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**ABOUT THE KINEMATICS OF SPINNING PARTICLES†**

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**ABSTRACT** – Inserting the correct Lorentz factor into the definition of the 4-velocity  $v^\mu$  for spinning particles entails new kinematical properties for  $v^2$ . The well-known constraint (identically true for scalar particles, but entering also the Dirac theory, and assumed *a priori* in all spinning particle models)  $p_\mu v^\mu = m$  is here derived in a self-consistent way.

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*“If a spinning particle is not quite a point particle, nor  
a solid three dimensional top, what can it be?”*

Asim O. Barut

## 1 The “extended-like” electron

Since the works by Compton,<sup>[1]</sup> Uhlenbeck and Goudsmith,<sup>[2]</sup> Frenkel,<sup>[2]</sup> and Schrödinger<sup>[3]</sup> till the present times, many classical theories —often quite different among themselves from a physical and formal viewpoint— have been advanced for spinning particles.<sup>#1</sup>

Following Bunge<sup>[4]</sup>, they can be divided into three classes:

I) strictly *point-like* particle models

II) actual extended-type particle models (“spheres”, “tops”, “gyroscopes”, and so on)

III) mixed models for “extended-like” particles, in which the center of the *point-like* charge  $Q$  results to be spatially distinct from the particle center-of-mass (CM).

Notice that in the theoretical approaches of type III —which, being in the middle between classes I and II, could answer the dilemma posed by Barut at the top of this paper— the motion of  $Q$  does not coincide with the motion of the particle CM. This peculiar feature has been actually found to be a characteristic for the kinematics of spinning particles, and is known as the zitterbewegung (zbw) motion.<sup>[5–10]</sup> The existence of such an internal motion is denounced, besides by the presence itself of spin, by the remarkable fact that in the ordinary Dirac theory the particle four-impulse  $p^\mu$  is *not* parallel to the four-velocity:  $v^\mu \neq p^\mu/m$ . Moreover, while  $[\mathbf{p}, H] = 0$  so that  $\mathbf{p}$  is a conserved quantity,  $\mathbf{v}$  is *not* a constant of the motion:  $[\mathbf{v}, H] \neq 0$  ( $\mathbf{v} \equiv \boldsymbol{\alpha} \equiv \gamma^0 \boldsymbol{\gamma}$  being the usual vector matrix of the Dirac theory). Let us explicitly notice that, for the models belonging to class III, assuming the zbw is equivalent<sup>[7–9]</sup> to splitting the

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<sup>#1</sup> Hereafter we shall often write “electron” or “spinning particle” instead of the more pertinent expression “spin- $\frac{1}{2}$  particle”.

motion variables as follows (the dot meaning derivation with respect to the proper time  $\tau$ ):

$$x^\mu \equiv \xi^\mu + X^\mu ; \quad \dot{x}^\mu \equiv v^\mu = w^\mu + V^\mu , \quad (1)$$

where  $\xi^\mu$  and  $w^\mu \equiv \dot{\xi}^\mu$  describe the *translational, external* or *drift* motion, i.e. the motion of the CM, whilst  $X^\mu$  and  $V^\mu \equiv \dot{X}^\mu$  describe the *internal* or *spin* motion. From an electro-dynamical point of view, the conserved electric current is associated with the trajectories of  $\mathcal{Q}$  (i.e., to  $x^\mu$ ), whilst the center of the particle Coulomb field—obtained<sup>[10]</sup> through a time average over the field generated by the quickly oscillating charge—is associated with the CM (i.e., with  $w^\mu$ ; and then, for free particles, to the geometrical center of the helical trajectory).<sup>#2</sup> In such a way, it is  $\mathcal{Q}$  which follows *the (total) motion*, whilst the CM follows the *mean motion* only. It is important to remark that the classical extended-like electron of type III is quite *consistent* with the standard Dirac theory. In fact the above decomposition for the total motion is the classical analogue of two well-known quantum-mechanical procedures, i.e., the so-called *Gordon decomposition* of the Dirac current, and the (operatorial) *decomposition of the Dirac position operator* proposed by Schrödinger in his pioneering works.<sup>[3]</sup> We shall show these points below.

The well-known Gordon decomposition of the Dirac current reads<sup>[13]</sup> (hereafter we shall choose units such that numerically  $c = 1$ ):

$$\bar{\psi}\gamma^\mu\psi = \frac{1}{2m} [\bar{\psi}\hat{p}^\mu\psi - (\hat{p}^\mu\bar{\psi})\psi] - \frac{i}{m}\hat{p}_\nu (\bar{\psi}S^{\mu\nu}\psi) , \quad (2)$$

$\bar{\psi}$  being the “adjoint” spinor of  $\psi$ ; quantity  $\hat{p}^\mu \equiv i\partial^\mu$  the 4-dimensional impulse operator; and  $S^{\mu\nu} \equiv \frac{i}{4}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$  the spin-tensor operator. The ordinary interpretation of eq.(2) is in total analogy with the decomposition given in eq.(1). The first term in

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<sup>#2</sup> From the classical-electrodynamics viewpoint, also in the free case, the charge, moving along a non-rectilinear (helical) path, should suffer a radiation-emission. Nevertheless, often this is assumed *not* to happen, in analogy with the stationary atoms orbits, and because of the fact that no external field is responsible for the accelerations of  $\mathcal{Q}$ , whose motion is “inertial”, in a way. However, it is also possible to regard the charge as actually radiating, and at the same time holding itself along stationary “light-like” orbits, because of a perfect balance (when time-averaging on stochastic fluctuations) between power emitted and power absorbed by any other accelerated charge in the universe. Starting from these assumptions, once known the numerical value of the cosmological Hubble constant, in a recent work<sup>[11]</sup> the value of the Planck constant has been deduced.

the r.h.s. results to be associated with the translational motion of the CM (*scalar* part of the current, corresponding to the traditional Klein–Gordon current). The second term in the r.h.s. results, instead, directly connected with the existence of spin, and describes the zbw motion.

In the abovequoted papers, Schrödinger started from the Heisenberg equation for the time evolution of the acceleration operator in Dirac theory

$$\mathbf{a} \equiv \frac{d\mathbf{v}}{dt} = \frac{i}{\hbar} [H, \mathbf{v}] = \frac{2i}{\hbar} (H\mathbf{v} - \mathbf{p}), \quad (3)$$

where  $H$  is equal as usual to  $\mathbf{v} \cdot \mathbf{p} + \beta m$  ( $\mathbf{v} \equiv \boldsymbol{\alpha}$ ). Integrating once this operator equation over time, after some algebra one can obtain:

$$\mathbf{v} = H^{-1}\mathbf{p} - \frac{i}{2}\hbar H^{-1}\mathbf{a}, \quad (4)$$

and, integrating it a second time, one obtains just the spatial part of the decomposition:

$$\mathbf{x} \equiv \boldsymbol{\xi} + \mathbf{X} \quad (5)$$

where (still in the operator formalism) it is

$$\boldsymbol{\xi} = \mathbf{r} + H^{-1}\mathbf{p}t, \quad (6)$$

related to the CM motion, and

$$\mathbf{X} = \frac{i}{2}\hbar \boldsymbol{\eta} H^{-1}, \quad (\boldsymbol{\eta} \equiv \mathbf{v} - H^{-1}\mathbf{p}), \quad (7)$$

related to the zbw motion.

Besides their consistency with the quantum theory, the type III models do easily entail the existence of spin, zbw and intrinsic magnetic moment for the electron, while these properties are hardly predicted by making recourse to the point-like-particle theories of class I. The “extended-like” electron models of class III are at present after fashion because of their possible generalizations to include supersymmetry and superstrings.<sup>[8]</sup> Furthermore, the “mixed” models seem to overcome the known non-locality problems involved by a relativistic covariant picture for extended-type (in particular *rigid*) objects of class II. Quite differently, the extended-like (class III) electron

is non-rigid and consequently variable in its “shape ”and in its characteristic “size”, depending on the considered dynamical situation. This is a priori consistent with the appearance in the literature of many different “radii of the electron”.#<sup>3</sup> Because of all those reasons, therefore, the spinning particle to which we shall refer in the next Section is described by a class III theory.

## 2 New kinematical properties for the extended-like spinning particles

We want now to analyze the formal and conceptual properties of a *new definition* for the 4-velocity of our extended-like electron. Such a new definition has been first adopted —but without any emphasis— in the papers by Barut et al. dealing with a recent, important model for the relativistic classical electron.<sup>[7,8,9?,10?]</sup> Let us consider the following. At variance with the procedures followed in the literature from Schrödinger’s till our days, we have to make recourse *not to the proper time of the charge  $\mathcal{Q}$ , but rather to the proper time of the center-of-mass*, i.e. to the time of the CM frame (CMF). #<sup>4</sup>

As a consequence, if we examine the definition for the 4-velocity, quantity  $\tau$  in the denominator of  $v^\mu \equiv dx^\mu/d\tau$  has to be the latter proper time. Up to now —with the exception of the above-mentioned papers by Barut et al.— in all theoretical frameworks the Lorentz factor has been assumed to be equal to  $\sqrt{1 - \mathbf{v}^2}$ , exactly as for the scalar particle case. On the contrary, into the Lorentz factor it has to enter  $\mathbf{w}^2$  instead of  $\mathbf{v}^2$ , quantity  $\mathbf{w} \equiv \mathbf{p}/p^0$  being the 3-velocity of the CM with respect to the chosen frame [ $p^0 \equiv \mathcal{E}$  is the energy]. By adopting the correct Lorentz factor, all the formulae containing it *have to be rewritten, and they get a new physical meaning*. In particular, we

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#<sup>3</sup> In his book *The Enigmatic Electron*,<sup>[13]</sup> M.H. McGregor lists at page 5 seven typical electron radii, from the Compton radius to the “classical” and to the “magnetic” one.

#<sup>4</sup> Let us recall that the CMF is the frame in which the kinetic impulse vanishes identically,  $\mathbf{p} = 0$ . For spinning particles, in general, it is *not* the “rest” frame, since the velocity  $\mathbf{v}$  is not necessarily zero in the CMF.

shall show below that the new definition does actually *imply*<sup>#5</sup> the following important constraint, which —holding identically for scalar particles— is often just *assumed* for spinning particles:

$$p_\mu v^\mu = m , \quad (8)$$

where  $m$  is the *physical* rest mass of the particle (and not an undefined mass-like quantity  $M$ ).<sup>#6</sup>

Our choice of the proper time  $\tau$  may be supported by the following considerations:

(i) The *light-like* zbw —when the speed of  $\mathcal{Q}$  is constant and equal to the speed of light in vacuum— is certainly the preferred one (among all the “a priori” possible internal motions) in the literature, and to many authors it appears the most adequate for a meaningful classical picture of the electron.<sup>#2</sup> In some special theoretical approaches,<sup>[5,6,11?,12?]</sup> the light speed is even regarded as the *quantum-mechanical* typical speed for the zbw. In fact, the Heisenberg principle in the relativistic domain<sup>[14?]</sup> implies (not controllable) particle–antiparticle pair creations when the (CMF) observation involves space distances of the order of a Compton wavelength. So that  $\hbar/m$  is assumed to be the characteristic “orbital” radius, and  $2m/\hbar^2$  the (CMF) angular frequency, of the zbw —as first noticed by Schrödinger—; and the orbital motion of  $\mathcal{Q}$  is expected to be *light-like*. Now, if the charge  $\mathcal{Q}$  travels at the light speed, *its proper time of  $\mathcal{Q}$  cannot exist*; while the proper time of the CM (which travels at sub-luminal speeds) does exist. Adopting as time the proper time of  $\mathcal{Q}$ , as often done in the past literature,

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<sup>#5</sup> For all plane wave solutions  $\psi$  of the Dirac equation, we have (labelling by  $\langle \rangle$  the corresponding *local mean value* or *local density*):  $p_\mu \langle \hat{v}^\mu \rangle \equiv p_\mu \psi^\dagger \hat{v}^\mu \psi \equiv p_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi \equiv p_\mu \bar{\psi} \gamma^\mu \psi = m$ .

<sup>#6</sup> Let make just an example, recalling that Paṽsić<sup>[8]</sup> derived, from a lagrangian containing an *extrinsic curvature*, the classical equation of the motion for a rigid  $n$ -dimensional world-sheet in a curved background spacetime. Classical world-sheets describe *membranes* for  $n \geq 3$ , *strings* for  $n = 2$ , and *point particles* for  $n = 1$ . For the special case  $n = 1$ , he found nothing but the traditional Papapetrou equation for a classical spinning particle; also, by “quantization” of the classical theory, he actually derived the Dirac equation. In ref.[8], however,  $M$  is not the observed electron mass  $m$ : and the relation between the two masses reads:  $m = M + \mu H^2$ , quantity  $\mu$  being the so-called *string rigidity*, while  $H$  is the second covariant derivative on the world-sheet.

automatically *excluded* a ligh-like zbw. In our approach, by contrast, such zbw motions are not excluded. Analogous considerations may hold for *Super-luminal* zbw speeds, without any problem since the CM (which carries the energy-impulse and the “signal”) is always endowed with a subluminal motion;

(ii) The independence between the center-of-charge and the center-of-mass motion becomes evident by our definition. As a consequence *the non-relativistic limit can be formulated by us in a correct, and univocal, way*. Namely, by assuming the correct Lorentz factor, one can immediately see that the zitterbewegung can go on being a relativistic (in particular, light-like) motion<sup>#7</sup> even in the non-relativistic approximation: i.e., when  $\mathbf{p} \rightarrow 0$ . In fact, in the non-relativistic limit, we have to take

$$w^2 \ll 1 ,$$

and *not* necessarily

$$v^2 \ll 1$$

as usually assumed in the past literature;

(iii) Our proposed definition for the 4-velocity agrees with the natural “classical limit” of the Dirac current. Actually, it has been used in those models which (like Barut et al.) define velocity *even at the classical level* as the bilinear combination  $\bar{\psi}\gamma^\mu\psi$ , via a direct introduction of *classical* spinors  $\psi$ . By the new definition, we shall be able to write the translational term as  $p^\mu/m$ , with the *physical* mass in the denominator, exactly as in the Gordon decomposition, eq.(2). Quite differently, in all the theories adopting as time the proper time of  $\mathcal{Q}$ , it appears in the denominator the already mentioned *variable* mass  $M$ , which depends on the internal zbw speed  $V$  (see below);

(iv) The choice of the CM proper time constitutes a natural extension of the ordinary procedure for relativistic scalar particles. In fact, for spinless particles in relativity the 4-velocity is known to be univocally defined as the derivative of 4-position with respect to the CMF proper time (which is the only one available).

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<sup>#7</sup> This is perhaps connected with the non-vanishing of spin in the non-relativistic limit, once we accept a correlation between spin and zbw.



The most important reason in support of our definition turns out to be the noticeable circumstance that *the old definition*

$$v_{\text{std}}^\mu = (1/\sqrt{1-v^2}; \mathbf{v}/\sqrt{1-v^2}) \quad (9)$$

*seems to entail a mass varying with the internal zbw speed.*

But let us explicitate our new definition for  $v^\mu$ . The symbols which we are going to use possess the ordinary meaning; the novelty is that now *the Lorentz factor*  $d\tau/dt$  *will not be equal to*  $\sqrt{1-v^2}$ , *but instead to*  $\sqrt{1-w^2}$ . Thus we shall have:

$$\begin{aligned} v^\mu &\equiv dx^\mu/d\tau \equiv (dt/d\tau; d\mathbf{x}/d\tau) \equiv \left( \frac{dt}{d\tau}; \frac{d\mathbf{x}}{dt} \frac{dt}{d\tau} \right) \\ &= (1/\sqrt{1-w^2}; \mathbf{v}/\sqrt{1-w^2}). \quad [\mathbf{v} \equiv d\mathbf{x}/dt] \end{aligned} \quad (10)$$

For  $w^\mu$  we can write:

$$\begin{aligned} w^\mu &\equiv d\xi^\mu/d\tau \equiv (dt/d\tau; d\xi/d\tau) \equiv \left( \frac{dt}{d\tau}; \frac{d\xi}{dt} \frac{dt}{d\tau} \right) \\ &= (1/\sqrt{1-w^2}; \mathbf{w}/\sqrt{1-w^2}); \quad [\mathbf{w} \equiv d\xi/dt] \end{aligned} \quad (11)$$

and for the 4-impulse:

$$p^\mu \equiv mw^\mu = m(1/\sqrt{1-w^2}; \mathbf{w}/\sqrt{1-w^2}). \quad (12)$$

[In presence of an external field such relations remain valid, provided that one makes the “minimal prescription”:  $p \rightarrow p - eA$  (in the CMF we shall have  $\mathbf{p} - e\mathbf{A} = 0$  and consequently  $\mathbf{w} = 0$ , as above)].

Let us now examine the resulting impulse–velocity scalar product,  $p_\mu v^\mu$ , which has to be a Lorentz invariant, both with our  $v$  and with the old  $v_{\text{std}}$ . Quantity  $p \equiv (\varepsilon; \mathbf{p})$  being the 4-impulse, and  $M_1, M_2$  two relativistic invariants, we may write:

$$p_\mu v^\mu \equiv M_1 \equiv \frac{\varepsilon - \mathbf{p} \cdot \mathbf{v}}{\sqrt{1-w^2}}, \quad (13)$$

or, alternatively,

$$p_\mu v_{\text{std}}^\mu \equiv M_2 \equiv \frac{\varepsilon - \mathbf{p} \cdot \mathbf{v}}{\sqrt{1-v^2}}. \quad (14)$$

If we refer ourselves to the CMF, we shall have  $\mathbf{p}_{\text{CMF}} = \mathbf{w}_{\text{CMF}} = 0$  (but  $\mathbf{v}_{\text{CMF}} \equiv V_{\text{CMF}} \neq 0$ ), and then

$$M_1 = \varepsilon_{\text{CMF}} \quad (15)$$

in the first case; and

$$\varepsilon_{\text{CMF}} = M_2 \sqrt{1 - V_{\text{CMF}}^2} \quad (16)$$

in the second case. So, we see that the invariant  $M_1$  is actually a *constant*, and —being nothing but the center-of-mass energy,  $\varepsilon_{\text{CMF}}$ — it can be identified, as we are going to prove, with the physical mass  $m$  of the particle. On the contrary, in the second case (the standard one), the center-of-mass energy results to be *variable* with the internal motion.

Now, from eq.(12) we have

$$p_\mu v^\mu \equiv m w_\mu v^\mu$$

and, because of eqs.(9-11),

$$p_\mu v^\mu \equiv m(1 - \mathbf{w}\mathbf{v})/1 - \mathbf{w}^2 . \quad (17)$$

Since  $\mathbf{w}$  is a vector component of the total 3-velocity  $\mathbf{v}$ , due to eq.(1), and  $\mathbf{w}$  is the *orthogonal projection* of  $\mathbf{v}$  along the  $\mathbf{p}$ -direction, we can write

$$\mathbf{w}\mathbf{v} = \mathbf{w}^2 ,$$

which, introduced into eq.(17), yields the constraint (8):

$$p_\mu v^\mu = m .$$

Quite differently, by use of the wrong Lorentz factor, we would have got

$$v^\mu = (1/\sqrt{1 - \mathbf{v}^2}; \mathbf{v}/\sqrt{1 - \mathbf{v}^2})$$

and consequently

$$\begin{aligned} p_\mu v^\mu &\equiv m(1 - \mathbf{w}\mathbf{v})/\sqrt{(1 - \mathbf{w}^2)(1 - \mathbf{v}^2)} \\ &= m\sqrt{1 - \mathbf{w}^2}/\sqrt{1 - \mathbf{v}^2} \neq m . \end{aligned}$$

By recourse to the correct Lorentz factor, therefore, we succeeded in showing that the important constraint  $p_\mu v^\mu = m$ , trivially valid for scalar particles, does hold for spinning particles too.

Finally, we want to show that —however— the ordinary kinematical properties of the Lorentz invariant  $v_\mu v^\mu$  do not hold any longer in the case of spinning particles, endowed with zitterbewegung. In fact, it is easy to prove that the ordinary constraint for scalar relativistic particles — $v^2$  constant in time and equal to 1— does *not* hold for spinning particles endowed with zbw. Namely, if we choose as reference frame the CMF, in which  $\mathbf{w} = 0$ , we have [cf. definition (10)]:

$$v_{\text{CMF}}^\mu \equiv (1; \mathbf{V}_{\text{CMF}}) , \quad (18)$$

wherefrom, it being

$$v_{\text{CMF}}^2 \equiv 1 - \mathbf{V}_{\text{CMF}}^2 , \quad (19)$$

one can deduce the following *new constraints*:

$$\begin{aligned} 0 < v_{\text{CMF}}^2(\tau) < 1 &\Leftrightarrow 0 < \mathbf{V}_{\text{CMF}}^2(\tau) < 1 && \text{("time-like")} \\ v_{\text{CMF}}^2(\tau) = 0 &\Leftrightarrow \mathbf{V}_{\text{CMF}}^2(\tau) = 1 && \text{("light-like")} \\ v_{\text{CMF}}^2(\tau) < 0 &\Leftrightarrow \mathbf{V}_{\text{CMF}}^2(\tau) > 1 . && \text{("space-like")} \end{aligned} \quad (20)$$

Notice that, since the square of the total 4-velocity is invariant and in particular it is  $v_{\text{CMF}}^2 = v^2$ , these new constraints for  $v^2$  will be valid in any frame:

$$\begin{aligned} 0 < v^2(\tau) < 1 &&& \text{("time-like")} \\ v^2(\tau) = 0 &&& \text{("light-like")} \\ v^2(\tau) < 0 . &&& \text{("space-like")} \end{aligned} \quad (21)$$

Let us examine the manifestation and consequences of such new constraints in a specific example: namely, in the already mentioned theoretical model by Barut–Zanghi<sup>[7]</sup> which

did implicitly adopt as time the proper time of the CMF. In this case we get that in general it is  $v^2 \neq 1$ . And in fact one obtains the important relation:<sup>[9,10]</sup>

$$v^2 = 1 - \frac{\ddot{v}_\mu v^\mu}{4m^2} . \quad (22)$$

In particular,<sup>[10]</sup> in the light-like case it is  $\ddot{v}_\mu v^\mu = 4m^2$  and therefore  $v^2 = 0$ .

Going back to eq.(19), notice that now quantity  $v^2$  is no longer related now to the *external* CM speed  $|\boldsymbol{w}|$ , but on the contrary to the *internal* zitterbewegung speed  $|\boldsymbol{V}_{\text{CMF}}|$ . Notice at last that, in general —and at variance with the scalar case— the value of  $v^2$  is *not constant in time* any longer, but varies with  $\tau$  (except when  $V_{\text{CMF}}^2$  is constant in time).

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