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HADRONIC MASS SPECTRA IN A UNIFIED APPROACH TO STRONG AND GRAVITATIONAL INTERACTIONS[1]

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ABSTRACT: The families of narrow resonances of the J/Ψ and Υ spectroscopies are commonly assumed to be bound states of *heavy* quark–antiquark pairs (and these heavy "quarkonia" play in a sense the role of basic 'atoms' of strong interaction physics). The problem of quantitative spectroscopy is, however, handicapped by the lack of a correct inter–quark potential derivable from fundamental principles. We have recently put forward a unified theoretical approach to strong and gravitational interactions based on the idea that our (gravitational) *cosmos* and (strongly interacting) *hadrons* —both considered as finite objects— can be systems internally governed by *similar* laws, differing only for the scale factor ρ which can carry the gravitational into the strong field. Within this framework, a quark–quark(antiquark) potential has been derived, which is used in this paper to predict the energy levels of the ground state and first few excited states of the Charmonium (J/Ψ) and Bottomonium (Υ) spectroscopies.

Using computer search for optimum parameters in the model, we compare the theoretical level spectra for the first few radial s states with the experimental values.

We take advantage of the present opportunity, moreover, to make known (both in the text, and in Appendix A) some other, related results obtained by two of us on a purely geometrical ground.

At last we mention possible applications of our theoretical approach to the *fireball* phenomenology (Centauro events, etc.).

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1. – INTRODUCTION

The enormous range of hadronic mass spectra from a few MeV to several GeV and the multiplicity of observed hadrons in strong contrast to that of leptons, as well as the lack of a unique potential source for the dynamics of the constituents —e.g., quarks,— indicate the possibly complex nature of the strong interaction force involved, as compared to that of ordinary matter which is so well explained by quantum electrodynamics (QED). The successful candidate theory of strong interactions is the so-called QCD —quantum chromodynamics,— according to which the hadronic masses correspond *grosso modo* to bound states of quarks and/or anti-quarks, moving under the influence of a potential due to colour forces; and the decay properties of these bound states are described by the usual rules of quantum mechanical transition probabilities. The formalism of QCD allows a set [#1] of coloured charged quarks[1] (Fig. 1), together with eight massless exchange particles called gluons which are themselves colour charged.

This colour force complicates the simple Coulomb–like potential of QED and further gives rise to a running coupling constant in oppositive behaviour to that found in QED. Both these aspects are encapsulated in the two basic characteristics of QCD —namely, confinement and asymptotic freedom. Although great success has been achieved under this framework by using heuristic potential models for heavy "quarkonia" —such as J/Ψ and Υ for $0.1 < r < 1$ fm—, as well as for light quarkonia with relativistic corrections —such as systems composed of the u,d,s quarks and anti-quarks for $r \gtrsim 1$ fm—, so far no knowledge of the effective potential has been obtained from *fundamental* principles and this restricts the credibility of the models when attempts are made to calculate the energies of ground and higher excited states, life-times, branching ratious, etc.

The quark masses assumed vary rather widely with typical mass values of $m_{\rm charm} \simeq 1.5$ GeV/c^2 and $m_{\rm bottom} \simeq 5.0$ GeV/c^2. The potential forms chosen are not unique and the interpolation between short & long distance behaviour involving free parameters is at best ambiguous. Even in the region of distances corresponding to the ground state sizes of charmonia and bottomonia, all common potentials have the same radial dependence, thus making it difficult to choose between them.

We have recently proposed a fundamental geometric approach[2] involving the unification of gravitational and strong interactions[3] that allows one, in a natural way, to incorporate the non-abelian gauge characteristics of QCD —confinement at large distances as well as asymptotic freedom at short distances— within the framework of a theoretical model.[4] We have made preliminary investigation into the mass-spectra of heavy quarkonia such as J/Ψ and Υ with reasonable sucess.[5] We refer here to a more detailed calculation

[#1] By considering quarks to be the real carriers of the strong charge (cf. Fig. 1), we can call "colour" the sign s_j of such strong charge. Namely, we can regard hadrons as endowed with a zero total strong charge, each quark possessing the strong charge $g_j = s_j|g_j|$ with $\Sigma s_j = 0$. Therefore, when passing from ordinary gravity to "strong gravity", we shall replace m by $g = n g_o$, quantity g_o being the average *magnitude* of the constituent quark rest–strong-charge, and n their number.

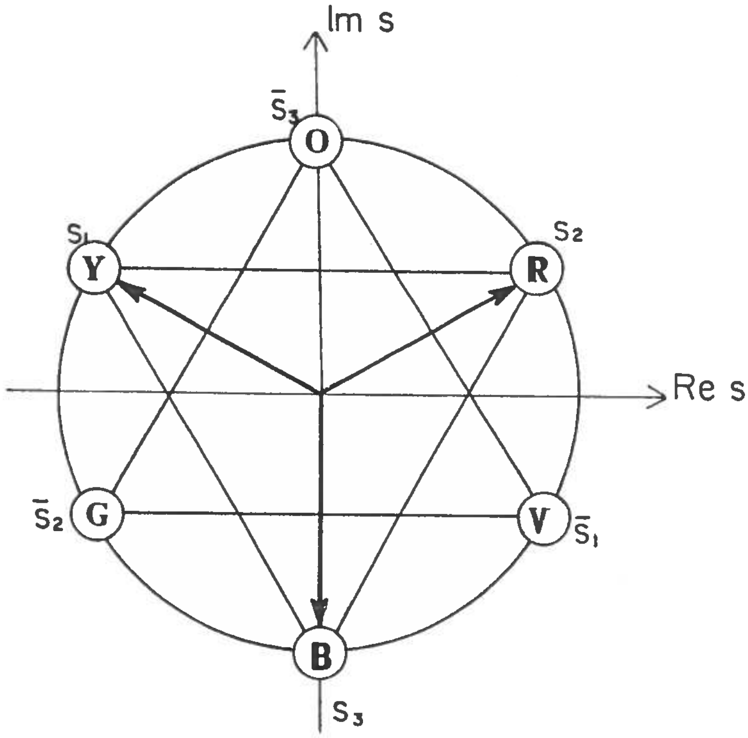

Fig. 1 – *"Coloured" quarks and their strong charge* – This scheme represents the complex plane[1] of the *sign s* of the quark strong-charges g_j in a hadron. These strong charges can have *three* signs, instead of two as in the case of the ordinary electric charge e. They can be represented, e.g., by $s_1 = (i - \sqrt{3})/2$; $s_2 = (i + \sqrt{3})/2$; $s_3 = -i$, which correspond to the arrows angularly separated by 120°. The corresponding anti-quarks will be endowed with strong charges carrying the complex conjugate signs $\bar{s}_1$, $\bar{s}_2$, $\bar{s}_3$. The three quarks are represented by the "yellow" (Y), "red" (R) and "blue" (B) circles; the three anti-quarks by the "violet" (V), "green" (G) and "orange" (O) circles. The latter colours are complementary to the correspondind former ones. Since in the real particles the inter-quark forces are saturated, hadrons are white. The white colour can be obtained either with three–quark structures, by the combinations YRB or VGO (as it happens in baryons and antibaryons, respectively), or with two–quark structures, by the combinations YV or RG or BO [which are actually quark–antiquark combinations], as it happens in mesons and their antiparticles. See also footnote #1.

of these levels using computer search for optimum parameters in the model, whose results are presented below.

2. – THE MODEL: In the Surroundings of a Hadron

Hadron structure and strong interactions are described by using the classical methods of General Relativity (GR) and by assuming (in the manner of Riemann, Clifford, and even Einstein[6]) that the appearance of the matter particles is due to a strong local curvature of space-time. Let us first recall that empirical observation shows[7] that the ratio R/r between the radius ($R \sim 10^{26}$m) of our cosmos and a typical hadron radius ($r \sim 10^{-15}$m) roughly equals the ratio between the strengths of the strong and gravitational field, the ratio ρ being of the order of 10^{41}.

In our geometrical approach, the strong field *surrounding* a hadron is described by a tensorial field $s_{\mu\nu}$ to be "added" to the usual gravitational field $e_{\mu\nu}$. Hadrons are attributed a "strong mass" (or "strong charge", g) which directly *deforms* the space-time in analogy to the gravitational mass (or "gravitational charge", m) but via the 'strong universal constant' $N \equiv \rho G \approx hc/m_\pi^2$, where G is the ordinary gravitational universal constant. Our new field equations

$$R_{\mu\nu} + \lambda s_{\mu\nu} = -\frac{8\pi}{c^4}\,[S_{\mu\nu} - \frac{1}{2}g_{\mu\nu}S^\rho_\rho]\;; \quad [S_{\mu\nu} \equiv NT_{\mu\nu}] \tag{1}$$

reduce to the usual Einstein equations far from the source–hadron, since they *imply* the strong–field to exist only in the hadron neighbourhood: so that (in suitable coordinates) $s_{\mu\nu} \rightarrow \eta_{\mu\nu}$ for $r >> 1$ fm. In eqs.(1) it is $\lambda \equiv \rho^2\Lambda$, quantity Λ being the ordinary (gravitational) cosmological constant. For instance, for $r \gtrsim 1$ fm, when our field equations can be linearized, the total metric $g_{\mu\nu}$ can be simply assumed to be the *sum* of the two metric fields; or, rather, $2g_{\mu\nu} = e_{\mu\nu} + s_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu}$. Quantity $s_{\mu\nu}$ may then be written as $s_{\mu\nu} \equiv \eta_{\mu\nu} + 2h_{\mu\nu}$, with $|h_{\mu\nu}| << 1$; so that, finally, $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$. Eqs.(1), let us repeat, do imply that $h_{\mu\nu} \rightarrow 0$ for $r >> 1$ fm. More precisely, in this case we get, at the static limit, that $V \equiv h_{oo} \equiv (s_{oo} - 1)/2 = g_{oo} - 1$ is just the Yukawa potential[#2]

$$V = -g\frac{\exp[-\sqrt{2|\lambda|}r]}{r} \simeq -\frac{g}{r}\exp[\frac{-m_\pi rc}{\hbar}]\,,$$

with the correct (within a factor 2) coefficient also in the exponential.[2,4]

The source–hadron can be regarded as an axially symmetric distribution of strong charge g: studying the metric around it leads us to deal with a problem of the Kerr-Newmann-deSitter (KNdS) type, and look for "*strong* black-hole"–type solutions. One

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[#2] We have considered $\lambda > 0$; in the case of negative λ (see the following), we ought[5] to have set $s_{\mu\nu} \equiv \eta_{\mu\nu} - 2h_{\mu\nu}$.

finds that hadrons apparently can be associated with such *strong black-holes* (SBH), which result to possess radii $r_S \simeq 1$ fm. For $r \to r_S$, we can adopt the approximation "opposite" to the linear one, putting $g_{\mu\nu} \simeq s_{\mu\nu}$, so that eq.(1) becomes

$$R_{\mu\nu} + \lambda g_{\mu\nu} \simeq -\frac{8\pi}{c^4} \left[S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\rho}_{\rho}\right] . \tag{2}$$

Following recent work by two of us, let us consider, e.g., the case $\lambda > 0$. In general one meets three horizons, i.e. three values of r_S. If we are interested in the hadrons stable w.r.t [with respect to] the strong interaction, we have to impose the SBH Temperature[8,5,4] (i.e., the *surface* field) to be vanishingly small. This requirement implies the coincidence of two, or more, of the strong horizons; and such a condition yields[4,5,9] some "Regge–type" relations among m, λ, N, q and J, where m, q, J are mass, electric charge and intrinsic angular momentum —respectively— of the considered hadron. Namely, once chosen q, J, λ and N, the present approach determines *mass* and *radius* of the corresponding stable hadron. Our theory is therefore a rare example of a formalism that —a priori, at least— allows finding out the stable particle [and even quark] masses. Further, interesting work in this direction has been performed by E.R. and V.T.Z.,[9,5] but it will be reported elsewhere. Here (Appendix A) we shall only mention a few results which refer to the strong coupling–constant.

Let us finally recall that a *preliminary* version of this theory was applied in ref.[10] to the fireball phenomenology (Centauro events, etc.): cf. Appendix B.

3. – THE MODEL: Inside a Hadron

When turning our attention to the *interior* of a hadron, eq.(2) will still hold; which can be rewritten as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\rho}_{\rho} - \lambda g_{\mu\nu} = -\frac{8\pi N}{c^4} T_{\mu\nu} \; ; \quad [N T_{\mu\nu} \equiv S_{\mu\nu}] \tag{3}$$

In other words, inside a hadron the ordinary Einstein equations (with cosmological term) will be valid, provided they are correctly scaled down. Namely, $S_{\mu\nu} \equiv N T_{\mu\nu}$ is now the strong–mass tensor, so as $G T_{\mu\nu}$ was the ordinary–mass tensor; and λ is the "strong cosmological constant" (or "hadronic constant"). The interior of a hadron, therefore, is regarded by us as a *micro-universe*,[4] in which the strong field dominates, instead of the gravitational field. Let us consider eq.(3) in the simple case of a spherically symmetric field created by a strong charge g' (which may for example be identified with a quark); it then admits a well-known Schwarzchild-deSitter–type solution, which in our case can be written as

$$\mathrm{d}s^2 = (1 - \frac{2Ng'}{c^2 r} + \frac{\lambda r^2}{3})\mathrm{d}t^2 - (1 - \frac{2Ng'}{c^2 r} + \frac{\lambda r^2}{3})^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2) \, . \tag{4}$$

Confining ourselves to the radial motion of a test–constituent of strong charge g'' in the field of the source–charge g', described by the metric (4), we get the geodesic equation (in vacuum), in the usual Schwarzshild-deSitter coordinates t, r, as:

$$\frac{\mathrm{d}^2r}{\mathrm{d}s^2} = -\Gamma^1_{\mu\nu}u^\mu u^\nu = -\frac{1}{2}[(1-\frac{2Ng'}{c^2r}+\frac{\lambda r^2}{3})(\frac{2Ng'}{c^2r^2}+\frac{2\lambda r}{3})u^0u^0$$
$$-(1-\frac{2Ng'}{c^2r}+\frac{\lambda r^2}{3})-1(\frac{2Ng'}{c^2r}+\frac{2\lambda r}{3})u^1u^1$$
$$-2r(1-\frac{2Ng'}{c^2r}+\frac{\lambda r^2}{3})u^2u^2+2r sin^2\theta\ (1-\frac{2Ng'}{c^2r}+\frac{\lambda r^2}{3})u^3u^3 \qquad (5)$$

At the static limit $[v << 1]$, eq.(5) writes

$$\frac{\mathrm{d}^2r}{\mathrm{d}t^2} = -\frac{1}{2}c^2(1-\frac{2Ng'}{c^2r^2}+\frac{\lambda r^2}{3})(\frac{2Ng'}{c^2r^2}+\frac{2\lambda r}{3})\hat{r}\ . \qquad (6)$$

Even in General Relativity, with some caution, a language can be introduced in terms of "forces" and "potentials"; for instance by defining#3 $F \equiv g"\mathrm{d}^2r/\mathrm{d}t^2$. For "intermediate distances" —i.e., at the newtonian limit— eq.(6) simplifies, yielding the force $F \simeq -\frac{1}{2}c^2g"(2Ng'/c^2r^2 + 2\lambda r/3)$, which is merely the sum of a newtonian and an elastic (*à la* Hooke) term. In this limit, incidentally, the last expression holds even when the test–constituent $g"$ is *not* endowed with a small strong mass, but is on the contrary a second quark. In general, our equations have an *approximate* validity when $g"$ too is a quark; nevertheless, they correctly describe some important characteristics of the hadron constituent behaviour, for both small and large values of r.

For large values of r $[r \gtrsim 1\ \mathrm{fm}]$, when referring to the simplest hadrons, one obtains a (*confining*) radial force proportional to r:

$$F \approx -g''c^2\lambda r/3 \propto -r\ ; \qquad (7)$$

so that the (confining) potential results to be $V \propto r^2$. This is reminiscent of a harmonic oscillator potential, which is well known to bind the constituents of a system to prescribed boundaries so that they are never "ionized", possibly explaining the so-called *confinement*: the non-observance of such constituentes —i.e., quarks and gluons— in experimental searches. Moreover, since the motion of $g"$ can be approximately regarded as harmonic, our approach can easily incorporate the interesting results got by various, different authors (e.g., in connection with the hadron mass spectra) just by *postulating* such kind of motion. Let us mention that we can have confinement even with negative λ;

#3 Notice that, in order to geometrize the strong field, we had to generalize the Mach principle: cf. refs.[2,4]

in fact, with less drastic approximations, we get $F \approx -\frac{1}{3}g"c^2\lambda(r + \lambda r^3/3 - Ng'/c^2)$, in which *the* λ^2 *term does dominate* for r large enough. Let us stress, however, that for non-simple hadrons [when λ, and even more N, can vary] other terms can become important, as the newtonian one, $-Ng'^2/r^2$, or even the constant one, $+N\lambda g'^2/3$, which corresponds to a linear potential. Let us observe, finally, that the last equation predicts two quarks to attract each other with the force of *a few tons* when the inter–quark distance is of the order of 1 fm.

Let us pass to consider the case of *not* too large distances, still at the static limit. It is then important to add to the radial potential the ordinary "kinetic energy term" (or centripetal potential) $(J/g")^2/2r^2$, in order to account for the orbital angular momentum of $g"$ w.r.t. g'. For the effective potential[4,5,9] acting between two constituents $g', g"$ we thus get the expression

$$V_{\rm eff} = \frac{1}{2}g"c^2[2(\frac{Ng'}{c^2})^2\frac{1}{r^2} - \frac{2Ng'}{c^2}\frac{1}{r} - \frac{2\lambda Ng'}{3c^2}r + \frac{\lambda}{3}r^2 + \frac{1}{2}(\frac{\lambda}{3})^2r^4] + \frac{(J/g")^2}{2r^2} \ ,$$

which, in the region in which GR reduces to the newtonian theory, does of course simplify into $V_{\rm eff} \approx -Ng'g"/r + (J/g")^2/2r^2$. In such a case the test–constituent $g"$ can stabilize itself (we shall come back to this point) at a distance r_e from the source–constituent g' for which V is minimum; i.e., at the distance $r_e = J^2/Ng'g"^2$. At this distance the "effective force" vanishes. We therefore get, for small distances, the so-called *asymptotic freedom*: for non-large distances (when the force terms proportional to r and r^3 become negligible) the hadron constituents behave as if they were (almost) free.

Let us go back, however, to the complete expression for $V_{\rm eff}$. Let us first observe that we can evaluate the radius for which the potential is minimum also when $J = 0$. In the case of the simplest quarks[9], we get always at least a solution: $r_e \approx 0.25$ fm. Passing to the case $J = \hbar$ we get, under the same conditions[9], the interesting value $r_e \simeq 0.9$ fm.

By recalling that *mesons* consists of two quarks ($q\bar{q}$), our approach suggests for mesons in their fundamental state —at least for $J = 0$— the model of two quarks *oscillating* around their equilibrium position. It is worth mentioning that for small oscillations (harmonic motions in space) the dynamical group becomes, then, SU(3) ! Let us notice also that the value $m_o = h\nu/c^2$ corresponding to the frequency $\nu = 10^{23}$ Hz yields the pion mass: $m_o \simeq m_\pi$.

4. – THE INTER–QUARK(ANTIQUARK) POTENTIAL $V_{q\bar{q}}$

Let us now apply the above formalism to the case of charmonium and bottomonium, where we consider the test as well as the source constituent of the hadron as quarks without the colour degree of freedom (i.e., the *sign* of their *strong charge*) specifically mentioned. In that case, as we already mentioned, the considerations above can be applied only *approximately.*

The inter–quark(antiquark) force, according to eq.(6), is

$$F_{q\bar{q}} \equiv g'' \frac{d^2 r}{dt^2} = -\frac{1}{2} c^2 g'' (1 - \frac{2Ng'}{c^2 r} + \frac{\lambda r^2}{3})(\frac{2Ng'}{c^2 r^2} + \frac{2\lambda r}{3}) \hat{r} \, .$$

By integration, we get the inter–quark(antiquark) potential:

$$V_{q\bar{q}} = g'' \frac{c^2}{2} [\frac{2g'^2 N^2}{c^4} \frac{1}{r^2} - \frac{2Ng'}{c^2} \frac{1}{r} - \frac{2g'\lambda N}{3c^2} r + \frac{\lambda}{3} r^2 + \frac{\lambda^2}{18} r^4] \; ; \tag{8}$$

the combined effect of all the terms can be shown to be roughly of the shape in Fig. 2.

5. – CALCULATING THE QUARKONIA MASS SPECTRA

Passing temporarily to the quantum mechanical language, after the usual manipulation, the radial dependence of the potential can be written:

$$V^J_{q\bar{q}}(r) = V_{q\bar{q}}(r) + \frac{\ell(\ell+1)\hbar^2}{2g^2} \frac{1}{r^2} \, , \tag{9}$$

where the centripetal potential has been properly included, quantity $\ell\hbar$ being the orbital angular momentum. We ignore the spin of the quarks in our approximation; and g is here the average quark strong-mass magnitude[9,5,4] $[g \equiv |g'| = |g''|]$. Let us insert such expression in the non-relativistic Schroedinger equation.

Before entering explicit calculations, let us recall the following.[9,5,4] Consider two identical particles endowed with both gravitational (m) and strong (g) mass, *i.e.* two identical hadrons, and the ratio between the corresponding strong (S) and gravitational (s) interaction–strengths. One finds that $S/s \equiv Ng^2/Gm^2 \simeq 10^{40 \div 41}$, thus verifying that $\rho \equiv R/r \simeq S/s$. For example, for $m = m_\pi$ one gets $Gm^2/\hbar c \simeq 1.3 \times 10^{-40}$; while $Ng^2/\hbar c \simeq 14$ or 3 (or 0.2) depending on the particular coupling constant considered: $pp\pi$ or $\pi\pi\rho$ (or quark-quark-gluon), respectively. At this point, it is important to observe —however— that the gravitational coupling constant $Gm^2/\hbar c$ (experimentally measured in the case of two "tiny components" of our particular cosmos) ought to be compared with the analogous coupling constant for the strong interaction of two tiny *components* (partons? partinos?) of the corresponding hadron, or rather of a constituent quark of its. Such a constant is not known to us. We know only, for the simplest hadrons, about the quark–quark–gluon coupling constant: $Ng^2/\hbar c \simeq 0.2$. As a consequence, if we now set[9] more carefully

$$N \equiv \rho_1 G \; ; \quad \lambda \equiv \rho^2 \Lambda \, , \tag{10}$$

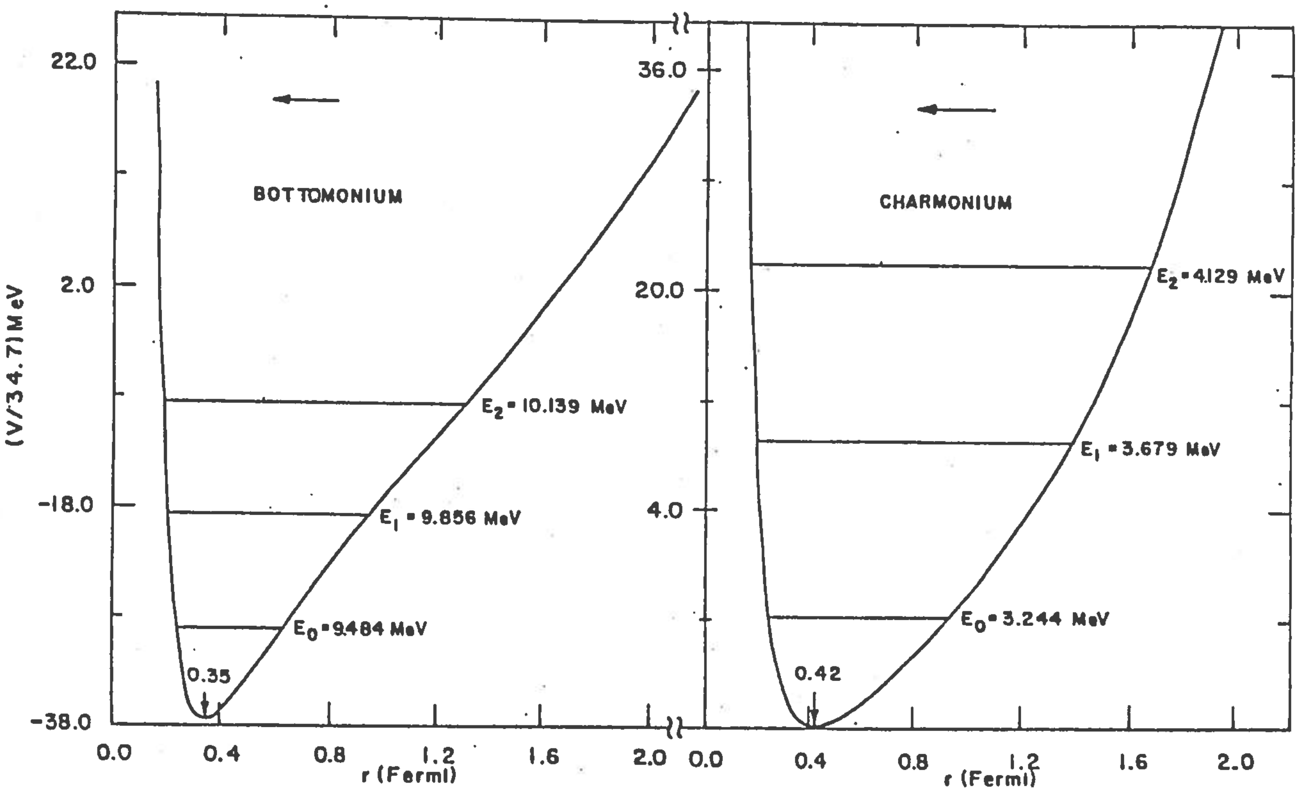

Fig. 2 – In this figure the shape is shown of two typical inter–quark potentials $V_{\rm eff}$ yielded by the present theoretical approach: cf. eq.(8). We show also the theoretical energy–levels *calculated* for the $1\!-\!{}^3s_1$, $2\!-\!{}^3s_1$ e $3\!-\!{}^3s_1$ states of "Bottomonium" and "Charmonium", respectively [by adopting for the *bottom* and *charm* quark the masses m(b)=5.25 and m(c)=1.68 GeV/c^2]. The comparison with experience is satisfactory:[11] see Chapt. 5.

the best value of ρ_1 that we can predict at the present time for those simple hadrons is $\rho_1 \simeq 10^{38} \div 10^{39}$; and actually 10^{38} is the value that yielded the results closer to experience. On the contrary, the values of the quantity ρ, related to λ, are theoretically expected to *not* vary very much (even when the particular hadron, or cosmos, under analysis does change[9]) w.r.t the "reference value" $10^{40 \div 41}$: for this reason, in eqs.(10) we distinguished ρ from ρ_1.

The Schroedinger equation for $V^J_{q\bar{q}}(r)$ [eq.(9)] is in spherical polar coordinates. It was solved, by using a finite difference method, for those r intervals for which the Hamiltonian yields $n = 5$ or 6 eigenvalues and eigenfunctions. Thus, we set n points for r and $V_{q\bar{q}}(r)$, giving a matrix of $n \times n$ dimensions to be solved. After this, the process was repeated with the same parameters ρ, ρ_1 and J for $2n$ values of r, and so on, until convergence is obtained. Always it has been correctly assumed $N \equiv \rho_1 G$ and $\lambda \equiv \rho^2 \Lambda$; with the costumary values $\Lambda = 10^{-52}\ \mathrm{m}^{-2}$ and $G = 6.67 \times 10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$. It is interesting that, due to the dependence of $V_{q\bar{q}}(r)$ on N and λ, *i.e.* on ρ_1 and ρ, we could verify also by such computer calculations that only values $\rho_1 \approx 10^{38}$ and $\rho \approx 10^{41}$ are actually consistent with $r \approx 1$ fm systems. This fact makes the terms in r^2 and r^4 very important.

The following Table I gives the comparison between theory and experimental results (for radial s-states; $J = 0$):

		Energy Level (MeV)		
Heavy Quarkonia	Ground State radius	Quantum State	Theory M	Experiment[3]
Charmonium	0.42 fm	$1\ ^3s_1$	3244	3096.9 ± 1
$\mathrm{m_{charm}} = 1.69\ \mathrm{GeV}/c^2$		$2\ ^2s_1$	3679	3686 ± 1
		$3\ ^3s_1$	4129	4028.7 ± 2.8
Bottomonium	0.35 fm	$1\ ^3s_1$	9484	9460.0 ± 0.3
$\mathrm{m_{bottom}} = 5.25\ \mathrm{GeV}/c^2$		$2\ ^2s_1$	9856	10023.4 ± 0.3
		$3\ ^3s_1$	10139	10035.5 ± 0.5

The charmonium spectrum has been obtained —by the computer fit— just in correspondence with the the expected, "standard" values $\rho = 10^{41}$ and $\rho_1 = 10^{38}$. The bottomonium spectrum in correspondence with $\rho = 0.5 \times 10^{41}$ and $\rho_1 = 0.5 \times 10^{38}$.

The agreement between theory and experiment[11] is good, considering the approximations made. Moreover, it can be interesting to notice that the ground–state sizes of charmonium and bottomonium are consistent with the asymptotic freedom in the sense that $\langle r \rangle_{\mathrm{char}}$ is larger than $\langle r \rangle_{\mathrm{bott}}$, as should be the case; both being less than 0.5 fermi.

APPENDIX A

The Strong Coupling–Constant

Following recent work still to be published,[9] let us add here that (in the case of a static, spherically symmetric metric, and for coordinates in which it is diagonal) the Lorentz factor is proportional to $\sqrt{g_{oo}}$, so that the *strong coupling–constant* $\alpha_S \equiv S$ assumes the form

$$\alpha_S(r) \simeq \frac{N}{\hbar c} \frac{{g'_o}^2}{1 - 2Ng'_o/c^2r + \lambda r^2/3} \,, \tag{11}$$

because of the fact that the value of any strong mass $g"$ varies with its speed,

$$g" = \frac{g"_o}{\sqrt{g_{oo}}} = \frac{g"_o}{\sqrt{1 - 2Ng'_o/r + \lambda r^2/3}} \,, \tag{12}$$

so as an ordinary relativistic mass. Our "constant" $\alpha_S(r)$ behaves analogously to the perturbative coupling constant of the "standard theory" (QCD): that is to say, $\alpha_S(r)$ decreases as the distance r decreases, and increases as it increases, once more justifying the phenomena both of confinement and of "asymptotic freedom". Let us recall that, when $g"_o = g'_o$, the definition of α_S is $\alpha_S \equiv S = N{g'}^2/\hbar c$.

The Schwarzschild–type coordinates $(t; r, \theta, \varphi)$ are known, however, to not correspond to any real observer. From a *physical* point of view it is therefore interesting to pass to local coordinates $(T; R, \theta, \varphi)$ associated with the observers *at rest* w.r.t. the metric[12] at each point (r, θ, φ) of space: $dT \equiv \sqrt{g_{tt}}dt$; $dR \equiv \sqrt{-g_{rr}}dr$, where $g_{tt} \equiv g_{oo}$ and $g_{rr} \equiv g_{11}$. These *local observers* measure a speed $U \equiv dR/dT$ (and strong–masses) such that[9] $\sqrt{g_{tt}} = \sqrt{1 - U^2}$, so that eq.(12) gets the transparent form

$$g" = \frac{g"_o}{\sqrt{1 - U^2}} \,. \tag{12'}$$

Once the speed U has been evaluated as a function of r (by the geodesic equation), it is then easy to verify, e.g., that for negative λ the minimum value of U^2 corresponds to $r = [3Ng'_o/|\lambda|]^{1/3}$. For positive λ, on the contrary, a similar expression, namely $r_o \equiv [6Ng'_o/\lambda]^{1/3}$, yields a (confining) limit–value of r which cannot be reached by any constituent.

Let us finally consider the case of a geodesic circular motion, as described by the "physical" observers, i.e. by our local observers (even if it is convenient to express every quantity as a function of the old Schwarzshild–deSitter coordinates). If a is the angular momentum for strong rest-mass unit, in the case of a test-quark orbiting around the source-quark one meets the interesting relation $g" = g'_o\sqrt{1 + a^2/r^2}$, which allows writing the strong coupling–constant in the particularly simple form:[9]

$$\alpha_S \simeq \frac{N}{\hbar c} g'_o (1 + \frac{a^2}{r^2}) \; . \tag{11'}$$

We can observe, e.g., that —if $\lambda < 0$— the specific angular momentum a vanishes in correspondence with the costumary geodesic $r \equiv r_{qq} = [3Ng'_o/|\lambda|]^{1/3}$; in such a case the test-quark can remain *at rest* at the distance r_{qq} from the source-quark. With the "typical" values $\rho = 10^{41}$; $\rho_1 = 10^{38}$, e $g'_o = m_p/3 \simeq 313$ MeV/c^2, one gets $r_{qq} \simeq 0.8$ fm.

APPENDIX B

On the Fireball Phenomenology

The data on hadron multiple production in very-high-energy interactions[13] have been shown to be compatible with a discrete mass spectrum of *fireballs*, which are formed in such collisions and trail the colliding hadrons after the interaction.[10] See Table II:

Name	Hadron multiplicity	$p_t(\gamma)$ (Gev/c)	Fireball estimated rest energy (GeV)	Other characteristics (all have n(π^o) $\approx$ 0)
Centauro	$\sim$ 100	0.35 ± 0.10	$200 \div 300$	—
Mini–Centauro	$\sim$ $15 \div 20$	0.35 ± 0.10	$20 \div 40$	—
Chiron	$\sim$ $10 \div 20$	2.5 ± 0.4	$\geq$ 200	mini-cluster
Geminion	$\sim$ 2	2.4 ± 0.4	$\geq$ 30	mini-cluster

These fireballs seem to possess different decay modes: either into pions only, or into baryons only. The decays are statistical and their temperature were derived; for instance, in the so-called "Chiron mode" the temperature appears to be about 10 GeV.

The most famous decay modes are associated with the *Centauro* events —multiple production of high p_t (transverse-momentum) 'baryons' accompanied by little or *no* pion emission,— which were first found by the Brazil–Japan Emulsion Chamber Collaboration about twenty years ago in cosmic ray $E_{lab} > 100$ TeV interactions.[13] Speculations into the nature of these puzzling events do not seem to have yet produced any reasonable insight within the existing theoretical framework of high energy physics.

Most of the mentioned speculations on the mechanism undergoing the Centauro events referred to the formation of a new type of hadronic matter, which would then decay in exotic ways. For example, Bjorken & McLerran suggested that in an extremely energetic interaction, high in the atmosphere, a metastable cluster of quarks —a "glob" of quark matter— is formed, and penetrates about 500 g m^{-2} of the atmosphere to the mountain level. Afterwards this glob decays, preferentially into baryons and antibaryons, with very little electromagnetic component (due to π^o's), above the detectors. Alternatively, Evans argued that the Centauro events could be explained in terms of baryon number violating processes, within the standard model. However, no matter what the production mechanism can be, the reported high p_t (~ 1 GeV/c) of the produced 'baryons' implies extremely short range (~ 0.2 fm) final state interactions, which *ought to* lead in any case to the appearance of pions. In particular, if heavy resonances were also formed, they would decay into nucleons and pions.

A reasonable mechanism that avoids pion production has been actually proposed by Sinha & Bandyopadhyay.[14] But in that model it is difficult to account for the Planck–type spectrum and the clustering property of the produced baryons.

Therefore, we regarded as worthwile applying a (preliminary) version of the new theory sketched in this paper to decay events like Centauros, Chirons and Geminions. Let us recall that this was done in ref.[10] Namely, those events were tentatively considered by us as "strong black-hole" *evaporations.*[8,4] Even if some difficulties are still to be solved, various positive results were thus obtained. For instance, our analysis seemed to suggest that in the considered collisions some phase transitions can take place, associated with the collapse of the colliding matter inside its possible "strong horizons". The horizon radii resulted in fair agreement with experience and yielded, in their turn, the transition temperatures through Bekenstein-Hawking–type relations;[4] the most interesting result being probably that, according to our model, *heavier* particle emission is expected to be *favoured.*[10]

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