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$K_{S,L}K_{L,S} \rightarrow 3\pi, \pi l \nu$ interferences at ϕ-factories

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Abstract

We study the $K_{S,L}K_{L,S} \rightarrow 3\pi, \pi l \nu$ time dependent interferences at a ϕ-factory. We find that, with $10^{10} - 10^{11}$ ϕ's as obtainable with a luminosity $\mathcal{L} = 10^{33} cm^{-2} s^{-1}$, it should be possible to measure the CP conserving amplitude of the decay $K_S \rightarrow \pi^+\pi^-\pi^0$ from the observation of such interferences. Furthermore, we point out that also the final state interaction phases of $K \rightarrow 3\pi$ could be reached experimentally in this way.

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For our understanding of low energy physics it is very interesting to study the $\Delta I = 3/2$ transitions in $K \to 3\pi$. The decay $K_S \to \pi^+\pi^-\pi^0$ is a pure $\Delta I = 3/2$ transition, and therefore its detection would be important in this regard. Since it is inhibited also by an angular momentum barrier, its branching ratio is expected to be rather small, $BR(K_S \to \pi^+\pi^-\pi^0) = (2-4) \times 10^{-7}$ [1,2,3,4], so that it has not been detected yet.

In what follows we investigate the possibility of measuring the $K_S \to \pi^+\pi^-\pi^0$ amplitude *via* its interference with $K_L \to \pi^+\pi^-\pi^0$ at a ϕ-factory [5,6]. This would represent a determination alternative to the direct measurement of the width. An advantage of this method is that also the final state interaction phases of $K \to 3\pi$ should be accessible. These phases are qualitatively expected to be small, of the order of $\delta \sim 0.1$ or so, due to the smallness of phase space. Nevertheless, they bring important information on the chiral structure of meson-meson interactions and, even more importantly, they determine the size of direct CP violation asymmetries in $K \to 3\pi$ [7,8,9], so that their experimental determination would be welcome. While in the width of $K \to 3\pi$ these phases appear quadratically (i.e. as $\cos\delta \sim 1 - \frac{\delta^2}{2}$) and therefore are quite difficult to be observed, in the time dependent interference mentioned above they appear linearly, in the $K_S - K_L$ mass oscillation factor $\sim \sin(\Delta m \cdot t + \delta)$. The latter could possibly be reconstructed from the data, leading to a direct determination of δ.

As observed by the authors of Ref.[10], statistics available at ϕ-factories might be not enough to measure CP violation in $K_{S,L} \to 3\pi$ through time-dependent asymmetries of the kind discussed in [11,12,13] for $K \to 2\pi$. We point out, however, that the CP conserving quantities $K_S \to \pi^+\pi^-\pi^0$ amplitude and the strong phases could be measurable from such time-dependent interferences, with a number of ϕ's between 10^{10} and 10^{11} as expected from a phi-factory with luminosity $\mathcal{L} = 10^{33} cm^{-2} s^{-1}$. Furthermore, compared to [10], we stress the crucial role of the explicit Dalitz distributions in computing rates and in making appropriate kinematical cuts to observe these interferences. This last observation has been shown to be very important on similar grounds also at CP-LEAR [14,15].

To develop these ideas for the ϕ-factory, we just outline the basic ideas underlying the results of [11]. Due to conservation of C in strong and electromagnetic interactions, at a ϕ-factory the $K^0\ \bar{K^0}$ pairs from $\phi \to K^0 \bar{K^0}$ are in a pure state with $J^{PC}(\phi) = 1^{--}$.

Thus, the initial $K\bar{K}$ state immediately after ϕ decay is represented in general by the following combination of K_S and K_L (we assume CPT conservation):

$$|i>\equiv|K^0\bar{K^0}(C=odd)>=\frac{|K_L(\hat{z})K_S(-\hat{z})>-|K_S(\hat{z})K_L(-\hat{z})>}{2\sqrt{2}pq}, \quad (1)$$

where

$$p=\frac{1+\bar{\epsilon}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}; \qquad q=\frac{1-\bar{\epsilon}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}. \quad (2)$$

In (1) $\hat{z}$ is the direction of the momenta of the kaons in the c.m. system, while in (2) $\bar{\epsilon}$ is the CP violating K^0-$\bar{K^0}$ mass mixing. In this situation the subsequent K_S and K_L decays are correlated, and their quantum interferences show up in relative time distributions and time asymmetries.

Specifically, we consider the transition amplitude for the initial state decay into the final states f_1 and f_2 at times t_1 and t_2 respectively:

$$<f_1(t_1,\hat{z}),f_2(t_2,-\hat{z})|i>= \frac{<f_1(t_1)|K_L><f_2(t_2)|K_S>-<f_1(t_1)|K_S><f_2(t_2)|K_L>}{2\sqrt{2}pq}. \quad (3)$$

Clearly, time dependent interferences between $<f_1(t_1)|K_L><f_2(t_2)|K_S>$ and $<f_1(t_1)|K_S><f_2(t_2)|K_L>$ can be observed. To maximize the effect of such interferences for $K_{S,L}\to 3\pi$, we choose $f_1=\pi^{\pm}l^{\mp}\nu$ and $f_2=\pi^+\pi^-\pi^0$, as also considered in [10]. Actually, to simplify the notation, in what follows we will denote f_2 simply by $f_2=3\pi$.

The time evolution of the initial quantum state state $|i>$ can be easily written down in terms of the exponential time dependences of the mass eigenstates K_S and K_L:

$$|K_{S,L}(t)>=e^{-i\lambda_{S,L}t}|K_{S,L}(0)>, \quad (4)$$

with

$$\lambda_{S,L}\equiv m_{S,L}-\frac{i}{2}\gamma_{S,L}. \quad (5)$$

Defining

$$t=t_1+t_2; \qquad \Delta t=t_2-t_1, \quad (6)$$

and the "intensity" $I(\Delta t)$:

$$I(\Delta t)\equiv\frac{1}{2}\int_{|\Delta t|}^{\infty}dt\,|<f_1(t_1,\hat{z}),f_2(t_2,-\hat{z})|i>|^2, \quad (7)$$

one finds for f_1 and f_2 chosen above, and neglecting CP violation which is not of interest here (so that $p = q = 1/\sqrt{2}$):

$$I(\pi^{\pm}l^{\mp}\nu, 3\pi; \Delta t < 0) = \frac{|<\pi^{\pm}l^{\mp}\nu|\begin{pmatrix}\bar{K^0}\\ K^0\end{pmatrix}>|^2}{8\gamma}\cdot\{|<3\pi|K_L>|^2 e^{-\gamma_S|\Delta t|}$$

$$+|<3\pi|K_S>|^2 e^{-\gamma_L|\Delta t|} \pm 2e^{-\gamma|\Delta t|}[Re(<3\pi|K_L>^*<3\pi|K_S>)\cos(\Delta m|\Delta t|)$$

$$+Im(<3\pi|K_L>^*<3\pi|K_S>)\sin(\Delta m|\Delta t|)]\} \qquad (8)$$

and

$$I(\pi^{\pm}l^{\mp}\nu, 3\pi; \Delta t > 0) = \frac{|<\pi^{\pm}l^{\mp}\nu|\begin{pmatrix}\bar{K^0}\\ K^0\end{pmatrix}>|^2}{8\gamma}\cdot\{|<3\pi|K_L>|^2 e^{-\gamma_L\Delta t}$$

$$+|<3\pi|K_S>|^2 e^{-\gamma_S\Delta t} \pm 2e^{-\gamma\Delta t}[Re(<3\pi|K_L>^*<3\pi|K_S>)\cos(\Delta m\Delta t)+$$

$$-Im(<3\pi|K_L>^*<3\pi|K_S>)\sin(\Delta m\Delta t)]\}. \qquad (9)$$

Here $\gamma = \frac{(\gamma_L + \gamma_S)}{2}$; $\Delta m = m_L - m_S$; and the dependences of the $K \to 3\pi$ amplitudes on pion momenta are implicit.

The isospin decomposition of $K_L \to \pi^+\pi^-\pi^0$ and $K_S \to \pi^+\pi^-\pi^0$ decay amplitudes, up to linear terms in pion momenta, can be written as follows [16,17,18]:

$$A(K_L \to \pi^+\pi^-\pi^0) = (\alpha_1 + \alpha_3)\, e^{i\delta_{1S}} - (\beta_1 + \beta_3)\, e^{i\delta_{1M}} Y, \qquad (10)$$

$$A(K_S \to \pi^+\pi^-\pi^0) = \frac{2}{3}\sqrt{3}\gamma_3 X e^{i\delta_2}. \qquad (11)$$

In (10), (11) Y and X are the Dalitz plot variables $Y = \frac{s_3 - s_0}{m_\pi^2}$ and $X = \frac{s_2 - s_1}{m_\pi^2}$, where $s_i = (p - p_i)^2$ with p and p_i ($i = 1, 2, 3$) the four-momenta of the kaon and of the pions respectively. In this notation $i = 3$ indicates che "odd charge" pion, i.e. the π^0 in our case, and $s_0 = \frac{1}{3}(s_1 + s_2 + s_3)$. The amplitudes α, β and γ correspond to the three possible (3π) isospin final states: $I = 1$ symmetric, $I = 1$ with mixed symmetry and $I = 2$. The subscripts $1, 3$ on α, β and γ refer to the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions respectively. The phases δ_{1S}, δ_{1M} and δ_2 account for final state strong interactions, and are needed in principle in order to fulfill unitarity. Finally, α, β and γ are real numbers if CP is conserved.

Using (10) and (11), we can replace in the right sides of (8) and (9) the following expansions (we denote $\alpha = \alpha_1 + \alpha_3$ and $\beta = \beta_1 + \beta_3$):

$$| < 3\pi|K_L > |^2 = \alpha^2 - 2\alpha\beta Y \cos(\delta_{1M} - \delta_{1S}) + \beta^2 Y^2 \tag{12}$$

$$| < 3\pi|K_S > |^2 = \frac{4}{3}\gamma_3^2 X^2 \tag{13}$$

$$Re(< 3\pi|K_L >^* < 3\pi|K_S >) = \frac{2}{\sqrt{3}}\gamma_3 X \left[\alpha \cos(\delta_2 - \delta_{1S}) - \beta Y \cos(\delta_2 - \delta_{1M})\right] \tag{14}$$

$$Im(< 3\pi|K_L >^* < 3\pi|K_S >) = \frac{2}{\sqrt{3}}\gamma_3 X \left[\alpha \sin(\delta_2 - \delta_{1S}) - \beta Y \sin(\delta_2 - \delta_{1M})\right]. \tag{15}$$

In practice, in the above equations one could simplify $\cos\delta \sim 1$ and $\sin\delta \sim \delta$ due to the expected smallness of strong interaction phases. Furthermore, for practical purposes one can consistently neglect the momentum dependence of those phases and fix them at e.g. their values at the centre of the Dalitz plot.

From Eqs.(8) and (9) one can notice that in the intensity of events for $\Delta t < 0$ the $| < 3\pi|K_S > |^2$ term is enhanced by $\gamma_L \ll \gamma_S$ with respect to $| < 3\pi|K_L > |^2$, a situation which is complementary to that of $K \to 2\pi$ [10]. Thus, for the total number of events, obtained by integrating (8) in $|\Delta t|$ and over the full $K \to 3\pi$ Dalitz plot (so that the interference terms do not contribute) we have:

$$N(\pi^\pm l^\mp \nu, 3\pi; \Delta t < 0) = N_{\phi \to K^0\bar{K^0}} \frac{1}{2} BR(K_L \to \pi l \nu)$$
$$\times \left[\left(\frac{\gamma_L}{\gamma_S}\right)^2 BR(K_L \to 3\pi) + BR(K_S \to 3\pi)\right] \times \Omega, \tag{16}$$

where, with the assumed luminosity, $N_{\phi \to K^0\bar{K^0}} \simeq 1.5 \times 10^{10}/year$. In (16) $\Omega < 1$ is a factor representing the experimental acceptance. For the predicted values of the $K_S \to 3\pi$ width the two terms in (16) are indeed comparable, and lead to about $3 \times 10^3 \times \Omega$ events/year.

The other important point is that the $K_S \to \pi^+\pi^-\pi^0$ amplitude γ_3 and the strong relative phases $\delta_2 - \delta_{1S}$ and $\delta_2 - \delta_{1M}$ appear linearly in the intensities of events through the interference terms (14) and (15), and are there multiplied by well defined, explicit time dependent coefficients. Thus in particular, by reconstructing the $\sin(\Delta m \Delta t)$ interference pattern one should have experimental access to the $K \to 3\pi$ strong phases.

To extract the interference terms we can define "weighted" integrals of the intensities (8) and (9) over the $K \to 3\pi$ Dalitz plot, with suitable cuts [15]. To this purpose it is useful to introduce polar variables r and ϕ, centered at the symmetric point of the Dalitz plot, such that [19]:

$$X = \frac{2m_K Q r \sin\phi}{\sqrt{3}m_\pi^2}; \qquad Y = -\frac{2m_K Q r \cos\phi}{3m_\pi^2} \tag{17}$$

where Q is the Q-value ($Q_{+-0} = 83.6\ MeV$). The $K \to 3\pi$ width can be expressed as:

$$\Gamma(K \to 3\pi) \equiv \frac{1}{(4\pi)^3 m_K}\frac{\sqrt{3}}{18}Q^2 \int\int r\ dr\ d\phi\ |A(r,\phi)|^2. \tag{18}$$

For our estimates the integration domain can be safely taken as the circle of unit radius (non-relativistic limit), so that (18) can be simplified to:

$$\Gamma(K_L \to \pi^+\pi^-\pi^0) = \frac{1}{(4\pi)^3 m_K}\frac{\sqrt{3}}{18}Q_{+-0}^2 \pi |\alpha_{exp}|^2 \tag{19}$$

with $|\alpha_{exp}| = 8.5 \times 10^{-7}$, and all integrals over the Dalitz plot needed in the following become trivial.

Specifically, to select $A(K_S \to \pi^+\pi^-\pi^0)$ and $\delta_2 - \delta_{1S}$ we can make a cut in X, by defining:

$$\Sigma_X(\Delta t) = \frac{1}{(4\pi)^3 m_K}\frac{\sqrt{3}}{18}Q_{+-0}^2 \int\int r\ dr\ d\phi\ \epsilon(X)\ I(\pi^\pm l^\mp \nu, 3\pi; \Delta t). \tag{20}$$

Using Eqs.(8)-(15):

$$\Sigma_X(\Delta t < 0) = \pm\frac{\gamma_L^2}{4\gamma}BR(K_L \to \pi l \nu)BR(K_L \to \pi^+\pi^-\pi^0)(\frac{2m_K Q_{+-0}}{\sqrt{3}m_\pi^2})\times$$
$$\frac{16\gamma_3}{3\sqrt{3}|\alpha_{exp}|\pi}e^{-\gamma|\Delta t|}[\cos(\Delta m|\Delta t|) + (\delta_2 - \delta_{1S})\sin(\Delta m|\Delta t|)], \tag{21}$$

and

$$\Sigma_X(\Delta t > 0) = \pm\frac{\gamma_L^2}{4\gamma}BR(K_L \to \pi l \nu)BR(K_L \to \pi^+\pi^-\pi^0)(\frac{2m_K Q_{+-0}}{\sqrt{3}m_\pi^2})\times$$
$$\frac{16\gamma_3}{3\sqrt{3}|\alpha_{exp}|\pi}e^{-\gamma\Delta t}[\cos(\Delta m \Delta t) - (\delta_2 - \delta_{1S})\sin(\Delta m \Delta t)]. \tag{22}$$

The part of the interference linear in $\delta_2 - \delta_{1M}$ can be determined by a cut in XY, defined as:

$$\Sigma_{XY}(\Delta t) = \frac{1}{(4\pi)^3 m_K}\frac{\sqrt{3}}{18}Q_{+-0}^2 \int\int r\ dr\ d\phi\ \epsilon(XY)\ I(\pi^\pm l^\mp \nu, 3\pi; \Delta t). \tag{23}$$

Explicitly:

$$\Sigma_{XY}(\Delta t < 0) = \pm \frac{\gamma_L^2}{4\gamma} BR(K_L \to \pi l \nu) BR(K_L \to \pi^+\pi^-\pi^0)\left(-\frac{4m_K^2 Q_{+-0}^2}{3\sqrt{3}m_\pi^4}\right)\times$$
$$\frac{2\beta\gamma_3}{\sqrt{3}|\alpha_{exp}|^2\pi} e^{-\gamma|\Delta t|}[\cos(\Delta m|\Delta t|) + (\delta_2 - \delta_{1M})\sin(\Delta m|\Delta t|)], \quad (24)$$

and

$$\Sigma_{XY}(\Delta t > 0) = \pm \frac{\gamma_L^2}{4\gamma} BR(K_L \to \pi l \nu) BR(K_L \to \pi^+\pi^-\pi^0)\left(-\frac{4m_K^2 Q_{+-0}^2}{3\sqrt{3}m_\pi^4}\right)\times$$
$$\frac{2\beta\gamma_3}{\sqrt{3}|\alpha_{exp}|^2\pi} e^{-\gamma\Delta t}[\cos(\Delta m \Delta t) - (\delta_2 - \delta_{1M})\sin(\Delta m \Delta t)]. \quad (25)$$

To assess the expected number of events at the ϕ-factory we integrate the above equations in $|\Delta t|$, and take the numerical values $\beta = -2.8 \times 10^{-7}$; $\gamma_3 = 2.3 \times 10^{-8}$ from the fit to $K \to 3\pi$ data of Ref.[3]. For the final state interaction phases at the centre of the Dalitz plot we adopt the value:

$$\delta_{1S} - \delta_{1M} = \frac{\delta_{1S} - \delta_2}{2} \simeq .07, \quad (26)$$

as predicted by model calculations using either chiral loops [8] or the non relativistic approximation [20]. From (21) and (22), γ_3 is determined by the combination

$$[N(\pi^+l^-\nu, 3\pi; \Delta t < 0) + N(\pi^+l^-\nu, 3\pi; \Delta t > 0)] - [(\pi^+l^-\nu) \to (\pi^-l^+\nu)], \quad (27)$$

while $\delta_2 - \delta_{1S}$ is determined by

$$[N(\pi^+l^-\nu, 3\pi; \Delta t < 0) - N(\pi^+l^-\nu, 3\pi; \Delta t > 0)] - [(\pi^+l^-\nu) \to (\pi^-l^+\nu)]. \quad (28)$$

Using the input values mentioned above, we would find about $490 \times \Omega$ and $70 \times \Omega$ events/year available to the determination of γ_3 and $\delta_2 - \delta_{1S}$ respectively, where Ω is the fiducial volume factor introduced in (16). Analogously, Eqs.(24) and (25) would give about $7 \times \Omega$ events/year for the determination of $\delta_2 - \delta_{1M}$.

These results indicate that the possibility to determine experimentally the CP conserving $K_S \to \pi^+\pi^-\pi^0$ amplitude at a ϕ-factory through the $K_L - K_S$ interference should be considered with some attention. Particularly appealing is the sensitivity of this method to the $K \to 3\pi$ final state interaction phases, which seem to be measurable for Ω not so far from unity (although, from the calculated number of events, probably only an upper limit could be derived for $\delta_2 - \delta_{1M}$). As anticipated, such a possibility directly relates to the explicit momentum dependence of Dalitz plot distributions, leading to Eqs.(21)-(25). Our discussion thus positively complements Ref.[10], which was limited to the consideration of CP violating $K \to 3\pi$.

Furthermore, we remark that one chooses $f_1 = \pi^{\pm} l^{\mp} \nu$ with respect to other channels as a convenient mode for tagging.

Clearly, the considerations above should be substantiated by further studies, taking into account experimental efficiencies, which here were taken equal to one. One manifest difficulty is due to the factor $e^{-\gamma|\Delta t|}$ in the interference term, which requires measurements at extremely short times. Naively then, the situation seems similar to that of the measurement of $Im\frac{\epsilon'}{\epsilon}$ in $K \to 2\pi$ [21]. The important difference, however, is that the measurement proposed here (if feasible) has the nice feature of being free from the background decay $\phi \to K^0 \bar{K}^0 \gamma$, leading to the $C = even$ state:

$$|K^0 \bar{K}^0 (C = even) >= \frac{|K_S(\hat{z}) K_S(-\hat{z}) > + |K_L(\hat{z}) K_L(-\hat{z}) >}{2\sqrt{2pq}}. \tag{29}$$

In fact, besides the overall suppression due to the small branching ratio of the originating process $\phi \to K^0 \bar{K}^0 \gamma$ [22,23], the contribution of the $|K_S K_S >$ state is further suppressed by tiny branching ratios, while that of the $|K_L K_L >$ state vanishes by the kinematical cuts in Eqs.(21)-(25).

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