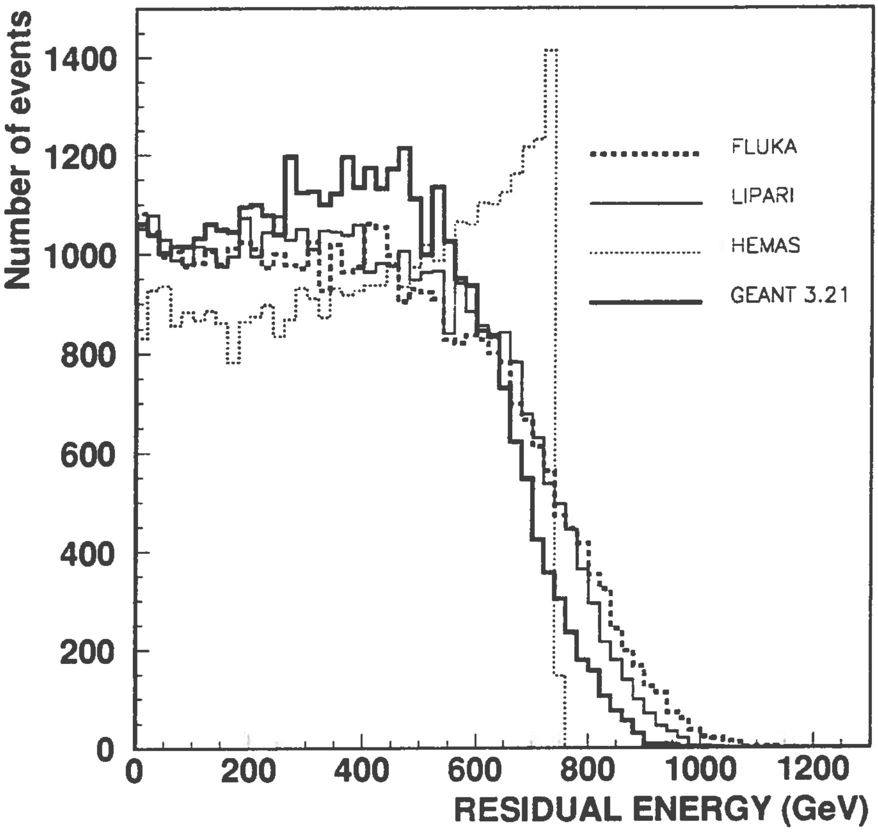

Figure 1: Muon residual energy distribution for the four considered codes.

Code	FLUKA no ioniz. fluct.	FLUKA with ioniz. fluct.	PROPMU	GEANT 3.21 (3.15)	HEMAS
Mean Energy (GeV)	380.9±1.2	386.2±1.2	378.±1.2	357.8±1.1) (401.1±1.1)	(395.6±1.2)
No. of Survived muons	$37494/5 \cdot 10^4$	$37249/5 \cdot 10^4$	$37620/5 \cdot 10^4$	$37996/5 \cdot 10^4$ $(39278/5 \cdot 10^4)$	$36063/5 \cdot 10^4$

Table 1: Mean residual energy and survived muons, for a total number of 50000 primary muons, computed with the different transport codes considered.

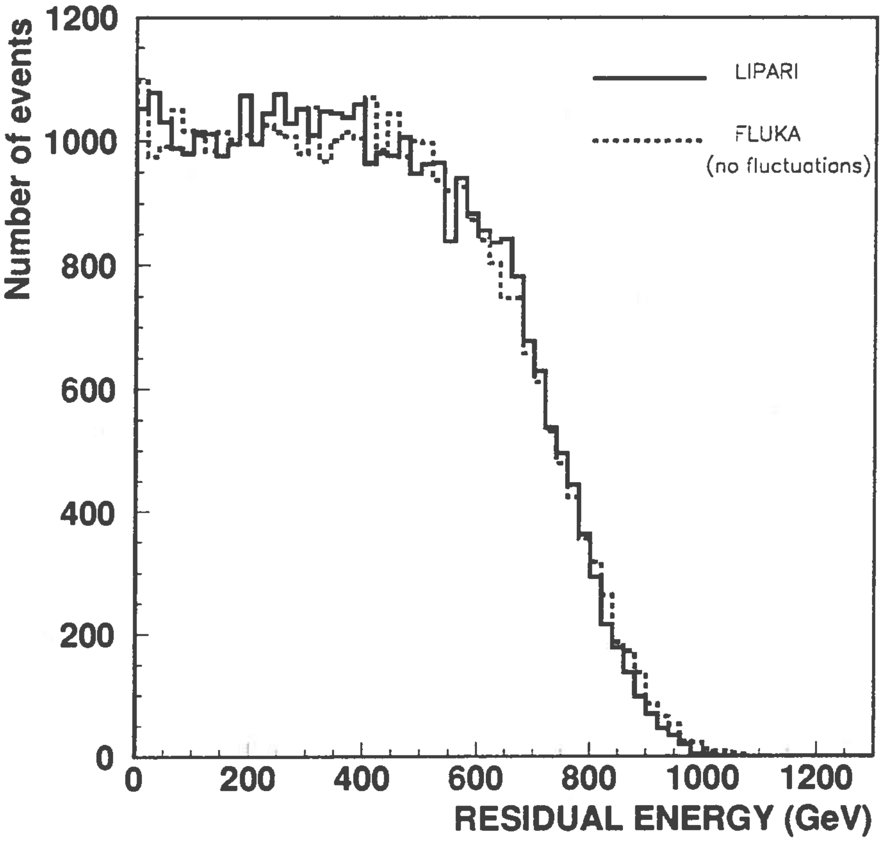

Figure 2: Muon residual energy distribution from PROPMU and FLUKA (without ionization energy loss fluctuation).

use of a too much very simplified treatment of the radiative processes. In particular, bremsstrahlung and photonuclear differential cross-sections are approximated like a $\simeq \frac{1}{v}$ function, where v=E_γ/E_μ, E_γ being the energy of the emitted (real or virtual) photon for single muon interactions at 3 TeV. These extreme simplifications affect both muon survival probability and muon residual energy distribution. The difference with respect to the other codes is so large that a further use of HEMAS in the systematic comparison appears useless.

In the following section we shall examine in more detail the treatment of radiative processes in the other codes.

3.2 Radiative processes

As discussed above, bremsstrahlung, pair production and photonuclear interaction are treated in FLUKA and PROPMU codes using the same cross sections, respectively from ref.[4, 5, 6]. Some improvements have been applied in FLUKA for bremsstrahlung together with a few corrections for the photonuclear cross sections: however their effect is small at 3 TeV and for light materials. In particular:

- the screening function for bremsstrahlung can be chosen according to the prescription of Petrukhin and Shestakov[4], Rozental[21], or Sakumoto et al.[22];
- following the prescription from Tsai[23], the Coulomb corrections in the relativistic approximation can be added[24];
- the photo-nuclear cross section from Bezrukov and Bugaev[6] has been integrated down to the lowest possible q^2 value (m_π^2), including the existing resonances in the γN cross section. This can be important for muons in the range of the tens of GeV.

In all our calculations, in order to be consistent with the choices of the other codes, we have used for bremsstrahlung the screening function of ref. [4], without asking for Coulomb corrections.

We computed the fractional energy spectrum of the emitted photon (real or virtual) with GEANT, FLUKA and PROPMU for each radiative process (pair production,bremsstrahlung and photonuclear interaction). The spectra has been compared with the expected shape analytically calculated from the respective differential cross sections. A cut has been imposed at E_γ=30 GeV.

To evaluate the total cross sections, we computed for each radiative process the muon mean free path, calculated from the injection point of 3 TeV muons up to the first interaction. In this case a cut has been applied at E_γ=15 GeV (v_{min} = 0.005 at the considered energy).

3.2.1 Bremsstrahlung

In Fig. 3 the distribution of of $v\frac{dN}{dv}$ (where $v=E_\gamma/E_\mu$) in the bremsstrahlung process is shown for E_μ=3 TeV. The agreement among the different codes is satisfactory.

Bremsstrahlung muon mean free path has been obtained switching off in GEANT all other processes. An exponential function has been fitted to the distribution of the first interaction path length. Fig. 4 reports the path length distribution; Table 2 shows a comparison between the mean free path for bremsstrahlung, and the other relevant

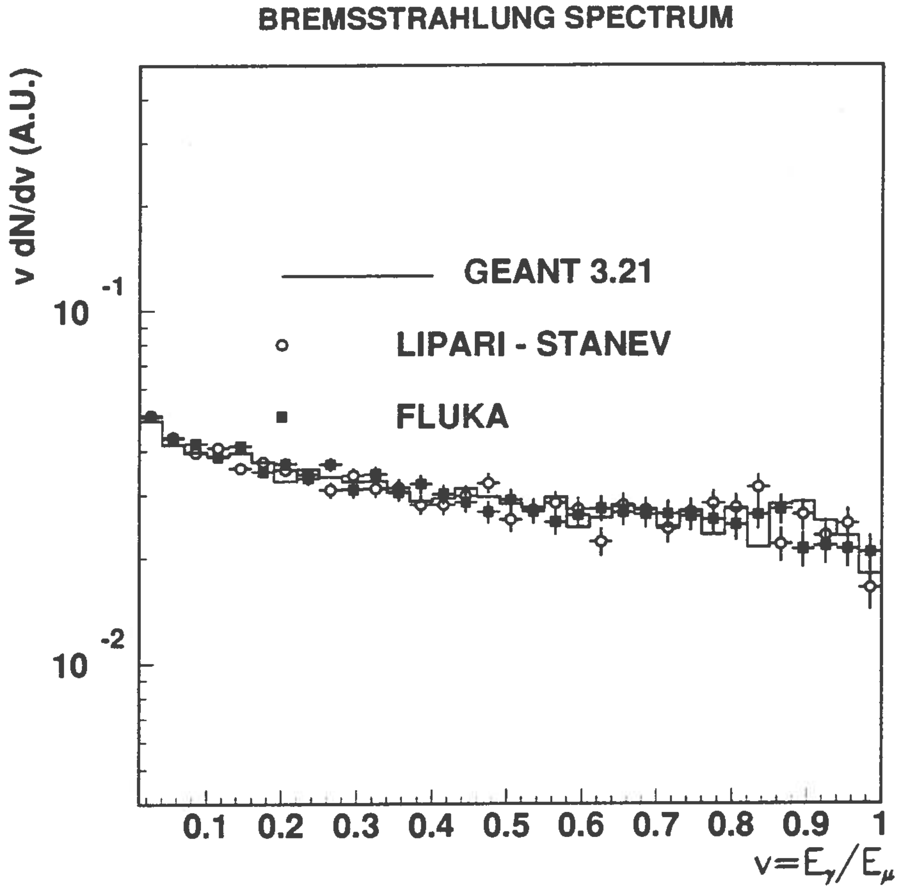

Figure 3: Bremsstrahlung spectrum distribution obtained using GEANT, FLUKA and PROPMU.

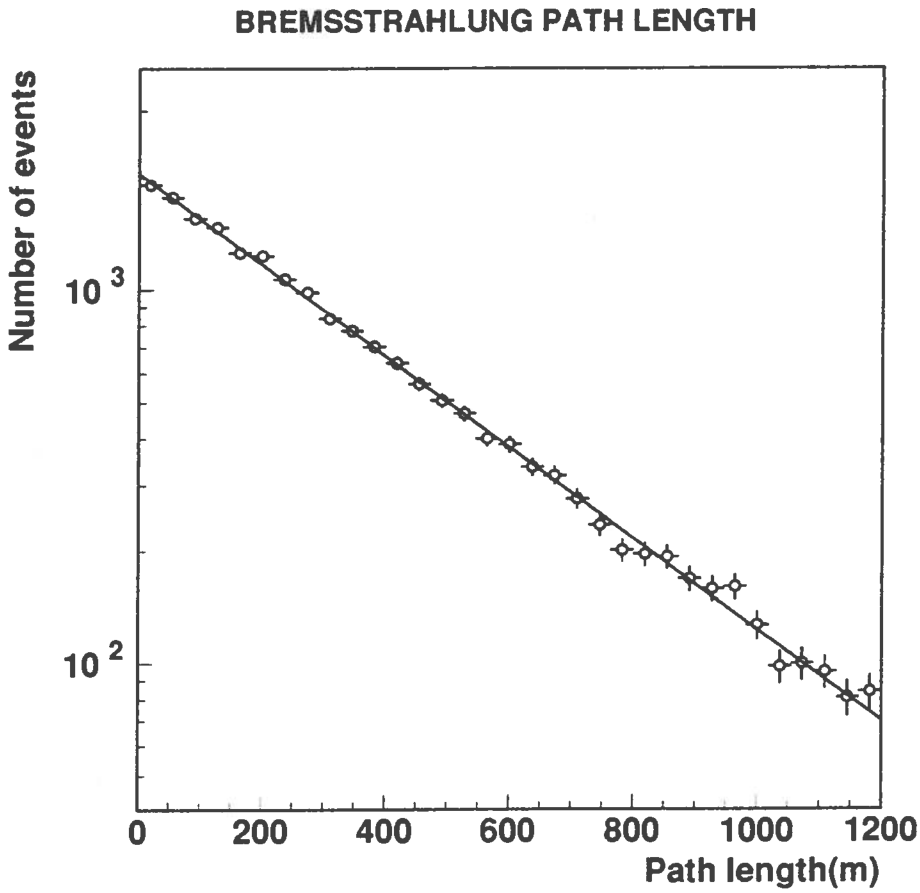

Figure 4: Bremsstrahlung path length distribution obtained using GEANT. The continuous line is an exponential fit.

interactions as well, computed with GEANT and those obtained from the formulas quoted in ref.[4, 3]. The values for bremsstrahlung are in agreement within 5%. The quality of the sampling algorithms can be evaluated from Fig. 5, where the shape of the sampled spectrum as obtained from FLUKA is compared to the analytical calculation from [4]. The other two codes are equivalent, as proved by Fig. 3.

3.2.2 Pair Production

As far as pair production is concerned, GEANT authors preferred to adopt a parameterization of formulas in ref.[5]. In Fig. 6, the distribution of $v\frac{dN}{dv}$ for pair production is presented. The agreement between FLUKA and PROPMU is still good, and well compatible with the expected shape. GEANT. instead, exhibits a slightly softer spectrum, although it makes reference to the same formula: the average of the GEANT distributions in Fig. 6 is lower by $(5.5 \pm 0.2)\%$ with respect to that of the other two codes.

Fig. 7 demonstrates that FLUKA (and hence also PROPMU) performs the correct sampling algorithm: there the shape of the sampled spectrum as obtained from FLUKA is compared to the analytical calculation from [5]. Mean free path for pair production (GEANT calculation is shown in Fig. 8) are also reported in Table 2. We find again an

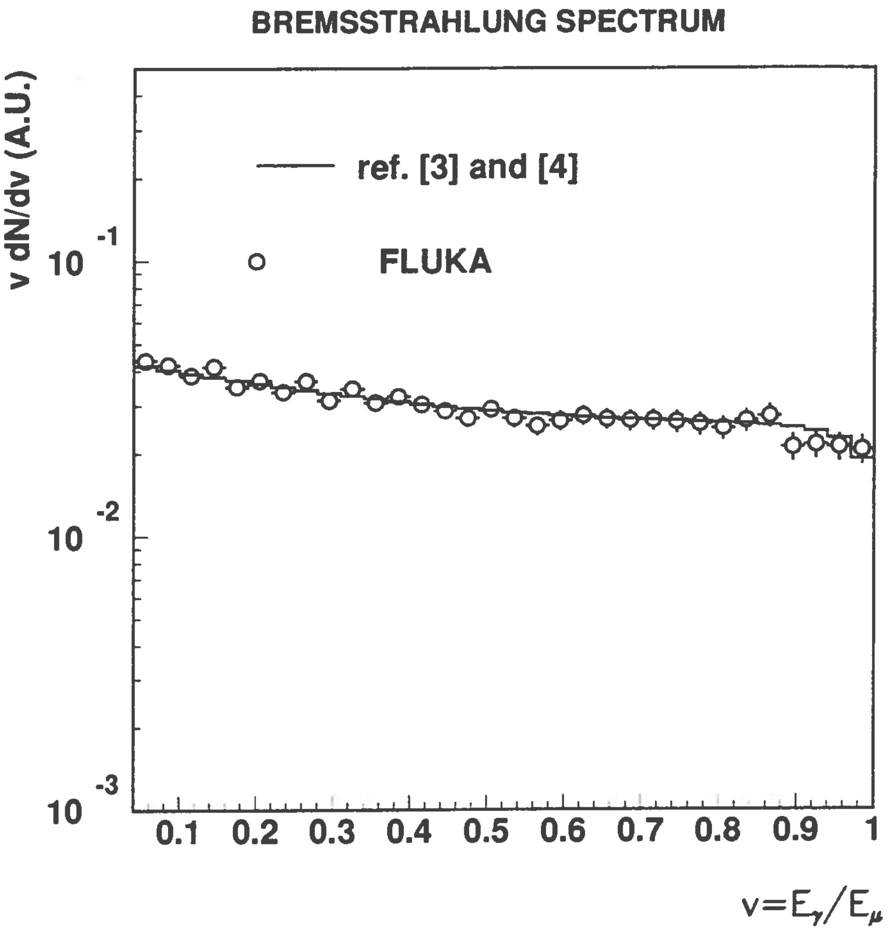

Figure 5: Comparison between bremsstrahlung spectrum obtained by FLUKA and the analytical expression from ref.[3, 4].

Process	Bremsstrahlung	Pair production	Photonuclear
Mean free path GEANT	(356±4) m	(42.9±0.3)m	(5680±570)m
Mean free path (Ref. [3])	361 m	43.5 m	751.7 m

Table 2: Mean free paths computed with GEANT and with formulas from Ref.[3].

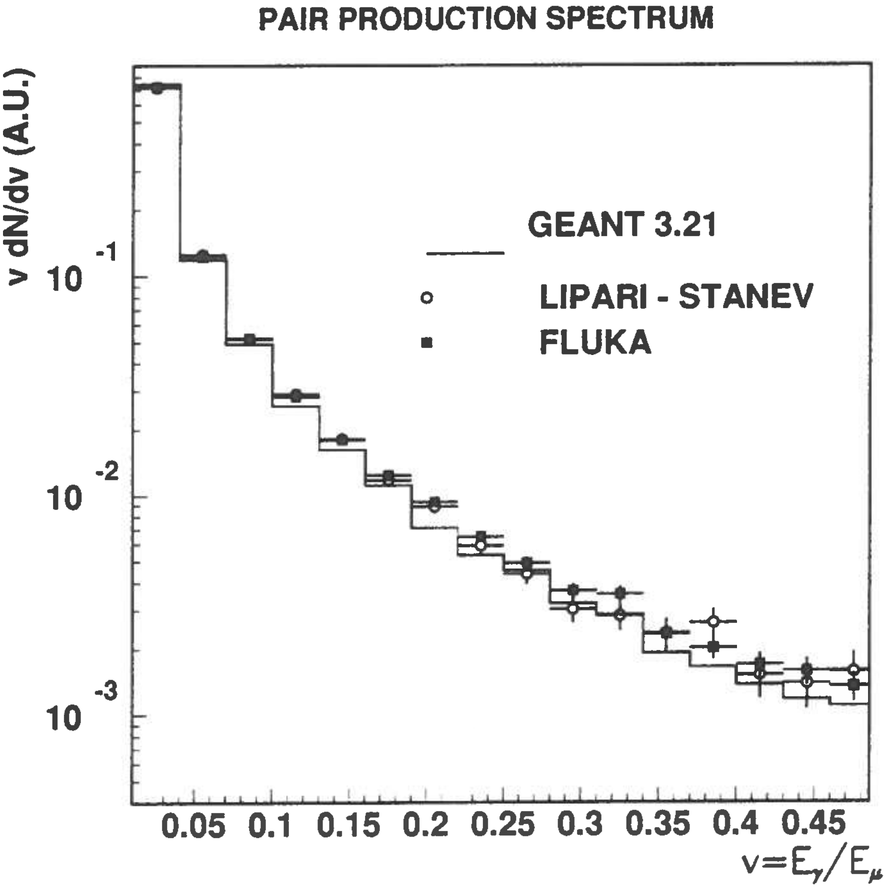

Figure 6: Pair production spectrum distribution obtained using GEANT, FLUKA and PROPMU.

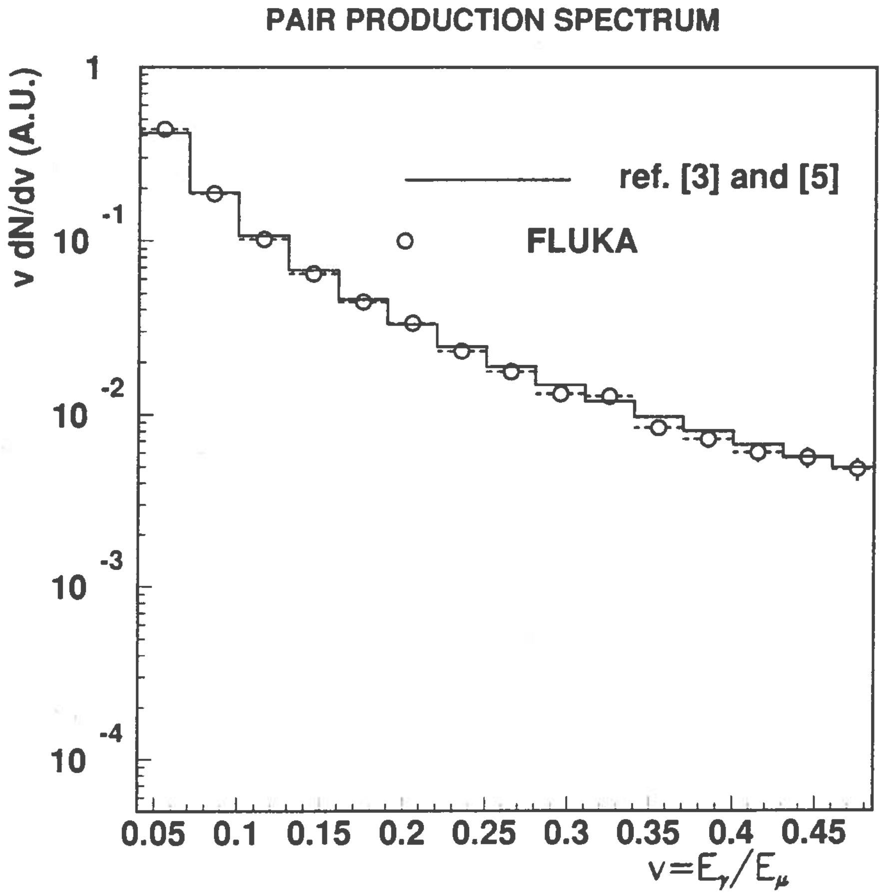

Figure 7: Comparison between pair production spectrum obtained by FLUKA and the analytical expression from ref.[3, 5].

agreement within 5% with those obtained from the formulas quoted in ref.[3].

3.2.3 Photonuclear interaction

As introduced in Section 2, this process is treated in a completely different manner inside GEANT with respect to the other codes, with substantial differences both in the total and the differential cross section. This is shown by the comparison of the sampled energy loss spectrum (Fig. 9) and by the resulting mean free path (Table 2 and Fig. 10).

The GEANT numbers are clearly different with those predicted by ref.[6, 3], which instead are well reproduced by FLUKA and PROPMU. Fig. 11 shows the case of the sampled spectrum in FLUKA, where the shape is compared to the analytical calculation from [6].

This mismatch is consistent with the total cross section quoted in GHEISHA[18] which at E_μ=3 TeV and for the standard rock parameters, predicts a value of $\sigma \simeq$0.0209 mbarn. Such a total cross section is by far lower than the cross section of ref.[6] used in FLUKA and PROPMU. From that formula, at E_μ=3 TeV and for standard rock, a value of σ $\simeq$0.183 mbarn is obtained. As a summary, in Table 3 we tabulate for different energies the macroscopic photonuclear cross section (*i.e.* $\sigma_{atom}\frac{N_A}{A}$)) computed with GHEISHA and the macroscopic cross sections obtained by integrating the differential cross section of [6].

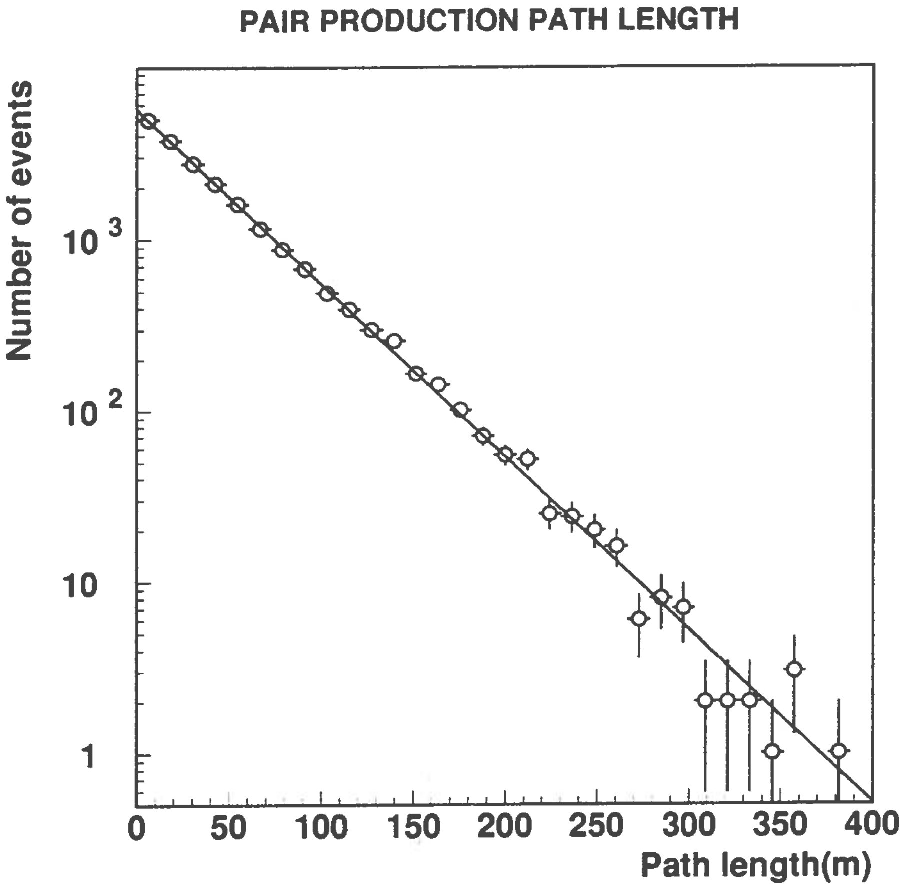

Figure 8: Pair production path length distribution obtained using GEANT. The continuous line is an exponential fit.

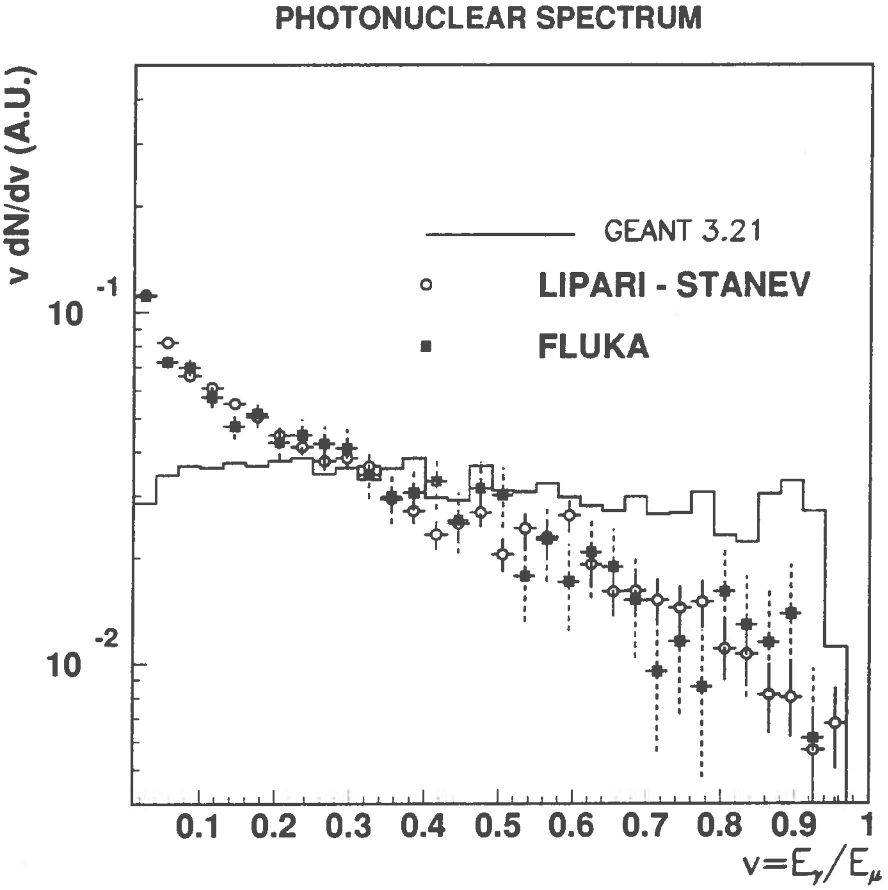

Figure 9: Photonuclear interaction spectrum distribution obtained using GEANT, FLUKA and PROPMU. It must be noticed that in order to bring the GEANT results on the same scale of the other two calculations (the total cross section used in GEANT is much smaller, see text), the distributions have been normalized at the same area.

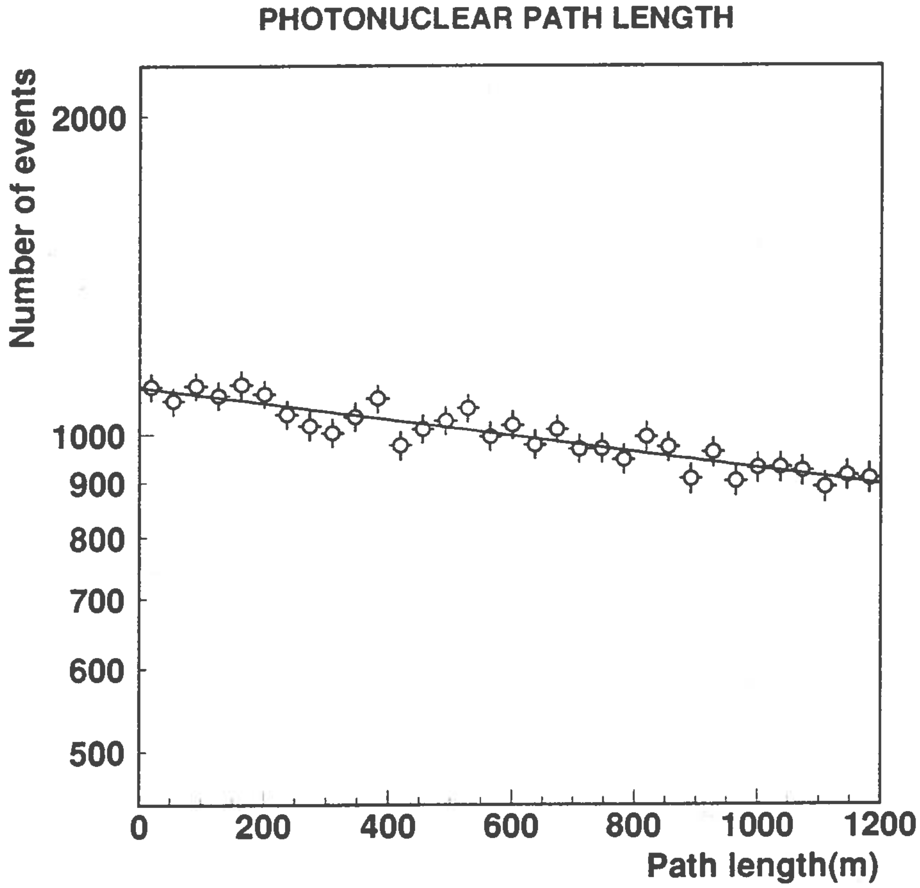

Figure 10: Photonuclear interaction path length distribution obtained using GEANT. The continuous line is an exponential fit.

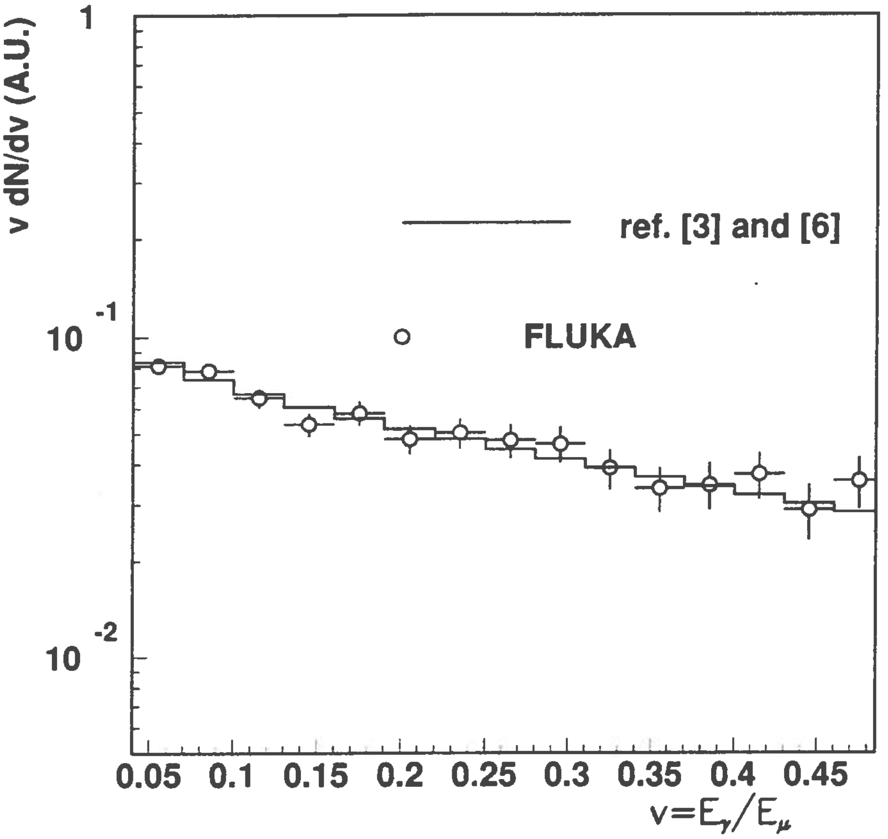

Figure 11: Comparison between photonuclear spectrum obtained by FLUKA and the analytical expression from ref.[3, 6].

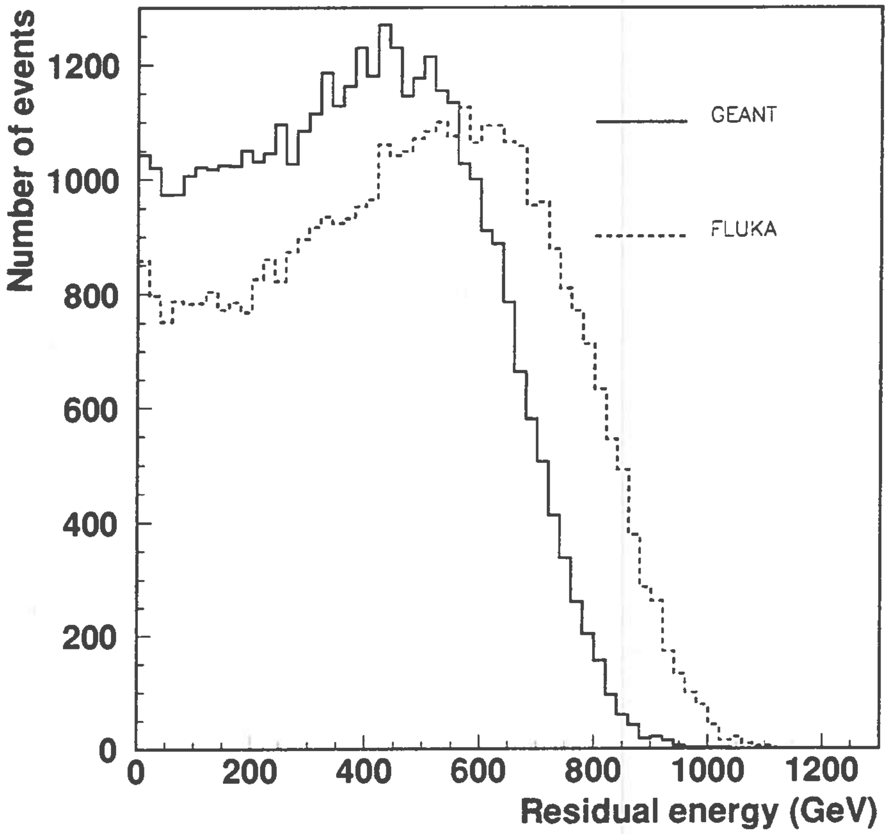

Figure 12: Residual energy distribution obtained switching off photonuclear interaction in GEANT and FLUKA.

The differences are important at all relevant energies. In the same table we also show the value of the macroscopic cross sections of the other process as calculated using the cross sections selected in the review of ref.[3].

We believe that the most important differences in the energy loss calculations between FLUKA and PROPMU with respect to GEANT (and HEMAS), arise from the discrete treatment of both the pair production and photonuclear interactions.

In order to check this hypothesis, we used FLUKA and GEANT, switching off photonuclear interaction and/or pair production. The comparison is summarised in Table 4.

Fig. 12 shows the residual energy distribution computed with FLUKA and with GEANT switching off photonuclear interaction. In comparison to Fig. 1, the GEANT distribution exhibits a smaller change than the FLUKA one, since the absolute contribution of the photonuclear interaction was already much smaller in GEANT. Fig. 13 shows the residual energy distributions when both photonuclear interaction and pair production are switched off. A decisive improvement in the agreement between the two codes is clearly visible (Table 4), thus demonstrating the important effects due to the different algorithms for the pair production, even when the theoretical reference is the same. Of course, both the pair and photonuclear processes are essential for Very High Energy muons.

Energy (GeV)	Σ_{ion} (cm^2/g) (Ref.[3])	Σ_{pair} (cm^2/g) (Ref.[5])	Σ_{brem} (cm^2/g) (Ref.[4])	Σ_{phn} (cm^2/g) (Ref.[6])	Σ_{phn} (cm^2/g) [18, 17])
1.00E+03	1.49E-05	8.41E-05	1.02E-05	1.55E-05	4.34E-07
1.12E+03	1.33E-05	8.45E-05	1.03E-05	1.59E-05	4.46E-07
1.26E+03	1.18E-05	8.48E-05	1.03E-05	1.62E-05	4.60E-07
1.41E+03	1.06E-05	8.52E-05	1.03E-05	1.66E-05	4.73E-07
1.58E+03	9.40E-06	8.55E-05	1.03E-05	1.69E-05	4.87E-07
1.78E+03	8.38E-06	8.58E-05	1.04E-05	1.73E-05	5.01E-07
2.00E+03	7.47E-06	8.60E-05	1.04E-05	1.77E-05	5.16E-07
2.24E+03	6.66E-06	8.63E-05	1.04E-05	1.81E-05	5.31E-07
2.51E+03	5.93E-06	8.65E-05	1.04E-05	1.84E-05	5.46E-07
2.82E+03	5.29E-06	8.67E-05	1.04E-05	1.89E-05	5.62E-07
3.16E+03	4.71E-06	8.69E-05	1.05E-05	1.92E-05	5.79E-07
3.55E+03	4.20E-06	8.71E-05	1.05E-05	1.96E-05	6.00E-07
3.98E+03	3.74E-06	8.73E-05	1.05E-05	2.00E-05	6.13E-07
4.47E+03	3.34E-06	8.74E-05	1.05E-05	2.04E-05	6.31E-07
5.01E+03	2.97E-06	8.76E-05	1.05E-05	2.08E-05	6.49E-07
5.62E+03	2.65E-06	8.77E-05	1.05E-05	2.12E-05	6.68E-07
6.31E+03	2.36E-06	8.78E-05	1.05E-05	2.17E-05	6.88E-07
7.08E+03	2.11E-06	8.79E-05	1.05E-05	2.21E-05	7.08E-07
7.94E+03	1.88E-06	8.80E-05	1.06E-05	2.25E-05	7.28E-07
8.91E+03	1.67E-06	8.81E-05	1.06E-05	2.30E-05	7.50E-07
1.00E+04	1.49E-06	8.82E-05	1.06E-05	2.34E-05	7.72E-07
1.12E+04	1.33E-06	8.83E-05	1.06E-05	2.38E-05	7.94E-07

Table 3: Macroscopic cross sections computed with formulas in Ref. [3, 4, 5, 6] and, in the last column, the photonuclear cross section computed with GHEISHA [18, 17].

Switched off Processes	Photonuclear interaction	Pair production and Photonuclear interaction
Survived muons GEANT 3.21 (3.15)	$38775/5 \cdot 10^4$ $(39702/5 \cdot 10^4)$	$44118/5 \cdot 10^4$ $(44629/5 \cdot 10^4)$
Survived muons (FLUKA)	$40041/5 \cdot 10^4$	$44390/5 \cdot 10^4$

Table 4: Survived muons using GEANT and FLUKA after switching off Photonuclear interaction and/or Pair Production.

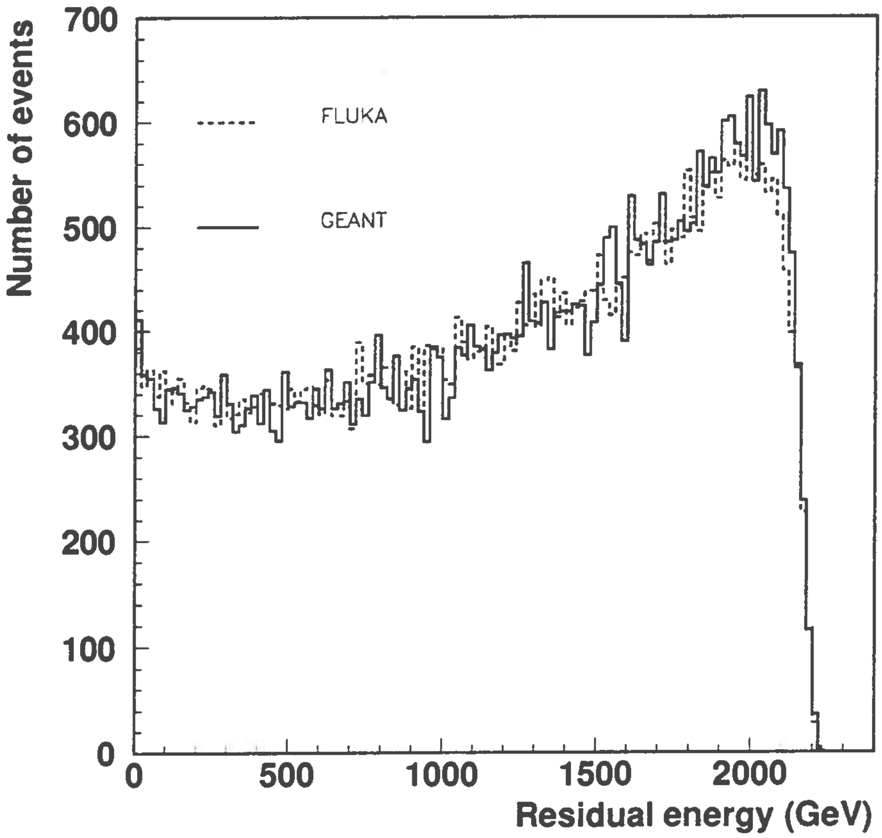

Figure 13: Residual energy distribution obtained switching off in GEANT and FLUKA both photonuclear interaction and pair production.

Code	FLUKA	PROPMU	GEANT 3.15	GEANT 3.21	H.Bilokon et al.[15]
E_μ=2 TeV	0.571± 0.002	0.577± 0.002	0.637± 0.002	0.577±0.002	0.55
E_μ=3 TeV	0.745± 0.002	0.745± 0.002	0.785± 0.002	0.760±0.002	0.70
E_μ=5 TeV	0.858± 0.002	0.861± 0.002	0.875±0.002	0.860±0.002	0.81
E_μ=10 TeV	0.924± 0.001	0.926± 0.001	0.931±0.001	0.929±0.001	0.886

Table 5: Survival probability for muon energy between 2 TeV and 10 TeV computed with the different transport codes considered.

We would like to mention that the agreement was not so good using the previous GEANT version (3.15). We are aware that this is due to the decisive upgrading of the simulation of ionization energy loss achieved in the 3.21 version.

It is interesting to look at results also for different energies. In Tab. 5, we compare the survival probabilities for muon energy between 2 TeV and 10 TeV computed with the different transport codes considered.

The fundamental results achieved at 3 TeV are confirmed: FLUKA and PROPMU exhibit a nice agreement while, for the same quoted physics input, the transport code of ref.[15] gives a lower survival probability everywhere. It is instead interesting to notice how the the last version of GEANT code has a large survival probability only below the 5 TeV region.

As a last point, one can verify the possible effect of the discussed differences on distribution of residual energy of muon underground. We have generated atmospheric muons distributed with an energy distribution ∝ $E^{-3.7}$ between 1 and 100 TeV. The energy distribution of muons surviving at 3500 hg/cm^2 in PROPMU, FLUKA and GEANT has then been fitted to the expression:

$$G(E_\mu, h)) = Ke^{-\beta h(\gamma-1)}\left(E_\mu + \epsilon\left(1 - e^{-\beta h}\right)\right)^{-3.7} \tag{2}$$

according to the suggestions of ref.[25], leaving ϵ and β as free parameters. The results are summarised in Table 6. It is interesting to notice how, although with some uncertainty, GEANT tends to give a slightly lower value for ϵ. Such a parameter represents a sort of critical energy above which the catastrophic radiative processes start to dominate the muon energy loss. The resulting trend is consistent with what has been already noticed, *i.e.* GEANT, even in the most recent version, somewhat underestimates with respect to the other two codes the soft contributions. These, beyond ionization losses, are strongly affected by the e^+e^- pair production mechanism. The ultimate effect of this,

Code	ϵ (GeV)	β cm^2/hg	$\chi 2/\nu$
FLUKA	599 ± 15	$(4.0 \pm 0.2) \cdot 10^{-4}$	73/93
PROPMU	581 ± 17	$(4.0 \pm 0.3) \cdot 10^{-4}$	100/93
GEANT (3.15)	542 ± 9	$(4.0 \pm 0.1) \cdot 10^{-4}$	183/93
GEANT (3.21)	563 ± 10	$(4.0 \pm 0.2) \cdot 10^{-4}$	91/93

Table 6: Fitted parameters of the residual energy distribution as calculated by the different codes at the depth of 3500 hg/cm^2, for an atmospheric muon spectrum following an $E^{-3.7}$ law.

apart from the considerations about the survival probabilities, is to distorce the residual energy spectrum, enhancing it below ϵ.

4 Discussion and conclusions

In summary, the following conclusions can be drawn. Looking at results reported in Tables 1,5 and Fig. 1, it is straightforward that the muon transport code implemented in HEMAS is not correct. It is indeed possible to verify that when dealing with a continuous energy spectrum, as that of atmospheric muons, the resulting effects on the residual energy distribution are not so visible in its shape, in contrast to the results of tests at fixed energy. However, the survival probability is significantly biased. Considering the principles at the basis of the original design of HEMAS, an important conclusion is the confirmation that continuous approximation of energy losses is not a valid choice for muon transport at high energies. As a good approximation, ionization losses can be an exception. We shall come back to this later.

We have found that the present version of GEANT can give a reasonable value for the survival probability of muons of few TeV, but due to the questions concerning the pair production and the photonuclear process, the residual energy is not well calculated. The previous GEANT version instead gave too large values for survival probabilities also in the TeV range. In any case a correction patch for the photonuclear process and for the pair production would be recommended. The two processes present two different kinds of problems. The question of pair production is a matter of consistency with respect to the quoted reference. The problem found here generates a bias in the results for muons as soon as pair production becomes relevant, and this happens already at few tens of GeV. The problem of photonuclear interaction is instead a matter of theoretical choice. It is true that there are uncertainties on the cross section of this particular process, but there is general agreement that the reference quoted in the review of Lohmann et al. constitutes still now the most reliable solution.

As a conclusion, some care has to be taken in using previous calculations, as those of ref.[16] which were based on GEANT 3.15, since they can lead to biased results. All this has been realized, for instance, by the MACRO collaboration at Gran Sasso, where in the analysis of ref.[26] they evaluate a systematic error due to the adopted survival probabilities ($\sim$ 5% in the reconstructed atmospheric muon flux as obtained by the underground vertical muon intensity). There are other areas of research which would demand an effort to improve the muon physics inside GEANT, like, for instance, the preparation of experiments for LHC.

FLUKA code turns out to be very accurate, but being a general Monte Carlo as GEANT, it might be even too much detailed and time consuming for underground applications. Whenever a fast, but precise enough, simulation of muon transport is requested, the philosophy of specialized codes (PROPMU is an example, but there are others) seems to be a good strategy. Referring again to the analysis performed by underground experiments, like those at the Gran Sasso laboratory, the PROPMU transport code can be easily interfaced to cosmic ray generators. In particular it is possible in this way to correct the existing problems in HEMAS. This is what has been done in the framework of MACRO experiment at Gran Sasso for the new analyses of both atmospheric muons[27] and muons induced by high energy neutrinos[28]. However, we have noticed how the fluctuations in energy losses (δ rays) give a measurable effect in the calculation of muon survival probabilities, if one aims at percent level. We think that a Landau distribution, or something equivalent, could be easily implemented in the PROPMU code. Probably a δ-ray production according to the high-energy approximation $F(E_{kin})/E_{kin}^2$ would be enough, where the spin dependent function $F(E_{kin})$ can be found in [29]. We have however to point out that also PROPMU cannot be a general purpose code for muon physics. For instance, in case of analyses concerning "stopping muons" with energies below 1 GeV, only fully detailed codes can be reliable.

As already mentioned in the introduction, some theoretical uncertainties are anyway still present in the treatment of the radiative processes adopted in the tested codes [7, 4, 22, 21, 23]. The consequences for underground physics of some of these choices, and in particular for those concerning bremsstrahlung, have been discussed also in ref.[30]. There it was shown how using the screening of ref.[21] the calculated underground muon flux was reduced by $\sim$6.6% at 3000 hg/cm^2 and by $\sim$30% at 10000 hg/cm^2 with respect to the use of the screening of ref.[4]. The choice of a different screening function has been supported by the experimental work of Sakumoto et al.[22]. It introduces a 10% enhancement of the bremsstrahlung process at TeV energies. From this point of view, we wish also to signal the experimental investigations of ref.[31, 32, 33], also containing discussions on these questions. We are also aware of a recent calculation of Kelner, Kokoulin and Petrukhin, where the authors put in evidence the possible relevance of exact calculations of muon bremsstrahlung on atomic electrons[34]. They also arrive at the conclusion that the work of Sakumoto et al.[22] contains fundamental errors. Furthermore, the same authors are working to arrive to a better evaluation of the effect of the pair production on the atomic electrons. A preliminary estimate suggests that such a contribution could introduce up to a 5% change in the muon energy loss[39].

As far as muon transport at high energy is concerned, other specialized codes have been developed. Of particular interest we quote MARS[35] and MUSIM[36]. Both have

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been developed at Fermilab. MARS was recently used at the SSC laboratory. It is based on the same cross sections used both by FLUKA and PROPMU, the only difference being that in MARS the screening function of Rozental[21] is used by default. MUSIM is the muon specialized version of the more general code CASIM[37]. Here the author recalculated himself all the relevant cross-sections[38]. Apparently he finds very similar results to those quoted in the CERN report of W. Lohmann et al.[3].

Acknowledgments

We wish to thank P. Lipari for providing PROPMU, the related software tools, and for the careful reading of the manuscript.

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