ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Trieste

INFN/AE-90/06

25 giugno 1990

C.A. Dominguez and N. Paver:

LEPTONIC DECAY CONSTANTS OF HEAVY FLAVOUR MESONS OF ARBITRARY MASS

Servizio Documentazione
dei Laboratori Nazionali di Frascati

LEPTONIC DECAY CONSTANTS OF HEAVY FLAVOUR MESONS OF ARBITRARY MASS

C.A. DOMINGUEZ
Institute of Theoretical Physics and Astrophysics
University of Cape Town, Rondebosch 7700, Cape, RSA

N. PAVER*
Dipartimento di Fisica Teorica, Università di Trieste, and
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, I-34100, Italy

Abstract

Leptonic decay constants of heavy flavour pseudoscalar and vector mesons of arbitrary mass are estimated in the framework of QCD sum rules. Predictions are compared with the scaling laws of the quark model.

*Also supported by MPI (Italian Ministry of Education).

At the present time the values of the leptonic decay constants of heavy flavour pseudoscalar and vector mesons are not known experimentally. Given their impact on weak hadronic physics it is, therefore, important to make reliable theoretical predictions, as model independent as possible, and using independent approaches. The most popular frameworks are: (a) numerical simulations of QCD on the lattice [1], (b) QCD-motivated potential models [2], and (c) QCD sum rules [3]. Results obtained so far for f_D and f_{D_s} from various calculations are in qualitative agreement, especially if one compares methods (a) and (c). On the contrary, the status of theoretical predictions for f_B and f_{B_s} is not as good, partly because current QCD lattices do not allow to treat reliably the propagation of a heavy quark such as beauty. To circumvent this problem a method has been recently proposed [4] to treat very heavy quark systems in the limit of an infinitely heavy quark mass. This should allow for more stringent tests and comparisons to be made in the future.

An interesting consistency check can be made by estimating the leptonic decay constants of hypothetical heavy flavour mesons of arbitrary mass. This is certainly possible in the framework of potential models, and to some extent also in QCD simulations within the capabilities of present day lattices. In this note we show that it is also possible in the framework of QCD sum rules, provided that the meson mass is not too large. We then estimate leptonic decay constants of pseudoscalar and vector mesons of arbitrary mass M in the range $M_D \leq M_P \lesssim 10$ GeV, and $M_{D^*} \leq M_{V^*} \lesssim 10$ GeV, respectively.

We begin by considering the leptonic decay constant of a heavy flavour pseudoscalar meson, defined as

$$< 0 \mid A_\mu \mid P(k) > = i\sqrt{2}\, f_P k_\mu, \tag{1}$$

where $A_\mu(x) =: \bar{q}(x)\ \gamma_\mu\ \gamma_5 Q(x) :$, with $q(x)(Q(x))$ being the light (heavy) quark field. The relevant two-point function in this case is

$$\psi_5(q) = i \int d^4x\, exp(iqx) < 0|T\left(\partial^\mu A_\mu(x),\, \partial^\nu A^\dagger_\nu(0)\right) \mid 0 >, \tag{2}$$

where $\partial^\mu A_\mu(x) = m_Q :\bar{q}(x) i\gamma_5 Q(x):$, neglecting the light quark mass m_q. In QCD, the function $\psi_5(q)$ satisfies a twice-subtracted dispersion relation, and the appropriate sum rules for heavy quark flavours are the Hilbert moments at $Q^2 \equiv -q^2 = 0$, i.e.

$$\varphi^{(n)}(0) \equiv \frac{(-)^n}{(n+1)!} \left(\frac{d}{dQ^2}\right)^{n+1} \psi_5(Q^2)\bigg|_{Q^2=0} = \int \frac{ds}{s^{n+2}} \frac{1}{\pi} \operatorname{Im} \psi_5(s), \tag{3}$$

with $n = 1, 2, \ldots$. The left hand side of Eq.(3) can be calculated in perturbative QCD at short distances. Non-perturbative effects are then introduced in the framework of the operator product expansion and parametrized in terms of quark and gluon vacuum condensates. The asymptotic freedom (AF) spectral function to two-loops reads [5]

$$\begin{aligned}\frac{1}{\pi} \operatorname{Im} \psi_5(x)\,|_{AF} &= \frac{3}{8\pi^2} m_Q^4 \frac{(1-x)^2}{x} \left\{1 + \frac{4\alpha_s}{3\pi}\left[\frac{18}{8} - 2Li_2\left(\frac{-x}{1-x}\right)\right.\right. \\ &- \ln\left(\frac{x}{1-x}\right)\ln\left(\frac{1}{1-x}\right) + \left(\frac{3}{2} - \frac{x}{1-x} - x\right)\ln\left(\frac{x}{1-x}\right) \\ &+ \left.\left.\frac{1}{1-x}\ln\left(\frac{1}{1-x}\right)\right]\right\},\end{aligned} \tag{4}$$

where $x \equiv m_Q^2/s$. The leading non-perturbative (NP) contributions to the first two Hilbert moments are [5]

$$\varphi^{(1)}(0)|_{NP} = \frac{C_4 < O_4 >}{m_Q^4} + \frac{3}{4}\frac{C_5 < O_5 >}{m_Q^5} - \frac{5}{3}\frac{C_6 < O_6 >}{m_Q^6}, \tag{5}$$

$$\varphi^{(2)}(0)\,|_{NP} = \frac{1}{m_Q^2}\left(\frac{C_4 < O_4 >}{m_Q^4} + \frac{3}{2}\frac{C_5 < O_5 >}{m_Q^5} - \frac{11}{3}\frac{C_6 < O_6 >}{m_Q^6}\right), \tag{6}$$

where

$$C_4 < O_4 > = < \frac{\alpha_s}{12\pi} G^2 - m_Q \bar{q} q >, \tag{7}$$

$$C_5 < O_5 > = < g_s \bar{q} i \sigma_{\mu\nu} \vec{G}_{\mu\nu} \cdot \vec{\lambda} q >, \tag{8}$$

$$C_6 < O_6 > = \pi\alpha_s < (\bar{q}\gamma_\mu \vec{\lambda} q) \sum_q \bar{q}\gamma^\mu \vec{\lambda} q >, \tag{9}$$

Next, the right hand side of Eq.(3) is calculated after parametrizing the hadronic spectral function by a pole term plus a continuum modelled by the AF expression Eq.(4) and starting at some threshold s_0, i.e.

$$\frac{1}{\pi}\operatorname{Im}\psi_5(s)\bigg|_{HAD} = 2 f_P^2 M_P^4 \delta(s - M_P^2) + \frac{1}{\pi}\operatorname{Im}\psi_5(s)\bigg|_{AF} \Theta(s - s_0). \tag{10}$$

The first two sum rules can then be written as

$$\frac{2f_P^2}{M_P^2} = \frac{1}{8\pi^2}\left[(1-a_1) + \alpha_s(0.751 - b_1)\right] + \varphi^{(1)}(0)\Big|_{NP}, \tag{11}$$

$$\frac{2f_P^2}{M_P^2} = \frac{M_P^2}{m_Q^2}\left\{\frac{1}{32\pi^2}\left[(1-a_2) + \alpha_s(1.706 - b_2)\right] + m_Q^2\,\varphi^{(2)}(0)\Big|_{NP}\right\}, \tag{12}$$

where $\varphi^{(1,2)}(0)|_{NP}$ are given by Eqs.(5)-(6), and $a_{1,2}$ and $b_{1,2}$ are functions of s_0 which represent the continuum corrections obtained by integrating Eq.(4) in the interval $0 \leq x \leq x_o \equiv m_Q^2/s_0$.

The two sum rules Eqs.(11)-(12) allow for a prediction of e.g. f_P and M_P, provided m_Q and s_0 are known. However, s_0 is not known a priori, except for the obvious loose bound $s_0 \geq M_P^2$, and hence one usually searches for values of s_0 which lead to a mass in agreement with experiment. In this case the estimate of f_P should be reasonably reliable. Predictions should, of course, be stable against generous changes in the value of s_0. For example, for D, D_s and B mesons $s_0(D) \simeq (2-3)M_D^2$, $s_0(D_s) \simeq (2-3)M_{D_s}^2$, and $s_0(B) \simeq (1.1-1.3)M_B^2$, lead to meson masses in agreement with experiment at the 5-10% level [3]. With increasing meson mass the continuum corrections in Eqs.(11)-(12) become increasingly important, and hence predictions are increasingly less accurate. This is an inherent limitation of the present approach.

We use now the known masses of the D and B mesons, and the charm and beauty quarks, to normalize the sum rules Eqs.(11)-(12) which can then be used to predict f_P as a function of M_P, the latter being a continuous variable. Empirically, $m_Q \simeq M_P - 0.6$ GeV fits both ends, i.e. $Q = c$ and $Q = b$ with $m_c = 1.3$ GeV, $m_b = 4.6$ GeV. Numerically, the procedure is then to input a value of M_P and search for the value of s_0 which will lead to a predicted mass in agreement with the input. Once s_0 is thus determined, f_P follows immediately. Variations of s_0 around its eigenvalue, and uncertainties in the vacuum condensates will account for the error in the predictions of $f_P(M_P)$. In Fig.1 we show the results for f_P as a function of M_P. The error bar indicates the typical error. In Fig.2 we plot $\sqrt{M_P}f_P$ versus M_P

and compare it with the non-relativistic quark model (NRQM) scaling law

$$\sqrt{M_P} f_P = \text{const} \tag{13}$$

which was adjusted to coincide with our results at $M_P = M_D$

In principle we could have adjusted Eq.(13) to coincide with our results at some other mass, e.g. $M_P = M_B$. However, since with increasing mass the continuum corrections become increasingly important it is much safer to choose $M_P = M_D$. At this value the non-perturbative and the continuum corrections are well under control. For these reasons we have chosen to truncate our predictions at $M_P \simeq 10$ GeV. In connection with Eq.(13) we notice that (modulo logs) it should be considered as a more fundamental asymptotic behaviour in the heavy quark mass, resulting from general properties of QCD [4],[6].

Turning to heavy flavour vector mesons we define their (dimensionless) leptonic decay constant as

$$< 0|J_\mu|V^*(k,\epsilon) >= \frac{M_{V^*}^2}{\sqrt{2}\gamma_{V^*}} \epsilon_\mu, \tag{14}$$

where $J_\mu(x) =: \overline{q}(x)\gamma_\mu Q(x):$. Although probably impossible to measure experimentally, this vector coupling is quite important for testing theoretical approaches to non-perturbative QCD because of its relation to the bound state wave function at the origin (just as f_P).

The appropriate two-point function in this case is

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i \int d^4x \, \exp(iqx) < 0|T\left(J_\mu(x), J_\nu^\dagger(0)\right)|0 > \\ &= -(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi^{(1)}(q^2) + q_\mu q_\nu \Pi^{(0)}(q^2). \end{aligned} \tag{15}$$

The $Q^2 = 0$ Hilbert moments of $Q^2\Pi^{(1)}(Q^2)$, which is free of kinematical singularities, become

$$\begin{aligned} \varphi^{(n)}(0) &\equiv \frac{1}{(n+1)!}\left(-\frac{d}{dQ^2}\right)^{n+1} \left[-Q^2\Pi^{(1)}(Q^2)\right]\Big|_{Q^2=0} \\ &= \int \frac{ds}{s^{n+1}} \frac{1}{\pi} \mathrm{Im}\Pi^{(1)}(s). \end{aligned} \tag{16}$$

The AF expression for $\Pi^{(1)}(s)$ is [5]

$$\frac{1}{\pi}\mathrm{Im}\,\Pi^{(1)}(x)|_{AF} = \frac{1}{8\pi^2}(1-x)^2(2+x)\left\{1+\frac{4\alpha_s}{3\pi}\left[\frac{13}{4}+2\mathrm{Li}_2(x)+\ln x\ln(1-x)\right.\right.$$
$$\left.\left.+\frac{3}{2}\frac{x}{(2+x)}\ln\left(\frac{x}{1-x}\right)-\ln(1-x)-\frac{(4-x-x^2)}{(1-x)^2(2+x)}x\ln x-\frac{5-x-2x^2}{(1-x)(2+x)}\right]\right\}, \quad (17)$$

and the leading NP contributions to the first two moments are [5]

$$\varphi^{(1)}(0)\Big|_{NP} = \frac{1}{m_Q^2}\left[\frac{-C_4 < 0_4 >}{m_Q^4}+\frac{3}{2}\frac{C_5 < 0_5 >}{m_Q^5}+\frac{5}{3}\frac{C_6 < 0_6 >}{m_Q^6}\right], \quad (18)$$

$$\varphi^{(2)}(0)\Big|_{NP} = \frac{1}{m_Q^4}\left[\frac{-C_4 < 0_4 >}{m_Q^4}+\frac{5}{2}\frac{C_5 < 0_5 >}{m_Q^5}+2\frac{C_6 < 0_6 >}{m_Q^6}\right], \quad (19)$$

where the vacuum condensates are the same as in Eqs.(7)-(9) except for a change of sign in the light quark condensate in Eq.(7). With a parametrization of the hadronic spectral function analogous to Eq.(10), the first two sum rules read

$$\frac{1}{2\gamma_{V^*}^2}\frac{1}{M_{V^*}^2} = \frac{1}{m_Q^2}\left\{\frac{3}{32\pi^2}\left[(1-c_1)+\alpha_s(1.140-d_1)\right]+m_Q^2\varphi^{(1)}(0)\Big|_{NP}\right\} \quad (20)$$

$$\frac{1}{2\gamma_{V^*}^2}\frac{1}{M_{V^*}^2} = \frac{M_{V^*}^2}{m_Q^4}\left\{\frac{1}{40\pi^2}\left[(1-c_2)+\alpha_s(1.582-d_2)\right]+m_Q^4\varphi^{(2)}(0)\Big|_{NP}\right\} \quad (21)$$

where $c_{1,2}$ and $d_{1,2}$ are the continuum corrections obtained from integrating Eq.(17) in the interval $0 \le x \le x_O$.

We normalize these sum rules by using the masses of the D^* and B^* mesons and the masses of the charm and beauty quark, thus obtaining $M_{V^*} = m_Q + 0.7$ GeV, consistent with our ansatz for the D and B mesons. By repeating the procedure followed above for f_P, we have calculated γ_{V^*} as a function of M_{V^*} with the results shown in Fig.3. Curves (a) and (b) correspond to having included the NP terms as in Eq.(18) and Eq.(19), and to having neglected them altogether, respectively. The spread in the curves provides then an idea of the uncertainties involved. In Fig.4 we plot $\gamma_{V^*}/M_{V^*}^{3/2}$ versus M_{V^*} for the above two cases and compare it with the NRQM scaling law (curve (c))

$$\frac{\gamma_{V^*}}{M_{V^*}^{3/2}} = \mathrm{const} \quad (22)$$

normalized to coincide with our result for γ_{D^*} .

Although there are obviously no known mesons between D and B, nor between D^* and B^*, the results of the present calculation can be of interest in connection with the dependence of f_P and γ_{V^*} on the heavy quark mass as predicted e.g. by the static quark model. As we mentioned already, both Eq.(13) and Eq.(22) should be considered as an asymptotic scaling law following directly from QCD; hence much more general than implied by the non-relativistic quark model. Clearly, as with all asymptotic relations, the mass value at which asymptotia starts is unknown a-priori. Our results, as displayed in Figs.1-4, indicate that the charmed quark could still be too light for the asymptotic scaling laws to apply, because Eqs.(13) and (22) do not appear to be verified in the neighbourhood of this point. For larger quark masses both $f_P\sqrt{m_Q}$ and $\gamma_{V^*}/\, m_Q^{3/2}$ seem consistent with an almost constant behaviour in m_Q, starting somewhere between $m_Q = m_c$ and $m_Q = m_b$, and continuing all the way up to $m_Q = 10$ GeV. As a matter of fact, in our approach such a behaviour is not directly connected with any explicit $1/\sqrt{m_Q}$ dependence of f_P. Instead, as may be seen from e.g. Eq.(11), it arises from the mismatch between a linear increasing dependence on m_Q and a decreasing effect from the s_0-dependent $a_{1,2}$ and $b_{1,2}$, as well as from the NP power corrections. The latter effects turn out to largely overwhelm the former. The resulting behaviour of f_P on m_Q is then an "effective" $1/\sqrt{m_Q}$ power dependence as shown in Figs. 1 and 2. A similar mechanism operates in the case of the vector constant γ_{V^*}, as shown in Figs. 3 and 4. This situation is reminiscent of the $1/Q^2$ behaviour of the pion form factor determined in the framework of QCD sum rules [7]. It would be interesting to compare our QCD sum rule predictions with results for f_p and γ_{V^*} as a function of m_Q from alternative non-perturbative approaches, in particular from lattice QCD calculations.

References

[1] For a recent review see e.g. S.R. Sharpe, University of Washington, Report No. 40423-11 (1989).

[2] For a recent review see e.g. S. Capstick and S. Godfrey, Carnegie Mellon Report No. CMU-HEP-89-20 [1989].

[3] C.A. Dominguez and N. Paver, Phys. Lett. **B197** [1987] 423; Z. Phys. **C41** [1988] 217. See also: L.J. Reinders, Phys. Rev. **D38** [1988] 423.

[4] E.Eichten, Nucl. Phys. **B(Proc. Suppl.)4** [1988] 170.

[5] D.J. Broadhurst, Phys. Lett. **B101** [1981] 423; D.J. Broadhurst and S.C. Generalis, Open University Report No. OUT-4102-8/R [1982]; S.C. Generalis, Ph.D. thesis, Open University Report No. OUT-4102-13 [1984].

[6] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. **45** [1987] 292; H.D. Politzer and M.B. Wise, Phys. Lett. **B206** [1988] 681; **B208** [1988] 504.

[7] V.A. Nesterenko and A.V. Radyushkin, Phys. Lett. **B115** [1982] 410.

Figure Captions

Fig.1 : Pseudoscalar meson decay constant f_P as a function of the mass. Vertical bar shows the typical theoretical error.

Fig.2 : The product $f_P\sqrt{M_P}$ as a function of M_P. Curve (a) is our prediction and curve (b) the scaling law (13) adjusted to coincide with our result at $M_P = M_D$.

Fig.3 : Vector meson decay constant γ_{V^*} as a function of the mass. Curves (a) and (b) correspond to having included non-perturbative corrections, and to having neglected them altogether, respectively.

Fig.4 : The ratio $\gamma_{V^*}/M_{V^*}^{3/2}$ as a function of M_{V^*}. Curves (a) and (b) have the same meaning as in Fig. 3, and curve (c) is the scaling law (22).

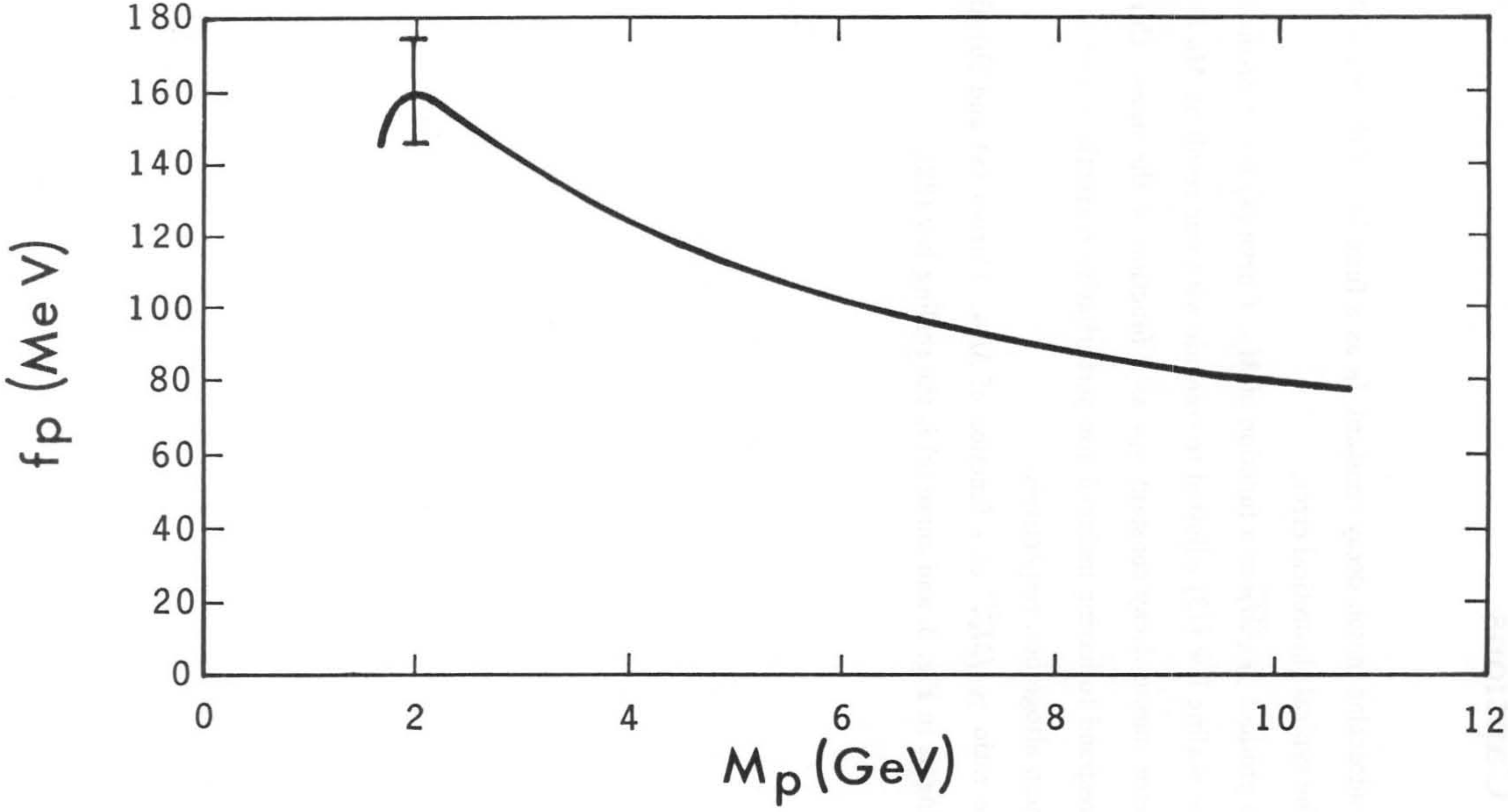

FIGURE 1

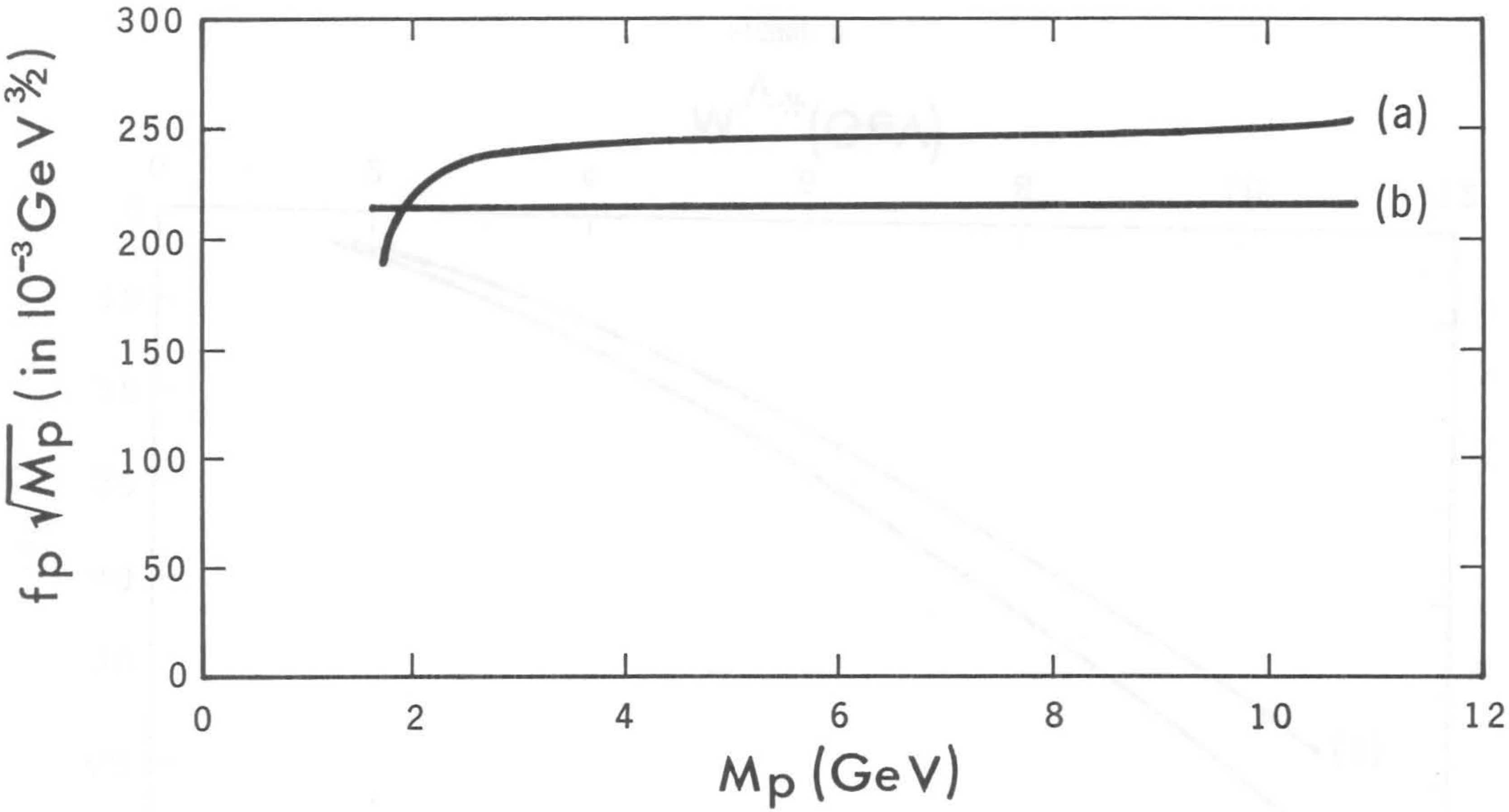
300
250
200
150
100
50
0
0
2
4
6
8
10
12
(a)
(b)
$f_p \sqrt{M_p}$ (in 10^{-3} GeV $^{3/2}$)
M_p (GeV)

FIGURE 2

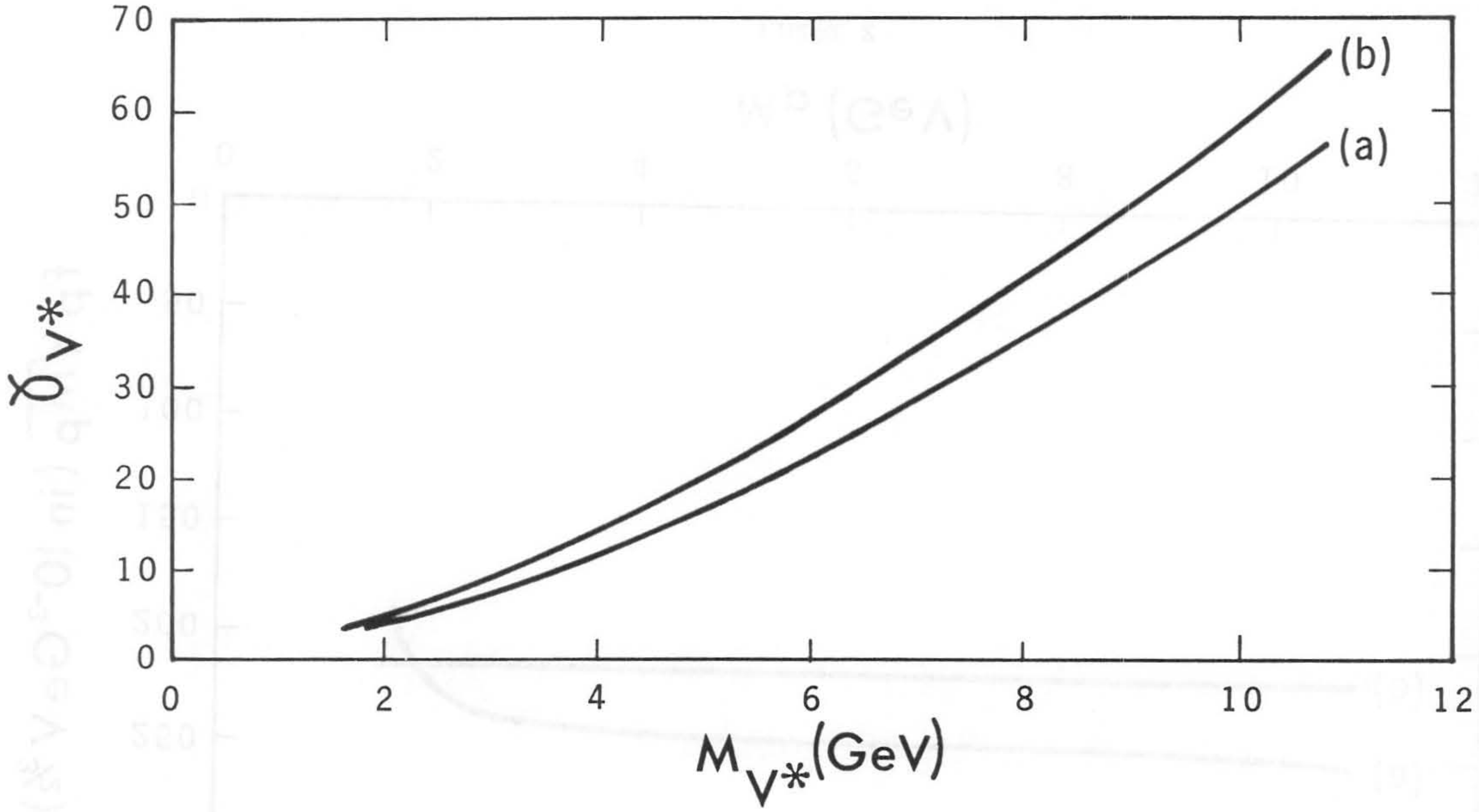

FIGURE 3

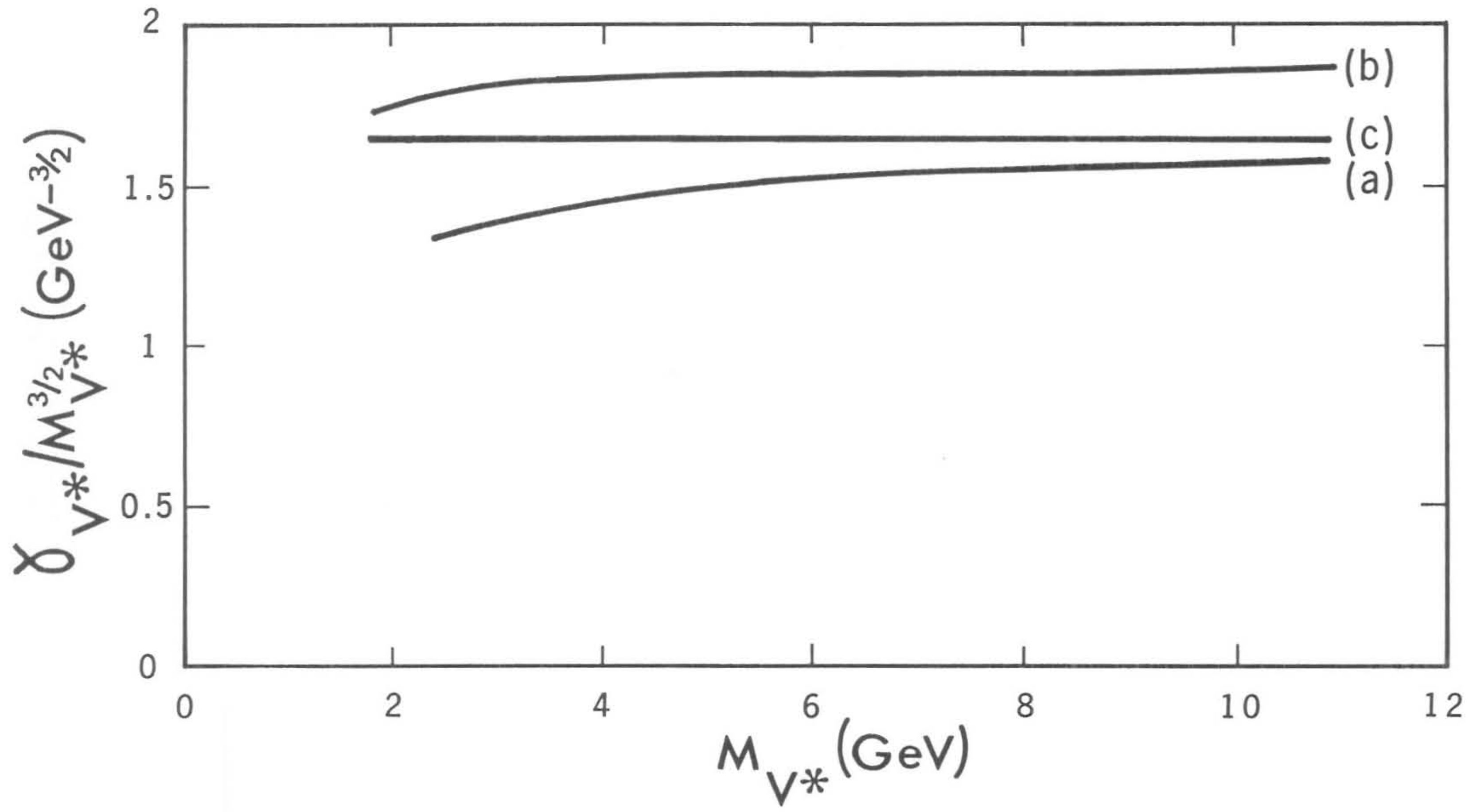

FIGURE 4