ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Genova

INFN/AE-94/17
5 Settembre 1994

E. Di Salvo, M. Pallavicini:

PREDICTIONS IN THE PSEUDOSCALAR CHANNEL OF CHARMONIUM BY MEANS OF QCD SUM RULES

PACS.: 11.50.Li; 14.40.Jz

To be published on Nuclear Physics B

INFN - Istituto Nazionale di Fisica Nucleare
Sezione di Genova

INFN/AE–94/17
5 Settembre 1994

PREDICTIONS IN THE PSEUDOSCALAR CHANNEL OF CHARMONIUM BY MEANS OF QCD SUM RULES

E. Di Salvo and M. Pallavicini

Dipartimento di Fisica – Universitá di Genova
Istituto Nazionale di Fisica Nucleare – sez. di Genova
Via Dodecaneso, 33 – 16146 Genova (Italy)

ABSTRACT

We use QCD sum rules for predicting the mass of the η_c' resonance and the partial decay width into two gammas of the η_c resonance, which are poorly known experimentally. To this end we elaborate two numerical algorithms. The former one allows us to extract the mass of the ground state in a given channel of Charmonium, if some parameters are known, and the mass of the first excited state, if the mass of the ground state and the coupling constant g_1 of the ground state to the proper quark current are known. The algorithm is applied to the vector channel for determining one free parameter which is common to all channels; moreover, closely examining the relation between g_1 and the mass of the first excited state in that channel suggests a second algorithm for determining g_1. Applying these algorithms to the pseudoscalar channel leads us to predicting $m_{\eta_c'} = (3575^{+30}_{-48})$ MeV and $\Gamma(\eta_c \longrightarrow \gamma\gamma) = (5585^{+490}_{-635})$ eV, in good agreement with other predictions and with presently available data.

1. Introduction

QCD sum rules constitute an important bridge between perturbative QCD and quark confinement physics. Such rules are founded on the assumption of quark-hadron duality and of analiticity in the energy squared s of the correlators, i. e. of the Fourier transforms of two-point functions of quark currents. The correlator in the time-like region, which depends on non-perturbative interactions responsible for confinement and may be put in relation to experimental data, is connected via a dispersion relation to the correlator in space-like region, which, for sufficiently large $|s|$, may be expanded perturbatively according to Wilson operator product expansion (OPE).

To be precise, strict OPE holds true for values of s which are too far from the region of resonances and it is not very useful for predicting particle masses. Therefore Shifman et al. (SVZ)[1-3] have proposed a nontrivial OPE extension - containing nonperturbative parameters like gluon and quark condensates - which is assumed to hold true also for values of s for which, in the spirit of duality, resonances give a significant contribution to the correlator.

It is customary to derive moment sum rules, by considering moments - either power moments[1,2,4,5] or exponential moments[3,6-10] - of both sides of the dispersion relation. Such sum rules have led to important and precise predictions about masses and widths of resonances[2,3].

Some authors[6-9] have argued, on the ground of different models, that power moment sum rules may lead to underestimating gluon condensates, since higher dimensional condensates may be important, as exhibited recently by means of sophisticated functional methods[11-13]. All that points out the opportunity of taking into account more terms in the nontrivial Wilson OPE of the correlators.

However power sum rules preserve some predictive power as regards heavy quarkonia. For example, these rules permitted SVZ[2] to predict to a high accuracy the mass of the η_c resonance, by comparison with the J/ψ resonance. This means that the systematic errors by which condensates may be affected do not seem to influence significantly the observable quantities predicted.

In this paper we use power moment sum rules in the version of Reinders, Rubinstein and Yazaki[4,5] (RRY) for determining the mass of the first excited state in the pseudoscalar channel of Charmonium. Usually QCD sum rules are not emploied to this end *. Here we show that the sum rule modification proposed by RRY is sensitive to the first excited state of a given channel, despite the small weight of this state in the dispersive integral.

We exploit the plateau method of RRY, fixing a precise criterion for optimizing the conditions under which to extract from QCD sum rules the mass of the ground state and of the first excited state in a channel of Charmonium. The latter mass turns out to depend critically on the coupling constant of the ground state to the charmed quark current with the right quantum numbers. We investigate this dependence in the vector channel, which allows us to elaborate an algorithm for determining both quantities. Moreover we fix free parameters so as to reproduce data in that channel. Some of these parameters we use as input in the pseudoscalar channel, which, as regards our analysis, exhibits quite an analogous behaviour to the vector channel. We apply the above mentioned algorithm for predicting to a good approximation the partial decay width of the η_c resonance into two gammas and the mass of the η_c' resonance, both of which are poorly known experimentally. In particular our prediction of the mass of η_c' is in fair agreement with experiment[14] and also with model predictions[15]. As to the partial width of η_c resonance, our prediction is in good agreement with the average value reported by Particle Data Book[16] (see also [17]) and with the predictions of other QCD sum rules[18-24] and of a potential model[25].

The paper is organized as follows.

* Bell and Bertlmann[6] studied predictions on first excited states by means of exponential moment sum rules, in the framework of a nonrelativistic potential model.

In sect. 2 we expose the QCD sum rules and, in particular, we describe the power moment method.

Sect. 3 is devoted to illustrating the plateau method and to defining a criterion which permits us to fix a procedure for extracting reliable values for masses of ground state and first excited state in a given channel of Charmonium.

In sect. 4 we apply the procedure described in sect. 3 to the vector channel. This leads us to the definition of an algorithm for determining the mass of the ground state, supposing some free parameters known; this algorithm yields also the mass of the first excited state, if the coupling constant g_1 of the ground state to the vector quark current is known. We fix the parameters so as to reproduce the experimental values of the masses of the J/ψ and ψ' resonances. Moreover we examine in detail the relation of the mass of the first excited state to g_1, from which we derive a second algorithm for determining g_1. We discuss errors by which fit parameters are affected.

In sect. 5 we apply the algorithms elaborated in the previous section to the pseudoscalar channel, in order to predict the mass of the η_c' resonance and the partial decay width of the η_c resonance into two gammas. We also compare our method and results with those by other authors.

Sect. 6 is devoted to a short conclusion.

2. QCD sum rules: power moments

The basic object of the QCD sum rules is the Fourier transform of the two-point function of the charmed quark current $\bar{c}\Gamma c$, where Γ is a Dirac operator with well defined tensor properties (vector, pseudoscalar, etc.). This is of the form[4]

$$\Xi_{\mu\nu\ldots}\Pi^{\Gamma}(Q^2), \qquad (Q^2 = -q^2),$$

where q is the overall four-momentum of the $c-\bar{c}$ pair, $\Xi_{\mu\nu\ldots}$ a tensor and Π^{Γ} a Lorentz invariant quantity, which depends on the quantum numbers of the channel considered, denoted again by Γ. As to Π^Γ, we may write the dispersion relation

$$\Pi^\Gamma(Q^2) = \frac{1}{\pi}\int_0^\infty \frac{Im\Pi^\Gamma(s)}{s+Q^2}ds. \tag{2.1}$$

Owing to the Zweig rule, the spectral function $Im\Pi^\Gamma(s)$ is not influenced by hadronic states other than in the channel Γ of Charmonium. The spectral function is in principle measurable; however

i) the behaviour in the neighbourhood of the continuum threshold is known only approximately;

ii) for large positive s we do not have direct measurements;

iii) in most channels of charmonium excited states have not yet been detected.

Therefore we assume, according to duality, that there is a threshold value s_0 (to be taken as a free parameter), such that for $s > s_0$ perturbative QCD is valid.

As to the l.h.s. of (2.1), for $Q^2 \geq 0$ we may assume a non-trivial Wilson operator product expansion (OPE) containing one or more gluon condensates, i. e.[9],

$$\Pi^\Gamma(Q^2) = \Pi^\Gamma_{pert}(Q^2) + \sum_{n\geq 2}\sum_{k=1}^{l_n}\frac{\Phi_{2n,k}}{Q^{2n}}, \tag{2.2}$$

where $\Pi^\Gamma_{pert}(Q^2)$ is the perturbative contribution to OPE and the $\Phi_{2n,k}$ are proportional to the gluon condensates of $2n$ mass dimension. We retain only the first addend in the second sum at the r.h.s. of (2.2). Moreover we expand $\Pi^\Gamma_{pert}(Q^2)$ to first order in the strong coupling constant α_s; however, as to the mass of the charmed quark, we consider a renormalized, gauge dependent mass in the Euclidean region[1], adopting the Landau gauge[1,4] and choosing the renormalization point so as to minimize higher order corrections[4]. Deriving n times both sides of (2.1), we obtain moment sum rules of the type

$$M^\Gamma_n(\xi) = \frac{1}{\pi}\int_0^\infty \frac{Im\Pi^\Gamma(s)}{(s+Q^2)^{n+1}}ds. \tag{2.3}$$

Here

$$M_n^\Gamma(\xi) = A_n^\Gamma[1 + \alpha_s a_n^\Gamma(\xi) \ + \Phi b_n^\Gamma(\xi)], \tag{2.4}$$

where $\alpha_s = \alpha_s[4(m_c^0)^2 + Q^2]$ is the running coupling constant ($\alpha_s[4(m_c^0)^2] \sim 0.3$, m_c^0 will be defined in a moment),

$$\xi = \frac{Q^2}{4(m_c^0)^2}$$

and the coefficients A_n^Γ, a_n^Γ and b_n^Γ are deduced from the Wilson OPE and have been tabulated for the most interesting channels by RRY[4]; we report their expressions in table I as regards vector and pseudoscalar channel, which are of interest in the present paper. There appears the running mass of the charmed quark, $m_c(\xi)$, where[4]

$$\frac{m_c(\xi)}{m_c^0} = 1 - \frac{\alpha_s}{\pi}[\frac{2+\xi}{1+\xi} ln(2+\xi) - 2ln2], \tag{2.5}$$

and

$$m_c^0 = m_c(p^2 = -m_c^2). \tag{2.6}$$

Φ is a dimensionless parameter, defined as

$$\Phi = \frac{\pi^2}{36(m_c^0)^4} < 0|\frac{\alpha_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu}|0 > \tag{2.7}$$

and $G_{\mu\nu}^a$ is the QCD strength tensor field; the quantity $< 0|\frac{\alpha_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu}|0 >$ is renormalization group invariant. As to $Im\Pi^\Gamma(s)$, we have explained above that it is influenced only by Charmonium resonances and continuum states, i. e.,

$$Im\Pi^\Gamma(s) = \frac{9\pi}{4} \sum_i \frac{m_i^2}{g_i^2} [m_i \Gamma_i \frac{\pi^{-1}}{(m_i^2 - s)^2 + (m_i \Gamma_i)^2}] + \frac{1}{4\pi}(1 + \frac{\alpha_s}{\pi})\theta(s - s_0), \tag{2.8}$$

m_i, Γ_i and g_i being, respectively, the mass, the total width and the coupling constant to $\bar{c}\Gamma c$ of the i-th resonance. The expression in square brakets reduces to a Dirac delta function for sufficiently thin resonances; this approximation - which, owing to the Zweig rule, may be adopted for any heavy quark resonance - will be assumed from now on.

The sum rule (2.1) does not hold true for values of q^2 near masses squared of resonances, since the perturbative expansion breaks down; on the other hand, for values of q^2 too far from the resonance region, the sum rule is not very sensitive to the contribution of resonances. Therefore, since higher moments correspond to higher terms in the Taylor expansion around a given Q^2, for a value of Q^2 where OPE holds true sum rules (2.3) are useful only in a limited, Q^2-dependent, range of values of n; moreover there is a critical value (again Q^2-dependent) of n, above which the parameters involved in the sum rules undergo a sudden exchange, owing to OPE breaking.

As can be seen from (2.3), the relative weights of higher-dimensional condensates (at the l.h.s.) and of the excited states in the channel considered (at the r.h.s.) may be triggered according to the value of Q^2 and according to the order of the moment. In particular, if we consider the Borel transform of both sides of the sum rule (2.1), that is, the limit

$$\lim_{n\to\infty}[(nM^2)^{n+1}M_n^\Gamma\ (nM^2)],$$

where M^2 is a parameter with dimensions of a mass squared, higher dimensional condensates are likely to have a negligible weight, as shown in non-relativistic models[6]. But also the weight of excited states becomes irrelevant and we loose any hope of determining the masses of such states.

We recover the method of power moments, trying to extract information on the mass of the first excited state in 0^- channel of Charmonium. It has been stressed[6-10] that this method may lead to an underestimation of Φ (although the "standard" value found by SVZ and by RRY cannot be completely excluded[12]); however the systematic error by which this parameter may be affected seems not to invalidate predictions as regards resonance masses. Indeed

i) the gluon condensate has been determined by SVZ so as to match sum rules in the vector channel to available data;

ii) on the other hand in the pseudoscalar channel sum rules have been successful in predicting the mass of the η_c resonance;

iii) lastly, resonance masses predicted by power moments turn out to agree very well with those calculated by exponential moments[7], which are believed to yield more reliable values of gluon condensates.

Incidentally, these arguments show that the power moment method is adherent to data, contrarily to what asserted in ref.[8].

3. The plateau method

Now we elaborate a numerical method for extracting the values of the masses of the ground state and of the first excited state of a given channel from the sum rules (2.3). To this end we invoke duality, owing to which we may include in the continuum the contribution of all resonances but the lowest one, or all but the lowest two. Therefore the sum in (2.8) may either be limited to a single term or run over two terms; the parameter s_0 is sensitive to this choice. In the former case we deduce

$$m_1^2 = \frac{\overline{M}_n^{\Gamma}}{\overline{M}_{n+1}^{\Gamma}} - Q^2, \tag{3.1}$$

where

$$\overline{M_n}^{\Gamma} = M_n{}^{\Gamma} - \frac{1 + \frac{\alpha_s}{\pi}}{4\pi^2 n (s_0 + Q^2)^n} \tag{3.2}$$

and M_n^{Γ} is defined by eq. (2.4).

On the contrary, in the latter case we have

$$m_2^2 = \frac{N_n^\Gamma}{N_{n+1}^\Gamma} - Q^2 \tag{3.3}$$

where

$$N_n^\Gamma = \overline{M}_n^\Gamma - \frac{9m_1^2}{4g_1^2(m_1^2 + Q^2)^{n+1}}. \tag{3.4}$$

As we have illustrated in the preceding section, for a given Q^2 there is only a short range of values of n where the the masses of resonances may be determined by means of the sum rules. In particular, we expect to find a value of n that optimizes the sensitivity of the sum rule to the mass of the ground state or of the first excited state. In correspondence of such a value, the resonance masses, as extracted from formula (3.1) or (3.3), are expected to be stationary with respect to n. In fact these mass formulae really yield a plateau in n for any ξ[4] (see fig. 1); moreover, as regards the vector and pseudoscalar channels, for reasonable values of m_c^0 and s_0, the values of the masses corresponding to this plateau are in satisfactory agreement with experimental data[4].

According to RRY, we can vary not only n - that is, the number of terms to be taken in consideration in the Taylor expansion in Q^2 - but also the point around which we take such an expansion. This permits us to trigger the width of the plateau of m_1 or of m_2 and to choose the optimal conditions for determining such masses. To this end we define a quantity that characterizes the width of the plateau, that is

$$D_i = m_i(\overline{n} - k) + m_i(\overline{n} + k) - 2m_i(\overline{n}), \quad i = 1, 2, \tag{3.5}$$

where $\overline{n}$ is the point where m_i is stationary and k a suitable positive integer. Roughly, D_i is proportional to the second derivative of m_i with respect to n at $n = \overline{n}$, therefore it seems quite proper to our end, since the stationary point is always a minimum. Our criterion consists in looking for a finite Q^2 which minimizes D_i. Indeed, as we shall see in the next section, the D_i always present some relative minima for not too large values of Q^2, so that our criterion is applicable. We may fix free parameters by imposing that results of QCD sum rules agree with data; furthermore, as we shall illustrate in detail, we may elaborate a method for predicting measurable quantities.

4. Applications to vector channel of Charmonium

Data availability in the vector channel allows us to fix the free parameters, m_c^0, s_0^v and Φ. In particular Φ has already been determined by SVZ to a great accuracy, using all the resonances known in the vector channel; the result of that analysis is

$$\Phi = 1.35 \quad 10^{-3}, \tag{4.1}$$

which will be adopted in the present paper; however we shall also explore the effects produced by varying Φ. As regards m_c^0 and s_0^v, initially we assume reasonable values, i.e., respectively, $1.25 - 1.28 \quad GeV$ and $14. - 14.4 \quad GeV^2$.

First of all we study the behaviour of D_1 (defined by formula (3.5)) against Q^2, i.e. against ξ. As to the ground state of the vector channel, in view of the behaviour of m_1 at varying n (fig. 1), it seems appropriate to choose $k = 2$ in the definition of D_1. The behaviour of D_1 versus ξ is shown in fig. 2, where, as one can see, it presents three relative minima. The procedure now described will be called, from now on, the "first algorithm". Now we are faced with the problem of choosing one of such minima. We require the minimum to be as low as possible, and moreover that the value of m_1 corresponding to this minimum be stable against small variations of the parameters s_0 and m_c^0. It may be shown that this last requirement is not met by the third minimum in fig. 2, whereas it is verified by the second one, which, therefore, we pick up.

This choice uniquely determines m_1 for any point of the three-dimensional space of the above mentioned parameters and selects, in such a space, the two-dimensional variety for which the mass m_1 coincides with the experimental value of $m_{J/\psi}$. In particular fixing Φ to the value (4.1) restricts the parameter space to the full line pictured in fig. 3. Incidentally, we note that for $m_c^0 = 1.28 \quad GeV$ our algorithm reproduces the results of the analysis by RRY.

We may fix a point on this line by imposing that m_2 (formula (3.3)) equal the mass of the first excited state. m_2 may be found by means of our first algorithm, as we are going to show. The coupling constant g_1^v may be determined from the partial decay width Γ_1^v of the J/ψ resonance into $e^+ - e^-$ through the following formula:

$$g_1^v = \alpha \sqrt{\frac{4\pi m_1}{3\Gamma_1^v}}, \tag{4.3}$$

where α is the fine structure constant. Since $\Gamma_1^v = 5.36 \pm .30 \ KeV$[4], it results $g_1^v = 11.36 \pm .30$. Therefore we may calculate m_2 by means of formula (3.3) and study the behaviour of such a quantity at varying n and ξ; this suggests that $k = 1$ is an appropriate value to be used in formula (3.5) relative to D_2. In fig. 4, which exhibits the behaviour of D_2 against ξ, three relative minima appear. However, if a minimum of D_2 occurs close to a minimum of D_1, it is to be considered spurious, since the function m_2 (eq. (3.3)) has a quite similar behaviour to the function m_1 (eq. (3.1)). Among the remaining local minima, we pick up the lowest one. In the specific case of fig. 4 we take the second minimum, since, by comparison with figure 2, the first and the third ones are to be discarded. Imposing that m_2 equal $m_{\psi'}$ restricts the space of parameters m_c^0 and s_0^v to the dashed line in fig. 3. The intersection point with the full line uniquely fixes the values of m_c^0 and of s_0^v for which our algorithm yields simultaneuosly the masses of the J/ψ and of the ψ' resonances, i. e.,

$$s_0^v = (15.83^{+1.17}_{-1.10}) \quad GeV^2, \qquad m_c^0 = (1.268^{+0.012}_{-0.008}) \quad GeV, \tag{4.4}$$

errors being induced by the statistical error on Γ_1^v.

Of course, as we have already observed, the value of s_0^v involved in matching m_2 to $m_{\psi'}$ is expected to be larger than the value required for matching m_1 to $m_{J/\psi}$, since in the former case we exclude the contribution of the first excited state from "continuum", that is, from the second addend of formula (3.2). Therefore the values of m_c^0 and of s_0^v are affected by a systematic error, that we may evaluate by calculating the contribution to "continuum" of the first excited state, against n and Q^2 (see fig. 5). We see that such a systematic error is of order 5 per cent of the value of s_0^v found above. As we shall see in the next section, a very similar effect occurs in the pseudoscalar channel, inducing on our predictions systematic errors which are, at most, 25% of the errors induced by statistical deviations of experimental quantities.

Now we investigate the predictive power of our method. First of all we observe that the coupling constant g_1 - of course an essential ingredient in formula (3.3) - may be determined from the sum rules (2.3), once m_1 is known. We find $g_1 = 10.87$, a good approximation to the experimental value, but not enough for predicting the mass of the first excited state to an acceptable precision, owing to the sensitivity of m_2 to g_1 in our formula. Therefore we elaborate a more complicated strategy. Preliminarily we observe that, according to the procedure described just above, to any value of Γ_1^v (say, $\Gamma_1^v = \overline{\Gamma}_1^v$) it corresponds a well determined pair of values of m_c^0 and s_0^v. Conversely, if, keeping m_c^0 and s_0^v fixed, we vary Γ_1^v and determine D_2 for any value of this quantity, we find that D_2 presents an oblique flex in correspondence of $\Gamma_1^v = \overline{\Gamma}_1^v$. The situation is illustrated in fig. 6 in the case of $\overline{\Gamma}_1^v = 5.36 KeV$. Actually more than one flex appears in fig. 6, but, except the one which occurs in correspondence of $\overline{\Gamma}_1^v$, they are unstable against small variations of the parameters s_0 and m_c^0. The procedure now described - to be named, from now on, the "second algorithm" - allows to determine m_2 and $\overline{\Gamma}_1$, once m_c^0 and s_0^v have been fixed; this algorithm will be used for predicting the mass of the first excited state in the pseudoscalar channel.

As to the parameter Φ, up to now we have kept it fixed; if we vary it within the interval

$$1.2 < \quad (10^3 \quad \Phi) \quad < 1.4,$$

the other parameters and resonance masses do not vary, only the locations of minima of D_1 and D_2 are shifted.

5. Predictions in the pseudoscalar channel

Now we apply the algorithms elaborated in the previous section to the pseudoscalar channel; this allows us to predict the mass of the η_c' particle and the partial decay width of the η_c particle into two gammas.

Firstly we take in consideration the ground state in the 0^- channel. Application of our first algorithm shows that the functions involved (m_1^2 vs n, D_1 vs ξ) behave quite similarly to the corresponding functions found in the 1^- channel. In particular, fig. 7 represents the graph of D_1 vs ξ, which presents three relative minima. Applying the same criterion as in the vector case (i. e., requiring stability of m_1 against small variations of the free parameters), we pick up the third minimum. Moreover, since we have already determined m_c^0, we may choose s_0^p in such a way that the mass m_1 coincide with the mass of the η_c particle. We get

$$s_0^p = (15.35^{+1.65}_{-1.45}) \quad GeV^2. \tag{5.1}$$

The left error on s_0^p is a consequence of the uncertainty on m_c^0, whereas the right error derives from imposing that s_0^p be smaller than the largest value which can be attributed to s_0^v according to formula (4.4).

We may use the first algorithm also for determining m_2 for any value of g_1. In fig. 8 it is represented the behaviour of D_2 at varying ξ, assuming for g_1 the same value as in the vector channel. According to the criterion described in the previous section - which consists in discarding local minima of D_2 that occur in correspondence of those of D_1 and choosing the lowest of the remaining ones - we must pick up the first minimum.

In order to make predictions on the first excited state, we apply our second algorithm. First of all, taking for m_c^0 the value (4.4) and for s_0^p the value (5.1), we draw the graph of m_2 and of D_2 versus Γ_1^p, where

$$\Gamma_1^p = \frac{16\pi}{9g_1^2} m_1 \alpha^2$$

is the electromagnetic decay width of η_c into two γ's. Analogously to the vector channel, the dashed line in fig. 9 presents some oblique flexes and again we discard those which are unstable against small variations of free parameters. Only the point signed by the arrow in fig. 9 is stable; in correspondence to this point, we find

$$m_2 = (3.575^{+.030}_{-.048}) \quad GeV; \qquad \Gamma_1^p = (5.585^{+.490}_{-.635}) \quad KeV, \qquad (5.2)$$

errors being due essentially to uncertainties on s_0^p, as given by (5.1). The corresponding value of g_1^p is $12.65^{+.490}_{-.635}$. (Direct application of moment formula yields $g_1^p = 10.50$).

As in the vector case, the value of s_0^p is affected by a systematic error, since we do not take into account the different values that this parameter assumes, according as we apply formula (3.1) or (3.3); this induces a systematic error in m_c^0, which in principle could be different from the value found in the vector channel. If we assume the nonrelativistic approximation[1,18], the difference is negligibly small. In a more conservative evaluation, taking into account the dependence of s_0^v and of s_0^p on m_c^0 (the former is represented in fig. 3, the latter has a similar behaviour), this difference could affect our predictions by, at most, 25% of the errors which appear in formula (5.2).

As a conclusion to our predictions, we observe that the value of m_2 which we have found occurs close to the experimental value of the mass of η_c' particle obtained by Edwards et al.[14] and is in very good agreement with potential model predictions[15]. As regards Γ_1^p, it is worth confronting our prediction with those by other authors, who use either sum rules[1,5,19-24], or potential models[25]. In the framework of sum rules, power moments[1,5,20-23] or exponential moments[24] of the three-current correlator at $Q^2 = 0$, or also power moments of the two-current correlator[1,19] have been used. By the way, other radiative decay widths of quarkonium states ($J/\psi \rightarrow \eta_c\gamma, \quad \chi_{c0}, \chi_{c2}, \eta_b \rightarrow \gamma\gamma$) have been calculated by means of the power moments of the three-current[26-28]; in particular, in ref.[28] - as well as in refs. [5] and [21] - the three-point sum rule has been divided by the two-point sum rule, in order to reduce the dependence on m_c^0. As regards the η_c radiative decay width, predictions range from $\approx 4.$ KeV to $\approx 7.$ KeV. This, on the one hand, leads us to concluding that all sum rule results - as well as the prediction of a potential model[25] - agree with existing experimental data. On the other hand, we point out that sum rule predictions are fluctuating, owing prevalently to the use of different values of the parameters ($m_c^0 = 1.25 \quad GeV$ to $1.28 \quad GeV$), or also to the different approximations adopted. This, in turn, indicates a relevant truncation error in OPE, which could be a consequence of having neglected higher condensates, as explained in the first two sections of the present paper. Our method is more likey to be free of such biases, since, unlike the other approaches, the parameters that enter the sum rule have been determined so as to match data in the vector channel, moreover the algorithm which yields g_1 has been tested successfully in this channel.

In connection with radiative decay widths of Charmonium states, it is worth pointing out some delicate and intriguing questions.

i) Nonrelativistic approximation predicts $\frac{\Gamma_1^p}{\Gamma_1^v} = \frac{4}{3}$, whence, according to our result, we conclude that nonperturbative contributions affect naive predictions by about 5%.

ii) Assuming the Appelquist-Politzer recipe[29,20,23],

$$\frac{\Gamma_{\eta_c \rightarrow \gamma\gamma}}{\Gamma_{\eta_c \rightarrow gg}} = \frac{9}{8} \frac{\alpha_s^2(Q^2)}{\alpha^2},$$

our prediction, together with the experimental value of the total width of the η_c particle, yields $\alpha_s = .296^{+0.071}_{-0.064}$, which is quite consistent with the assumption by RRY, adopted in our paper.

iii) QCD sum rules (together with vector dominance assumption) yield a decay width of the J/ψ particle into $\eta_c\gamma$ which is considerably higher than the experimental value. Shifman[20] proposes this fact as a crucial QCD test. Alternatively this could indicate a gluon admixture in η_c[27]. In this sense it would be interesting to determine experimentally the radiative decay width of χ_{c0} into $\gamma\gamma$, in order to compare it with QCD prediction[27]: indeed, in this case the $c-$ quarks are in a P-wave and interact with the gluon condensate. If the hypothesis of a gluon admixture were confirmed, we could also expect a lower value of the decay width of η_c into $\gamma\gamma$[27].

iv)The gluon condensate could be determined more accurately from precise measurements of radiative decay widths of heavy quarkonia, by confronting them with predictions of QCD sum rules with three-point functions[30].

6. Conclusion

Let us briefly recall the procedure and the main results of the present paper.

A) Starting from QCD sum rules, we have stated a criterion, based on the plateau method, which has led us to elaborating a first algorithm for determining the mass of the ground state in a given channel, once the parameters m_c^0, s_0 and Φ are fixed. This algorithm yields also the mass of the first excited state, if we assign m_1 and the coupling constant g_1.

B) We have determined m_c^0, Φ and s_0^v by imposing that the results of our first algorithm should match data in the vector channel.

C) The behaviour in the vector channel of the quantity D_2 (eq. (3.5)) at varying g_1^v - while keeping m_c^0 and s_0^v fixed - has suggested us a second algorithm for extracting g_1.

D) We have applied the first algorithm to the pseudoscalar channel, determining s_0^p in such a way that m_1 equal the mass of the ground state; successively we have used the second algorithm for predicting g_1^p and $m_{\eta_c'}$ in that channel. Our results are in good agreement with available data.

Lastly we stress that, besides QCD, no particular assumption has been introduced in our algorithm, therefore experimental verification of our predictions would constitute a good test of QCD.

ACKNOWLEDGMENTS

The authors are thankful to their fiends profs. C.M. Becchi, A. Di Giacomo, N.Paver and A. Santroni for useful suggestions. One of them (E.D.S.) is grateful to prof. H.R. Rubinstein for the stimulating discussion during a visit to ICTP (Trieste).

TABLE I

In the following we have set $\rho = \frac{\xi}{1+\xi}$, $m = m_c$ (see formula (2.5)) and $F(a,b,c;x) = {}_2F_1(a,b,c;x)$.

a) Vector channel

$$A_n^v = \frac{3}{4\pi^2}\frac{2^n(n+1)(n-1)!}{(2n+3)!!\,[(4m^2)(1+\xi)]^n}F(n,\frac{1}{2},n+\frac{5}{2};\rho)$$

$$a_n^v = \frac{(2n+1)!!(2n+3)}{3\cdot 2^{n-1}n!(2n+2)}[F(n,\frac{1}{2},n+\frac{5}{2};\rho)]^{-1}$$

$$\cdot\{\pi - [\frac{\pi}{3} + \frac{1}{2}(\frac{\pi}{2} - \frac{3}{4\pi})]\ F(n,1,n+2;\rho)(n+1)^{-1}$$

$$+[3(n+1)(n+2)]^{-1}(\frac{\pi}{2} - \frac{3}{4\pi})F(n,2,n+3;\rho)\}$$

$$-(\frac{\pi}{2} - \frac{3}{4\pi}) - 2n\frac{log(2+\xi)}{\pi} \; \frac{2+\xi}{(1+\xi)^2} \, \frac{F(n+1, \frac{1}{2}, n+\frac{5}{2}; \rho)}{F(n, \frac{1}{2}, n+\frac{5}{2}; \rho)}$$

$$b_n^v = -\frac{n(n+1)(n+2)(n+3)}{(2n+5)(1+\xi)^2} \; \frac{F(n+2, -\frac{1}{2}, n+\frac{7}{2}; \rho)}{F(n, \frac{1}{2}, n+\frac{5}{2}; \rho)}$$

b) Pseudoscalar channel

$$A_n^p = \frac{3}{8\pi^2} \frac{2^n(n-1)!}{(2n+1)!! \; [(4m^2)(1+\xi)]^n} F(n, \frac{1}{2}, n+\frac{3}{2}; \rho)$$

$$a_n^p = \frac{(2n+1)!!}{3 \cdot 2^{n-1} n!} \; [F(n, \frac{1}{2}, n+\frac{3}{2}; \rho)]^{-1}$$

$$\cdot[\pi - \frac{1}{2(n+1)}(\frac{\pi}{2} - \frac{3}{4\pi}) \; F(n, 1, n+2; \rho)] - (\frac{\pi}{2} - \frac{3}{4\pi})$$

$$+\frac{1}{\pi}[\frac{8}{3} - \frac{4}{n} \; \frac{F(n, \frac{3}{2}, n+\frac{3}{2}; \rho)}{F(n, \frac{1}{2}, n+\frac{3}{2}; \rho)} - \frac{5}{6}(n+\frac{3}{2})^{-1} \; \frac{F(n, \frac{3}{2}, n+\frac{5}{2}; \rho)}{F(n, \frac{1}{2}, n+\frac{3}{2}; \rho)}]$$

$$-2n\frac{log(2+\xi)}{\pi} \; \frac{2+\xi}{(1+\xi)^2} \, \frac{F(n+1, \frac{1}{2}, n+\frac{3}{2}; \rho)}{F(n, \frac{1}{2}, n+\frac{3}{2}; \rho)}$$

$$b_n^p = -\frac{n(n+1)(n+2)(n+3)}{(2n+3)(1+\xi)}$$

$$\cdot[\frac{F(n+1, -\frac{3}{2}, n+\frac{5}{2}; \rho)}{F(n, \frac{1}{2}, n+\frac{3}{2}; \rho)} - \frac{6}{n+3} \, \frac{F(n+1, -\frac{1}{2}, n+\frac{5}{2}; \rho)}{F(n, \frac{1}{2}, n+\frac{3}{2}; \rho)}]$$

REFERENCES

[1] V.A.Novikov, L.B.Okun, M.A.Shifman, A.I.Vainshtein, M.B.Voloshin and V.I.Zakharov: Phys.Rep. *41* , 1 (1978)

[2] M.A.Shifman, A.I.Vainshtein, M.B. Voloshin and V.I.Zakharov: Phys.Lett. B *77* , 80 (1978)

[3] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov: Nucl.Phys. B *147* , 385, 448, 519 (1979)

[4] L.J.Reinders, H.R.Rubinstein and S.Yazaki: Nucl.Phys. B *186* , 109 (1981)

[5] L.J.Reinders, H.R.Rubinstein and S.Yazaki: Phys.Rep. *127* , 1 (1985)

[6] J.S.Bell and R.A.Bertlmann: Nucl.Phys. B *177* , 218 (1981)

[7] R.A.Bertlmann: Nucl.Phys. B *204* , 387 (1982)

[8] J.Fischer and P.Kolar: Z.Phys. C - Particles and Fields *34* , 375 (1987)

[9] J.Marrow, J.Parker and G.Shaw: Z.Phys. C - Particles and Fields *37* , 103 (1987)

[10] C.A.Dominguez and N.Paver: Phys.Lett. B *293* , 197 (1992)

[11] R.A.Bertlmann, C.A.Dominguez, M.Loewe, M.Perrottet and E.deRafael: Z.Phys. C - Particles and Fields *39* , 231 (1988)

[12] M.B.Causse and G.Mennessier: Z.Phys. C - Particles and Fields *47* , 611 (1990)

[13] I.Caprini: Phys.Rev. D *44* , 1569 (1991)

[14] C.Edwards et al.: Phys.Rev.Lett. *48* , 70 (1982)

[15] M.Baker, J.S.Ball and F.Zachariasen: Phys.Rev. D *47* , 3021 (1992); A.Martin and J.M.Richard: Phys.Lett. B *115* , 323 (1982)

[16] Particle Data Group (M.Aguilar-Benitez et al.): Phys.Rev. D *45* , (1992)

[17] DM2 Collaboration (D.Bisello et al.): Nucl.Phys. B *350* , 1 (1991)

[18] K.T.Chao: Nucl.Phys. B *317* , 597 (1989)

[19] K.T.Chao: Nucl.Phys. B *335* , 101 (1990)

[20] M.A.Shifman: Z.Phys. C - Particles and Fields *4* , 345 (1980)

[21] L.J.Reinders, H.R.Rubinstein and S.Yazaki: Phys.Lett. B *113* , 414 (1982)

[22] R.Kirschner and A.Schiller: Z.Phys. C - Particles and Fields *16* , 141 (1982)

[23] T.M.Aliev: Yad.Fiz. *37* , 403 (1983) (Sov.Jou.Nucl.Phys. *37* , 242 (1983))

[24] V.A.Beilin and A.V.Radyushkin: Yad.Fiz. *45* , 546 (1987) (Sov.Jou.Nucl.Phys. *45* , 342 (1987))

[25] T.Appelquist et al.: Ann.Rev.Nucl.Part.Sci. *28* , 387 (1978)

[26] T.M.Aliev: Z.Phys. C - Particles and Fields *26* , 275 (1984)

[27] L.S.Dul'yan, A.G.Oganesyan and Yu.Khodzamiryan: Yad.Fiz. *44* , 746 (1986) (Sov.Jou.Nucl.Phys. *44* , 483 (1986))

[28] L.S.Dul'yan, A.D.Magakyan and Yu.Khodzamiryan: Yad.Fiz. *51* , 492 (1990) (Sov.Jou.Nucl.Phys. *51* , 314 (1990))

[29] T.Appelquist and H.D.Politzer: Phys.Rev.Lett. *34* , 43 (1975)

[30] A.G.Oganesyan and Yu.Khodzamiryan: Yad.Fiz. *55* , 229 (1992) (Sov.Jou.Nucl.Phys. *55* , 128 (1992))

FIGURE CAPTIONS

[Fig.1] - Vector channel: the behaviour of m_1 versus n for $\xi = .5$.

[Fig.2] - Vector channel: the behaviour of D_1 (eq. (3.5)) versus ξ. The arrow indicates the stationary value which yields the right value of m_1.

[Fig.3] - Vector channel: the full line represents the relation between m_c^0 and s_0^v which derives from imposing that $m_1 = m_{J/\psi}$. The dashed line derives from imposing that $m_2 = m_{\psi'}$.

[Fig.4] - Vector channel: behaviour of D_2 (eq. (3.5)) versus ξ. The arrow indicates the stationary value which yields the right value of m_2.

[Fig.5] - Vector channel: the difference between the two "threshold" invariant energies squared at varying ξ, for five different values of n.

[Fig.6] - Vector channel: behaviour of m_2 (full line) and of D_2 (dashed line, eq. (3.5)) versus Γ_1. The oblique flex which occurs in correspondence of the vertical dashed line yields the correct experimental value of Γ_1 and, therefore, of m_2.

[Fig.7] - Pseudoscalar channel: same as fig. 2.

[Fig.8] - Pseudoscalar channel: same as fig. 4.

[Fig.9] - Pseudoscalar channel: same as fig. 6; the arrow indicates the oblique flex which corresponds to our predictions on $m_{\eta_c'}$ and on Γ_{η_c}.

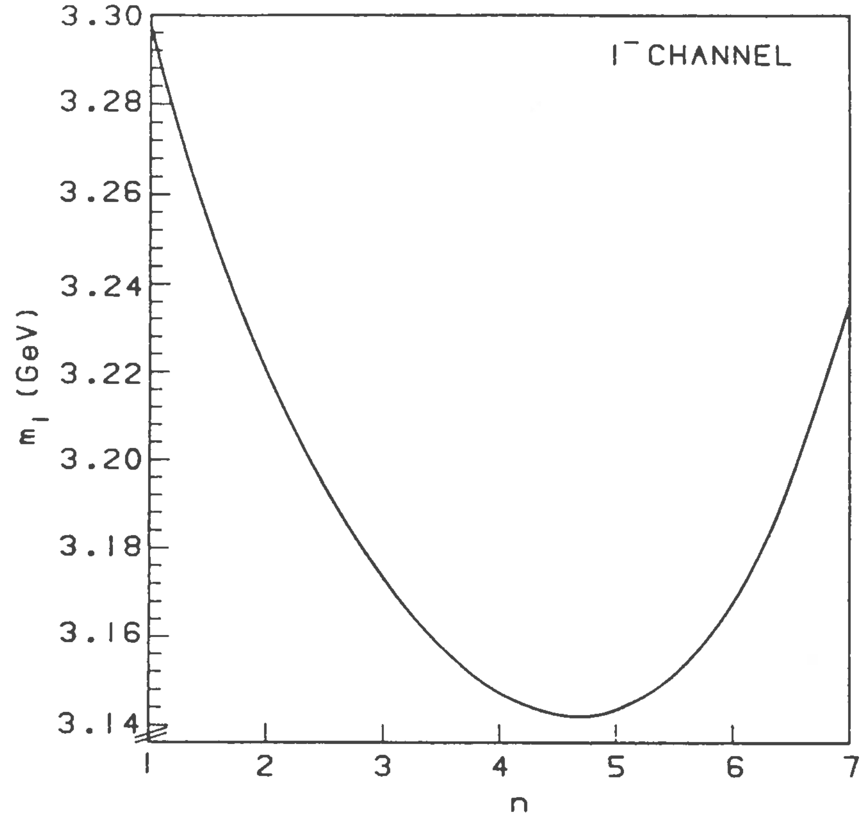

1⁻ CHANNEL
3.30
3.28
3.26
3.24
3.22
3.20
3.18
3.16
3.14
m_1 (GeV)
1
2
3
4
5
6
7
n

FIG.1

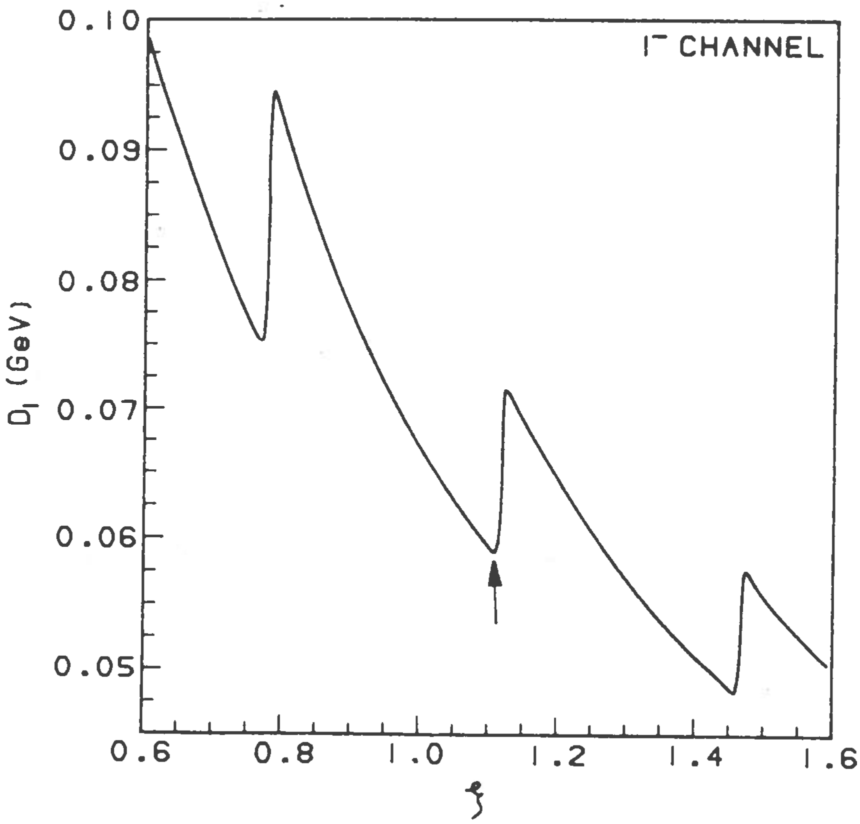

1⁻ CHANNEL
0.10
0.09
0.08
0.07
0.06
0.05
D_1 (GeV)
0.6
0.8
1.0
1.2
1.4
1.6
ξ

FIG.2

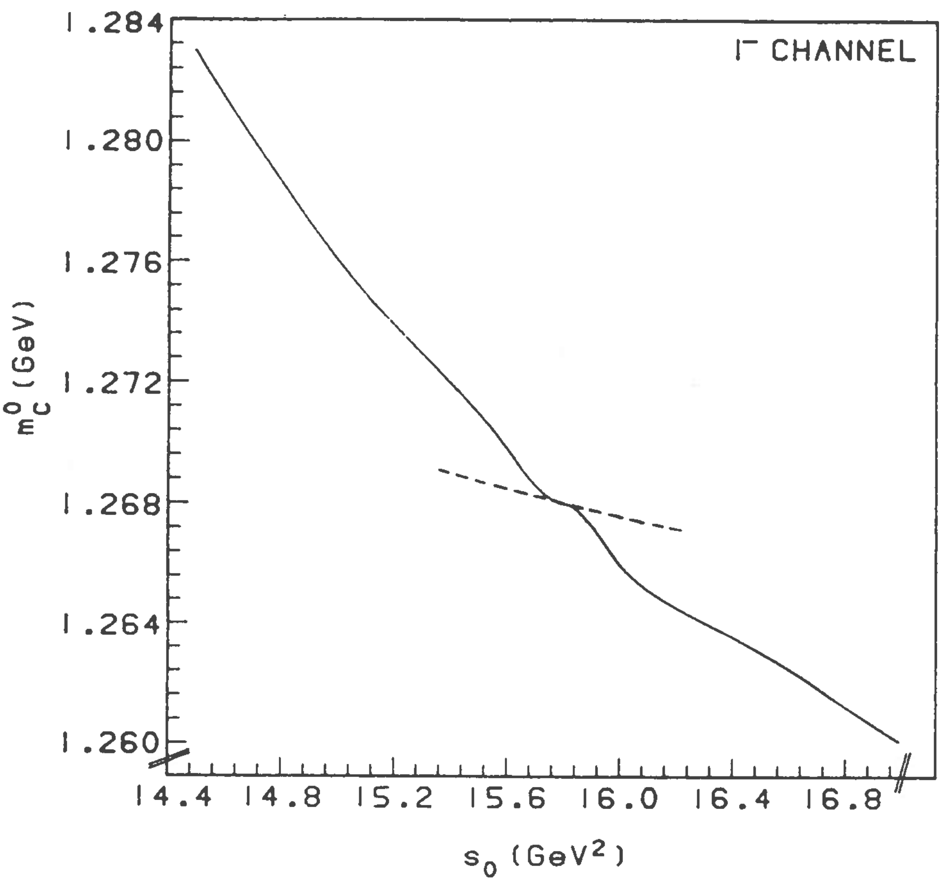

1.284
1.280
1.276
1.272
1.268
1.264
1.260
1^- CHANNEL
m_c^0 (GeV)
14.4
14.8
15.2
15.6
16.0
16.4
16.8
s_0 (GeV^2)

FIG.3

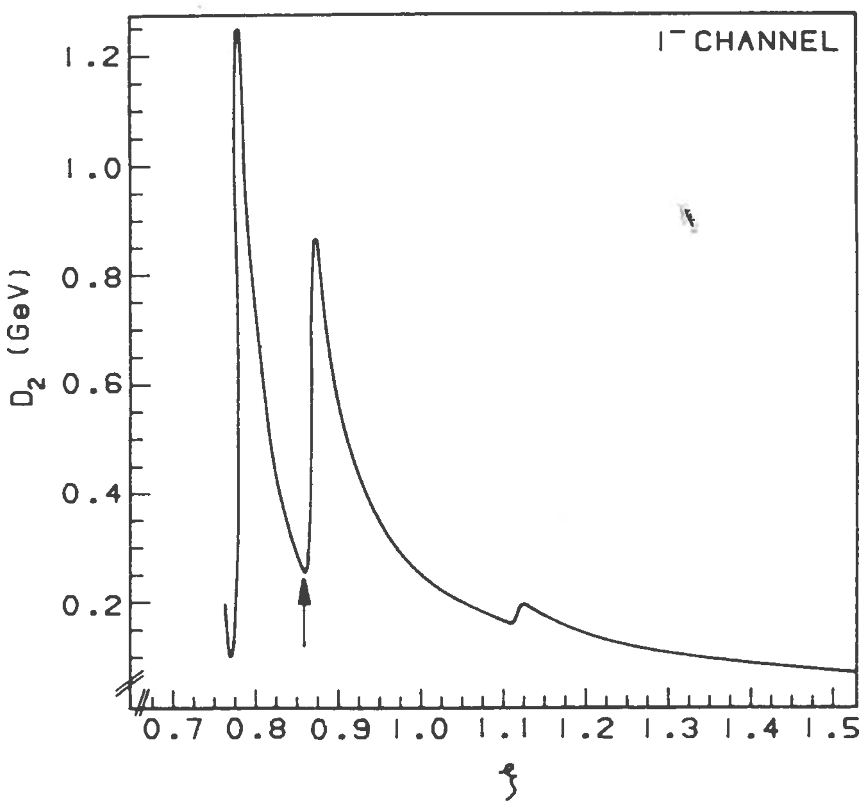

1.2
1.0
0.8
0.6
0.4
0.2
1^- CHANNEL
D_2 (GeV)
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
ξ

FIG.4

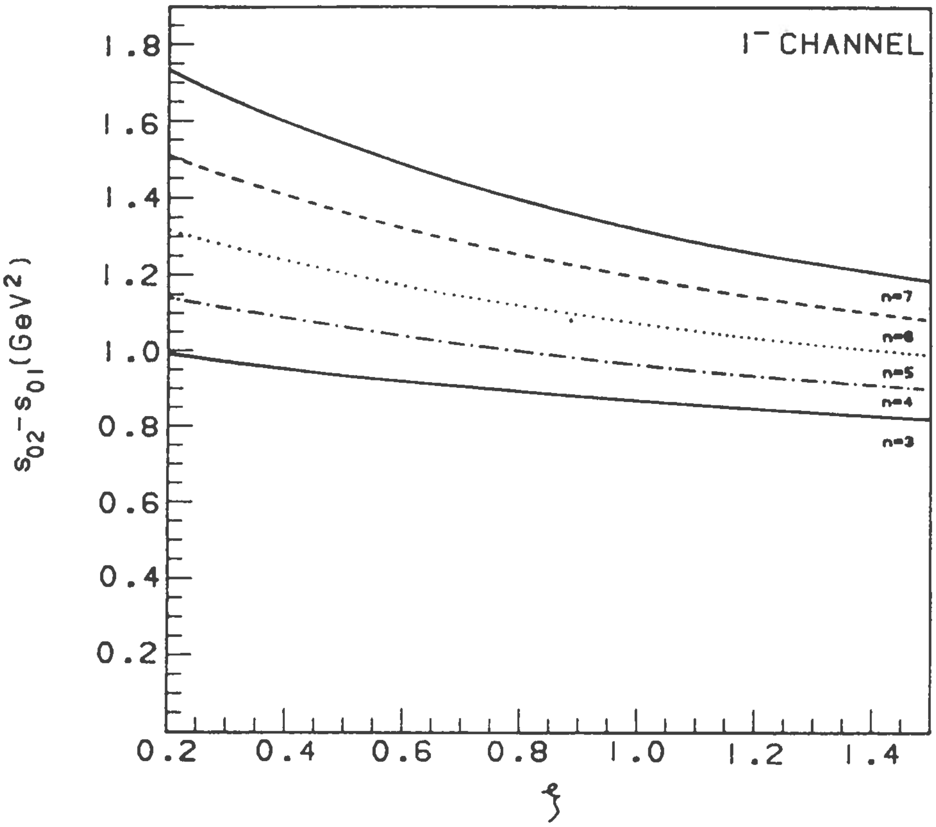

1⁻ CHANNEL
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
s02−s01(GeV²)
n=7
n=6
n=5
n=4
n=3
0.2
0.4
0.6
0.8
1.0
1.2
1.4
ξ

FIG.5

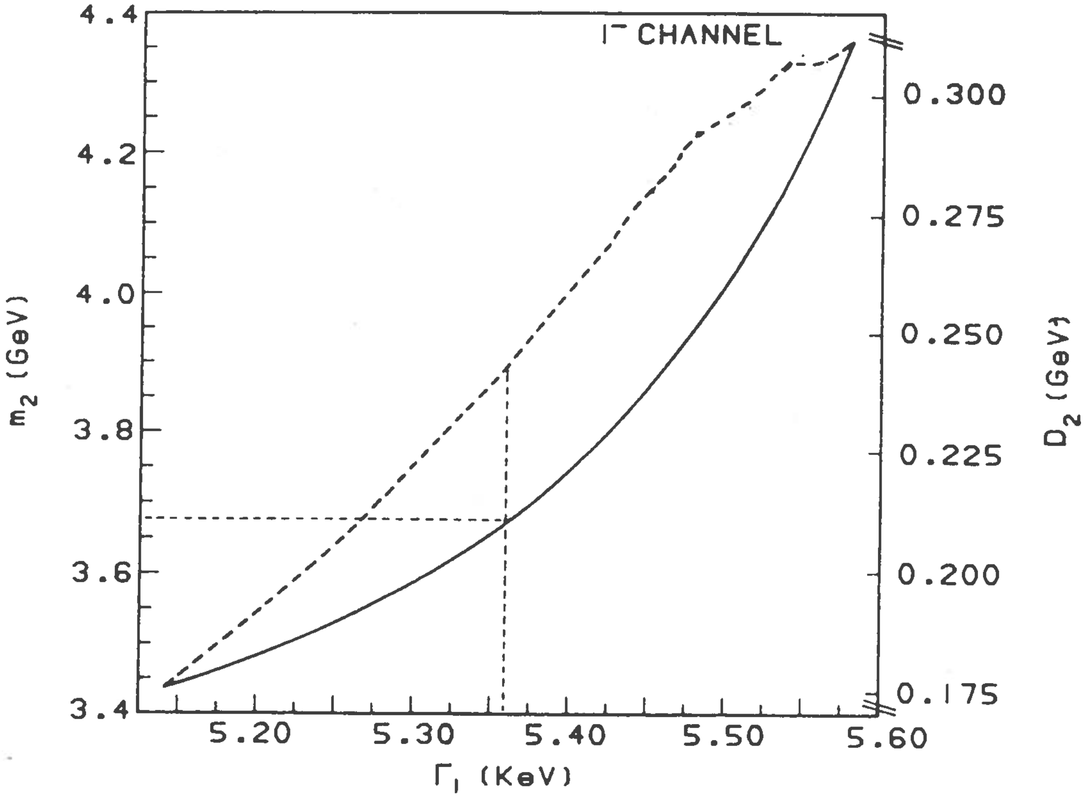

1⁻ CHANNEL
4.4
4.2
4.0
3.8
3.6
3.4
m2 (GeV)
0.300
0.275
0.250
0.225
0.200
0.175
D2 (GeV)
5.20
5.30
5.40
5.50
5.60
Γ1 (KeV)

FIG.6

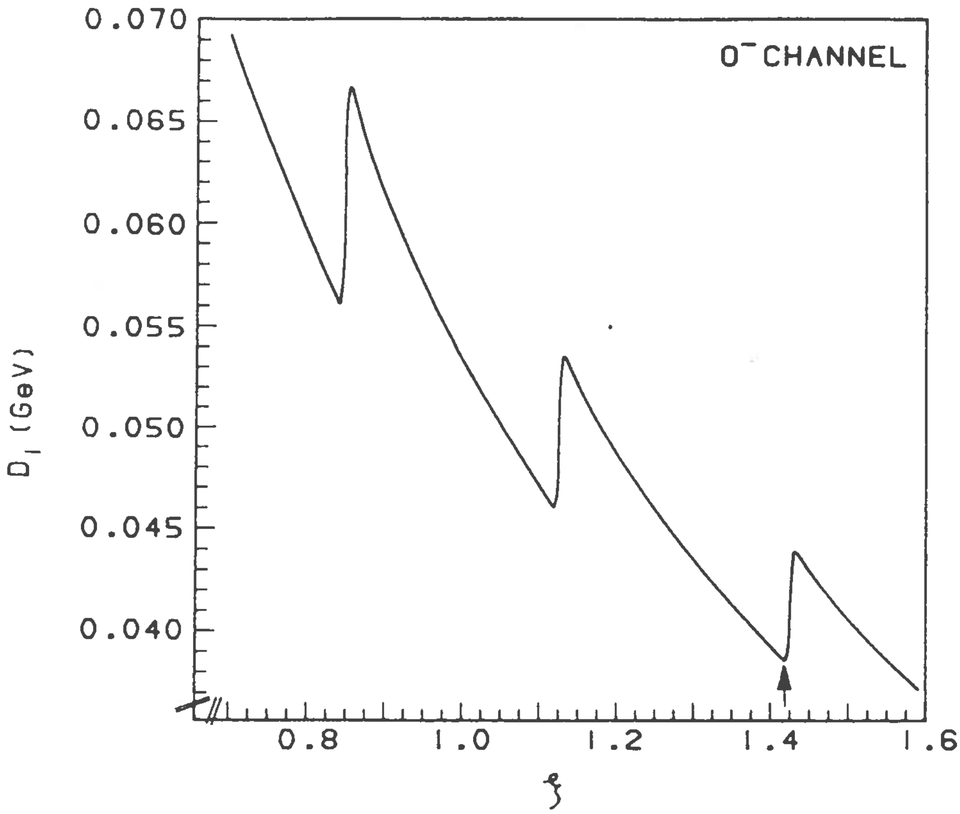

0.070
0.065
0.060
0.055
0.050
0.045
0.040
D_1 (GeV)
0⁻ CHANNEL
0.8
1.0
1.2
1.4
1.6
ξ

FIG.7

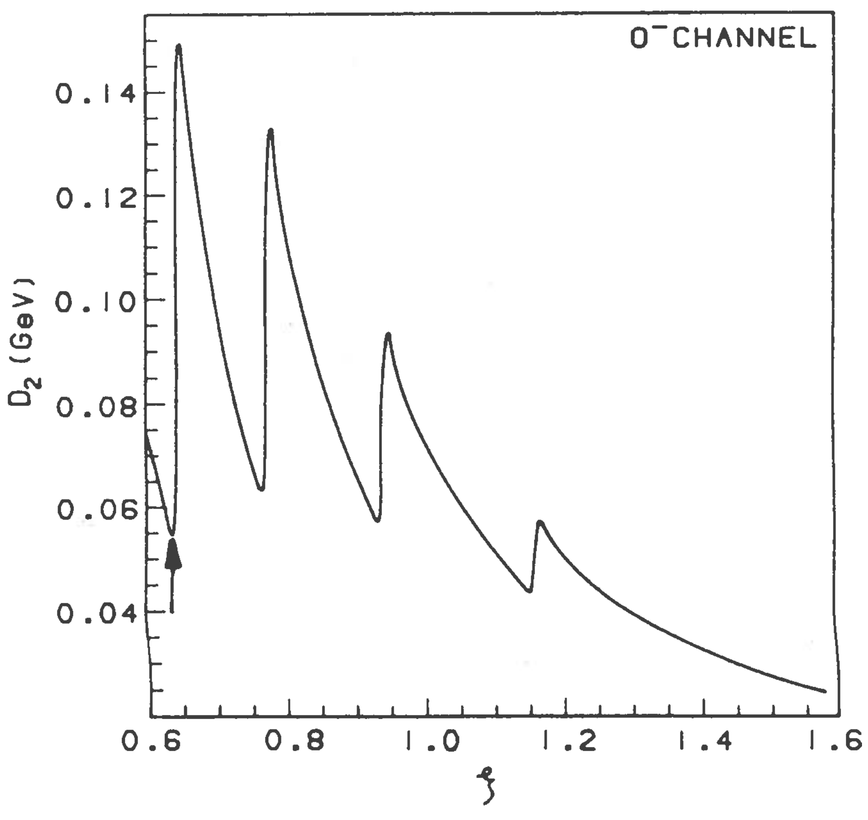
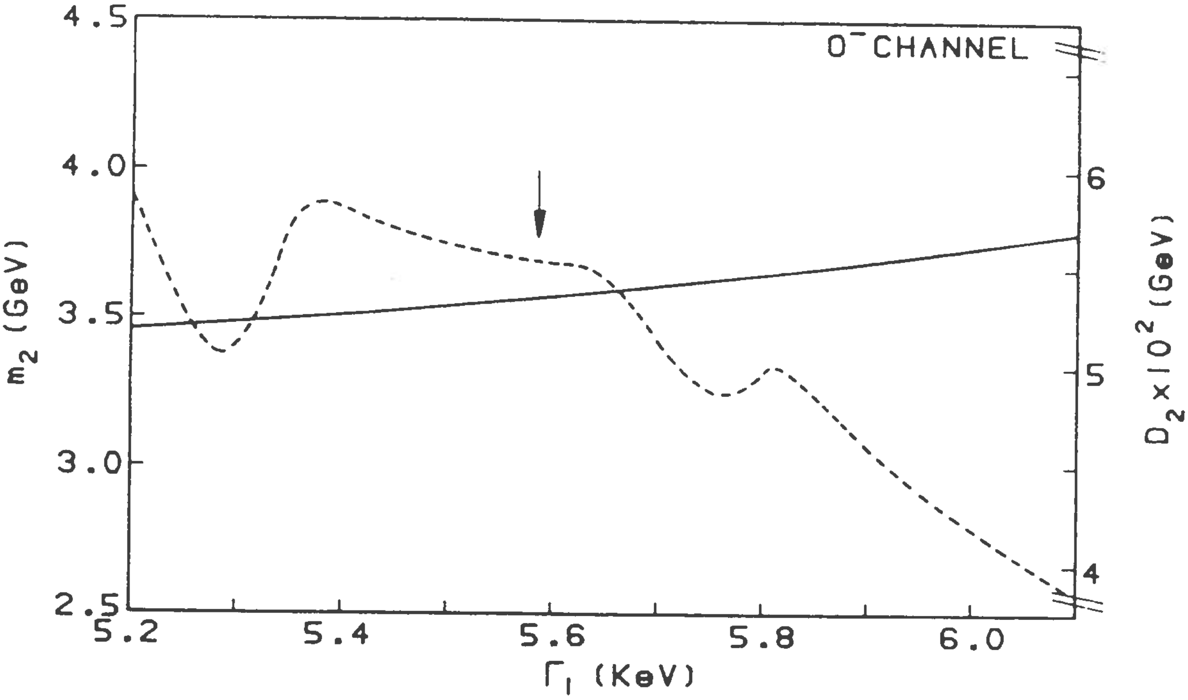

0.14
0.12
0.10
0.08
0.06
0.04
D_2 (GeV)
0⁻ CHANNEL
0.6
0.8
1.0
1.2
1.4
1.6
ξ

FIG.8



4.5
4.0
3.5
3.0
2.5
m_2 (GeV)
0⁻CHANNEL
6
5
4
$D_2 \times 10^2$ (GeV)
5.2
5.4
5.6
5.8
6.0
Γ_1 (KeV)

FIG.9