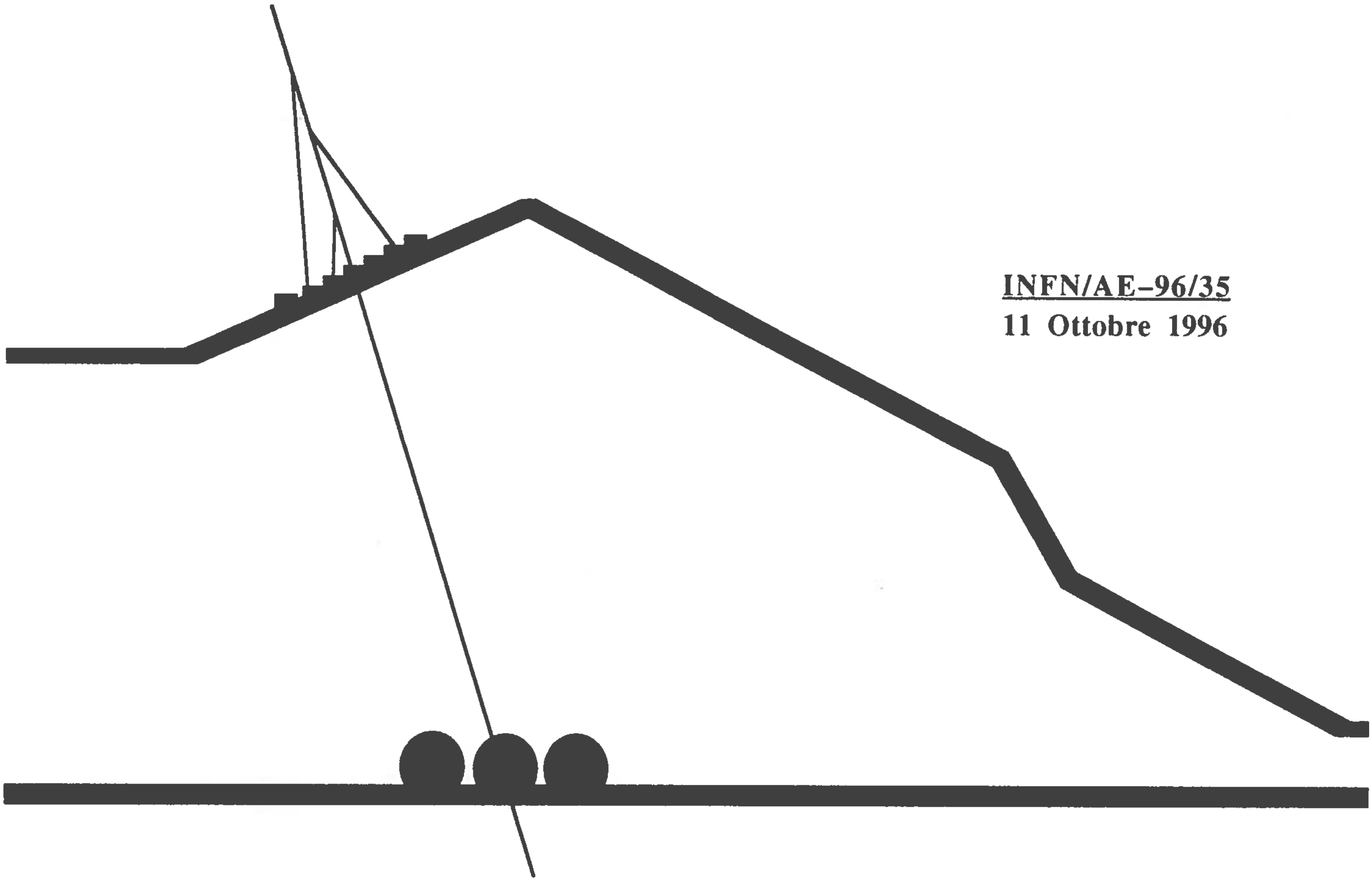

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# Energy loss of supermassive magnetic monopoles and dyons in main sequence stars

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As pointed out by some authors ( [12]), monopoles might form bound states with protons or heavier nuclei by means of magnetic moment interaction. However it is likely that if monopoles capture protons or nuclei, they will also capture a number of electrons, by the Coulomb force, which will nearly neutralize the electric charge. Similarly, if monopoles captures electrons, Coulomb repulsion will likely prevent them from capturing a large number. Thus, if capture is possible, the resultant state will not have a net electric charge much greater than that of a proton or an electron. Since monopole-electron and electric charge-electron scatterings are of a transverse and longitudinal nature respectively, the stopping power of a dyon would be the incoherent sum of the individual monopole and electric charge stopping powers. In the calculations which follow, we will consider the problem of the energy loss of both bare supermassive magnetic and electric charges. The stopping power for dyons should then be given by the sum of these two.

## III. CALCULATION OF STOPPING POWER IN A CLASSICAL ELECTRON GAS

As we have stated previously, the treatement of stopping power in [10] overestimates the energy loss in non-degenerate electron gases such as is found in main sequence stars. Electrons in non-degenerate gases can collide with the supermassive charged particle in both head-on and overtaking collision resulting in a transfer of energy both to and from the electrons. The motion of such a particle through the gas results in a surfeit of head-on collisions an thus a net loss of energy to the gas. This mechenism was first proposed by Fermi [13] [14] to describe the acceleration of cosmic rays through multiple interactions with massive, slowly moving, magnetic regions in the interstellar medium. The calculations of the stopping power of massive charged particles in a non-degenerate gas is similar, except here we concentrate on the energy loss of the massive particle rather than on the acceleration of the gas particles.

Consider that a *particle* of mass $M \gg m$ ($m$ is the electron mass) moves with a velocity $V$ through a nonrelativistic nondegenerate electron gas of temperature $T$, and suppose that the *particle* has either magnetic charge $g$ or electric charge $q = ze$, where $z$ could be fractional, such as in the case of the fractional electric charged particles with mass of the Planck scale in certain superstring model [15]. If we choose the $+z$ axis along the *particle's* trajectory, we can write the electron momentum distribution in the *particle's* rest frame as

$$n(\vec{p})d^3\vec{p} = (2\pi mkT)^{-3/2} N e^{-mV^2/2kT} e^{-p^2/2mkT} e^{-p_z V/kT} d^3\vec{p} \ . \tag{3.1}$$

Here $N$ is the electron density, $k$ is Boltzman's constant, and $\vec{p}$ is the electron momentum in the particle's rest frame. The time rate of change of energy transferred to the *particle* by scattered electrons with initial (final) rest frame momentum $\vec{p}_{\rm i}$ ($\vec{p}_{\rm f}$) is

$$\frac{dE}{dt} = \int_{\vec{p}_{\rm i}} \int_{\Omega} \Delta T(\theta_{\rm i}, \theta_{\rm f}, p)\, \frac{p}{m}\, n(\vec{p}_{\rm i})\, d^3\vec{p}_{\rm i}\, \frac{d\sigma}{d\Omega}(\psi)\, d\Omega \ . \tag{3.2}$$

Here $p = |\vec{p}_{\rm i}| = |\vec{p}_{\rm f}|$, $p/m$ is the magnititude of electron velocity in the *particle's* rest frame, $d\Omega = \sin\psi d\psi d\alpha$, $\psi$ is the angle between $\vec{p}_{\rm i}$ and $\vec{p}_{\rm f}$, $\alpha$ is the corresponding azimuthal angle, $\theta_{\rm i}$ ($\theta_{\rm f}$) is the angle between the $z$ axis and $\vec{p}_{\rm i}$ ($\vec{p}_{\rm f}$), $d\sigma/d\Omega$ is the differential scattering cross section for electrons on the *particle*, and $\Delta T$ is the lab frame energy transfer to the *particle* by a single electron. Since the amount of the lab frame energy that an electron gains in a scattering is $Vp\,(\cos\theta_{\rm f} - \cos\theta_{\rm i})$, the energy conservation law gives

$$\Delta T = -Vp\,(\cos\theta_{\rm f} - \cos\theta_{\rm i}) \ . \tag{3.3}$$

We now choose a new (primed) coordinate system (see Fig.2) which also moves along with the *particle* but which is rotated so that the $z'$ axis lies along $\vec{p}_{\rm i}$ and the $z$ axis lies in the $x'z'$ plane. The unit vector $\hat{r}$ which lies in the direction of $\vec{p}_{\rm f}$ can be expressed in terms of the unit vectors of the new coordinate system as $\hat{r} = \sin\psi\cos\alpha\,\hat{\imath}' + \sin\psi\sin\alpha\,\hat{\jmath}' + \cos\psi\,\hat{k}'$. Similarly, we have $\hat{k} = \sin\theta_{\rm i}\,\hat{\imath}' + \cos\theta_{\rm i}\,\hat{k}'$ so that we can now express $\cos\theta_{\rm f}$ in terms of $\theta_{\rm i}$, $\psi$ and $\alpha$ as

$$\cos\theta_{\rm f} = \hat{r}\cdot\hat{k} = \cos\psi\cos\theta_{\rm i} + \sin\psi\cos\alpha\sin\theta_{\rm i} \ . \tag{3.4}$$

We can now rewrite eq. (3.2) as

$$\frac{dE}{dt} = -\int_{\vec{p}_{\rm i}} d^3\vec{p}_{\rm i}\, n(\vec{p}_{\rm i})\, V\frac{p^2}{m} \int_{\Omega} \sin\psi d\psi d\alpha\, \left[(\cos\psi - 1)\cos\theta_{\rm i} + \sin\psi\cos\alpha\sin\theta_{\rm i}\right] \frac{d\sigma}{d\Omega}(\psi) \ . \tag{3.5}$$

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# Energy loss of supermassive magnetic monopoles and dyons in main sequence stars

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We estimate the energy loss rate for a supermassive magnetic monopole or dyon in the interior of a main sequence star. For this purpose, the medium is shown to behave as a non-degenerate classical electron gas, the nuclear contribution to the stopping power being negligible. A linear dependence of the energy loss from the velocity of the particle is obtained. This result would provide a basic input to detailed calculations of the dynamics of monopoles and dyons in our galaxy.

## I. INTRODUCTION

Attempts to unify the forces of the nature have led to the prediction of supermassive magnetically charged particles. It has been demonstrated that any unified gauge theory in which $U_{em}(1)$ is embedded in a spontaneously broken semisimple gauge group *necessarily* contains magnetic monopoles with masses of the order of $10^{17}$ GeV [1] [2]. Cosmological theories forsee the value, for the abundance of such particle in the Universe, which is either too large to be in agreement with experimental observations or to low to be detectable (see [3] or [4] for a review on monopoles). A number of astrophysical arguments have been used to obtain several upper limits to the magnetic monopole flux. The most popular one is the so called Parker bound [5] (see also the Extended Parker Bound in [6]) which is based upon the assumption for the galactic magnetic field not to be destroyed in accelerating magnetic charges. In the following we will estimate the energy loss suffered by a supermassive monopole or dyon crossing a main sequence star. In particular, we will evaluate the stopping power for both supermassive electrically and magnetically charged particles, then we will obtain the correponding expression for a dyon. This will be very important for a complete treatement of the dynamics of both monopoles and dyons in our galaxy.

## II. ENERGY LOSS MECHANISMS OF MAGNETIC MONOPOLES IN MATTER

Ahlen [9] has summarized the status of monopole stopping power calculations prior to 1979. Most of these were similar to calculations of the stopping power of fast electrically charged particles and were valid only for velocities $V >> v_0$ where $v_0$ is a typical electron velocity in the stopping material. In this regime it is reasonably accurate to obtain the stopping power of monopoles from that for electric charges of the same velocity and charge $Q$ by replacing $Q$ with $g\beta$, where $\beta = V/c$ and $g$ is the magnetic charge of the monopole. This prescription does not work for $V < v_0$ as is shown by Ahlen and Kinoshita [10] in their calculations of the stopping power of slow monopoles in a degenerate electron gas. It is found that the stopping power of slow monopoles is proportional to $V$, which is the same behaviour displayed by electric charges. This is in contrast to the case of large velocities where the stopping power of electric particles is proportional to $1/V^2$ while that for monopoles is roughly constant.

It is not possible to take results from [10] and apply them to the Sun and the main sequence stars[1]. The reason for this is that the electrons in this case are at a sufficiently high temperature and low density that the interparticle separation is larger than the thermal De Broglie wavelength. Thus, the electrons are distinguishable and form a classical ideal gas to a good approximation (see Fig.1) The electron gas considered in [10] was degenerate. Thus, in any monopole-electron collision in [10] the monopole could not gain energy since the electron gas was in its ground state. For a classical electron gas, this is not the case and the electrons can either lose or gain energy in collisions with monopoles. This leads to a reduced value of stopping power compared to what would be obtained from the results of [10]. In the next section we will show that is not difficult to take this effect into account.

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[1] It was done in [8] and [11].

The integral over $\alpha$ of the second term in square brackets is zero since $d\sigma/d\Omega$ is a function only of the scattering angle $\psi$. The same integral for the first term is just $2\pi$. The stopping power $dE/dx$, where $x$ is the pathlength in cm, can be obtained from $dE/dt$ by dividing by the *particle* velocity as follows:

$$\frac{dE}{dx} = \frac{1}{V}\frac{dE}{dt} = \frac{2\pi}{m}\int_{\vec{p}_{\rm i}} d^3\vec{p}_{\rm i}\, n(\vec{p}_{\rm i})\, p^2 \cos\theta_{\rm i} \int_{\psi} d\psi \, \sin\psi(1-\cos\psi)\frac{d\sigma}{d\Omega}(\psi) \ . \tag{3.6}$$

### A. Energy loss of electric charge.

Suppose that the *particle* has an electric charge $q$. The differential cross section of electrons scattering off the *particle* follows the well-known Rutherford formulae:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm E} = \frac{m^2 q^2 e^2}{4p^4 \sin^4(\psi/2)} \ . \tag{3.7}$$

Substituting this cross section into eq. (3.6), we first evaluate the integral over $\psi$ and we obtain

$$\int_{\psi_{\rm min}}^{\pi} \frac{\sin\psi(1-\cos\psi)}{\sin^4(\psi/2)} d\psi = -8 \ln\sin\frac{\psi_{\rm min}}{2} \ , \tag{3.8}$$

where $\psi_{\rm min}$ is the minimum scattering angle. We will return to the evaluation of $\psi_{\rm min}$ later. The integral over $\vec{p}_{\rm i}$ is evaluated in the Appendix as denoted by $I_{\rm E}$, from which we obtain

$$\left(\frac{dE}{dx}\right)_{\rm E} = \frac{8\sqrt{\pi}q^2e^2Na}{3kT} C_{\rm E}(a) \ln\sin\frac{\psi_{\rm min}}{2} \ , \tag{3.9}$$

where $a = \sqrt{mV^2/2kT} = 17.2\beta/T_7^{1/2}$, $\beta = V/c$ and $T_7$ is the temprature in unit of $10^7$ K, $C_{\rm E}(a)$ is defined in eqs. (A20a) and (A21) and plotted in Fig.3.

### B. Energy loss of magnetic monopole

Let's now consider the case of the magnetic monopole, i.e., the *particle* with a magnetic charge $g$. Kazama, Yang and Goldhaber [16] have solved the Dirac equation for electron-monopole scattering thereby including the dynamical effects of the electron spin. They obtain

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm M} = \frac{g^2e^2 f(\psi,g)}{4p^2c^2\sin^4(\psi/2)} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm R} f(\psi,g) \ , \tag{3.10}$$

where $(d\sigma/d\Omega)_{\rm R}$ is the Rutherford differential cross section for an electron scattering off a nucleus with electric charge $Q = gv/c$, $v = p/m$ is the electron velocity in the monopole's rest frame, $f(\psi,g)$ is a function of $\psi$ and $g$. For the Dirac monopole [17] $g = \pm g_D = \pm\hbar c/2e$ and the Schwinger monopole [18] $g = \pm 2g_D = \pm\hbar c/e$, the function $f(\psi,g)$ is tabulated in ref. [16], and it varies between 1 and 2 for scattering angle between 0° and 180°. With this cross section substituted, the integral over $\psi$ in eq. (3.6) is evaluated in eq. (3.8) for $f(\psi,g)=1$. If we perform this integral over $\psi$ exactly, we can obtain a multiplicative correction factor $F(\psi_{\rm min},g)$ given by

$$F(\psi_{\rm min},g) = \frac{-1}{2\ln\sin(\psi_{\rm min}/2)}\int_{\psi_{\rm min}}^{\pi}\frac{f(\psi,g)}{\tan(\psi/2)}d\psi \ . \tag{3.11}$$

A graph of $F(\psi_{\rm min},g)$ evaluated as a function of $\psi_{\rm min}$ for $g = g_D$ and $g = 2g_D$ appears in Fig.4. For $\psi < 20°$ this correction amounts to $\lesssim 10\%$ increase in the stopping power. The integral over $\vec{p}_{\rm i}$ in eq. (3.6) is evaluated in the Appendix as denoted by $I_{\rm M}$, from which we obtain

$$\left(\frac{dE}{dx}\right)_{\rm M} = \frac{16\sqrt{\pi}g^2e^2Na}{3mc^2} C_{\rm M}(a) F(\psi_{\rm min},g) \ln\sin\frac{\psi_{\rm min}}{2} \ , \tag{3.12}$$

where $a$ is the same as in the case of electric charge energy loss, $C_{\rm M}(a)$ is defined in eqs. (A13a) and (A13b) and plotted in Fig.3.

### C. Determination of $\psi_{\min}$

In order to calculate the energy loss $dE/dx$, we now need only to estimate $\psi_{\min}$, the minimum scattering angle. Since $\psi_{\min}$ appears in a logarithmic term, $dE/dx$ is insensitive to the actual value of $\psi_{\min}$, the crude evaluation given below should suffice. It is well known that for the case of electrically charged particles the Rutherford cross section diverges at small angles due to the long range nature of the Coulomb force. In an ionized medium, such as is found in stellar interiors, the ions are shielded by the collective action of electrons resulting in an effective cutoff for the Coulomb interaction [19] at radius $R_Z$, given by $\frac{4}{3}\pi R_Z^3 N = Z$, where $Z$ is the charge of the ion. In the case that the *particle* is electrically charged, we can effectively treat $R_Z$ as the cutoff for Coulomb interaction between the electrons and the *particle*. In the case of a magnetic monopole, no shielding of the magnetic charge by electrons can occur, but the presence of the ions limits the effective lateral extent, $\Lambda$, of the electron wave packets to roughly the electron mean free path $\bar{l}$. By assuming that the electron-ion scattering cross section is approximately $\pi R_Z^2$, it can be shown that $\bar{l} \approx R_Z$. Since the uncertainty principle relates $\Lambda$, or $R_Z$, to the minimum allowed transverse momentum of the electron via $p_{\perp\min} \approx \hbar/R_Z$, we have $\psi_{\min} = \hbar/(mvR_Z)$, where $v$ is the electron velocity in the *particle's* rest frame. Since the typical electron velocity in the lab frame $v_l$ in a main sequence star is much larger than the the lab frame velocity of the incident *particle* $V$, we have $v \approx v_l$. For a fully ionized gas where the atomic charge to mass ratio $A/Z \approx 2$ for $Z \geq 2$, the electron number density is

$$N = \rho N_{\mathrm{A}}(1 + X_{\mathrm{H}})/2 \; , \tag{3.13}$$

where $\rho$ is the mass density of the gas, $N_{\mathrm{A}} = 6.02 \times 10^{23}\,\mathrm{g}^{-1}$ is the Avagadro's number and $X_{\mathrm{H}}$ is the hydrogen mass fraction. If we use the peak of the thermal distribution, $v_l \approx (3kT/m)^{1/2}$, as a representative velocity, we find that

$$\psi_{\min} \approx 3.4^{\circ} \left[\frac{\rho(1 + X_{\mathrm{H}})}{T_7^{3/2} Z}\right]^{1/3} \; . \tag{3.14}$$

This result is the same as obtained in ref. [19] where this minimum angle is used to calculate the conductivity in stellar interiors. When more than one species of ion is present, $Z$ can be replaced by the number average atomic charge $<Z>$. Inside a main sequence star, any element other than hydrogen and helium is of negligible abundance, thus the atomic charge avraging over hydrogen and helium is $<Z> = 2(X_{\mathrm{H}} + 1)/(3X_{\mathrm{H}} + 1)$. Inserting this result into eq. (3.14), we obtain that

$$\psi_{\min} \approx 2.7^{\circ} \rho^{1/3} (1 + 3X_{\mathrm{H}})^{1/3} T_7^{-1/2} \; . \tag{3.15}$$

which is reported, as a function of the distance from the center of the Sun, in Fig.5.

### D. Simplified expressions

Simplified Expressions for the stopping power can be obtained by using the fine structure constant $\alpha = e^2/\hbar c = 1/137$ and the Dirac quantization condition $g_D e/\hbar c = 1/2$, by using the previous expression for $N$, and by dividing $dE/dx$ by $\rho$ to obtain $dE/dX$ where $X$ is the pathlength in g/cm$^2$. When this is done and the constants are evaluated, we obtain the stopping power of a magnetic monopole with magnetic charge $g$

$$\left(\frac{dE}{dX}\right)_{\mathrm{M}} = 9.32\beta \left(\frac{g}{g_D}\right)^2 (1 + X_{\mathrm{H}}) T_7^{-1/2} \ln \sin \frac{\psi_{\min}}{2} F(\psi_{\min}, g) C_{\mathrm{M}}(17.2\beta T_7^{-1/2}) \;\mathrm{GeV\,g^{-1}cm^2} \; , \tag{3.16}$$

and that of a supermassive *particle* with electric charge $q$

$$\left(\frac{dE}{dX}\right)_{\mathrm{E}} = 0.589\beta \left(\frac{q}{e}\right)^2 (1 + X_{\mathrm{H}}) T_7^{-3/2} \ln \sin \frac{\psi_{\min}}{2} C_{\mathrm{E}}(17.2\beta T_7^{-1/2}) \;\mathrm{GeV\,g^{-1}cm^2} \; . \tag{3.17}$$

As stated above, the stopping power of a dyon,$(\frac{dE}{dX})_D$, should be simply given by the the sum of $(\frac{dE}{dX})_M$ and $(\frac{dE}{dX})_E$. In particular

$$\frac{(dE/dX)_D}{(dE/dX)_M} - 1 = 6.3 \times 10^{-2} \left(\frac{q/e}{g/g_D}\right)^2 \frac{T_7^{-1}}{F(\psi_{min}, g)} \frac{C_{\mathrm{E}}(17.2\beta T_7^{-1/2})}{C_{\mathrm{M}}(17.2\beta T_7^{-1/2})} \tag{3.18}$$

The eq.(3.16) was obtained by two of the authors of the present paper (S.P.A. & G.T.) with a missing factor of two [20]. This incorrect result was used in [7] to evaluate the dynamics of supermassive monopoles in main sequence stars. The monopole stopping power in a classical electron gas was estimated in [24] in the first order approximation for $V/v$, resulting in a value, for $(dE/dX)_M$, which is one order of magnitude smaller than the result obtained here.

## IV. LIMITATIONS OF THE CALCULATION

In the preceding calculations we have assumed that the stopping medium was a pure non-relativistic non-degenerate electron gas. In Fig.1 we display the conditions of density and temperature under which this assumption is valid and also the range of conditions one might expect to find throughout the Sun and the center of other main sequence stars. The horizontal line separating the relativistic from the non-relativistic region denotes the temperature, $T = mc^2/(3k)$, at which typical electron thermal energies are comparable to the electron rest energy. Above this temperature the calculation would have to be modified to include the correct expressions for the relativistic electron momentum distribution and energy transfer. As the De Broglie wavelength of the electrons become larger than the collision mean free path, the phase space available to the electrons will be restricted and the gas will become degenerate. A rough dividing line between the region of electron degeneracy and non-degeneracy can be identified by setting the classical electron gas pressure, $NkT$, equal to the pressure, $(\frac{h^2}{20m})(\frac{3}{\pi})^{2/3}N^{5/3}$, calculated for a completely degenerate electron gas. When we substitute the previous expression for N, this leads to the relation

$$\frac{(1+X_H)\rho}{2} \simeq 2.4 \times 10^{-8} T^{3/2} \tag{4.1}$$

represented in the diagonal in Fig.1. Above this line the stopping power formulas presented here are valid, while below this line the calculation in [10] would be more apppropriate. As one can clearly see, the conditions existing in main sequence stars, are ideal for the application of the stopping power expression obtained here.

Throughout the calculation we have neglected nuclear contribution to the stopping power. The De Broglie wavelength of the massive ions will always be smaller than that for the electrons while the collision mean free path will be comparable. The ion gas will, therefore, be non-degenerate whenever conditions favor non-degeneracy. As a result, we can use eqs.(3.16) and (3.17) to evaluate the nuclear stopping power for a supermassive particle provided that both $(\frac{dE}{dX})$ and $\psi_{min}$ are multiplied by the factor $\sqrt{m/m_I}$, where $m_I$ is the ion mass, to account for the increased mass and thermal momentum of the ion. For a wide range of conditions existing in main sequance stars, the contribution of the nuclear stopping power is small. For the Sun the ratio of nuclear to electronic stopping power varies from $\sim 6.6\%$ at the center to $\sim 4\%$ near the surface. No large errors would be incurred by neglecting the nuclear stopping power completely. It should be noted that this discussion of nuclear stopping power applies for velocities $V$ smaller than $\sim 10^{-2}c$. For large $V$, exact results can be obtained for nuclear stopping power by replacing $m$ by $m_I$ in all the above formulae. As $V \to c$, the relative stopping power of ions to that of electrons goes as $m/m_I$.

Hamilton and Sarazin [21] have claimed that, in addition to the velocity dependent stopping power which we have calculated here, there is a velocity independent contribution to the stopping power of monopoles in classical gases. This is supposedly due to collective motions of the plasma induced by the magnetic field of the passing monopole. However collective effects were shown [22] to contribute insignificantly to the stopping power of monopoles in electron gases with exception of superconductors and even then only for extremely small velocities ($\beta < 10^{-4}$). Other calculations ( [23] and [24]), anyway, gave as a result an energy loss, for collective effects, of about one order of magnitude smaller than the one due to binary collisions reported in this paper.

## APPENDIX

In this Appendix, we shall evaluate the following two integrals:

$$I_{\rm M} \equiv \int_{\vec{p}} n(\vec{p}) \cos\theta d^3\vec{p} \,, \tag{A1}$$

$$I_{\rm E} \equiv \int_{\vec{p}} \frac{n(\vec{p})\cos\theta}{p^2} d^3\vec{p} \,, \tag{A2}$$

where $n(\vec{p})$ is described in eq. (3.1). Here we have dropped the subscript i of the variables $\vec{p}_{\rm i}$ and $\theta_{\rm i}$ for simplicity. These two integrals over the electron momentum space are needed to calculate the energy loss of a supermassive particle, either with a magnetic charge or with an electric charge. We define a set of new dimensionless variables:

$$a \equiv \sqrt{\frac{mV^2}{2kT}} \quad \text{and} \quad \vec{x} \equiv \frac{\vec{p}}{\sqrt{2mkT}} \,, \tag{A3}$$

then $p_z V/kT = 2ax\cos\theta$, where $x = |\,\vec{x}\,|$. Using these new variables we may rewrite eq. (3.1) as

$$n(\vec{p})d^3\vec{p} = \pi^{-3/2}Ne^{-a^2}e^{-x^2}e^{-2ax\cos\theta}d^3\vec{x} \ . \tag{A4}$$

Therefore eqs. (A1) and (A2) can be rewritten as

$$I_{\rm M} = 2\pi^{-1/2}Ne^{-a^2}\int_0^\infty x^2e^{-x^2}I_\theta(ax)dx \ , \tag{A5}$$

$$I_{\rm E} = \frac{N}{\sqrt{\pi mkT}}e^{-a^2}\int_0^\infty e^{-x^2}I_\theta(ax)dx \ , \tag{A6}$$

where

$$I_\theta(ax) \equiv \int_0^\pi e^{-2ax\cos\theta}\cos\theta\sin\theta d\theta \ . \tag{A7}$$

This integral over $\theta$ can be easily evaluated if we define a new variable $y = \cos\theta$:

$$\begin{aligned} I_\theta(ax) &= \int_{-1}^1 ye^{-2axy}dy \\ &= (2a^2x^2)^{-1}\sinh(2ax) - (ax)^{-1}\cosh(2ax) \ . \end{aligned} \tag{A8}$$

*a. We first evaluate the integral* $I_{\rm M}$. Substituting this $I_\theta(ax)$ into eq. (A5), we have

$$\begin{aligned} I_{\rm M} &= \frac{N}{2\sqrt{\pi}a^2}\int_0^\infty \left[(1-2ax)e^{-(x-a)^2} - (1+2ax)e^{-(x+a)^2}\right]dx \\ &= \frac{N}{2\sqrt{\pi}a^2}\left[M(a) - M(-a)\right] \ . \end{aligned} \tag{A9}$$

Evaluate $M(a)$ we have

$$\begin{aligned} M(a) &\equiv \int_0^\infty (1-2ax)e^{-(x-a)^2}dx \\ &= \int_{-a}^\infty (1-2a^2-2ax)e^{-x^2}dx \\ &= 2^{-1}\sqrt{\pi}\left(1-2a^2\right)\left[1+{\rm erf}(a)\right] - ae^{-a^2} \ , \end{aligned} \tag{A10}$$

where ${\rm erf}(a) = 2\pi^{-1/2}\int_0^a e^{-t^2}dt$ is the standard error function [25]. Thus we obtain

$$I_{\rm M} = -N\left[\frac{1}{\sqrt{\pi}a}e^{-a^2} + (1-\frac{1}{2a^2}){\rm erf}(a)\right] \ . \tag{A11}$$

We can rewrite it as

$$I_{\rm M} = -\frac{4Na}{3\sqrt{\pi}}C_{\rm M}(a), \tag{A12}$$

where the magnetic correction factor

$$C_{\rm M}(a) \equiv \frac{3\sqrt{\pi}}{4a}\left[\frac{1}{\sqrt{\pi}a}e^{-a^2} + \left(1-\frac{1}{2a^2}\right){\rm erf}(a)\right] \ . \tag{A13a}$$

Alternatively, we can expand this $C_{\rm M}(a)$ into a power series as

$$C_{\rm M}(a) = 3\sum_{n=0}^\infty \frac{(-1)^n}{n!(2n+1)(2n+3)}a^{2n} \ . \tag{A13b}$$

Either of the above two expressions of $C_{\rm M}(a)$ can be easily calculated numerically. A curve of $C_{\rm M}(a)$ as a function of $a$ is plotted in Fig.3, from which one can see that $C_{\rm M}(a) = 1$ is an excellent approximation for $a \lesssim 0.1$.

*b. We now evaluate the integral* $I_{\rm E}$. Substituting eq. (A8) into eq. (A6), we have

$$I_{\rm E} = \frac{Ne^{-a^2}}{2\sqrt{\pi}mkTa^2} E(a) \ , \tag{A14}$$

where

$$E(a) \equiv \int_0^\infty e^{-x^2} \left[x^{-2}\sinh(2ax) - 2ax^{-1}\cosh(2ax)\right] dx \ . \tag{A15}$$

If we differentiate $E(a)$ over $a$, we can integrate the differentiated integrand to a closed form expression.

$$\frac{dE(a)}{da} = -4a\int_0^\infty e^{-x^2}\sinh(2ax)dx \tag{A16a}$$

$$= -2\sqrt{\pi}ae^{a^2}\mathrm{erf}(a) \ , \tag{A16b}$$

where the integral in eq. A16a is evaluated using the same approach as the evaluation of $I_{\rm M}$ and $M(a)$ in eqs. (A9) and (A10). From eq. (A15) one can see that $E(a=0)=0$. Thus we have

$$E(a) = -2\sqrt{\pi}\int_0^a ze^{z^2}\mathrm{erf}(z)dz \tag{A17a}$$

$$= -4\int_0^a dz \int_0^z dt\, ze^{z^2}e^{-t^2} \tag{A17b}$$

$$= -4\int_0^a dt \int_t^a dz\, ze^{z^2}e^{-t^2} \tag{A17c}$$

$$= -2\int_0^a e^{-t^2}\left(e^{a^2} - e^{t^2}\right) dt \tag{A17d}$$

$$= -2\left[2^{-1}\sqrt{\pi}e^{a^2}\mathrm{erf}(a) - a\right] \ . \tag{A17e}$$

Therefore we obtain that

$$I_{\rm E} = -\frac{N}{\sqrt{\pi}mkTa^2}\left[\frac{\sqrt{\pi}}{2}\mathrm{erf}(a) - ae^{-a^2}\right] \ . \tag{A18}$$

Similar to $I_{\rm M}$, we can rewrite $I_{\rm E}$ as

$$I_{\rm E} = -\frac{2Na}{3\sqrt{\pi}mkT}C_{\rm E}(a) \ , \tag{A19}$$

where the electric correction factor

$$C_{\rm E}(a) = \frac{3}{2a^3}\left[\frac{\sqrt{\pi}}{2}\mathrm{erf}(a) - ae^{-a^2}\right] \ . \tag{A20a}$$

Just like $C_{\rm M}(a)$, the above expression can be expanded into a power series:

$$C_{\rm E}(a) = 3\sum_{n=0}^\infty \frac{(-1)^n}{n!(2n+3)}a^{2n} \ . \tag{A21}$$

Again similar to the case of the magnetic correction factor $C_{\rm M}(a)$, both of the above two expressions for the electric correction factor are easy to evaluate. And a curve of $C_{\rm E}(a)$ as a function of $a$ is plotted in Fig.3, from which one can see that $C_{\rm E}(a) = 1$ is an excellent approximation for $a \lesssim 0.1$ .

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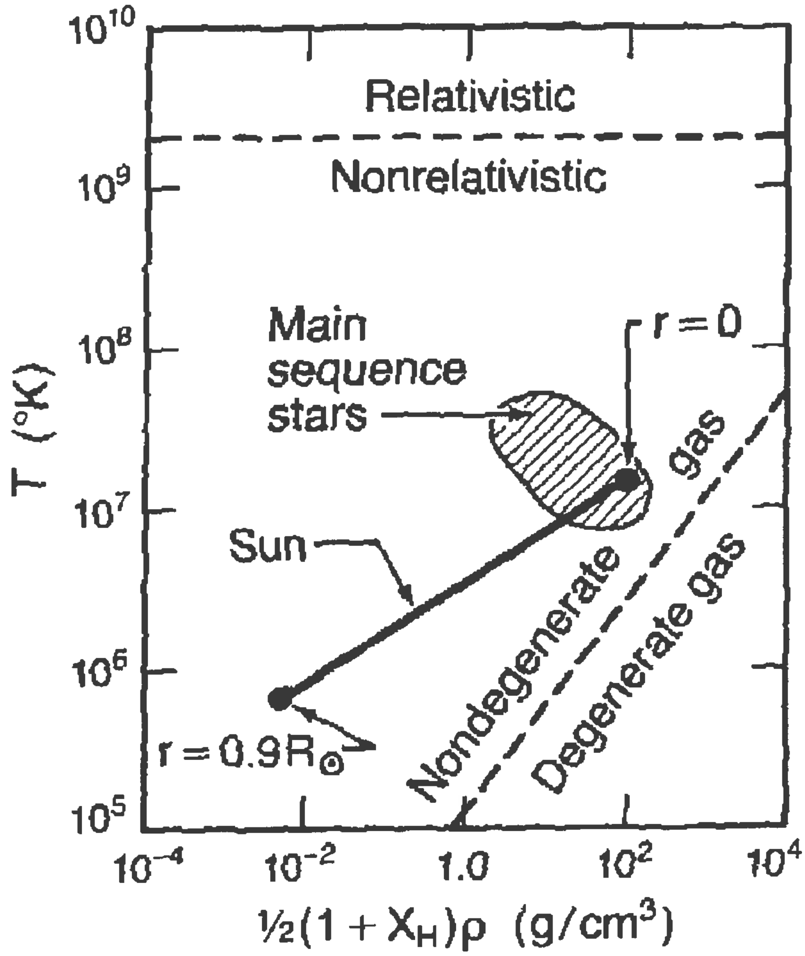


FIG. 1. Temperature-density plot showing that in the Sun and in all main sequence stars the electron gas can be considered both non-relativistic and non-degenerate (see text).

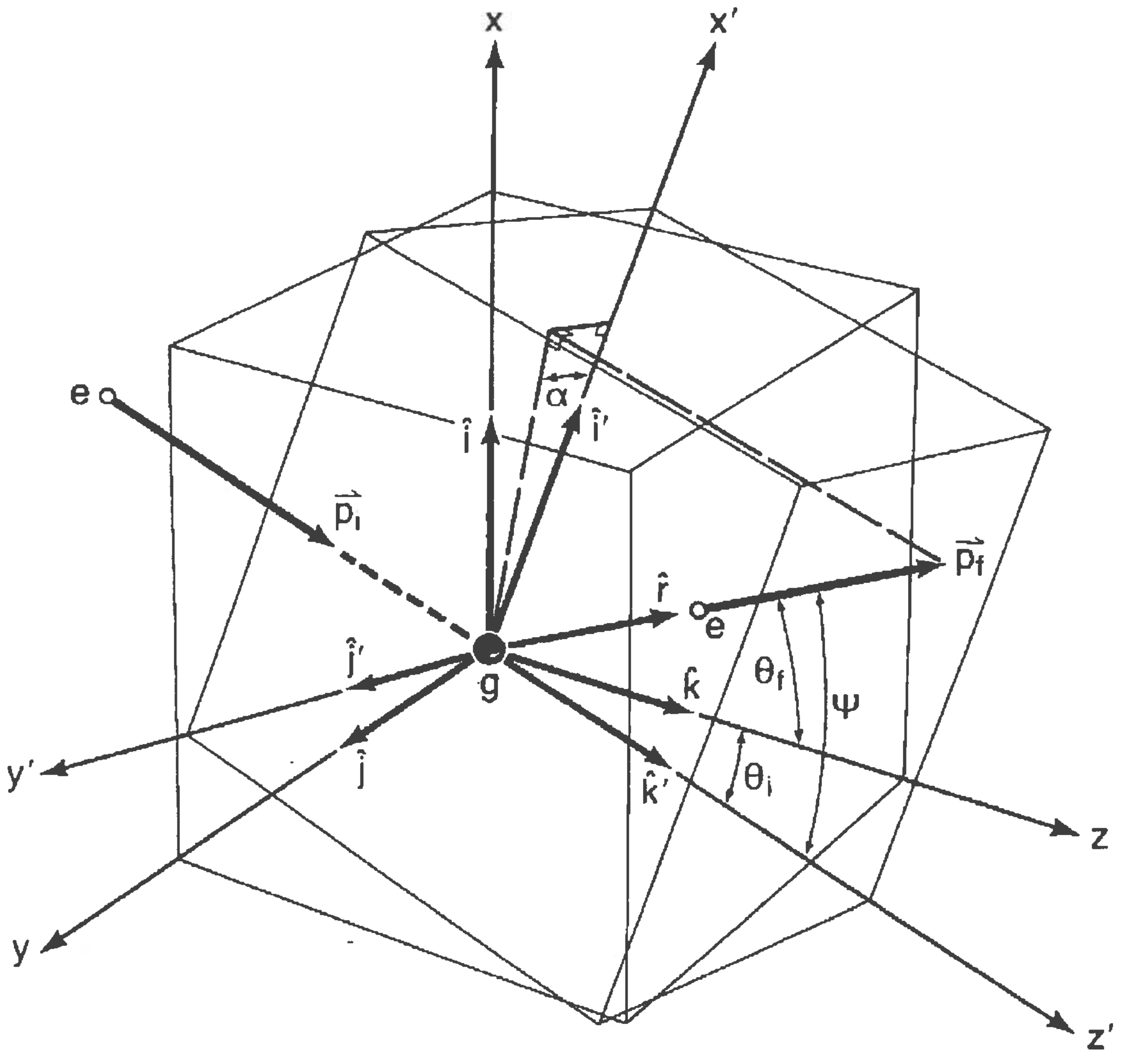


FIG. 2. References frames used in the stopping power calculation.

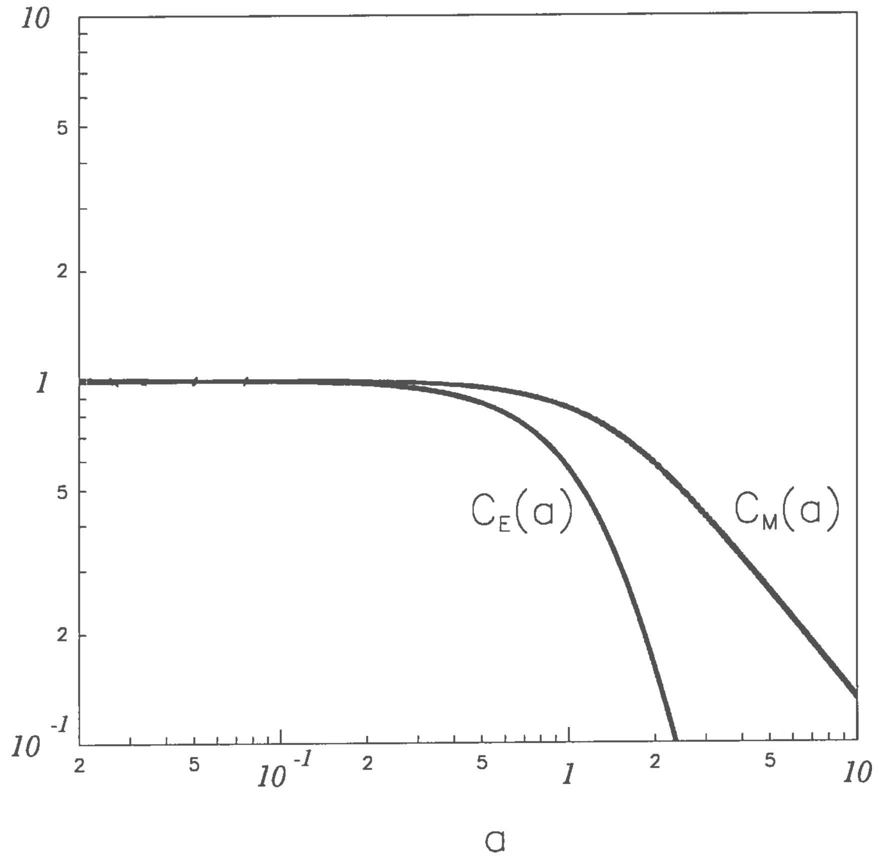


FIG. 3. The factors $C_E(a)$ and $C_M(a)$ as a function of $a$. In both cases taking these factors as equal to 1 is an excellent approximation for $a \leq 0.1$.

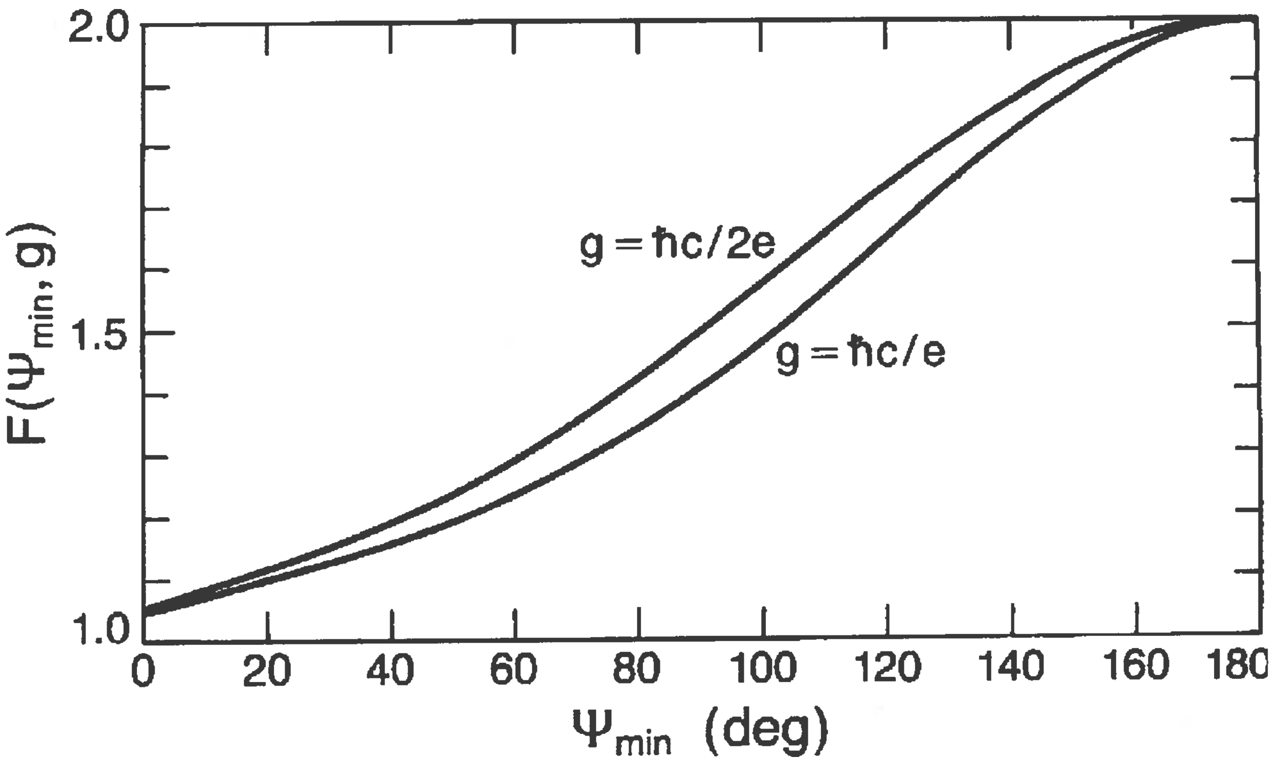


FIG. 4. The correction factor $F$ as a function of the minimum scattering angle $\psi_{min}$ for a couple of values of the monopole's magnetic charge.

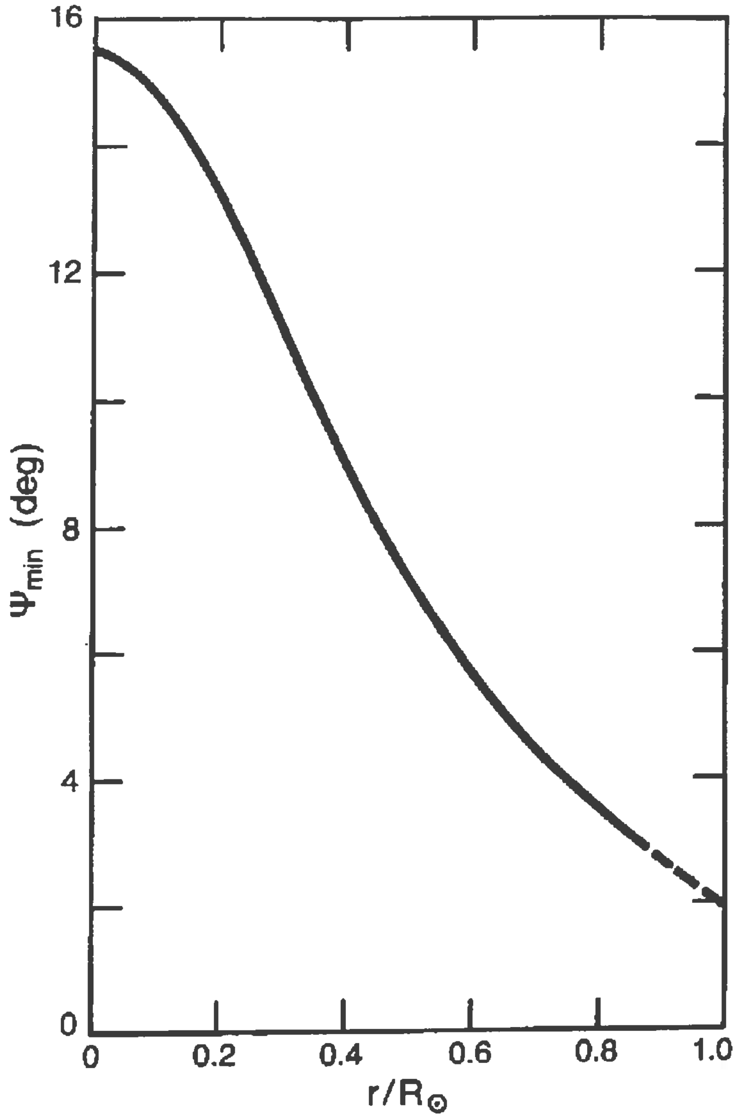


FIG. 5. The minimum scattering angle $\psi_{min}$ as a function of the distance from the center of the Sun.