ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Catania

INFN/AE-91/03
20 Marzo 1991

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Sezione di Catania

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THE STRONG COUPLING CONSTANT: ITS THEORETICAL DERIVATION FROM A GEOMETRIC APPROACH TO HADRON STRUCTURE.(*)

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ABSTRACT – Since more than a decade, a *bi-scale*, unified approach to strong and gravitational interactions has been proposed, that uses the geometrical methods of general relativity, and yielded results similar to "strong gravity" theory's. We fix our attention, in this note, on hadron structure, and show that also the strong interaction strength α_S, ordinarily called the "(perturbative) coupling–constant square", can be evaluated within our theory, and found to decrease (increase) as the "distance" r decreases (increases). This yields both the confinement of the hadron constituents [for large values of r], and their asymptotic freedom [for small values of r inside the hadron]: in qualitative agreement with the experimental evidence. In other words, our approach leads us, on a purely theoretical ground, to a dependence of α_S on r which had been previously found only on phenomenological and heuristical grounds. We expect the above agreement to be also quantitative, on the basis of a few checks performed in this paper, and of further work of ours about calculating meson mass–spectra.

(*) Work partially supported by CNPq and FAPESP, and by INFN, M.P.I. and CNR.

Introduction. – Since 1978, a unified approach to strong and gravitational interactions was proposed[1–4], which used the geometrical methods of general relativity; and assumed covariance of physical laws under global (discrete) dilations. It yielded results similar to those given by the "strong gravity" theory[5] (even if the starting point is quite different).

Within such an approach, and in connection with hadron structure, we came in particular to associate hadron constituents with suitable stationary, axisymmetric solutions of certain new Einstein–type equations, supposed to describe the strong field inside hadrons. Those Einstein–type equations are nothing but the ordinary Einstein equations (with cosmological term) suitably scaled down[2]. As a consequence, the cosmological constant Λ and the gravitation universal constant G (or the masses M) result, in our theory, to be scaled up and transformed into a "hadronic constant" λ and into a "strong universal constant" N (or into "strong masses" g), respectively.[1,2]. Our field equations, to be valid inside a hadron, are therefore:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\rho}_{\rho} + \lambda g_{\mu\nu} = -KNT_{\mu\nu}\ ; \quad [K \equiv \frac{8\pi}{c^4}], \tag{1}$$

where, because of simple dimensional considerations[2],

$$\lambda \simeq {\rho_1}^2 \Lambda\ ; \quad N \equiv \rho G\ ; \quad \rho \simeq \rho_1 \simeq 10^{41}\ .$$

If we adopt for the ordinary cosmological constant the value $\Lambda \simeq 10^{-52}\mathrm{m}^{-2}$, then we get for the "strong cosmological constant" (hadronic constant) the value $\lambda \simeq 10^{30}\mathrm{m}^{-2} = (1\ \mathrm{fm})^{-2}$. Notice that in our present units, for which $N = \rho G$, mass M and strong mass (or "strong charge") g of the same particle become equal ($M \equiv g$).

Throughout this paper, we shall choose the signature -2. When convenient, we shall use units such that it be also $c = 1$.

The *simplest* solution of eqs.(1) is the Schwarzschild–de Sitter's, corresponding to the metric generated by a central, static, spherically–symmetric distribution of strong charge; *i.e.*, to the metric generated by a hadron constituent (say, a quark) when neglecting its electric charge and intrinsic angular momentum:

$$\begin{aligned} ds^2 &\equiv g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = \\ &= (1 - \frac{2Ng_{\circ}}{r} - \frac{\lambda r^2}{3})\mathrm{d}t^2 - (1 - \frac{2Ng_{\circ}}{r} - \frac{\lambda r^2}{3})^{-1}\mathrm{d}r^2 - r^2(\sin^2\theta \mathrm{d}\varphi^2 + \mathrm{d}\theta^2) \end{aligned} \tag{2}$$

where $g_{\circ}$ is the strong charge of the considered constituent, and (t, r, θ, φ) are spherical (Schwarzschild–type) coordinates. Let us stress once more that, in the present units, $g_{\circ}$ is equal to the rest–mass $M_{\circ}$ of the hadron constituent. This metric may be trivially transformed into the Reissner–Nordström–de Sitter one by adding the term ke^2/r^2; $k \equiv (4\pi\varepsilon_{\circ})^{-1}$ into $g_{\circ\circ}$ and g_{11}.

Strong–charge and its dependence on r – Let us now consider the geodesic motion of a test–particle in the metric (2). [Notice that, if the test–particle is replaced by a second quark, then the evaluations performed below assume an indicative value *only*, since we are not going to involve ourselves here with the (general relativistic) twobody problem]. Our test particle, when free–falling, will be endowed[6] with a constant total–energy $E_o \equiv g'_o c^2$, which in the previous coordinates can be written

$$E_o \equiv g'_o c^2 = g_{tt} p^t \; ; \qquad p^t \equiv p^o \equiv \frac{\mathrm{d}t}{\mathrm{d}s} \, , \tag{3}$$

where $g_{tt} \equiv g_{oo}$, and g'_o is the (*rest*) strong–mass of the test particle.

Since the Schwarzschild–type coordinates do not correspond to any physical observer, let us pass —however— to the *local coordinates* (T, R, θ, φ), associated with observers *at rest* w.r.t. [with respect to] the metric at each point (r, θ, φ) of the considered space:

$$\mathrm{d}T \equiv \sqrt{g_{tt}}\mathrm{d}t \; ; \qquad \mathrm{d}R \equiv \sqrt{-g_{rr}}\mathrm{d}r \, ,$$

where $g_{rr} \equiv g_{11}$. The local observers measure a new total–energy E_ℓ for the considered test particle. Quantity E_ℓ is no longer a constant of the motion and is related to E_o through the relation

$$E_\ell \equiv \frac{\mathrm{d}T}{\mathrm{d}s} \equiv p^T = \sqrt{g_{tt}}\, p^t \, , \tag{4a}$$

that is to say

$$E_\ell \equiv g' c^2 = \frac{g'_o c^2}{\sqrt{g_{tt}}} \, . \tag{4b}$$

The interesting point is that (in the static case) $\sqrt{g_{tt}} = \sqrt{1 - V^2}$, provided that V is measured by the local observers.

The physical meaning of eqs.(4) is more evident if, instead of setting $M \equiv g$ and $N = \rho G$, we put $N = G = 1$ so that (in such new units) for the strong charge it holds

$$g = \sqrt{\rho} M \; ; \quad g_o = \sqrt{\rho} M_o . \tag{5}$$

Incidentally, in connection with eqs.(5), let us recall[1,2] that, for $M \simeq m_\pi$, one gets $g =$ Planck–mass; that is to say, the strength of the interaction between two (strongly interacting) quarks is equal to the strength of the interaction between two (gravitationally interacting) particles endowed with the Planck mass.

In whatever units, eqs.(4) tell us that the strong charge g' of the test–particle does change with its speed V, w.r.t. the local observers, as follows:

$$g' = \frac{g'_o}{\sqrt{1-V^2}} \; ; \tag{6a}$$

where $V \equiv \mathrm{d}R/\mathrm{d}T$ (and g') are measured —let us repeat— in the local reference–frames: actually, it is in these frames that they have a *direct* physical meaning.[7] In the case of generic motion, we are left, of course, with the relation

$$g' = \frac{g'_o}{\sqrt{g_{tt}}} \; . \tag{6b}$$

In other words, the strong charge (or strong mass) of a particle does depend, inside a hadron, on the particle speed exactly as the ordinary gravitational mass does in our space-time.

Notice that eqs.(6) allow us to express the value of the strong charge g' as a function, *e.g.*, of its radial coordinate r relative to the source–quark. Namely, in the case of eq.(6a) one has

$$V^2 = 2Ng_o/r + \lambda r^2/3 \; ,$$

and therefore, from eq.(6b):

$$g' = \frac{g'_o}{\sqrt{1 - 2Ng_o/r - \lambda r^2/3}} \; , \tag{6c}$$

where, let us recall it, g_o is the rest strong–mass (or rest strong–charge) of the source–quark.

Quark–quark coupling costant – In analogy with the electromagnetic case, in which $\alpha_E = (ke^2)/(\hbar c)$, the strong interaction strength is defined[1,2] as

$$\alpha_S \equiv \frac{Ng'^2}{\hbar c}$$

which is also a pure number and —passing to the field-theoretical language— corresponds to the dimensionless square of the vertex coupling-constant. Let us recall that g' is measured by the local observer. From eq.(6c) we obtain $\alpha_S = (N/\hbar c)g_o'^2(1-2Ng_o/r-\lambda r^2/3)^{-1}$, where g_o, g'_o are the (rest) strong mass of the source–quark and the test–constituent, respectively. In the case when also g' is a quark, we have:

$$\alpha_S \simeq \frac{N}{\hbar c} \; \frac{{g_o}^2}{1 - 2Ng_o/r - \lambda r^2/3} \; . \tag{7}$$

Therefore, the strong interaction strength α_S, which in elementary particle physics is ordinarily called the "(perturbative) coupling–constant square", is predicted by our approach to decrease (increase) as the "distance" r decreases (increases). This yields both the *confinement* of the constituents (for large values of r, of the order of 1 fm), and their so-called *asymptotic freedom* (for small, but not too small, values of r inside the hadron): in qualitative agreement with the experimental evidence. In other words, our approach leads us —on purely theoretical grounds— to a dependence of α_S on r which was previously found, within the perturbative QCD[8], only on phenomenological and heuristical grounds.(*)

Before going on, let us stress the following point, which is essential when performing explicit calculations. To evaluate ρ, at the beginning we tacitly compared[1] the gravitational interaction strength $Gm_o{}^2/\hbar c$ with the value $Ng_o{}^2/\hbar c \simeq 14$ corresponding to the $pp\pi$ coupling constant square. However, the gravitational interaction strength (which is experimentally measured for the interaction between two "tiny components" of our cosmos: two pions, or two nucleons, for instance) should be compared with the analogous strength for the interaction between two small *components* of the corresponding ("reference") hadron, or rather of a constituent quark of its. Such a strength is unknown. We know, however, the quark–quark–gluon coupling constant square[9] for the simplest hadrons: $Ng_o{}^2/\hbar c \simeq 0.2$. As a consequence, the best value of ρ that we can work out, for calculations inside such hadrons, is $\rho \simeq 10^{38} - 10^{39}$. Actually, the value 10^{38} is the one that yielded the results closest to the experimental data[2,10].

Further remarks – In connection with eq.(6a), it is interesting to write down the explicit dependence of g' on the radial coordinate r, by expressing $V \equiv \mathrm{d}R/\mathrm{d}T$ as a function of r starting directly from the geodesic equation.

Since in our metric the geodesic motion is always a motion in a plane, let us fix $\theta = \pi/2$. From the geodesic equation $\mathrm{d}^2x^\mu/\mathrm{d}\tau^2 + \Gamma^\mu{}_{\nu\sigma}(\mathrm{d}x^\nu/\mathrm{d}\tau)(\mathrm{d}x^\sigma/\mathrm{d}\tau) = 0$, in which τ is an affine parameter [*e.g.*, the proper time: $\tau = s$], one gets

$$(\mathrm{d}r/\mathrm{d}s)^2 = 1/H^2 - (1 - 2Ng_o/r - \lambda r^2/3)(1 + a^2/r^2) \; ,$$

where $1/H$ and a are rest-energy and angular momentum, respectively, for unit rest-mass; notice that H is an integration constant. The last equation yields

$$V^2 \equiv (\frac{\mathrm{d}R}{\mathrm{d}T})^2 = 1 - H^2(1 - \frac{2Ng_o}{r} - \frac{\lambda r^2}{3}) \; . \tag{8}$$

(*) Let us explain in this note what do we mean by "not too small" distances. The horizon radii for metric (2) are the roots of the equation

$$g_{tt} \equiv 1 - 2Ng_o/r - \lambda r^2/3 = 0 \; .$$

The physically interesting case[4] is the one in which this equation has two real positive solutions r_1, r_2 (the third solution being always a negative real number). We suppose the effective radius r_q of the source–quark to be much *larger* than r_1 [and much smaller than r_2], so that the "space" available to the other constituents is the one between r_q and r_2.

Let us observe that, for $\lambda > 0$, the minimum of V^2 is got for $r = (3Ng_o/\lambda)^{1/3}$.

When $H = 1$, one is simply left with $V^2 = 2Ng_o/r + \lambda r^2/3$. Let us remark that eqs.(3),(4),(6) were just written down with the choice $H = 1$. It is moreover worthwhile to notice that, for $\lambda < 0$, one meets a limiting value of r, namely $r_o = (3Ng_o/|\lambda|)^{1/3}$, which cannot be reached by any internal constituent: $V(r_o) = 0$; so that one obtains again a *confinement* of the costituents.

By substituting eq.(8) into eq.(6a), we get once more eq.(6c), as expected. In connection with eq.(6c), let us here emphasize that [for $\rho_1 = 10^{41}$; $\rho = 10^{38}$, and $g_o = m_p/3 \simeq 313$ MeV/c^2] the minimum of g', namely $g' \simeq 1.5$, is obtained at $r \simeq 0.6$ fm. It can be easily verified, moreover, that the plot of $(1 - 2Ng_o/r - \lambda r^2/3)^{-1}$ as a function of r has the shape of a *confining potential.*

Let us now consider, in particular, the case of *circular motion.* By imposing $dr/ds = 0$ we get, w.r.t. the local observers, the orbital velocity

$$r^2(\frac{d\varphi}{dT})^2 = \frac{Ng_o/r - \lambda r^2/3}{1 - 2Ng_o/r - \lambda r^2/3} . \tag{9}$$

By substituting eq.(9) into eq.(6a), and taking into account the fact that for geodesic circular motion [r = constant along each geodetics] it holds

$$a^2/r^2 = \frac{2Mg_o/r - \lambda r^2/3}{1 - 3Ng_o/r} ,$$

we obtain the interesting relation

$$g' = g_o\sqrt{1 + a^2/r^2} \tag{10}$$

which allows writing the strong interaction strength (for a test-quark orbiting around the source-quark) in the particularly simple form

$$\alpha_S \simeq \frac{N}{\hbar c}{g_o}^2(1 + \frac{a^2}{r^2}) . \tag{11}$$

At last, one can observe that —if $\lambda > 0$— the angular momentum per unit rest-mass, a, vanishes (*i.e.*, $V = 0$) in correspondence with the geodetics $r \equiv r_{qq} = (3Ng_o/\lambda)^{1/3}$; in such a case the test-quark remains at rest, at a distance r_{qq} from the source-quark. For instance [for the same set of values $\rho_1 = 10^{41}$; $\rho = 10^{38}$, and $g_o = m_p/3 \simeq 313$ MeV/c^2] we get the interesting value $r_{qq} \simeq 0.8$ fm.

Conclusion – We have seen that the strong interaction strength, α_S, ordinarily called the "(perturbative) coupling–constant square", can be evaluated within our theory, and found to decrease (increase) as the "distance" r decreases (increases). This yielded the confinement of the constituents (for large values of r), as well as their asymptotic freedom (for small values of r inside the hadron): in qualitative agreement with the experimental evidence. In other words —as we already mentioned— our approach led us, on a purely theoretical ground, to a dependence of α_S on r which had previously been found only on phenomenological and heuristical grounds.

We hope the abovementioned agreement to be quantitative, and not only qualitative, on the basis of the few checks performed here and, even more, of our work [past, and in progress] for calculating, *e.g.*, the meson mass spectra.[3,4,10]

Acknowledgements: Useful discussions are acknowledged with R.H.A. Farias, E. Giannetto, L. Lo Monaco, G.D. Maccarrone, W.A. Rodrigues Jr. and particularly with P. Ammiraju, L.A.B. Annes, A. Insolia, A. Italiano, J.A. Roversi and S. Sambataro.

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