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THE DELPHI VERTEX DETECTOR ALIGNMENT: A PEDAGOGICAL STATISTICAL EXERCISE

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ABSTRACT

We describe a statistical method for the Delphi MicroVertex detector alignment using interaction tracks. The general principles the method relies on are discussed and Montecarlo results are also shown.

Inside the DELPHI collaboration (Ref. 1) it has been conceived a detector that can guarantee a highly precise measurement of a charged particle impact point. This is the so called VERTEX DETECTOR (VD, from now on). It is placed as close as possible to the interaction point, its nominal intrinsic resolution is about 5 μm and it measures 2 points for each charged track in the plane orthogonal to the beam ($R\phi$ plane).

A high intrinsic resolution is a necessary but not a sufficient condition to guarantee a very precise measurement: if a local observer (on the detector itself) can perform good measurements but its position is poorly known with respect to the other tracking devices then the precision is degraded.

The Vertex Detector has been described elsewhere (Ref. 2). Here we just recall its features (Cap. 1) pertaining the second item, i.e. its alignment. We describe the general ideas about it (Cap. 2) and particularly the statistical method based on the informations coming from reconstructed charged particle trajectories (Cap. 3).

Results coming from Monte Carlo simulation are shown (Cap. 4).

Before we proceed any further we have 2 remarks:

* trying to define this procedure we have learned many basic concepts; so in this paper we will try to be pedagogical. Maybe it will help someone facing similar problems
* the DELPHI Vertex Detector is by now fully operational and the procedure we describe is going to be applied.

1. - THE DELPHI VERTEX DETECTOR

The DELPHI experiment has the typical structure of an apparatus conceived for colliding particle-antiparticle accelerated beams: the central volume detectors have cylindrical symmetry and the "end caps" guarantee an acceptance over the full solid angle (Fig. 1). Fig. 2 shows a projection of the central volume detectors in the plane othogonal to the beams (the so called "Rϕ plane"). The TPC (Time Projection Chamber) is the main tracking device: it can reconstruct track elements in three dimensions and the separation among trajectories is large enough in its sensitive volume to minimize the ambiguities. The global track fit starts from informations coming by the TPC. The OD (Outer Detector) mainly guarantees a better resolution in the particle momentum measurement while the ID (Inner Detector) increases the precision on the track fit. These tracking devices are "gas detectors": charged particle trajectories correspond to ionization trails in gas; the track is recontructed sampling and localizing the ionization.

The VD (Vertex Detector) is located between the ID and the beam pipe, inside a cylindrical volume with minimum radius 8.5 cm and a maximum one 11.5 cm. The impact point of charged particles on two sensitive layers can be reconstructed with a 5 μm precision in the plane orthogonal to the beam direction. These informations allow a better extrapolation of the fitted track to the interaction point. A quality factor for the track fit is the error on the impact parameter, i.e. the minimum distance of the extrapolated track to the interaction point; Fig. 3 shows the impact parameter error with\without the VD as a function of the track momentum.

The VD is a solid state detector: ionization is localized inside fully depleted semiconductor layers. Localization occurs with a microstrip configuration of the diodes (Fig. 4): the weighted mean of the signal on neighbouring strips around the particle track allows the measurement of the impact point in the coordinate transverse to the strip direction.

An assonometric view of VD and a schematic projection on the Rϕ plane are shown in Fig. 5a, b. The main features of the design are:

1) a cylindrical symmetry; the axis coincides with the beam line (z axis in the standard DELPHI conventions); the dimensions of the detector correspond to a full azimuthal acceptance and a polar one in (45°; 135°);
2) the chamber consists of 2 half shells (separated along the vertical plane), to allow its insertion after the beam pipe has been mounted;
3) the volume is defined by 24 sectors (15° acceptance each);
4) for each sector we have 2 parallel sensitive modules, defining the so called inner and outer layer. We have to remark the modules are asymmetric with respect to the normal line going through the centre of DELPHI (i.e. the origin of the general DELPHI reference frame, Fig. 5 b).
5) Within each module there are 4 detector plates, with strips parallel to the z axis (Fig. 5c). So the measured coordinate is the transverse one, corresponding to the Rϕ plane;

6) the 2 detector plates in the +(-) z hemisphere are electrically connected, strip by strip, and the signals are read out by custom designed chip (Ref. 3). The strip pitch is 25 μm, the readout pitch is 50 μm. There are 512 x 2 readout channels on an inner layer module, 640 x 2 for an outer layer one.

The modularity has been chosen to reduce the degradation of the intrinsic resolution by inclined tracks The asymmetry with respect to the normal is the optimal solution to let the DELPHI magnetic field (parallel to the beam axis) shrink the ionization charge distribution for inclined tracks.

2. - THE ALIGNMENT QUESTION

Any unknown deformation of the mechanical structure and deviation from the nominal position inside DELPHI implies degraded measurements with respect to the intrinsic resolution of the microstrip detectors. So mapping and recovering the position of the Vertex Detector is a task as important as building the chamber itself, since uncertainties at the micron level have to be guaranteed as well.

A rather articulated procedure has been defined to achieve this goal and two different aspects can be distinguished:

* **mapping of the structure**. Once the detector is ready, before the insertion in DELPHI, a full mapping of the surfaces has to be done, looking for any disagreement between nominal and actual characteristics of the design. The position of the detector plates in a module and fiducial marks on it are measured by a microscope; then the position of the module on the mechanical structure of the Vertex Detector is defined by a three dimensional metrological machine (Ref. 4).
* **alignment of the Vertex Detector**. After the insertion in DELPHI, we have to localize the chamber inside its fiducial volume and we have to monitor the stability of its position, tracing a time evolution. Since the general DELPHI reference frame is defined by the TPC, we will often refer to this problem as to the alignment (with respect to the TPC) question.

The alignment procedure has to be flexible enough to face a wide range of possible situations, determined by the STABILITY TIME INTERVAL of the detector position. Breaking of a stability situation can come by temperature variations, mechanical vibrations, deformations in the Inner Detector inner wall (where the VD insertion rails are glued) and other unforeseeable effects.

So the first element in the recipe has to be a hardware device or a software method working as an alarm bell when a steady state situation breaks down. Then, within a stability time interval, we can think of a statistical method to recover the actual position of the detector.

A canonical statistical way relies on charged particle tracks. If a trajectory is reconstructed irrespective of the detector to be aligned then it can be extrapolated (or interpolated) to its reference surface. The residue between the extrapolated point and the one reconstructed by the current detector has a functional dependence on the geometrical misalignment and the track parameters. A careful analysis of the residue distribution allows the calculation of the actual position of a detector with a precision limited by the available statistical sample. Of course the achievable precision for an equal number of degrees of freedom depends on the precision of the extrapolated informations and the detector intrinsic resolution.

Anyhow, if alignment uncertainties have to induce just a few micron error on a measured charged particle impact point we reasonably need a number of tracks per module corresponding to 24-48 hours of data taking (see below). Since it is very hard to state "a priori" whether such a stability time interval makes sense, the "alarm bell" devices have been conceived themselves as real position sensors.

There are 2 complementary hardware survey systems:

1) an array of infrared laser spot. They are attached to the Inner Detector support tube and they shine directly on the outer layer detectors (Fig. 6). The variation of the centroid coordinate of the spot marks a break of the steady state
2) a system of probes measuring local value of capacity (Ref. 5). The sensors are placed on the VD supporting endrings (Fig. 7) and the corresponding ground electrodes are glued on the ID wall (Fig. 8). Since the capacity of the probe-electrode system depends on geometrical parameters (mainly the gap and the overlapped surface), we can recover global informations on the VD position variation from local capacity measurements. The procedure is quite complex and a careful analysis on systematic effects is needed (Ref. 6).

Both systems can provide a few micron precision on a single point and measurements can be done at a 1 Hz frequency, thus they guarantee a high sensitivity to any "alarm" situation. Furthermore the set of measurements can be quite helpful to reconstruct any global VD position variation. Anyhow they are affected by basic limitations such as:

* they can monitor just relative positions and an independent "bootstrap" procedure is needed anyway
* the number of surveyed points does not allow a good quality fit at the single module level, thus a rigid body behaviour has to be assumed for the whole detector and local effects can not be recovered.

So we can say that the stability time interval determines the relative importance of any statistical method versus the instrumental one and the "granularity" of the alignment procedure. At one extreme we could have to face mechanical vibrations at the Hertz level; then we just might use the informations from the survey systems, assuming the VD as a rigid body. On the other hand we could have a weekly long steady state or more; then we can really trust the statistical method, even distinguishing details in a module.

3. - FUNDAMENTALS OF THE RESIDUE ANALYSIS

As we already said, alignment criteria can be based on the comparison among informations on charged particle trajectories independently reconstructed in DELPHI and the Vertex Detector.

We can get an analytical solution of the alignment question assuming "small" deviations from the nominal position: we linearize the functional dependence of the residue on the geometrical parameters of the actual position with respect to the nominal one (first order in the McLaurin expansion).

This has to be a reasonable assumption when the actual position of every single module in VD is independently looked for, irrespective of the systematics by the whole chamber localization in the DELPHI reference frame. And indeed we assume "mapping" has been succesful and we already "bootstrapped" the procedure determining the Vertex Detector as a whole position. The procedure we are going to describe can be applied for this very first step too, once a single set of six parameters determines the misposition of every local reference frame. The major challenge in this case comes from the possibility of having finite and not infinitesimal parameter values. If it is so then numerical solution to the χ^2 minimization have to be looked for.

3.1 - Preliminary Considerations

The projection of one of the horizontal VD modules in the plane orthogonal to the beam is shown in Fig. 9. For any other module the same representation applies in a reference frame rotated by a suitable multiple of 15°. The measured coordinate in the module reference frame is x', defined as the transverse to the strip direction.

It is worthwhile to underline that an unknown deviation from the nominal position induces an **error** on the assigned impact point in the DELPHI frame **depending on either the track geometry and the actual position of the module with respect to the assumed one** (this is an important remark expressed in Ref. 7, the very first paper where the alignment question has been faced in DELPHI) .

In Fig. 10, we show three situations corresponding to Δx, Δy translations and an α rotation around the module symmetry point. Corresponding to unknown x translations the impact point of a particle does not change and the systematic error is Δx, irrespective of the track direction. On the other hand, the effect of a y translation is an impact point variation, from P to $P_{\Delta y}$. So if mispositioning is neglected we assign the coordinate $x^{ii}{}_{P\Delta y}$ at the nominal y rather then $x^{i}{}_{P}$ and the systematic error is

$$\frac{\Delta y}{tg(\phi)} = x'_{P} - x^{ii}{}_{P\Delta y}$$

depending on the track projected direction. If we consider an α rotation around the z axis then we have either a variation of the impact point and a change in the local x axis, so the error is:

$$x^{iii}p_\alpha - x'p = x'p \left[\frac{1}{\cos(\alpha) - \sin(\alpha)\, tg(\frac{\pi}{2} - \phi)} - 1 \right]$$

A second remark concerns the residue distribution. For each reconstructed charged particle track we can extrapolate the trajectory onto the VD module sensitive surface. The residue is defined as the difference between the coordinate transverse to the strip of the extrapolated impact point (x'_{extra}) and the one measured by the VD itself (x'_{VD}). If the VD current module is not affected by any misalignment, the residue distribution will reasonably be gaussian with $< x'_{extra} - x'_{VD} > = 0$; the variance should be $\sigma^2_{RES} = \sigma^2_{x'(VD)} + \sigma^2_{x'(extra)}$, i.e. the sum in quadrature of the VD intrinsic resolution and the error on the extrapolated intersection point.

Because of a neglected deviation from the ideal position the average value of the residue distribution is shifted and the widht increased, i.e. there are a **systematic average error** and a **degraded measurement precision**. As we will see below, we can distinguish geometrical parameters of the actual position affecting just the average value shift or the broadening or both. As example, we consider the effect of a Δy neglected translation.

Since

$$< tg\, \omega > = \frac{\int_0^\beta d\omega\, tg(\omega)}{\int_0^\beta d\omega} = \frac{1}{\beta} \int_0^{tg(\beta)} dx \frac{x}{1+x^2} = \frac{1}{2\beta} \ln(1 + tg^2(\beta))$$

$$< tg^2\, \omega > = \frac{\int_0^\beta d\omega\, tg^2(\omega)}{\beta} =$$

$$= \frac{1}{\beta} \int_0^{tg(\beta)} dx \frac{x^2}{1+x^2} = \frac{tg(\beta)}{\beta} - \frac{brctg(\, tg(\beta)\,)}{\beta} = \frac{tg(\beta)}{\beta} - 1$$

where

$$\omega = \frac{\pi}{2} - \phi; \quad \beta = 15°$$

we have an extra-contribution to the distribution r.m.s.

$$\sigma_{res} = \Delta y \left[\frac{tg(\beta)}{\beta} - 1 - \left(\frac{1}{2\beta} \ln(1+tg^2(\beta)) \right)^2 \right]^{1/2}$$

i.e. about 7 μm for Δy =100 μm uncertainty in the y position of the module. The effects by Δx, Δz translations and by the three rotations can be calculated in a similar way.

So we can get rid of the average systematic error by an unknown misalignment just recovering the shift of the most probable value of the residue distribution. But to fully exploit the intrinsic resolution of the detector we have to determine the values of the 6 correlated geometrical parameters defining the actual position of the module and the residue has to be corrected on a track by track basis.

3.2. - Explicit Calculation of the Residue

The module position is identified by its local reference frame (LRF, Fig. 11). The LRF origin is in the symmetry point of the module and the axis orientation is defined such as x_{loc} corresponds to the measured coordinate, y_{loc} is the normal direction to the module, z_{loc} has still the beam direction (parallel to z_{DELPHI}). In the ideal geometry the LRF origins stay on a circumpherence having a radius

R_{inner} = 8.740 cm
R_{outer} = 10.891 cm

respectively for the inner and outer layer modules.

The actual position is parametrized with respect to the ideal LRF; since we assume a rigid body movement, we have to recover 6 unknown quantities:

* three translations (Δx, Δy, Δz) defining the actual local reference frame origin with respect to the ideal one.

* three rotations (ε_x, ε_y, ε_z) defining the axis orientation. In the actual position the axis are defined with same criteria as for the ideal LRF.

Instead of the Euler angles we perform rotations around the coordinate axis; this choice is more suitable for "small" angle approximation. The rotation matrix from the ideal to the actual module reference frame is

$$\begin{bmatrix} c(\varepsilon_y)c(\varepsilon_z) & c(\varepsilon_z)s(\varepsilon_x)s(\varepsilon_y)+s(\varepsilon_z)c(\varepsilon_x) & s(\varepsilon_z)s(\varepsilon_x)-s(\varepsilon_y)c(\varepsilon_x)c(\varepsilon_z) \\ -s(\varepsilon_z)c(\varepsilon_y) & -s(\varepsilon_x)s(\varepsilon_y)s(\varepsilon_z)+c(\varepsilon_x)c(\varepsilon_z) & s(\varepsilon_z)s(\varepsilon_y)c(\varepsilon_x)+s(\varepsilon_x)c(\varepsilon_z) \\ s(\varepsilon_y) & -s(\varepsilon_x)c(\varepsilon_y) & c(\varepsilon_x)c(\varepsilon_y) \end{bmatrix}$$

where c stands for cos( ) and s for sin( ); for small angles it reduces to

$$\begin{bmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{bmatrix}$$

From now on we drop the "loc" subscript to identify the ideal LRF and we label the actual one as "act" if it is needed. We can write the actual module indefinite plane equation Σ in the ideal LRF as

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{x_0}) = 0$$

where

$$\mathbf{n} = M^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} n_x &= -\cos(\varepsilon_y)\ \sin(\varepsilon_z) \\ n_y &= -\sin(\varepsilon_x)\ \sin(\varepsilon_y)\ \sin(\varepsilon_z) + \cos(\varepsilon_x)\ \cos(\varepsilon_z) \\ n_z &= \sin(\varepsilon_x)\ \cos(\varepsilon_z) + \cos(\varepsilon_x)\ \sin(\varepsilon_y)\ \sin(\varepsilon_z) \end{aligned}$$

is the normal versor to the module plane and

$$\begin{aligned} x_0 &= \Delta x \\ y_0 &= \Delta y \\ z_0 &= \Delta z \end{aligned}$$

identifies the origin.

We have now to extrapolate the reconstructed trajectory by the outward tracking chambers to this indefinite plane. As a result of the track fit we get the charge, the mass, the intersection point coordinates $\mathbf{x}_{ref}$ and the 3-momenta p on reference cylindrical surfaces at different radii. As far as the VD is concerned the reference surfaces are at 9 and 11 cm, and they are intersected by the module planes. We then consider the straight line t passing by

$$(x_{ref}, y_{ref}, z_{ref})$$

and having the direction corresponding to

$$(p_x, p_y, p_z)$$

(in the ideal module reference frame); its equation can be written as:

$$y - y_{ref} = \frac{p_y}{p_x} (x - x_{ref})$$

$$x - x_{ref} = \frac{p_x}{p_z} (z - z_{ref})$$

and we calculate the intersection between Σ and t. Approximating the helix trajectory to a straight line over a few hundred microns for tracks with more than 1 GeV momentum is an acceptable approximation; furthermore we linearize the intersection point dependence on the 6 geometrical parameters.

So at zero-th order we get

$$x_{int}(0) = x_{ref} - \frac{p_x}{p_y} y_{ref}$$

$$y_{int}(0) = 0$$

$$z_{int}(0) = z_{ref} - \frac{p_z}{p_y} y_{ref}$$

corresponding to the intersection with the ideal VD module; the first order corrections can be expressed through the following derivatives

* x coordinate:

$$\frac{\partial x_{int}}{\partial(\Delta x)} = 0; \qquad \frac{\partial x_{int}}{\partial(\Delta y)} = 0; \qquad \frac{\partial x_{int}}{\partial(\Delta z)} = 0;$$

$$\frac{\partial x_{int}}{\partial(\varepsilon x)} = \frac{p_x}{p_y}\left[\frac{p_z}{p_y} y_{ref} - z_{ref}\right];$$

$$\frac{\partial x_{int}}{\partial(\varepsilon y)} = 0; \qquad \frac{\partial x_{int}}{\partial(\varepsilon z)} = -\left(\frac{p_x}{p_y}\right)^2 \left(y_{ref} - \frac{p_y}{p_x} x_{ref}\right)$$

* y coordinate

$$\frac{\partial y_{int}}{\partial(\Delta x)} = 0; \qquad \frac{\partial y_{int}}{\partial(\Delta y)} = 1 \qquad \frac{\partial y_{int}}{\partial(\Delta z)} = 0;$$

$$\frac{\partial y_{int}}{\partial(\varepsilon x)} = \frac{p_z}{p_y} y_{ref} - z_{ref}; \qquad \frac{\partial y_{int}}{\partial(\varepsilon y)} = 0;$$

$$\frac{\partial y_{int}}{\partial(\varepsilon z)} = -\frac{p_x}{p_y}\left(y_{ref} - \frac{p_y}{p_x} x_{ref}\right)$$

* z coordinate

$$\frac{\partial z_{int}}{\partial(\Delta x)} = 0; \qquad \frac{\partial z_{int}}{\partial(\Delta y)} = \frac{p_z}{p_y} \qquad \frac{\partial z_{int}}{\partial(\Delta z)} = 0;$$

$$\frac{\partial z_{int}}{\partial(\varepsilon x)} = \frac{p_z}{p_y}\left(\frac{p_z}{p_y} y_{ref} - z_{ref}\right);$$

$$\frac{\partial z_{int}}{\partial(\varepsilon z)} = \frac{p_z}{p_x}\left(\frac{p_x}{p_y}\right)^2 \left(y_{ref} - \frac{p_y}{p_x} x_{ref}\right)$$

As a last step we need to calculate the x coordinate of the intersection point in the actual module LRF, the only number that can be really compared to the VD measured quantity:

$$x^{act}_{int} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \end{pmatrix} \begin{pmatrix} x_{int} \\ y_{int} \\ z_{int} \end{pmatrix} - \Delta x$$

and we finally write the residue as:

$$x^{act}_{int} - x^{act}_{vd} = Res = Res(0) + \Omega_k \theta_k;$$

where

$$\Omega = \left[-1 \; ; \; \frac{p_x}{p_y} ; \; 0 \; ; \; \frac{p_x}{p_y}\left(\frac{p_z}{p_y} y_{ref} - z_{ref}\right) ; \; -\left(z_{ref} - \frac{p_z}{p_y} y_{ref}\right) ; \; -\left(\frac{p_x}{p_y}\right)^2 \left(y_{ref} - \frac{p_y}{p_x} x_{ref}\right) \right]$$

$$\theta = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \qquad \mathrm{Res}(\theta_j = 0) = \left(x_{ref} - \frac{p_x}{p_y} y_{ref}\right) - x_{vd}$$

The Ω vector expresses the **residue sensitivity to every misalignment parameter** and it is track dependent. The distribution of the Ω_i for trajectories within the module acceptance is shown in Fig. 12 a-d.

Now we can point out a few remarks already introduced in a qualitative way:

* a Δx translation changes the residue irrespective of the track direction. So an x uncertainty strongly determines the average residue shift but it does not affect the distribution width.
* Δy influence is determined by p_x/p_y. Since the module acceptance is 15 °, we have $0 < p_x/p_y < 0.27$, so a normalized y uncertainty changes the residue less than a x one
* we have no first order effect by Δz. This is quite natural since z is the strip direction and the transverse coordinate is the measured one.
* as far as rotations are concerned, we have a hierarchy too, determined by the p_x/p_y factor in the corresponding $\Omega_{\varepsilon i}$:

$$\varepsilon_y \;\text{----->}\; -\left(z_{ref} - \frac{p_z}{p_y} y_{ref}\right)$$

$$\varepsilon_x \;\text{----->}\; \frac{p_x}{p_y}\left(\frac{p_z}{p_y} y_{ref} - z_{ref}\right)$$

$$\varepsilon_z \;\text{----->}\; -\left(\frac{p_x}{p_y}\right)^2 \left(y_{ref} - \frac{p_y}{p_x} x_{ref}\right)$$

* since $< \Omega_{\varepsilon y} > = < \Omega_{\varepsilon x} > = 0$, neither ε_x nor ε_y contribute to the average residue shift, they just influence the distribution width.

3.3 - Parameter Estimate

Once a suitable sample of tracks have been collected (see below), the parameters $\theta = (\Delta x, \Delta y, \varepsilon_x, \varepsilon_y, \varepsilon_z)$ can be estimated minimizing the sum in quadrature of the residues:

$$\chi^2 = \sum_{j=1}^{N} w_j \left(Res(0) + \Omega_k \theta_k \right)_j^2$$

If we define the matrices

$$A = \begin{pmatrix} Res(0)_1 \\ . \\ . \\ . \\ . \\ Res(0)_N \end{pmatrix} ; \qquad V = \begin{pmatrix} \sigma^2_1 & 0 & 0 & 0 \\ 0 & \sigma^2_2 & 0 & 0 \\ . & . & . & . \\ 0 & 0 & 0 & \sigma^2_N \end{pmatrix}$$

$$B = \begin{pmatrix} \Omega_{11} & ... & \Omega_{15} \\ . & .... & . \\ . & .... & . \\ \Omega_{N1} & ... & \Omega_{N5} \end{pmatrix}$$

then the quadratic sum reads

$$\chi^2 = (A - B\theta)^T V^{-1} (A - B\theta)$$

Minimization corresponds to the solution of the linear system

$$(B^T V^{-1} B)\, \theta = B^T V^{-1} A$$

where

$$B^T V^{-1} B = \sum_i \Omega^T{}_i\, w_i\, \Omega_i \qquad 5x5$$

$$B^T V^{-1} A = \sum_i \Omega^T{}_i\, w_i\, Res_i(0) \qquad 5x1$$

so, if the LHS matrix is not singular, we get:

$$\theta = (B^T V^{-1} B)^{-1} (B^T V^{-1} A)$$

And the covariance matrix of the estimated parameters is:

$$V_{fit} = (B^T V^{-1} B)^{-1}$$

Apart from the pure technical procedure to get the estimated parameters, we think there are a few important remarks:

1) **alignment error.** If the association between a VD measured point and an extrapolated track is defined onto the ideal position plane, then the VD information to be considered is

$$(x_{act}(\text{measured}) - \eta), \quad \eta = \Omega_i \theta_i .$$

So the error is

$$\sigma^2_{VD} = \sigma^2_{intrinsic} + \sigma^2_\eta = \sigma^2_{intrinsic}\left(1 + \frac{\sigma^2_\eta}{\sigma^2_{intrinsic}}\right)$$

i.e. the sum in quadrature of the intrinsic resolution and the alignment precision. Since

$$\sigma^2_\eta = \Omega\, V_{fit}\, \Omega^T$$

the full covariance matrix of the estimated parameters enters the η error. The number of tracks to be collected is determined by the condition

$$\frac{\sigma^2_\eta}{\sigma^2_{intrinsic}} << 1$$

and it corresponds to different precision on the single parameters, depending on their impact on the residue (see below).

2) **convergence of the estimated parameters.** Using V_{fit} we can check the consistency between the estimated parameters and the foreseen statistical error.

First of all we can rely on the diagonal elements $V_{ii} = \sigma^2_i$, the second order momenta of the marginal distributions.We do expect the ratios θ_k/σ_k, calculated as the number of degrees of freedom increases, to be distributed as a normal. Indeed σ_k's are also determined by correlation effects (see below) but if we wish to take into account the influence on the current parameter value of the other estimated ones we have to refer to the conditional distributions: their first and second order momenta do depend on the actual parameter values, on the σ^2_i and on their true value.

Let us consider 2 correlated variables with gaussian probability distribution function (p.d..f):

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{|V|}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T\, V^{-1}\, (\mathbf{x}-\mu)\right)$$

where

$x = (x_1, x_2);$ $\mu = (\mu_1, \mu_2)$ $V = \begin{pmatrix} \sigma^2{}_1 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma^2{}_2 \end{pmatrix}$

ρ = correlation coefficient $= \frac{\sigma_{xy}}{\sigma_x\sigma_y}$

The conditional p.d.f. $F(x_2 \mid x_1{}^{fixed})$ is defined as the distribution of the possible x_2 values once $x_1 = x_1{}^{fixed}$:

$$F(x_2 \mid x_1 \text{ fixed}) = \frac{f(x_1 \text{ fixed}, x_2)}{h(x_1 \text{ fixed})}$$

where $h(x_1)$ is the marginal p.d.f. Since

$$h_1(x_1) = \int_{-\infty}^{+\infty} dx_2 \, f(x_1, x_2) =$$

$$= \frac{1}{\sqrt{2p}\,\sigma_1} \exp\left(- \frac{1}{2} \left(\frac{x_1 - \mu_1}{\sigma_1} \right)^2 \right)$$

F simply reads

$$F(x_2 \mid x_1 \text{ fixed}) = \frac{1}{\sqrt{2\pi}\,\sigma_{eff}} \exp\left(- \frac{1}{2} \left(\frac{x_2 - \mu_{eff}}{\sigma_{eff}} \right)^2 \right)$$

so it corresponds to a normal distribution with squared mean

$$\sigma^2{}_{eff} = \sigma^2{}_2 \, (1-\rho^2)$$

and average value

$$\mu_{eff} = \mu_2 + \rho \frac{\sigma_2}{\sigma_1} (x_1 \text{ fixed} - \mu_1)$$

and the correlation among the measured quantities is expressed by the functional dependence of the x_2 conditional p.d.f. parameters on the actual x_1 value and its marginal distribution quantities.

Extending the definition to the 5 parameters situation, we can write

$$F(x_1 \mid x_{2f}, x_{3f}, x_{4f}, x_{5f}) = \frac{f(x_1, x_{if};\ i=2,5)}{h(x_{kf};\ k=2,5)}$$

where the marginal distribution h is characterized by a covariance matrix $V_{reduced}$ deducted from V, cancelling row i and column j. So the generalized conditional p.d.f. has a variance

$$\sigma^2_{i,conditional} = \frac{|V|}{|V_{reduced}|}$$

while the exponential function argument reads

$$\left[(\mathbf{x}-\mu)^T{}_m \; V^{-1}{}_{mn} \; (\mathbf{x}-\mu)_n \right] - (\mathbf{x}-\mu)^T{}_l \; V^{-1}{}_{lq} \; (\mathbf{x}-\mu)_q \; | \; l,q \neq i$$

Statistical significance and parameter convergence have been checked on MonteCarlo data using both marginal and conditional distributions (see chapt. 4). In the latter we have artificially set μ_i to the true values; of course this is possible just in the simulated world, where nothing can in principle escape our willing. Since Nature likes to joke on our guess, inspite of our wisdom we should better limit ourselves to the marginal distributions when real data have to be faced.

3) **statistical weight of each residue**. It has been simply calculated as the inverse of the error on the residue at null misalignment parameters. Since

$$Res(\theta_j=0) = (x_{ref} - \frac{p_x}{p_y} y_{ref}) - x_{vd}$$

then through the error propagation we get

$$\sigma^2_{Res(0)} = \sigma^2_{xref} + \sigma^2_{xvd} + \left(\frac{p_x}{p_y}\right)^2 \sigma^2_{yref} + \left(\frac{y_{ref}}{p_y}\right)^2 \sigma^2_{px} +$$

$$+ \left(\frac{p_x y_{ref}}{p^2_y}\right)^2 \sigma^2_{py} + 2\left(\frac{p_x y_{ref}}{p^2_y}\right) \sigma_{px,yref} -$$

$$-2\left(\frac{y_{ref}}{p_y}\;\frac{p_x y_{ref}}{p^2_y}\right)\sigma_{px;py} - 2\left(\frac{p_x}{p_y}\;\frac{p_x y_{ref}}{p^2_y}\right)\sigma_{py,yref}$$

3.4 - The 2 Parameter Case

In order to get a better insight into some numerical results, we considered also an oversimplified situation where we allow just Δx, Δy translations. Being so, the residue reads

$$Res(\Delta x, \Delta y) = Res(0, 0) - \Delta x + \frac{p_x}{p_y} \Delta y$$

$$Res(0, 0) = R = (x_{ref} - \frac{p_x}{p_y} y_{ref}) - x_{vd}$$

and the quantity to be minimized is

$$\chi^2 = \sum_i w_i \left(R_i - \Delta x + \alpha_i \Delta y \right)^2; \qquad \alpha_i = \left(\frac{p_x}{p_y} \right)_i$$

$$\frac{\chi^2}{N} = \langle wR^2 \rangle + \langle w \rangle \Delta x^2 + \langle w\alpha^2 \rangle \Delta y^2 - 2\langle wR \rangle \Delta x -$$

$$- 2 \langle w\alpha \rangle \Delta x \Delta y + 2 \langle wR\alpha \rangle \Delta y$$

The parameter estimates proceeds through the first order derivative calculation, defining the linear system to be solved:

$$\frac{\partial \chi^2}{\partial(\Delta x)} = 2\langle w \rangle \Delta x - 2\langle wR \rangle - 2\langle w\alpha \rangle \Delta y = 0$$

$$\frac{\partial \chi^2}{\partial(\Delta y)} = 2\langle w\alpha^2 \rangle \Delta y - 2\langle w\alpha \rangle \Delta x + 2\langle wR\alpha \rangle = 0$$

The solutions are

$$\Delta x = D_1/D \qquad \Delta y = D_2/D$$

where

$$D = \det \begin{pmatrix} \langle w \rangle & -\langle w\alpha \rangle \\ \langle w\alpha \rangle & -\langle w\alpha^2 \rangle \end{pmatrix}$$

$$D_1 = \det \begin{pmatrix} \langle wR \rangle & -\langle w\alpha \rangle \\ \langle wR\alpha \rangle & -\langle w\alpha^2 \rangle \end{pmatrix} \qquad D_2 = \det \begin{pmatrix} \langle w \rangle & \langle wR \rangle \\ \langle w\alpha \rangle & \langle wR\alpha \rangle \end{pmatrix}$$

If we assume a constant w_i, i.e. all the tracks in the sample have the same statistical weight, then

$$D = - w \left(\langle \alpha^2 \rangle - \langle \alpha \rangle^2 \right) = -w \, \sigma^2_\alpha$$

so we have a determined system unless the sensitivity factor distribution is delta-like, i.e. the detector to be aligned has a very limited geometrical acceptance or particles in a narrow cone are selected. If it is so then we have an ill posed problem indeed, since it is $D_1 = D_2 = 0$ too. Whenever it happens we have no chance of distinguishing Δx, Δy and we just recover the average shift of the residue distribution.

In our situation $\alpha = p_x/p_y$ and it is uniformly distributed in (0, 0.27), with $\sigma_\alpha = 0.27/\sqrt{12} = 0.078$. So, apart from numerical problems, the system is determined and the solutions are

$$\Delta x = \frac{<\alpha^2><R> - <\alpha><\alpha R>}{<\alpha^2> - <\alpha>^2}$$

$$\Delta y = \frac{<\alpha><R> - <\alpha R>}{<\alpha^2> - <\alpha>^2}$$

We can realize the estimated parameter values scale by an α factor, reflecting the different contribution to the residue. We will see below this effect again in the covariance matrix.

As far as the 5 parameter case is concerned, we have to say that the matrix $B^T V^{-1} B$ is not singular but the determinant is very small so to get reliable results in its inversion all the calculations have been done in double precision.

We can now turn to the error analysis; the questions we asked ourselves were:

a) how the poissonian trend scales because of the residue sensitivity to the current parameter

b) when we go from one to n parameters how the correlation changes the values of the diagonal elements of the covariance matrix.

Let us start assuming we have just one misalignment parameter, that is to say:

$$\chi^2 = \sum_k \frac{1}{\sigma^2_k} \left(Res_k(0) + \alpha_k \omega \right)^2$$

The error on its estimated value is

$$\sigma^2_\omega = \left[\sum_k \frac{\alpha^2_k}{\sigma^2_k} \right]^{-1} = \left[N < \frac{\alpha^2}{\sigma^2} > \right]^{-1} = \frac{1}{N} < \frac{\sigma^2}{\alpha^2} >$$

so if we presuppose measurement errors and sensitivity are statistically independent (this could not be true since σ is strongly determined by the extrapolation error, depending on the same quantities defining α) then

$$\sigma_\omega = \frac{1}{\sqrt{N}} \sqrt{\frac{<\sigma^2>}{<\alpha^2>}}$$

So the Poisson behaviour scales linearly with $< \sigma^2 >$ and inversely with the sensitivity r.m.s. Because of this we can state for example that, in the single parameter case or for geometrical acceptances corresponding to uncorrelation, the Δx error will be smaller than the Δy one since $\alpha = p_x/p_y$.

If both Δx and Δy are considered, then

$$\Omega^T{}_k\,\Omega_k = \begin{pmatrix} 1 & -\alpha_k \\ -\alpha_k & \alpha^2{}_k \end{pmatrix}$$

and we define the covariance matrix by the inversion of

$$V^{-1} = \sum_k \frac{1}{\sigma^2{}_k}\Omega^T{}_k\,\Omega_k = \begin{pmatrix} \sum_k \frac{1}{\sigma^2{}_k} & -\sum_k \frac{\alpha_k}{\sigma^2{}_k} \\ -\sum_k \frac{\alpha_k}{\sigma^2{}_k} & -\sum_k \frac{\alpha^2{}_k}{\sigma^2{}_k} \end{pmatrix}$$

assuming again $\sigma^2{}_k = < \sigma^2{}_{meas} >$, then

$$V^{-1} = \frac{N}{< \sigma^2{}_{mis} >}\begin{pmatrix} 1 & -<\alpha> \\ -<\alpha> & <\alpha^2> \end{pmatrix}$$

and

$$V = \frac{< \sigma^2{}_{mis} >}{N}\,\frac{1}{<\alpha^2>-<\alpha>^2}\begin{pmatrix} <\alpha^2> & <\alpha> \\ <\alpha> & 1 \end{pmatrix}$$

$$\sigma^2{}_{\Delta x} = \frac{<\sigma^2{}_{mis}>}{N}\,\frac{<\alpha^2>}{<\alpha^2>-<\alpha>^2} > \frac{<\sigma^2{}_{mis}>}{N}$$

$$\sigma^2{}_{\Delta y} = \frac{<\sigma^2{}_{mis}>}{N}\,\frac{1}{<\alpha^2>-<\alpha>^2} > \frac{<\sigma^2{}_{mis}>}{N}\,\frac{1}{<\alpha^2>}$$

and we can see how correlation increases the values of the diagonal elements of the covariance matrix. By the way, $< \alpha > = 0$, i.e. uncorrelated translations, simply corresponds to a symmetric module acceptance with respect to the DELPHI (y, z) plane (or a rotated plane by a suitable multiple of 15 °). As we stated before, the asymmetric geometry has been chosen to let the Lorentz force induced by the DELPHI magnetic field shrink the charge carrier distribution for inclined tracks.

On the other hand, we can see how the correlation effect does not change the average shift error, as expected:

$$\sigma^2{}_{<\eta>} = \Omega\,V\,\Omega^T =$$

$$= \frac{<\sigma^2_{mis}>}{N} \frac{1}{<\alpha^2>-<\alpha>^2} \left(-1 \quad <\alpha>\right) \begin{pmatrix} <\alpha^2> & <\alpha> \\ <\alpha> & 1 \end{pmatrix} \begin{pmatrix} -1 \\ <\alpha> \end{pmatrix} =$$

$$= \frac{<\sigma^2_{mis}>}{N}$$

but we also recall that to get the nominal resolution back we have to correct the residue on a track by track basis and then the full covariance matrix enter the η_k calculation.

Correlation effects are clear in the 5 parameter case too. Table I shows the estimated parameter values and their marginal variance for different configurations. We can clearly distinguish two uncorrelated subset of correlated parameters:

* ε_y, ε_z
* Δx, Δy, ε_z

We see how the second subset corresponds to the parameter defining the average shift of the residue distribution while the first set ones are responsible just for its broadening.

Table I - Estimated values of the geometrical parameters and their variances for different configurations; the results refer to NDF=1500 and the aim of the exercise is highlighting the correlation among subset of parameters.

# of parameters					
#	2	3	3	4	5
Δx	1	0.7	1.7	0.62	1.8
σ_x	2.1	2.1	3.98	2.1	3.98
Δy	8.9	5.4	18.7	5.0	19
σ_y	16.1	16	43	16.1	43
ε_y	.	0.04	.	0.008	0.008
$\sigma_{\varepsilon y}$	.	0.02	.	0.05	0.05
ε_x	.	.	.	0.25	0.25
$\sigma_{\varepsilon x}$	.	.	.	0.405	0.405
ε_z	.	.	-1.3	.	-1.3
$\sigma_{\varepsilon z}$	.	.	3.7	.	3.7

4. - Numerical results from a MonteCarlo calculation

The described procedure has been numerically tested through the DELPHI apparatus simulation (Ref. 7) and analysis program (Ref. 8). The assumed hypothesis are:

1) A 35 μm most probable measurement error on the null-parameter residue. We have a fixed 5 μm contribution by the VD intrinsic resolution and an extrapolation error; its distribution is not symmetric but it shows a long tail to larger values, corresponding to badly reconstructed tracks.
2) We have generated and tracked through DELPHI isolated charged particles, to skip any topological problem. In the following we will estimate the fraction of tracks by hadronic final states that guarantee safety conditions. We assumed a flat momentum distribution in the interval (2; 45) GeV/c; lower momentum particles have strongly influenced trajectories by Coulomb multiple scattering.
3) True geometrical parameter values are zeroes, that is to say we aimed to check the measurement precision of the nominal position using the described method. We have to remark this is a quite easy assumption; for non zero parameters a major challenge is the identification of bias in the associated tracks. In order to get rid of this problem we have to rely on very simple topologies where associations can not be ambiguous and we have to carefully cross check the experimental distributions of the sensitivity coefficients (Fig. 12) with the Montecarlo ones.

The first numerical evaluation we did concerns the 2 parameter case; for 7920 degrees of freedom (NDF) we got

$$\sigma_x(1) = 0.890\ \mu m$$
$$\sigma_y(1) = 6.931\ \mu m$$

while for NDF = 4680 we had

$$\sigma_x(2) = 1.162\ \mu m$$
$$\sigma_y(2) = 9.052\ \mu m$$

and we can remark three points:

a) $\dfrac{\sigma_x(1)}{\sigma_x(2)} = 0.766; \quad \dfrac{\sigma_y(1)}{\sigma_y(2)} = 0.766; \quad \sqrt{\dfrac{N(2)}{N(1)}} = 0.769$

so we numerically check the poissonian trend.

b) $\dfrac{\sigma_x(1)}{\sigma_y(1)} = \dfrac{\sigma_x(2)}{\sigma_y(2)} = 0.128$

since $\alpha \in (0.017;\ 0.213)$ then

$$\langle\alpha\rangle = 0.115$$

$$\sigma_\alpha = 0.196/\sqrt{12} = 5.66 \times 10\text{-}2$$

thus

$$\langle\alpha^2\rangle = 0.128$$

in full agreement with the numerical result

c) σ_y value has to be

$$\sigma_y = \frac{1}{\sqrt{N}} \frac{\sigma_{mis}}{\sigma_\alpha} = \begin{cases} 6.95 & N = 7920 \\ 9.04 & N = 4680 \end{cases}$$

as we find from the numerical experiment.

Stepping to the 5 parameter case, we started with the consistency check between the estimated values and the covariance matrix. Fig. 13 show the distribution of the parameters normalized to the corresponding variance of the marginal and conditional distribution. The histograms have been filled with values after NDF=100, for 100 cycles. Furthermore, in Fig. 14 we have the trend

$$\theta_k/\sigma_k \text{ vs NDF}$$

up to NDF=5000 and sampling frequency 1/50. They show the absence of errors in the code and check the self consistency of the procedure.

For a few NDF values, we collect in Table II the estimated parameters, their uncertainty, η and σ_η as well as their own variance. This last quantity is meaningful because it is related to the effect we mentioned since the very beginning, i.e. the dependence of the residue on the track direction too; η and σ_η distributions within a module acceptance are shown in Fig. 15 for NDF= 300, 700, and 3000.

We then considered the full covariance matrix for NDF=3000 (linear quantities in μm, angular ones in milliradians):

$$V = \begin{pmatrix} 1.728 & 76.43 & 0.402\times10^{-3} & -0.2428\times10^{-1} & -5.847 \\ * & 864.7 & 0.561\times10^{-2} & -0.3194\times10^{-1} & -69.95 \\ * & * & 0.1367\times10^{-2} & -0.9435\times10^{-2} & -0.4886\times10^{-3} \\ * & * & * & 0.8033\times10^{-1} & 0.2263\times10^{-2} \\ * & * & * & * & 6.624 \end{pmatrix}$$

it is clear how the variances scale with the "importance" of each parameter in the residue definition, according to what we remarked starting with the 2 parameter case. We can also build up a correlation Table, according to the definition

$$\rho_{ij} = V_{ij}/\sigma_i\sigma_j$$

so we have

	Δx	Δy	ε_x	ε_y	ε_z
Δx	·	0.963	4×10^{-3}	-3×10^{-2}	-1.0
Δy		·	5×10^{-3}	-4×10^{-3}	-0.924
ε_x			·	-0.901	-5×10^{-3}
ε_y				·	3×10^{-3}
ε_z					·

The Table confirms the possibility of defining two subsets of uncorrelated parameters, as we previusly stated.

TABLE II

	NDF 700	NDF 1500	NDF 3000
x (µm)	14	1.8	0.6
σ_x(µm)	5.7	3.9	2.7
y(µm)	152	19	6.0
σ_y(µm)	61	43	29
ε_y(mrad)	0.02	0.83×10^{-2}	0.85×10^{-2}
$\sigma\varepsilon_y$(mrad)	0.08	0.54x10-1	0.37x10-1
ε_x(mrad)	0.24	0.26	0.13
$\sigma\varepsilon_x$(mrad)	0.6	0.40	0.28
ε_z(mrad)	-9.6	-1.3	-0.66
$\sigma\varepsilon_z$(mrad)	5.4	3.7	2.7
$<\eta>$ (µm)	3.5	0.4	0.07
$\sqrt{<\eta^2>-<\eta>^2}$ (µm)	8.4	1.98	1.0
$<\sigma_\eta>$(µm)	4.0	2.9	1.9
$\sqrt{<\sigma^2_\eta>-<\sigma_\eta>^2}$ (µm)	2.0	1.2	0.7

Last but not least, we can estimate (Ref. 10) the time we need to collect about 1500 tracks per module, a reasonable number to get a σ_η value small enough with respect to the VD intrinsic resolution.

The fraction of tracks from hadronic events useful for alignment purposes can be calculated taking into account the following reduction factors:

1) VD acceptance (f_1=0.8)
2) dead angles in its acceptance (f_2=0.94)
3) separations between neighbouring trajectories larger than 1 mm at the reference surfaces (f_3=0.85)
4) momenta larger than 1 GeV/c (f_4=0.60)
5) single module acceptance (f_5=0.041)

Hadronic events correspond to 68% of the interactions and the average number of charged particle is about 20. If we assume a luminosity L = 6×10^{30} cm^{-2} s^{-1} and a Z° production cross section σ = 30 nbarn at its peak, then we have an interaction rate 0.18 events/s. Being so, we should collect about 3500 useful tracks per module a day, leptonic events included.

Even considering an extra 50% reduction factor, we should be able to have enough statistics in 24 - 48 hours; but we have to remark the very strong assumption we made concerning the extrapolation error about 35 µm; the larger it will be in reality the longer the period we need to integrate over.

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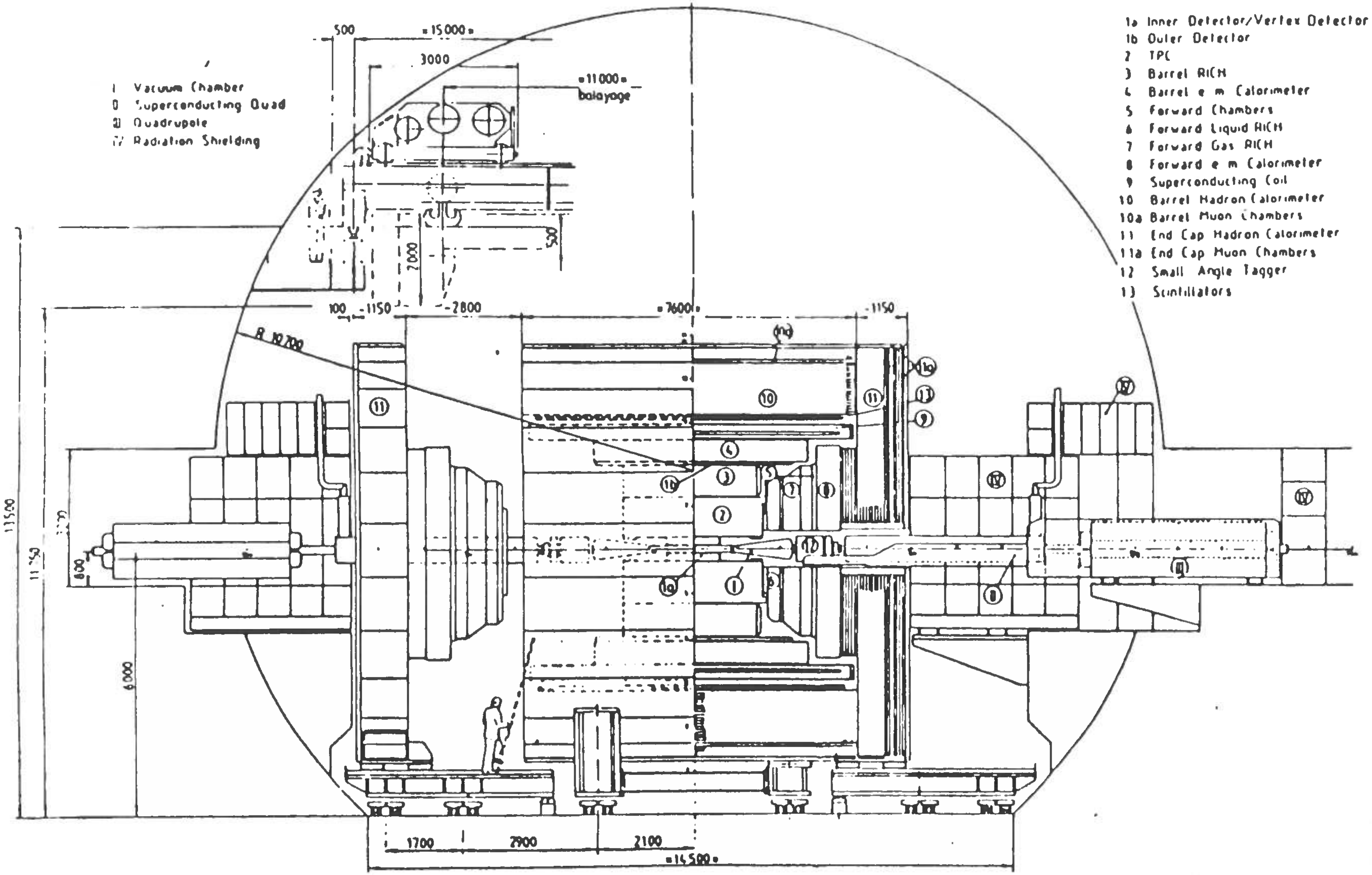


FIG. 1 - Layout of the DELPHI detector

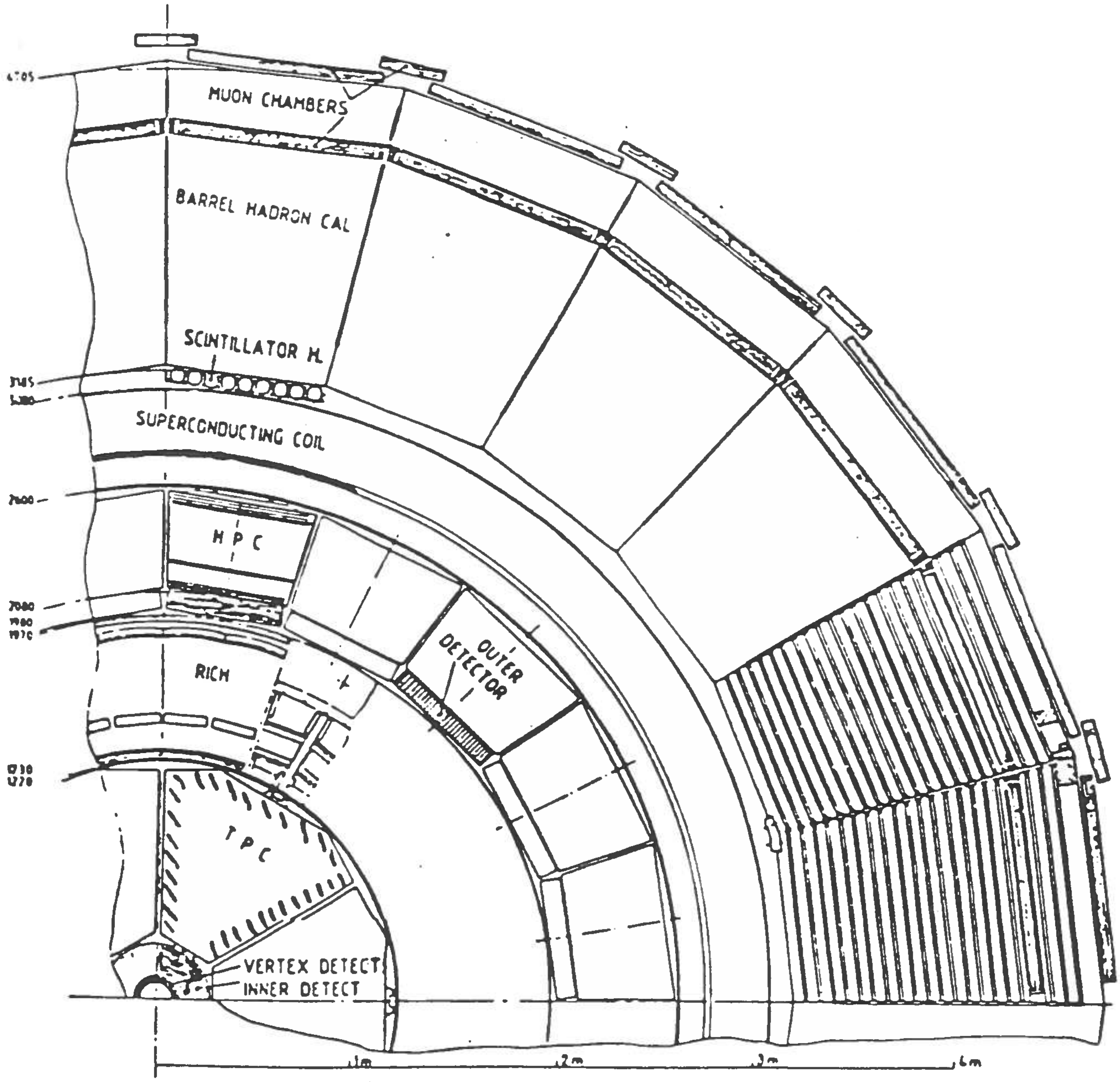


FIG. 2 - Transverse (with respect to the beam direction) view of the DELPHI barrel detectors.

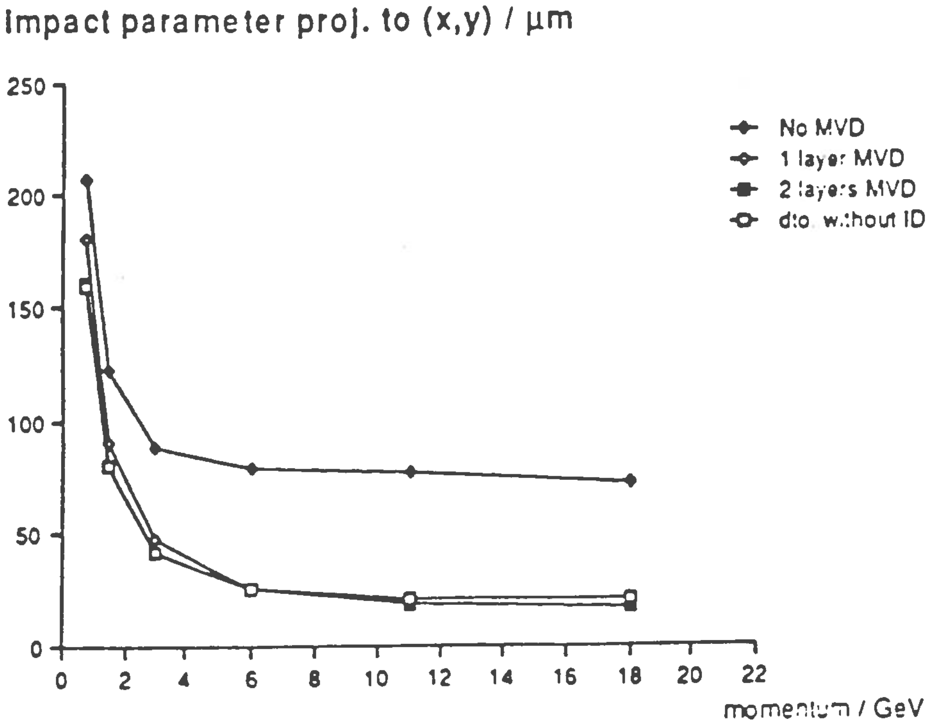


FIG. 3 - Impact parameter error in the plane orthogonal to the beams for different detector configurations as a function of the particle momentum; the assumed VD intrinsic resolution is 5 µm.

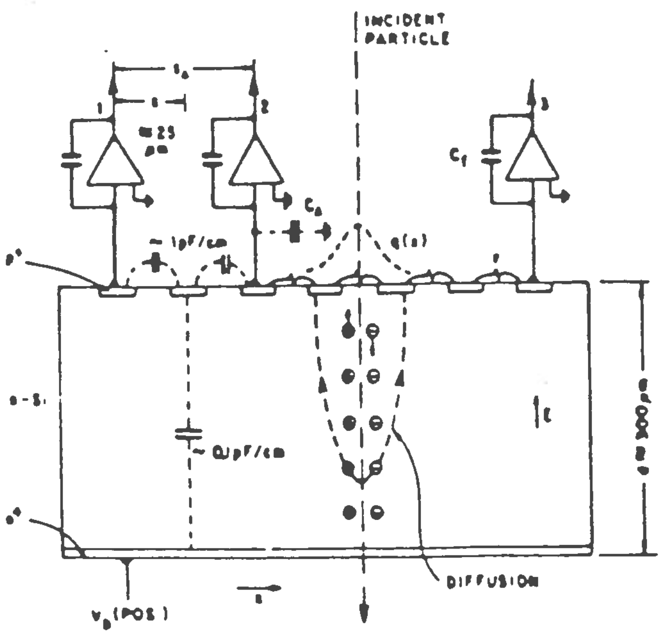


FIG. 4 - Layout of the diode configuration in the Silicon microstrip detectors.

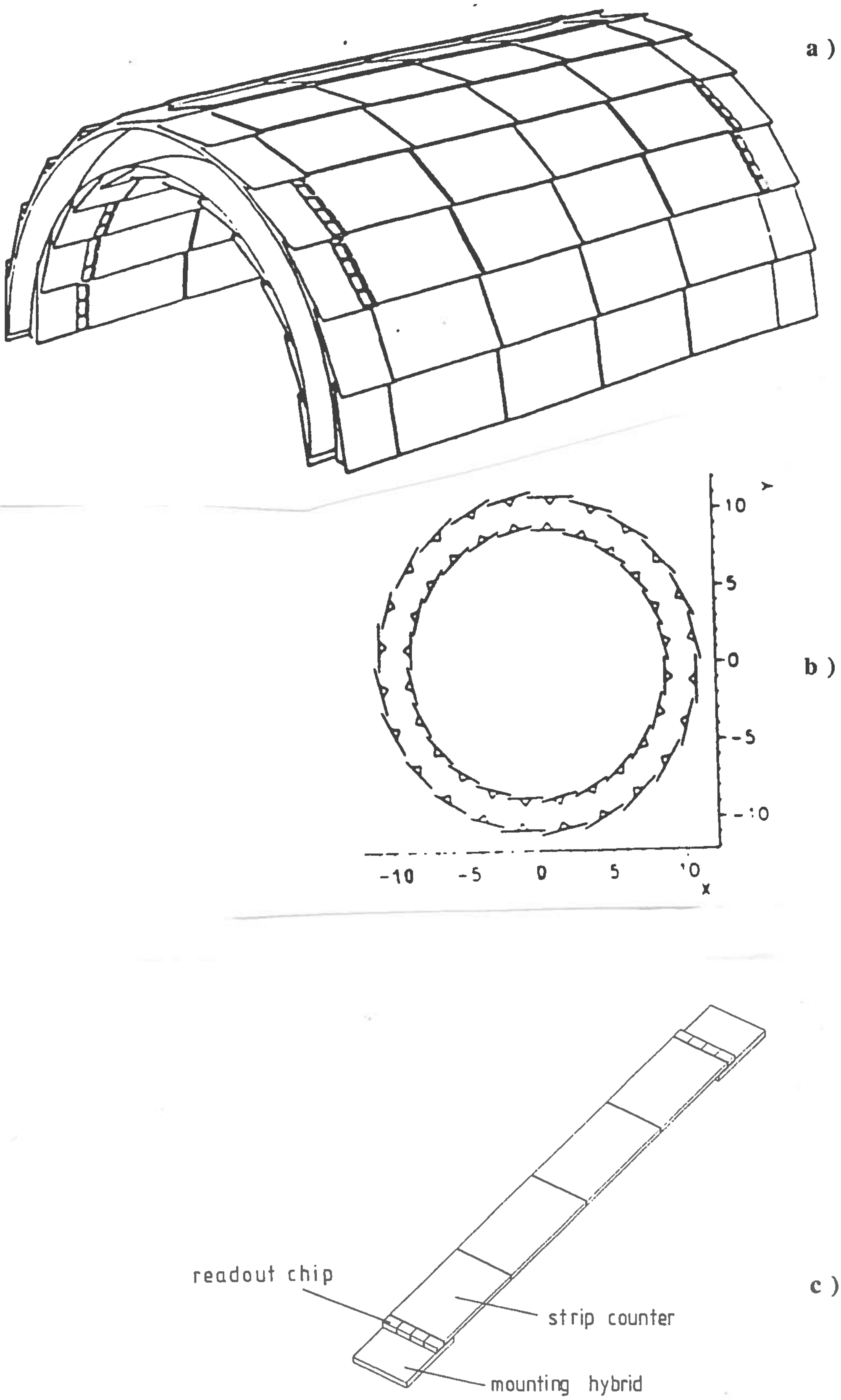


FIG. 5 - A VD half shell (a); a schematic projection of the chamber in the plane orthogonal to the beam (b) showing the module position; a single module layout (the readout chip position is also shown) (c).

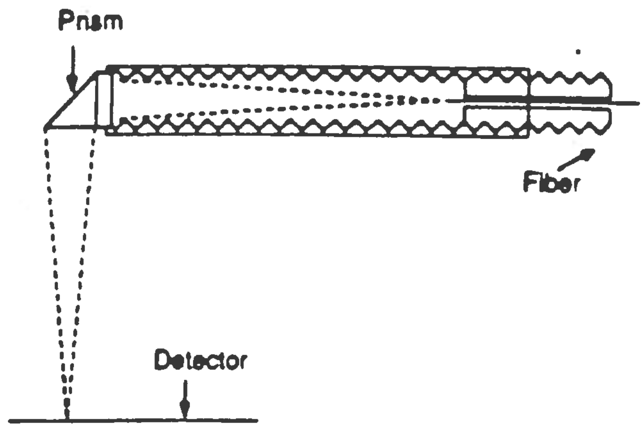


FIG. 6 - Schematic drawing of the optical fiber + microprisma system bringing laser light onto the outer layer detectors in 24 doublets of spots.

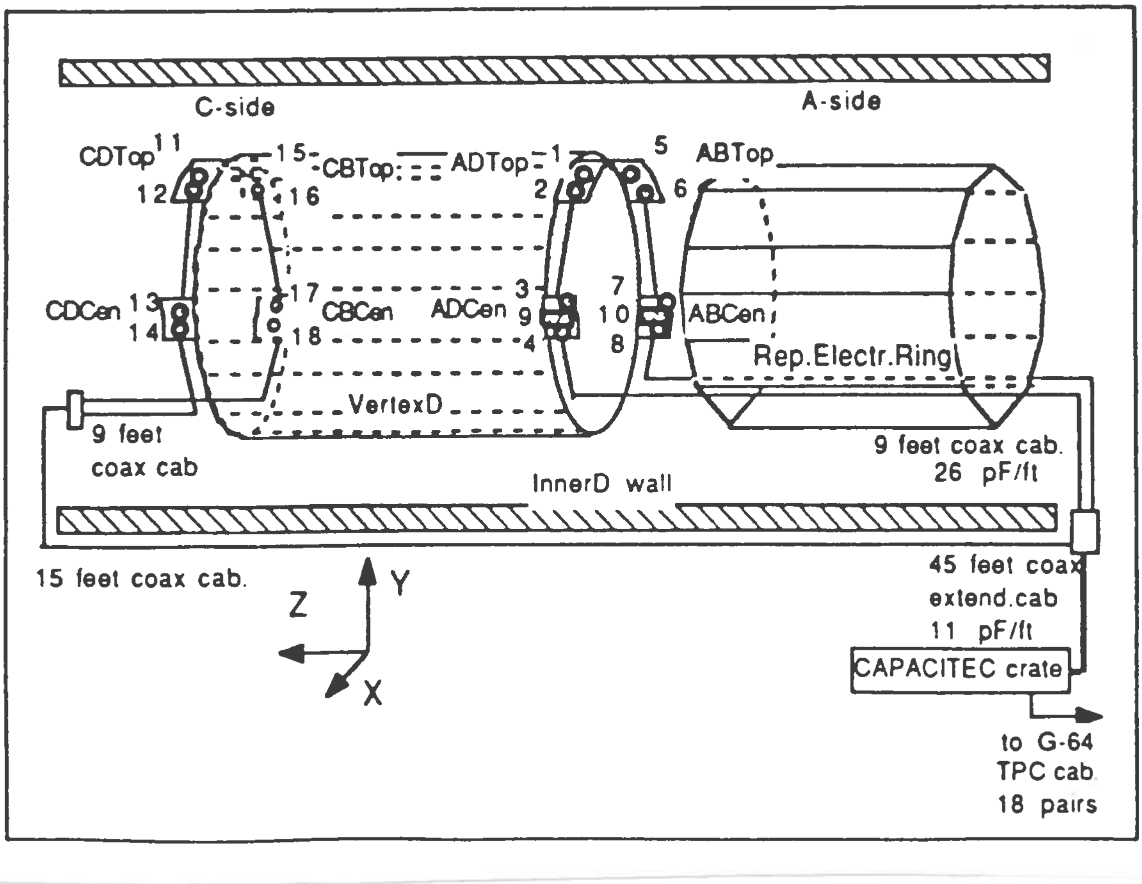


FIG. 7 - Layout of the Capacitive Probe setup on the VD.

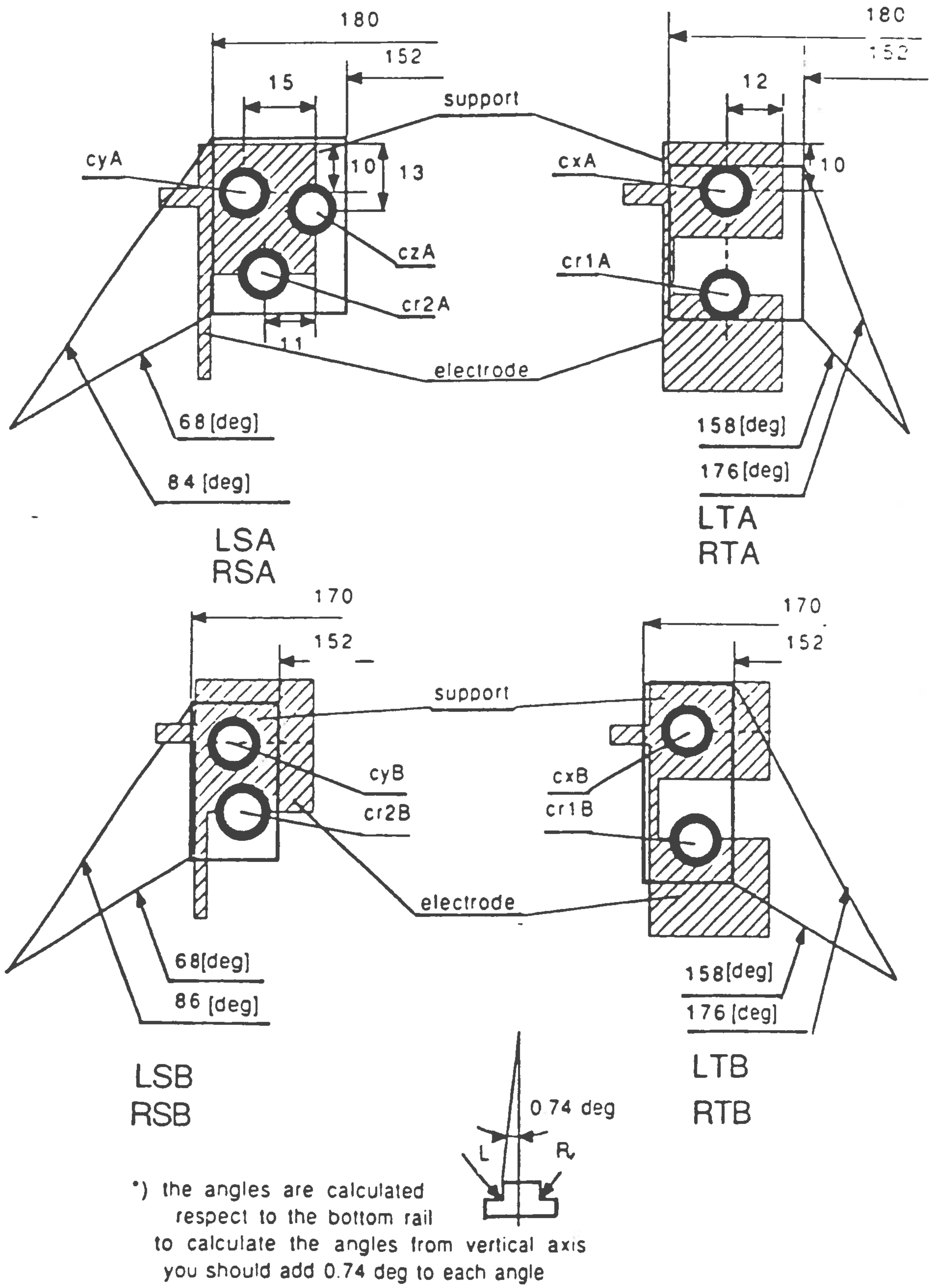


FIG. 8 - Relative position of probes and ground electrodes for the system mounted on VD.

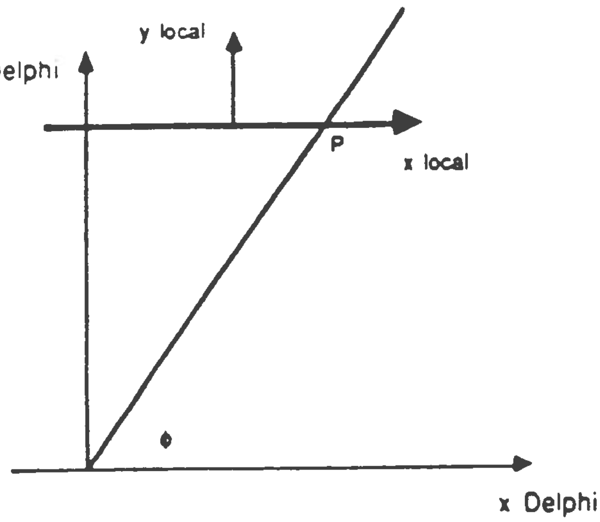


FIG. 9 - Rϕ projection of one of the horizontal VD modules

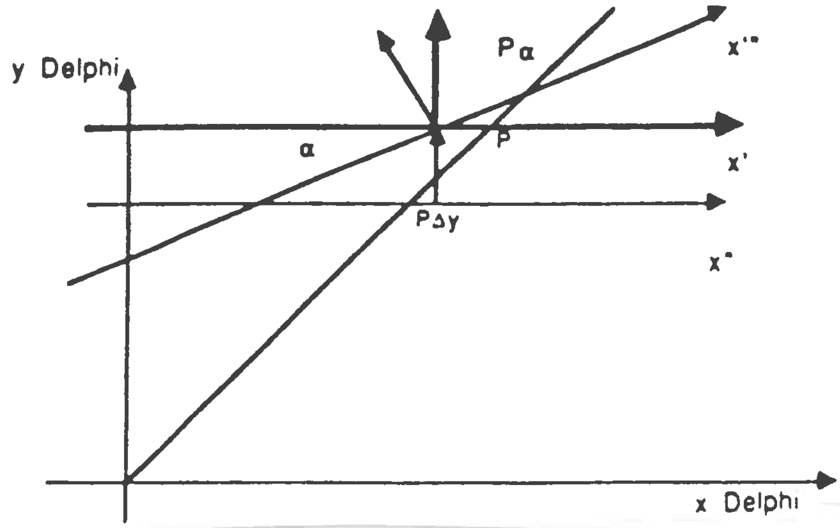


FIG. 10 - Actual position of the module local reference frame for a misalignment corresponding to a Δy translation and an α rotation around the z_{loc} axis.

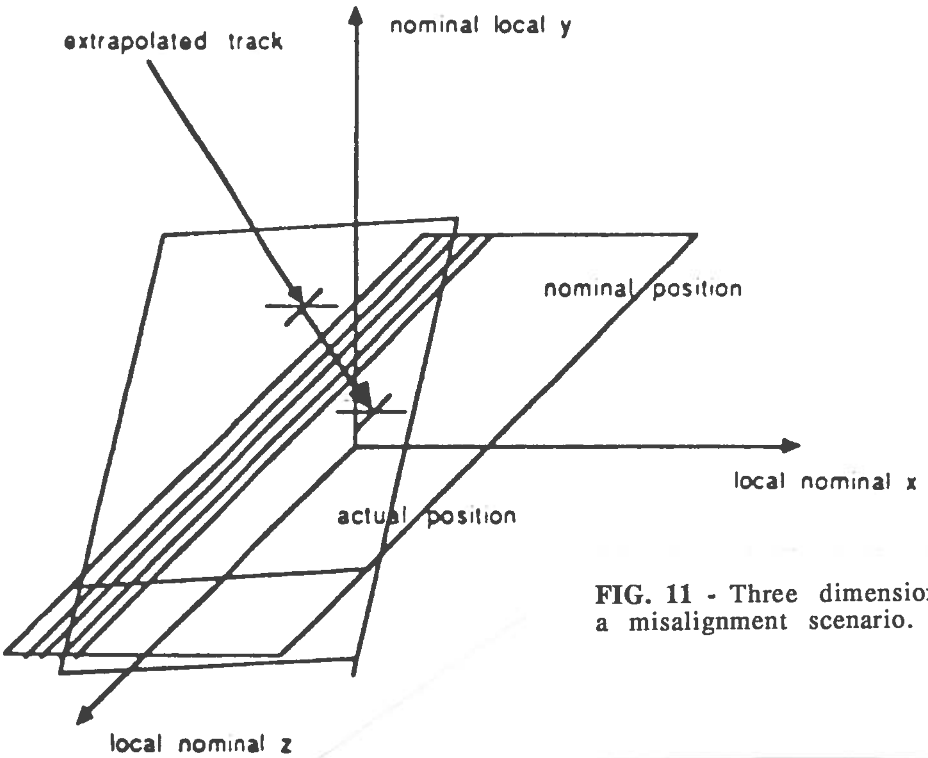


FIG. 11 - Three dimensional sketch of a misalignment scenario.

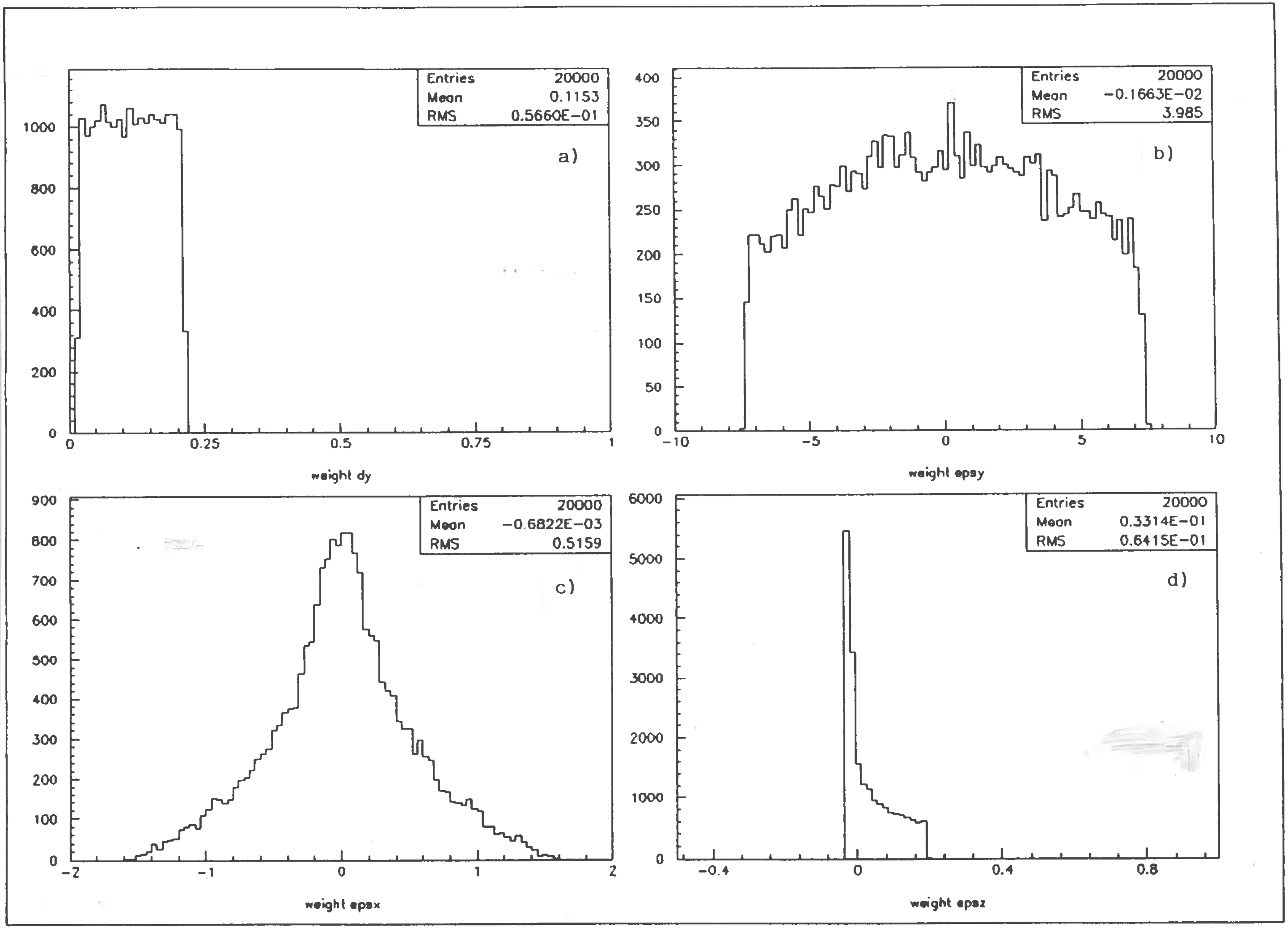


FIG. 12 - Distribution within the module acceptance of the residue sensitivity to the different misalignment parameters: a) Δy translation, b) ε_y rotation, c) ε_x rotation , d) ε_z rotation.

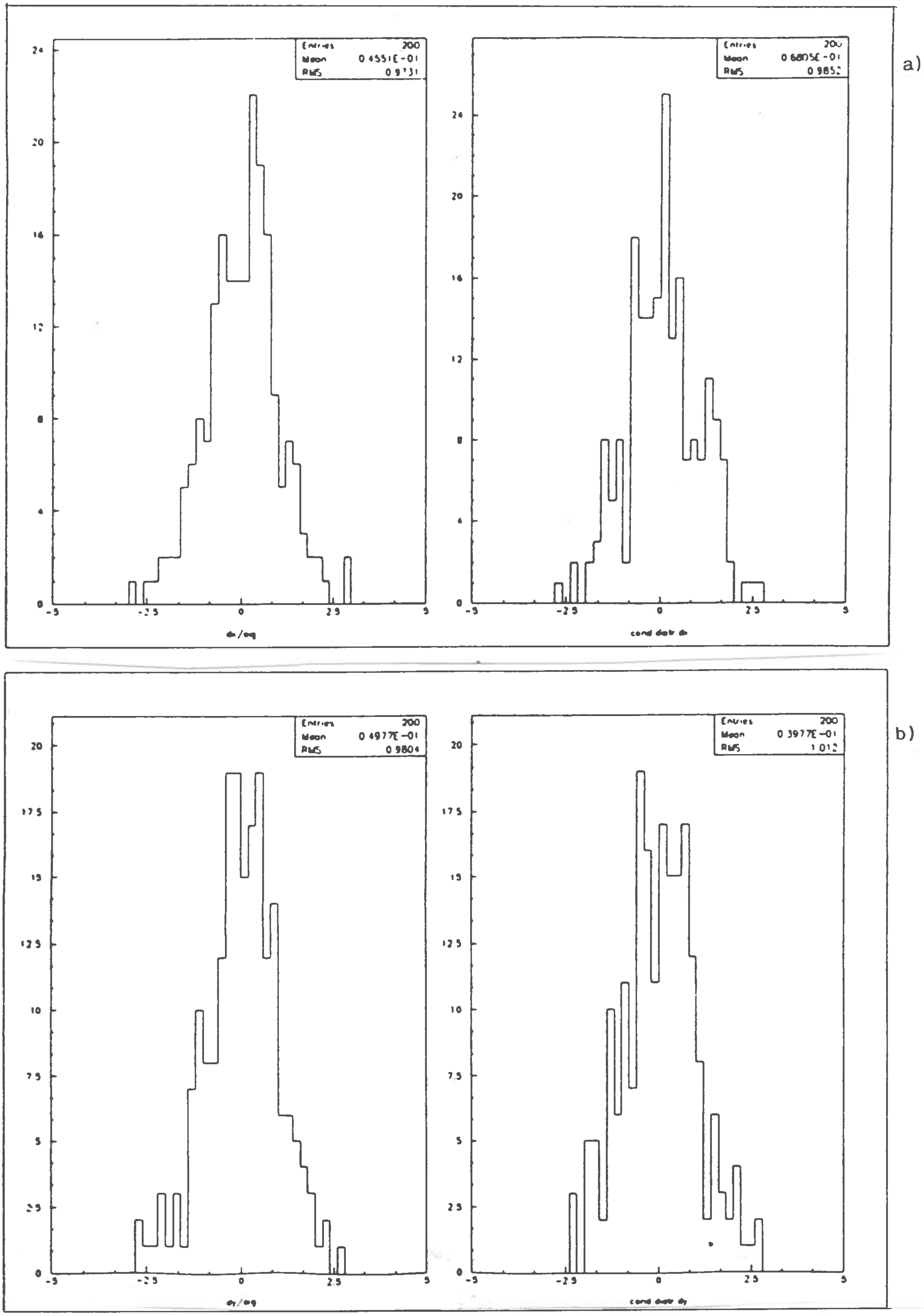
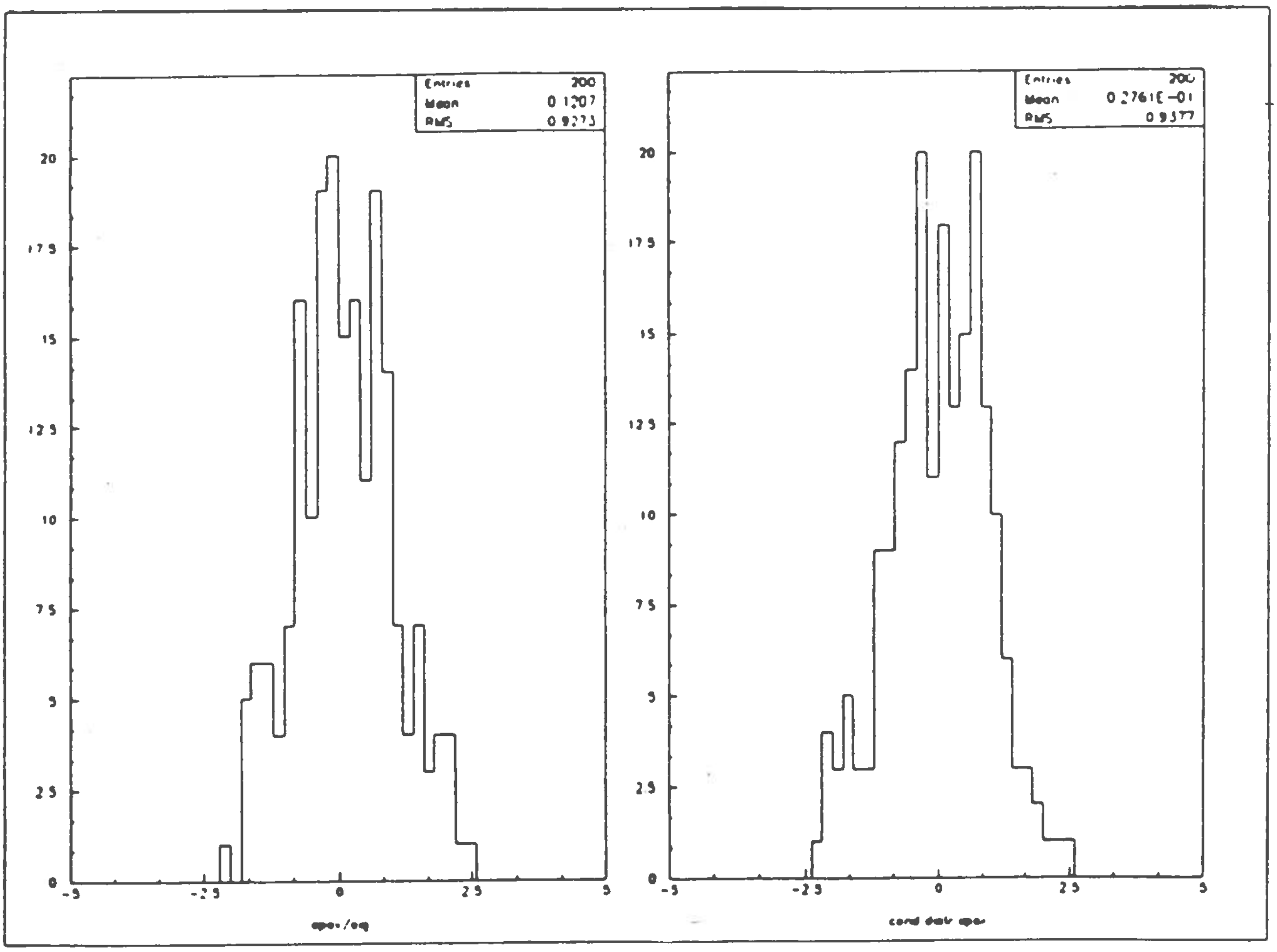
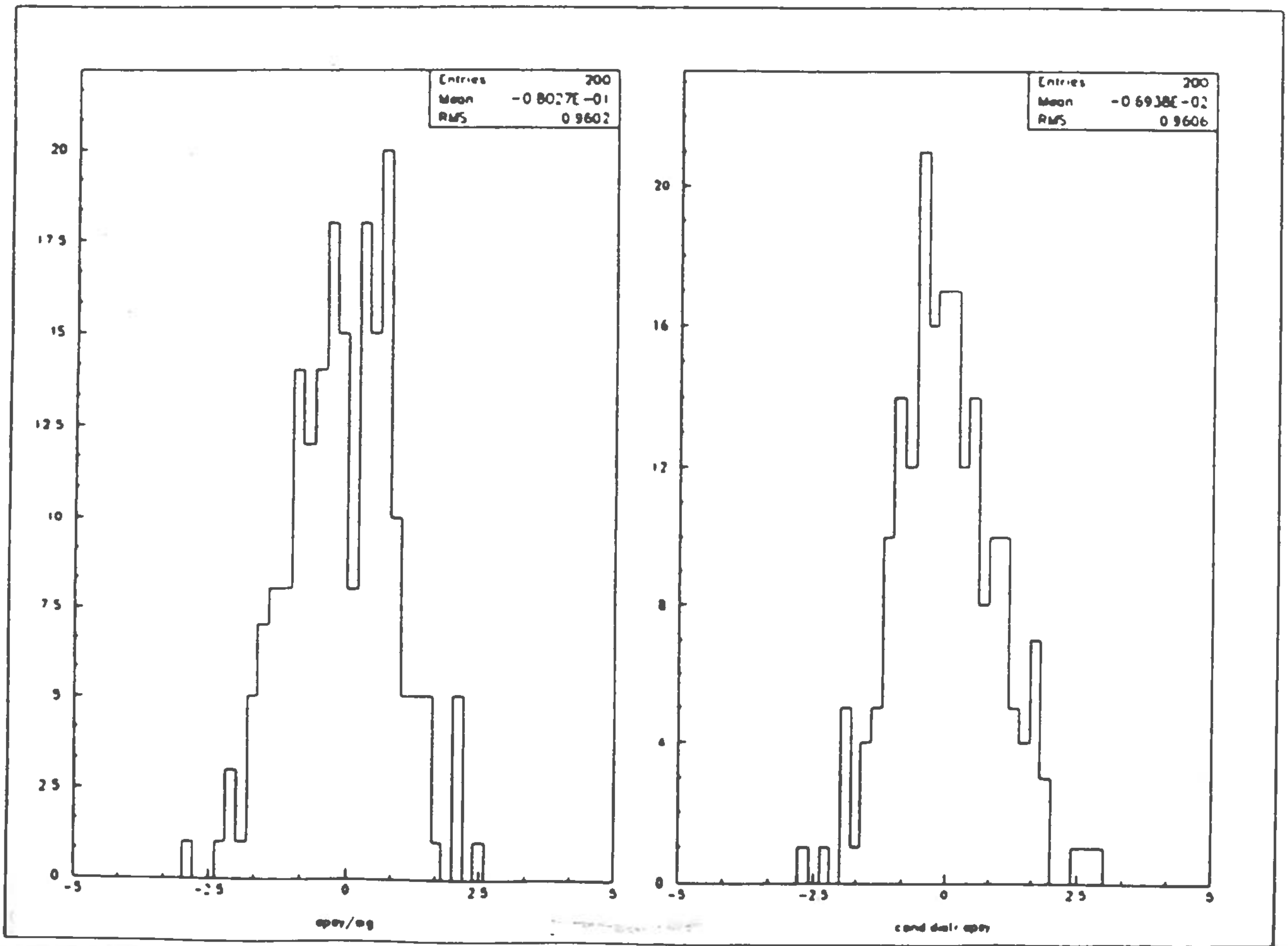

a)
Entries 200
Mean 0.4551E-01
RMS 0.9731
dx/σq
Entries 200
Mean 0.6805E-01
RMS 0.9852
cond distr dx
b)
Entries 200
Mean 0.4977E-01
RMS 0.9804
dy/σq
Entries 200
Mean 0.3977E-01
RMS 1.012
cond distr dy

FIG. 13


Entries 200
Mean 0.1207
RMS 0.9273
Entries 200
Mean 0.2761E-01
RMS 0.9377

c)


Entries 200
Mean -0.8027E-01
RMS 0.9602
Entries 200
Mean -0.6938E-02
RMS 0.9606

d)

FIG. 13

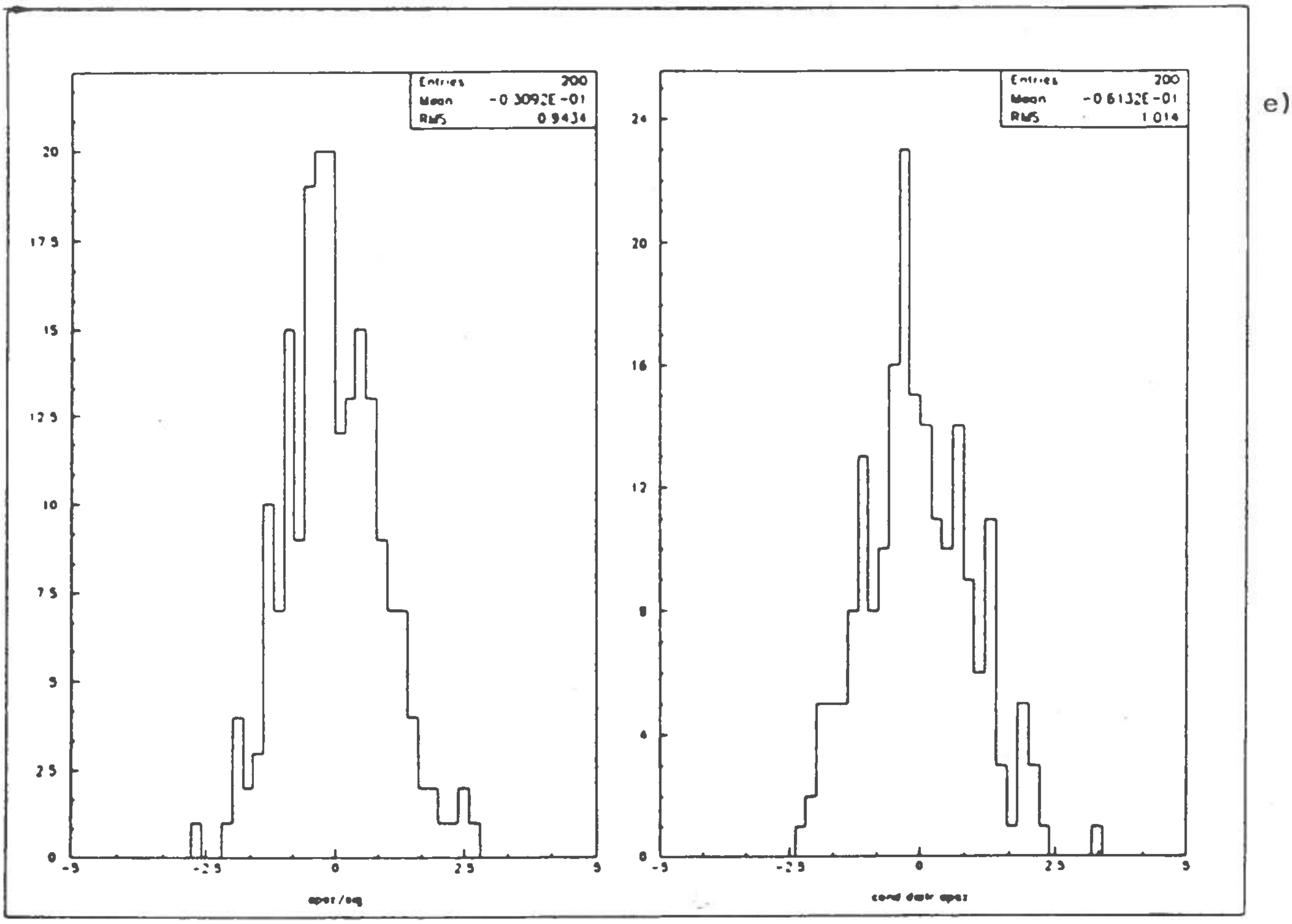


FIG. 13 - Normalized estimated parameter values; the normalization corresponds to the marginal distribution r.m.s for the LHS plots and to the conditional distribution r.m.s for the RHS ones; figures refer to Δx (a), Δy (b), ε_x (c), ε_y (d), ε_z (e).

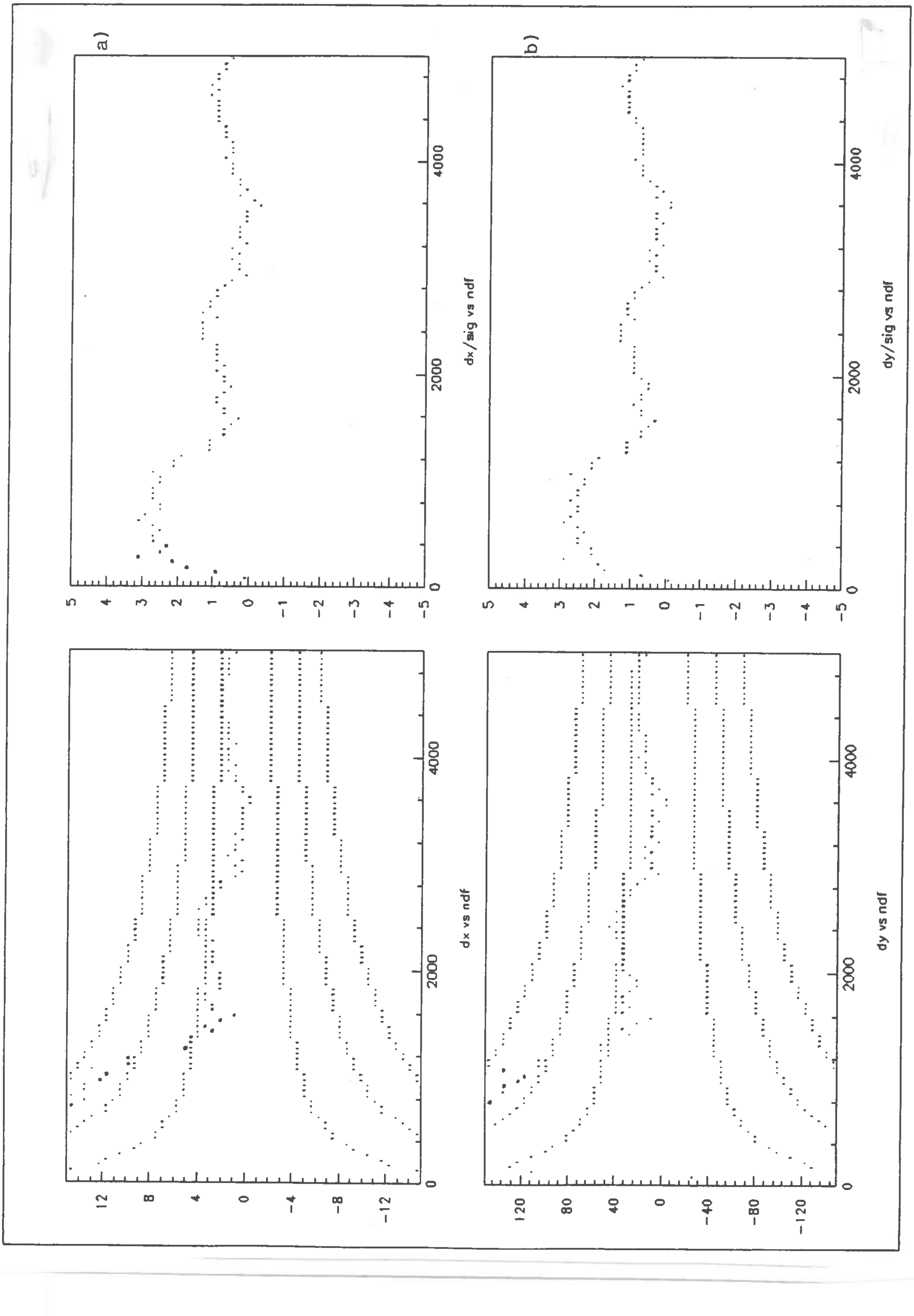


FIG. 14

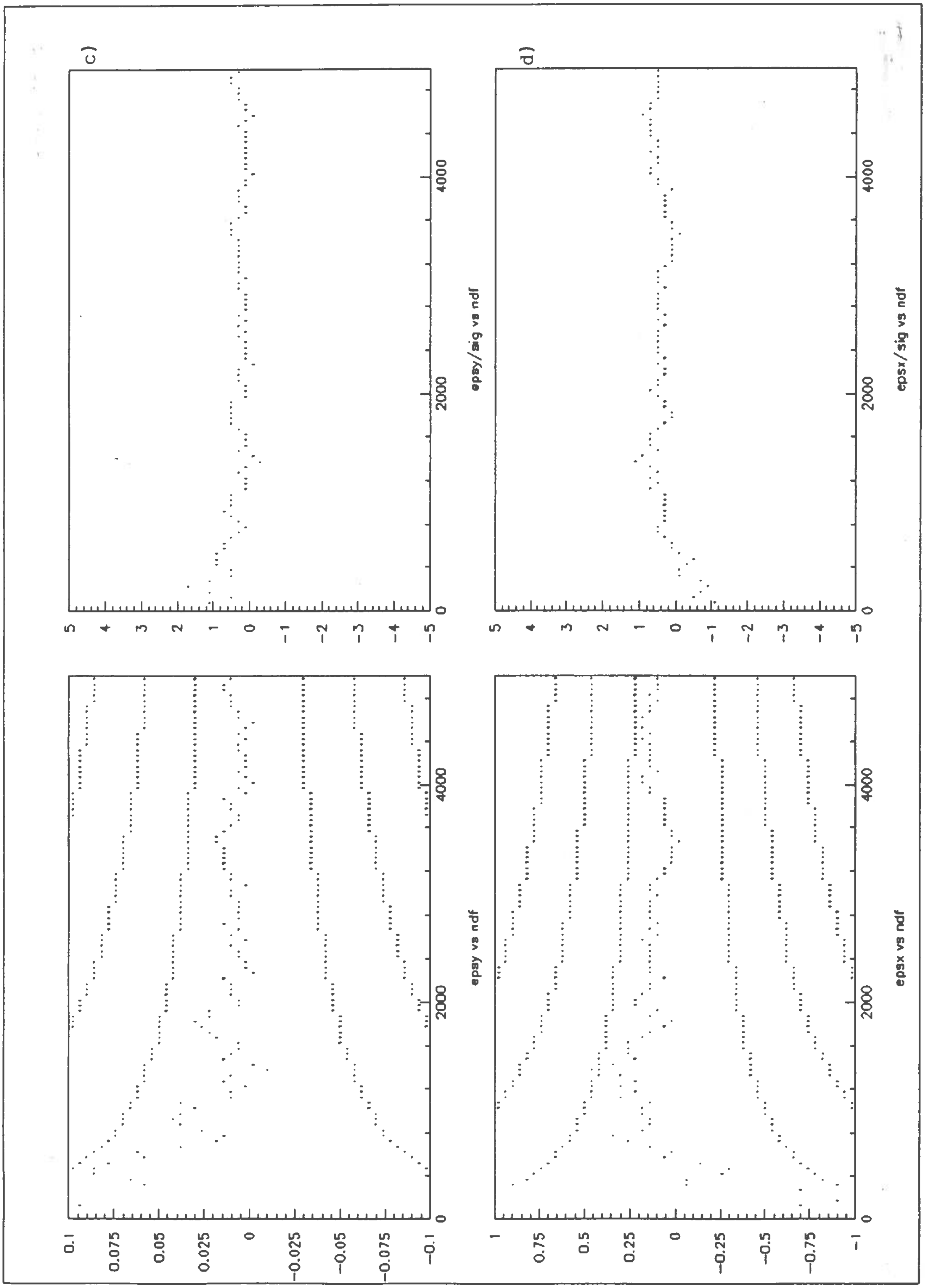

c)
d)
epsy vs ndf
epsy/sig vs ndf
epsx vs ndf
epsx/sig vs ndf
0
2000
4000
0.1
0.075
0.05
0.025
-0.025
-0.05
-0.075
-0.1
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
-1
5
4
3
2
-1
-2
-3
-4
-5

FIG. 14

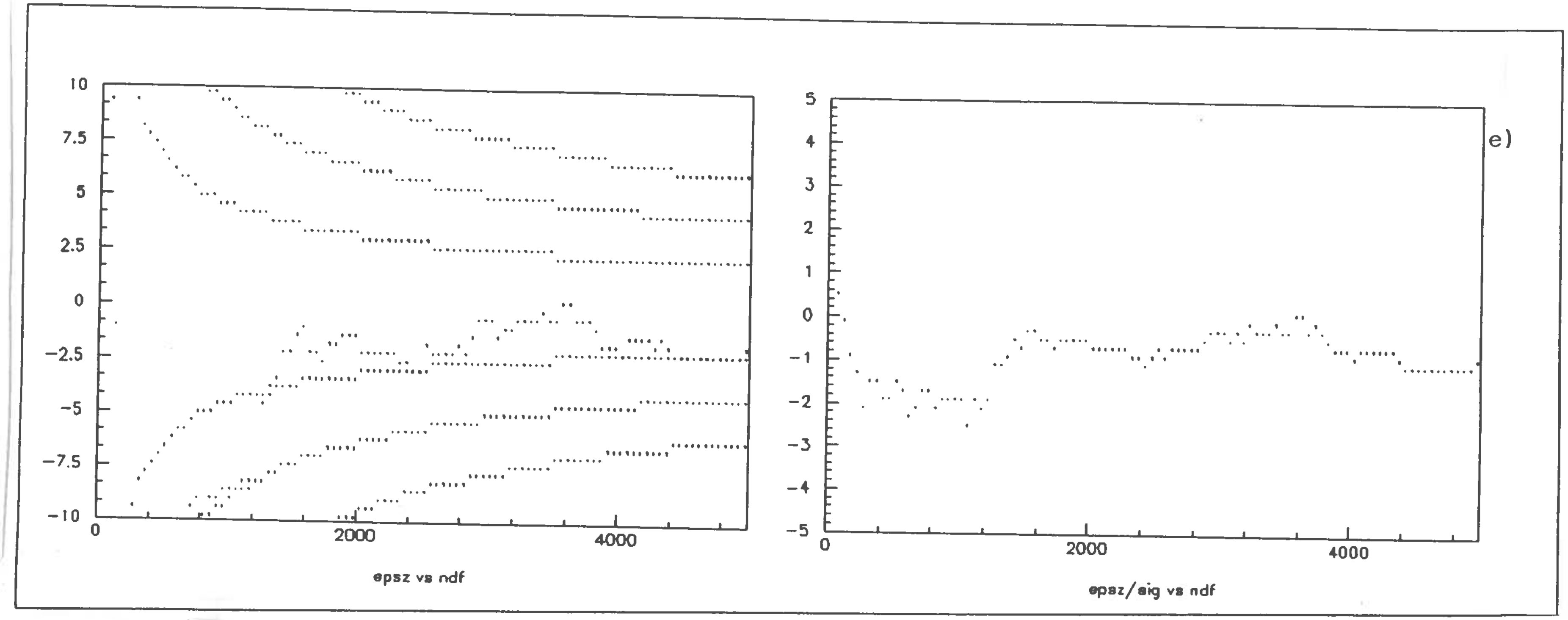


FIG. 14 - Estimated parameter value vs NDF; absolute value (LHS) and normalized to the marginal distribution r.m.s (RHS); figures refer to Δx (a), Δy (b), ε_x (c), ε_y (d), ε_z (e).

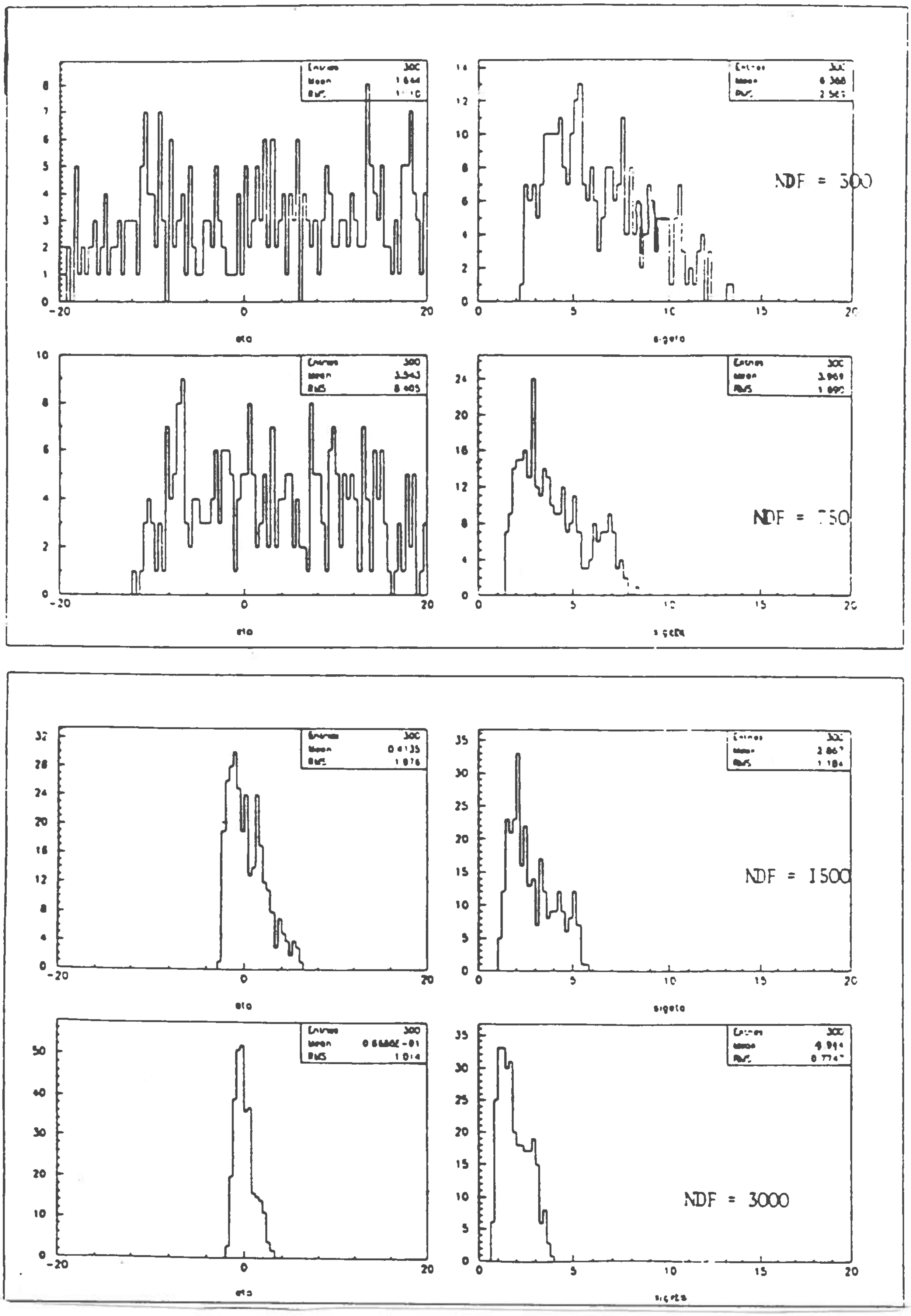


FIG. 15 - η and σ_η distribution for different NDF values.