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EXCLUSIVE $b \to s$ RARE DECAYS OF BEAUTY

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ABSTRACT

The exclusive beauty decays $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$ are studied using a hadronic model based on QCD sum rules embedded in a general Vector Meson Dominance framework. Both the short and the long distance contributions to the above decays are considered. As a byproduct, the rates for $B \to K\Psi$, $B \to K^*\Psi$, $B \to K\Psi'$ and $B \to K^*\Psi'$ are obtained.

$$A_i = C_i^{box} + C_i^Z - s_w^2(F_1^i + 2C_i^Z);\quad B_i = -\, s_w^2(F_1^i + 2C_i^Z). \tag{3}$$

In eq. (3):

$$C_i^{box} = \frac{3}{8}(x_i - 1)^{-1}\left[(x_i \ln x_i)/(x_i - 1) - 1\right], \tag{4}$$

$$C_i^Z = \frac{1}{4}x_i - \frac{3}{8}(x_i - 1)^{-1} + \frac{3}{8}(2x_i^2 - x_i)(x_i - 1)^{-1}\ln x_i\,, \tag{5}$$

$$F_1^i = \frac{4}{9}(\ln x_i)/(x_i - 1) - \frac{1}{6}x_i\,(x_i - 1)^{-1}\left[\frac{41}{3} + \frac{13}{6}(x_i - 1)^{-1} - (x_i - 1)^{-1}\right]$$

$$-\frac{1}{6}(x_i \ln x_i)\,(x_i - 1)^{-1}\left[1 - \frac{89}{6}(x_i - 1)^{-1} - \frac{5}{3}(x_i - 1)^{-2} + (x_i - 1)^{-3}\right], \tag{6}$$

$$F_2^i = \frac{2}{3}x_i\,(x_i - 1)^{-1}\left[1 + \frac{21}{8}(x_i - 1)^{-1} + \frac{3}{4}(x_i - 1)^{-2} - \frac{9}{4}\left(x_i - \frac{2}{3}\right)(x_i \ln x_i)\,(x_i - 1)^{-3}\right], \tag{7}$$

where $x_i = m_i^2/M_w^2$.

With the Lagrangian (1) the decay rate is given by:

$$\Gamma(b \to s l^+ l^-) = \frac{G_F^2 m_b^5}{192\pi^3}\left(\frac{\alpha}{4\pi s_w^2}\right)^2 .$$

$$\left[|V_i A_i|^2 + |V_i B_i|^2 + 4s_w^2\, V_j(A_j + B_j)\,(V_i F_2^i) + 16 s_w^4\, |V_i F_2^i|^2 \left(\ln\frac{m_b}{2m_l} - \frac{2}{3}\right)\right]. \tag{8}$$

QCD corrections enhance F_2 and hence the rate eq. (8). The QCD corrected expression is

$$\tilde{F}_2(m_t) = \left(\frac{\alpha_s(M_w^2)}{\alpha_s(m_b^2)}\right)^{16/23}\left\{\frac{116}{135}\left[\left(\frac{\alpha_s(m_b^2)}{\alpha_s(M_w^2)}\right)^{10/23} - 1\right] + \frac{58}{189}\left[\left(\frac{\alpha_s(m_b^2)}{\alpha_s(M_w^2)}\right)^{28/23} - 1\right] + F_2(m_t)\right\}. \tag{9}$$

Turning to the exclusive modes, one needs to sandwich the effective Lagrangian eq. (1) between hadronic states. Hence, the problem becomes that of estimating the relevant form factors. In the sequel we discuss separately the decays $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$.

2a) $B\to K l^+ l^-$

For $B\to K l^+ l^-$ the current matrix elements can be expressed as(*)

$$\langle K(k)|\bar{s}\gamma_\mu b|B(p)\rangle = (p+k)_\mu f_+(q^2) + (p-k)_\mu f_-(q^2), \quad (10)$$

$$\langle K(k)|\bar{s}\sigma_{\mu\nu}q_\nu b|B(p)\rangle = \left[q^2(p+k)_\mu - (M_B^2-m_K^2)q_\mu\right] f_T(q^2). \quad (11)$$

For $l = e, \mu$ terms proportional to q_μ can be ignored as their contribution to the transition probability is of order m_l^2/m_b^2. In the Vector Meson Dominance (VMD) framework the form factors above can be expressed as (see Fig. 1):

$$f_+(q^2) = \frac{M^2_{B_s^*}}{\sqrt{2}\gamma_{B_s^*}} \frac{g_{B_s^*BK}}{M^2_{B_s^*} - q^2} F_{B_s^*}(q^2), \quad (12)$$

$$f_T(q^2) = \frac{M_{B_s^*}}{\gamma_T} \frac{g_{B_s^*BK}}{M^2_{B_s^*} - q^2} F_{B_s^*}(q^2), \quad (13)$$

where B_s^* is the $1^-(b\bar{s})$ vector meson resonance, $g_{B_s^*BK}$ is the strong coupling constant, and the leptonic couplings $\gamma_{B_s^*}$ and γ_T are defined through

$$\langle 0|\bar{s}\gamma_\mu b|B_s^*(q,\varepsilon)\rangle = \frac{M^2_{B_s^*}}{\sqrt{2}\gamma_{B_s^*}} \varepsilon_\mu, \quad (14)$$

$$\langle 0|\bar{s}\sigma_{\mu\nu} b|B_s^*(q,\varepsilon)\rangle = \frac{M_{B_s^*}}{\gamma_T} (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu). \quad (15)$$

In eqs. (12), (13) the form factor $F(q^2)$ takes into account potential corrections to single pole dominance, presumably arising from radial excitations of the B_s^*. For instance, in the case of ρ-dominance of the pion form factor $F_\rho(0) \cong 0.8$ from experiment $(g_{\rho\pi\pi}/\sqrt{2}g_\rho|_{exp} = 1.22\pm0.03)$.

The leptonic couplings in eqs. (14) and (15) have been determined from QCD sum rules in [18] and in [13] respectively.

(*) We use the definition of form factors adopted in [11].

Briefly summarizing the general ideas of this method [19], one considers the two-point correlator

$$T(q^2) \;=\; i\int d^4x\, e^{iq\cdot x} \langle 0|TJ(x)J^{+}(0)|0\rangle, \tag{16}$$

where the current $J(x)$ is either $\bar{s}\gamma_\mu b$ or $\bar{s}\sigma_{\mu\nu}b$. The above Green functions can be reliably calculated via perturbative QCD at short distances, and then extrapolated to longer distances, i.e. closer to the hadronic dimensions, by means of the operator product expansion. Such an extrapolation introduces power corrections in the inverse of the typical large mass scale, represented in this case by the heavy quark mass m_b. These corrections are parametrized in terms of quark and gluon vacuuum condensates whose values can be extracted from data on e^+e^-, τ-decay, etc.

From analiticity of eq. (16) in q^2 the QCD expression so obtained can be related through a dispersion relation to the hadronic spectral function involving the matrix elements (14) or (15). The result of such an analysis is [13], [18]:

$$\gamma_{B_s^*} \cong \gamma_{B_d^*} \;=\; 22 \pm 4\,, \tag{17}$$

$$\gamma_T \equiv \sqrt{2}\,\gamma_{B_s^*}\,. \tag{18}$$

Using eq.(18) in eqs. (12) and (13) leads to:

$$\frac{f_+(q^2)}{f_T(q^2)} \;=\; M_{B_s^*}\,, \tag{19}$$

valid for all q^2. This is an important simplification as it allows to express the transition matrix element of $B\to K l^+ l^-$ in terms of a single form factor.

Next, the strong coupling constant $g_{B_s^* BK}$ may be estimated by relating it to the coupling $g_{B^*B\pi}$ through an SU(3) relation. An estimate of the latter can also be obtained from first principles, e.g. by using a well-convergent current algebra sum rule plus PCAC. The result is [18]:

$$g_{B^*B\pi} \cong \frac{M_{B^*}}{\sqrt{2}\, f_\pi} \tag{20}$$

which is compatible with an independent estimate based on a constituent quark model incorporating PCAC [20]. Corrections to the above result can also be obtained, by introducing the contribution of the B^*-radial excitations, which are expected to be close in mass to the ground state.

Accounting for SU(3) corrections, namely using f_K instead of f_π (with $f_K/f_\pi \cong 1.2$), we finally obtain:

$$g_{B_s^* BK} \cong \frac{M_{B_s^*}}{\sqrt{2}\, f_K} \left(1 + \frac{M^2_{B_s^*}}{M^2_{B_s^{*\prime}}}\right)^{-1/2} . \tag{21}$$

This particular parametrization of higher states leads in turn to the following result for the form factor $F_{B_s^*}(q^2)$ in eqs. (12)-(13):

$$F(q^2) = 1 - \frac{M^2_{B_s^{*\prime}}}{M^2_{B_s^*}} \frac{\gamma_{B_s^*}}{\gamma_{B_s^{*\prime}}} \left(\frac{M^2_{B_s^*} - q^2}{M^2_{B_s^{*\prime}} - q^2}\right) . \tag{22}$$

Making use of the mass formula of [21] we find

$$\frac{M_{B^{*\prime}}}{M_{B^*}} \cong 1.14 . \tag{23}$$

The ratio $\gamma_{B^*}/\gamma_{B^{*\prime}}$ may be related to the respective ratio of wavefunctions at the origin:

$$\frac{|\gamma_{B^*}|}{|\gamma_{B^{*\prime}}|} \cong \frac{|\Psi_{2s}(0)|}{|\Psi_{1s}(0)|} \left(\frac{M_{B^*}}{M_{B^{*\prime}}}\right)^{3/2} \cong 0.65 , \tag{24}$$

which qualitatively results at $q^2 = 0$ into $F_{B_s^*}(0) \cong 0.35$. This should not come as a surprise on account of the rather long extrapolation from $q^2 = M^2_{B^*}$ to $q^2 = 0$, and of the fact that for ρ-dominance the correction is $F_\rho(0) \cong 0.8$ as already mentioned. Also, we may notice that such a value of $F_{B_s^*}(0)$ implies, in the treatment of $B \to K^*\gamma$ presented in [13], the ratio $R = \Gamma(B \to K^*\gamma)/\Gamma(b \to s\gamma) \cong 0.15$. Since $M_{B_s^*}$ has not been measured yet we shall take $M_{B_s^*} \cong M_{B^*} = 5.32$ GeV.

Collecting the results above the form factors (12) and (13) can be expressed as follows:

1. Introduction

Beauty decays proceeding through the flavour changing neutral current transition $b \to s$ have been identified as a valuable source of information on the Standard Model and its extensions [1]. In general these decays are described by an effective interaction Hamiltonian in terms of quark and gluon fields, the so-called penguin one-loop amplitude, which accounts for short distance electroweak dynamics at the hadron constituent level. Such a short distance H_{eff} can depend rather sensitively on KM angles, on the top quark mass, and eventually on a fourth quark generation. In some cases perturbative QCD corrections [2]-[4] and SUSY virtual particle exchanges [5] may give rise to rate enhancements.

Matrix elements of H_{eff} between hadronic states are needed to predict the rates for the various exclusive channels, and thus to probe the underlying quark physics by a comparison with experimental data. Since our present ability to compute nonperturbative matrix elements from first principles is somewhat limited one has to adopt for that purpose some specific hadronization scheme. This introduces a considerable model dependency of the theoretical predictions which is difficult to assess quantitatively.

A further complication is represented by long-distance contributions to the decay amplitude, which occur at the hadronic level and can potentially obscure the short-distance component one is interested in. Indeed, the assumed dominance of the latter should be checked phenomenologically in each case, by considering the complete situation where both the short- and the long-range effects are taken into account.

Clearly, accurate experimental data, presumably to become available at B factories, will be extremely useful in order to put the various theoretical schemes under stringent scrutiny. Hopefully, this should improve our knowledge and ability to make reliable estimates of hadronic matrix elements. Obviously, it is important to stress in this regard the need for developing alternative theoretical models to confront with experiment.

Particularly significant to such a programme should be the decays $B \to K^* \gamma$, $B \to K^* l^+ l^-$ and $B \to K l^+ l^-$, which are controlled by form factors of the corresponding electroweak currents. Form factors are conceptually the simplest kind of hadronic matrix elements. Some of their properties should be a priori known, or at least it should be possible to find a reasonable phenomenological parametrization. Thus, in principle we may expect the situation concerning hadronization effects to be under better control than in the case of e.g. the nonleptonic decay channels.

One popular framework which has been extensively applied to the decays above is the Constituent Quark Model [6]-[12]. In a recent paper we have introduced an alternative hadronic model based on QCD sum rules embedded in a general Vector Meson Dominance framework and applied it to the decays $B\to K^*\gamma$ and $B\to Q\gamma$ [13]. In the present paper we would like to extend that calculation to the processes $B\to K^* l^+l^-$ and $B\to K l^+l^-$, including in addition to the short-distance form factors the contributions from long-range effects. The latter have been estimated to be small in the case of $B\to K^*\gamma$ [14]-[16]. In the case of $B\to K l^+l^-$ and $B\to K^* l^+l^-$ the only significant long-distance effect corresponds to the (quark-spectator) processes $B\to K\Psi(\Psi')$ and $B\to K^*\Psi(\Psi')$ followed by $\Psi(\Psi')\to l^+l^-$. Such transitions are allowed to occur via real $\Psi(\Psi')$ by phase space. In our approach the needed nonleptonic amplitude $B\to K\Psi$ etc. are determined in terms of the same physical parameters which enter the estimate of the form factors. Thus, comparison with the available experimental data for those nonleptonic transitions already represents a significant consistency test for the framework we are proposing.

The paper is organized as following. In Section 2 we present the necessary formulas for the short-distance H_{eff}. After that we write the most general structure of vector and tensor current matrix elements between B and $K(K^*)$ states in terms of form factors and discuss their parametrization in a Vector Dominance framework. In Section 3 we discuss the long-distance contributions to $B\to K l^+l^-$ and $B\to K^* l^+l^-$ due to $\Psi(\Psi')\to l^+l^-$ conversion. In this context we present our results for the exclusive decays $B\to K\Psi(\Psi')$ and $B\to K^*\Psi(\Psi')$, and compare them with the present experimental situation. In this same Section we give the differential decay rate for $B\to K(K^*)l^+l^-$, as well as the expected branching ratios resulting from the short- plus the long-distance contributions. Finally Section 4 contains a discussion of the implications of our calculation and some concluding remarks.

2. Effective Lagrangian and current matrix elements for $B\to K(K^*)l^+l^-$

At the quark level the processes of interest correspond to $b\to s l^+l^-$, which proceeds through the one-loop penguin diagrams involving the photon and the Z boson, and the W^+W^- exchange box diagram. The effective Lagrangian may be written as [17]:

$$L_{eff}(b\to s l^+l^-) = \frac{G_F}{\sqrt{2}}\left(\frac{\alpha}{4\pi s_w^2}\right)\Sigma_i V_i\left(A_i \bar{s} L_\mu b \bar{l} L_\mu l + B_i \bar{s} L_\mu b \bar{l} R_\mu l + \frac{2 m_b s_w^2}{q^2} F_2^i \bar{s} T_\mu b \bar{l} \gamma_\mu l\right), \quad (1)$$

where

$$L_\mu = \gamma_\mu(1-\gamma_5); \quad R_\mu = \gamma_\mu(1+\gamma_5); \quad T_\mu = -i\sigma_{\mu\nu}(1+\gamma_5)q_\nu , \quad (2)$$

$l = e, \mu$, $V_i = U_{is}^* U_{ib}$ $(i = u,c,t)$ with U_{ij} the KM matrix elements , and

$$f_+(q^2) = \frac{f_+(0)}{1 - q^2/M^2_{B_s^*}} \frac{F(q^2)}{F(0)}; \; f_+(0) \cong 0.29, \tag{25}$$

and $f_T(q^2)$ is given by eq. (19). One can notice that the form factor (25) has a q^2-behaviour which deviates from the single pole-dominated form. This should reflect itself into a modified shape of the differential rate. It is an interesting aspect which should be subject to experimental test once precise enough data become available. This will clearly help to tune the model parameters of the present approach. For comparison purposes we shall later quote results using (25) as well as a single-pole dominated form factor obtained from (25) by setting $F(q^2)/F(0) = 1$ and the same value for $f_+(0)$.

2b) $B \to K^* l^+ l^-$

Turning to $B \to K^* l^+ l^-$, this process involves a priori six independent form factors defined through the following matrix elements ($q = p - k$):

$$\langle K^*(k,\varepsilon)|\bar{s}\gamma_\mu b|B(p)\rangle = i\varepsilon_{\mu\nu\rho\sigma}\,\varepsilon_\nu (p+k)_\rho (p-k)_\sigma\, V(q^2), \tag{26}$$

$$\langle K^*(k,\varepsilon)|\bar{s}\gamma_5\gamma_\mu b|B(p)\rangle = \varepsilon_\mu (M_B^2 - M_{K^*}^2) A_1(q^2) - (q\cdot\varepsilon)(p+k)_\mu\, A_2(q^2) + (q\cdot\varepsilon) q_\mu A_P(q^2), \tag{27}$$

$$\langle K^*(k,\varepsilon)| -i\,\bar{s}\sigma_{\mu\nu}(1+\gamma_5) q_\nu b|B(p)\rangle = i\varepsilon_{\mu\nu\rho\sigma}\,\varepsilon_\nu (p+k)_\rho (p-k)_\sigma\, F_1(q^2) +$$

$$+ \left[\varepsilon_\mu (M_B^2 - M_{K^*}^2) - (q\cdot\varepsilon)(p+k)_\mu\right] F_2(q^2) + \varepsilon_\nu \left[g_{\mu\nu}\, q^2 - q_\mu q_\nu\right] F_3(q^2). \tag{28}$$

Note that the last term in (28) vanishes for a real photon. Concerning the axial-vector form factors eq. (27) we have retained the scalar form factor $A_P(q^2)$. Being multiplied by q_μ this term is not relevant for the rate in case of a produced lepton pair $l = e, \mu$. However, it does play a considerable role in the Ward identity for the axial-vector current $A_\mu = \bar{s}\gamma_5\gamma_\mu b$ which we are going to employ in relating the various form factors in eqs. (27) and (28). The same is true of the form factor F_3.

Specifically, the Ward identities have the form

$$\langle K^*(k,\varepsilon)|\bar{s}\, i\sigma_{\mu\nu} q_\nu \gamma_5 b|B(p)\rangle =$$

$$= (m_b - m_s)\langle K^*(k,\varepsilon)|\bar{s}\gamma_\mu\gamma_5 b|B(p)\rangle + (p+k)_\mu \langle K^*(k,\varepsilon)|\bar{s}\gamma_5 b|B(p)\rangle, \tag{29a}$$

$$- i \; q_\mu \langle K^*(k,\varepsilon)|\bar{s}\gamma_\mu\gamma_5 b|B(p)\rangle =$$

$$= \langle K^*(k,\varepsilon)|\partial_\mu(\bar{s}\gamma_\mu\gamma_5 b|B(p)\rangle = (m_b+m_s)\langle K^*(k,\varepsilon)|\bar{s}i\gamma_5 b|B(p)\rangle \; . \qquad (29b)$$

Defining

$$\langle K^*(k,\varepsilon)|\bar{s}\gamma_5 b|B(p)\rangle = (q\cdot\varepsilon)\, F_p(q^2), \qquad (29c)$$

and substituting eqs. (27)-(28) into eqs. (29a,b,c) we obtain:

$$(M_B^2 - M_{K^*}^2)\, F_2(q^2) + q^2 F_3(q^2) = (M_B^2 - M_{K^*}^2)\; [(m_b-m_s)A_1(q^2)], \qquad (30a)$$

$$F_2(q^2) = (m_b-m_s)\, A_2(q^2) + F_P(q^2)\, , \qquad (30b)$$

$$- F_3(q^2) = (m_b-m_s) A_P(q^2), \qquad (30c)$$

$$(M_B^2 - M_{K^*}^2)\, [A_1(q^2) - A_2(q^2)] + q^2 A_P(q^2) = (m_b+m_s) F_P(q^2). \qquad (30d)$$

One can note that these equations are mutually consistent in the approximation where $M_B^2 - M_{K^*}^2 \approx m_b^2 - m_s^2$. We now assume that $A_P(q^2)$ and $F_P(q^2)$ are dominated by the 0^- B_s-meson pole:

$$A_P(q^2)\Big|_{Pole} = \frac{2\sqrt{2}\, f_{B_s}\, g_{B_sBK^*}}{M_{B_s}^2 - q^2} \qquad (31a)$$

$$(m_b+m_s)\, F_P(q^2)\Big|_{Pole} = \frac{2\sqrt{2}\, f_{B_s} M_{B_s}^2\, g_{B_sBK^*}}{M_{B_s}^2 - q^2} \; . \qquad (31b)$$

Substituting into eqs. (30) we obtain:

$$(m_b-m_s)\, A_1(0) = F_2(0)\, , \qquad (32)$$

$$(M_B^2 - M_{K^*}^2)\, (A_1(0) - A_2(0)) = 2\sqrt{2}\, f_{B_s}\; g_{B_sBK^*}\, \mathcal{F}_{B_s}(0)\, , \qquad (33)$$

where we have multiplied the B_s-pole contribution by the usual suppression factor $\mathcal{F}_{B_s}(0) \cong 0.35$. An estimate of the strong coupling constant $g_{B_sBK^*}$ can be attempted by K^*

vector-dominating the matrix element $<B_s|V_\mu^{(K^*)}|B>$, where $V_\mu^{(K^*)}$ is the SU(3) current such that in the SU(3) limit $<B_s|Q^{(K^*)}|B> = -1$, where $Q^{(K^*)}=\int d\vec{x}\, V_0^{(K^*)}(\vec{x},0)$. We have

$$\frac{\sqrt{2}\, f_{K^*} g_{B_sBK^*}}{M_{K^*}^2} \cong -1, \tag{34}$$

a relation reminescent of ρ-dominance and which should hold to the same degree of accuracy.

Thus finally eq. (33) becomes

$$[A_1(0) - A_2(0)] = -\frac{2\, M_{K^*}^2}{M_B^2 - M_{K^*}^2}\left(\frac{f_{B_s}}{f_{K^*}}\right) F_{B_s}(0)\,, \tag{35}$$

where we shall use $f_{B_s}/f_{K^*} \cong f_{B_d}/f_\rho$, with $f_{B_d} \cong$ (105-150) MeV from QCD sum rules [22,23] and $f_\rho \cong 0.12$ GeV^2 form the leptonic ρ width.

Analogously:

$$F_3(q^2) = \frac{2(m_b - m_s)}{(1 - q^2/M_{B_s}^2)}\left(\frac{M_{K^*}}{M_{B_s}}\right)^2\left(\frac{f_{B_s}}{f_{K^*}}\right) F_{B_s}(q^2). \tag{36}$$

Turning to $F_1(q^2)$ and $F_2(q^2)$, one has from the $\sigma_{\mu\nu}(1+\gamma_5)$ structure of eq. (28) that

$$F_1(q^2) = F_2(q^2), \tag{37}$$

where in VMD analogously to eq. (19):

$$F_1(q^2) = M_{B_s^*}\, V(q^2). \tag{38}$$

In VMD the form factor $V(q^2)$ can be written as

$$V(q^2) = \frac{M_{B_s^*}^2}{\sqrt{2}\gamma_{B_s^*}}\,\frac{g_{B_s^*BK^*}}{M_{B_s^*}^2 - q^2}\,\frac{1}{2}\, F_{B_s^*}(q^2), \tag{39}$$

which should be compared with the vector form factor $f_+(q^2)$ appearing in $B \to K l^+ l^-$, eq. (12). The only difference is the strong coupling constant $g_{B_sBK^*}$. This may be related to $g_{B^*B\rho}$ by an SU(3) rotation , i.e.

$$g_{B_s^*BK^*} \cong \sqrt{2} g_{B^*B\rho} \cong \sqrt{2}\cdot 11\ GeV^{-1}, \tag{40}$$

where $g_{B^*B\rho^0}$ has been estimated in [18]. With this value one finds

$$V \cong \frac{1}{M_{B_s^*}} \; \mathcal{F}_{B_s^*}(0), \tag{41}$$

where we have defined

$$V(q^2) = \frac{V}{1 - q^2/M_{B_s^*}^2} \; \frac{\mathcal{F}_{B_s^*}(q^2)}{\mathcal{F}_{B_s^*}(0)} \; . \tag{42}$$

Similarly, introducing the suppression factor $\mathcal{F}_{B_s}(q^2)$ we write

$$A_i(q^2) = \frac{A_i}{(1 - q^2/M_{B_s^*}^2)} \; \frac{\mathcal{F}_{B_s^*}(q^2)}{\mathcal{F}_{B_s^*}(0)} \; , \tag{43}$$

where now the A_i are determined through eqs. (32)-(35). They are given by

$$A_1 \cong \frac{F_1(0)}{m_b} = \frac{M_{B_s^*}}{m_b} \; V \tag{44a}$$

$$A_2 \cong A_1 + \frac{2M_{K^*}^2}{M_B^2 - M_{K^*}^2} \left(\frac{f_{B_s}}{f_{K^*}}\right) \mathcal{F}_{B_s}(0), \tag{44b}$$

where V is given in eq. (41). Also(*):

$$\frac{F_3(0)}{F_1(0)} \cong 2m_b \left(\frac{M_{K^*}}{M_{B_s}}\right)^2 \left(\frac{f_{B_s}}{f_{K^*}}\right) \cong 0.3 \; . \tag{45}$$

We have used $M_{B_s} \cong M_{B_s^*} \cong M_{B^*} = 5.32$ GeV, $m_b = 5.1$ GeV, and have neglected the small strange quark mass m_s in the spirit of the approximations adopted so far. This completes our discussion of the form factors of the short-distance transition amplitude eq. (1).

(*) Actually the contribution of F_3 to the differential rate is quite small compared to the other form factors.

3. Long-distance contributions to $B \to K(K^*) l^+ l^-$

As discussed in [14]-[16], the main source of long-distance (l. d.) contributions to the processes of interest is represented by the $B \to K(K^*)\Psi$ weak transition followed by $\Psi \to "\gamma" \to l^+ l^-$ conversion. This is governed by the nonleptonic weak Hamiltonian

$$H_w = \frac{G_F}{\sqrt{2}} U^*_{cs} U_{bc} (c_- + \frac{c_+}{3}) \left[\bar{s}\gamma_\mu (1-\gamma_5) b\right]\left[\bar{c}\gamma_\mu (1-\gamma_5) c\right], \qquad (46)$$

where $c_\pm$ are QCD coefficients. Actually the combination of c_+ and c_- in eq.(46) is very sensitive to Λ_{QCD} as the two terms tend to cancel. As suggested in [9] we use in what follows $(c_- + \frac{c_+}{3}) \simeq -0.24$ i.e. the value obtaianed in the $N_c \to \infty$ limit. In the VMD approach followed here the l.d. contributions are represented by the diagrams depicted in Fig. 2, where H_w is given in eq.(46). As in the previous section we discuss $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$ separately.

3a) $B \to K l^+ l^-$

By comparing Fig. 2 with Fig. 1 and using the explicit expressions of eqs.(10) and (12), one can easily see that the l.d. contribution to $B \to K l^+ l^-$ can be written as

$$T^{l.d.} = \frac{G_F}{\sqrt{2}} (4\pi\alpha)\, U^*_{cs} U_{cb} (c_- + \frac{c_+}{3}) \cdot \frac{2}{3} \frac{f^2_\Psi}{M^2_\Psi} f_+(q^2) \frac{1}{M^2_\Psi - q^2 - i M_\Psi \Gamma_\Psi} \cdot$$

$$\cdot (p+k)_\mu \frac{1}{2} \left[\bar{l} L_\mu l + \bar{l} R_\mu l \right] , \qquad (47)$$

while from eqs.(1) and (10) - (19):

$$T^{s.d.} = \frac{G_F}{\sqrt{2}} \left(\frac{\alpha}{4\pi s^2_w} \right) f_+(q^2)\, (p+k)_\mu \cdot \sum_i V_i \left[(A_i + \frac{2m_b}{M_{B^*_s}} s^2_w F^i_2)\, \bar{l}\, l_\mu l + B_i\, \bar{l} R_\mu l \right]. \qquad (48)$$

In eq. (47) f_Ψ is the Ψ-photon junction

$$<o|\, J_\mu^{em}\, |\Psi> = \frac{2}{3} <o|\bar{c}\,\gamma_\mu c\,|\Psi> = \frac{2}{3} f_\Psi . \tag{49}$$

This parameter appears in the determination of the mixing matrix element $<B_s^*|H_w|\gamma>$ of Fig. 2 via the vacuum saturation approximation. One can also check that the mixing of Fig. 2 vanishes for a real photon in accord with gauge invariance (the same is true for the case of $B \to K^*$) [15, 24].

Thus, combining the s.d. and the l.d. contributions of eqs. (47) and (48) one can write the differential spectrum $d\Gamma/dq^2$ for $B \to K l^+ l^-$ as:

$$\frac{d\Gamma}{dq^2} = \frac{1}{192\,\pi^3 M_B^3}\left(\frac{G_F}{\sqrt{2}}\right)^2\left(\frac{\alpha}{4\pi s_w^2}\right)^2 \left(|H|^2 + |C|^2\right) .$$

$$\cdot\, |f_+(q^2)|^2 \left[\left(M_B^2 + m_K^2 - q^2\right)^2 - 4 M_B^2 m_K^2\right]^{3/2} , \tag{50}$$

where

$$H = \sum_i V_i \left(\tilde{A}_i + \tilde{B}_i + \frac{2 m_b}{M_{B_s^*}} s_w^2\, F_2^i\right) , \tag{51a}$$

$$C = \sum_i V_i \,(A_i - B_i) , \tag{51b}$$

and where

$$\tilde{A}_i = A_i + s_w^2 \left(c_- + \frac{c_+}{3}\right) \frac{16\pi^2}{3} \sum_{v = \Psi, \Psi'} \frac{f_v^2}{M_v^2}\, \frac{1}{M_v^2 - q^2 - i\, M_v \Gamma_v}\, \delta_{ic} , \tag{52a}$$

$$\tilde{B}_i = B_i + s_w^2 \left(c_- + \frac{c_+}{3}\right) \frac{16\pi^2}{3} \sum_{v = \Psi, \Psi'} \frac{f_v^2}{M_v^2}\, \frac{1}{M_v^2 - q^2 - i\, M_v \Gamma_v}\, \delta_{ic} , \tag{52b}$$

with A_i , B_i and F_2^i defined in eqs. (3) - (7).

In eqs. (52a, b) we have included the Ψ' in addition to the Ψ. The values of the corresponding constants obtained from the measured leptonic Ψ and Ψ' widths are

$$f_\Psi = 1.2\ \mathrm{GeV}^2 ; \quad f_{\Psi'} = 1.02\ \mathrm{GeV}^2 . \tag{53}$$

In Fig. 3 we illustrate the resulting spectrum $d\Gamma/dq^2|_{s.d.+l.d.}$, for $f_+(q^2)$ as given by eq. (25). In Tab. 1 we summarize the values of the various parameters used in the calculation. We have also used a single-pole parametrizatiom of $f_+(q^2)$ with the same $f_+(0)$ as in (25). Both curves are hardly distinguishable with the scales chosen in Fig. 3. The difference can be appreciated in Fig. 4, where we show instead $d\Gamma/dq^2|_{s.d.}$, i.e. the spectrum without the l.d. contribution, for the same two possibilities considered in Fig. 3.

As a byproduct of the l.d. calculations above we may estimate the transition matrix elements for $B \to K\Psi$ and $B \to K\Psi'$. These are defined as ($V = \Psi, \Psi'$):

$$T(B(p) \to K(k)V(q,\varepsilon)) = F_{B\to KV} \cdot (2p \cdot \varepsilon(q)), \tag{54}$$

such that

$$\Gamma(B \to KV) = \frac{|F_{B\to KV}|^2}{16\pi M_B^3 M_V^2} \left[(M_B^2 - M_V^2 - m_K^2)^2 - 4 M_V^2 m_K^2\right]^{3/2}. \tag{55}$$

In the VMD we are considering

$$F_{B\to KV} = G\, f_+(M_V^2)\,, \tag{56}$$

where

$$G = \frac{G_F}{\sqrt{2}} U_{cs}^* U_{cb} (C_- + \frac{C_+}{3}) f_V\,. \tag{57}$$

The couplings in eq.(54) are thus determined in terms of the same parameters given in Tab. 1 and used for Figs. 3 and 4. Accordingly the rates eq.(55) become a prediction, representing an important internal consistency test of the approach followed here. The predicted branching fractions for $B \to K\Psi(\Psi')$ are given in Tab. 2, where the comparison with experimental data and with other calculations is also presented.

3b) $B \to K^* l^+ l^-$

This case can be treated following exactly the same procedure as in the preceding case, only that the kinematics is more complicated. Thus, in place of eq. (47) we will have for $T^{l.d.}$ an analogous expression containing the Ψ plus the Ψ' propagators, with $(p+k)_\mu f_+(q^2)$ replaced by the full tensorial structures of eqs. (26)-(28) and the corresponding form factors. Accordingly, the differential decay distribution for $B \to K^* l^+ l^-$ is given by:

$$\frac{d\Gamma}{dq^2} = \frac{1}{96\pi^3 M_B^3} \left(\frac{G_F}{\sqrt{2}}\right)^2 \left(\frac{\alpha}{4\pi s_w^2}\right)^2 \left[(q^2-M_B^2-M_{K^*}^2)^2 - 4M_B^2 M_{K^*}^2\right]^{1/2} \cdot$$

$$\cdot \Big\{q^2\left[(M_B^2-M_{K^*}^2-q^2)^2 - 4q^2M_{K^*}^2\right] \cdot 2|J_1|^2 + \frac{1}{4M_{K^*}^2}(M_B^2-M_{K^*}^2)^2\left[(M_B^2-M_{K^*}^2-q^2)^2 +\right.$$

$$\left. + 8q^2M_{K^*}^2\right] \cdot |J_2|^2 + \frac{1}{4M_{K^*}^2}\left[(M_B^2-M_{K^*}^2-q^2)^2 - 4q^2M_{K^*}^2\right] \cdot |J_3|^2 - \frac{1}{4M_{K^*}^2} \cdot \tag{58}$$

$$\cdot (M_B^2-M_{K^*}^2)(M_B^2-M_{K^*}^2-q^2)\left[(M_B^2-M_{K^*}^2-q^2)^2 - 4q^2M_{K^*}^2\right] 2\mathrm{Re} J_2 J_3^* + (J_i \to J_i')\Big\},$$

where

$$J_1 = a_1\, V(q^2) + a_2\, F_1(q^2)/q^2$$

$$J_2 = a_1\, A_1(q^2) + a_2\, \tilde{F}_2(q^2)/q^2 \tag{59}$$

$$J_3 = a_1\, A_2(q^2) + a_2\, \tilde{F}_2(q^2)/q^2,$$

with

$$a_1 = \sum_i V_i \tilde{A}_i; \quad a_2 = \sum_i V_i (m_b s_w^2 F_2^i); \quad b_1 = \sum_i V_i \tilde{B}_i, \tag{60}$$

while the J_i' are obtained from the J_i by changing a_1 to b_1. In eqs. (59) and (60) $\tilde{F}_2(q^2) = F_2(q^2) + q^2 F_3(q^2)/(M_B^2-M_{K^*}^2)$, the various quantities have been defined in eqs. (3)-(7) and (52a,b), and finally the form factors $V(q^2)$, $A_1(q^2)$ and $F_i(q^2)$ have been determined in eqs. (42)-(44).

In Fig. (5) we show the resulting differential decay rate $d\Gamma/dq^2|_{s.d.+l.d.}$ for the form factor estimates of Section 2b as well as for the case of a single-pole dominance parametrization. In Fig. (6) we display for comparison only $d\Gamma/dq^2|_{s.d.}$ for the two cases above.

Turning to the transition matrix elements for $B \to K^* \Psi(\Psi')$, they can be written as $(V = \Psi, \Psi')$:

$$T(B(p) \to K^*(k,\eta) V(q,\varepsilon)) = i\, \varepsilon_{\mu\nu\rho\sigma}\, \epsilon_\mu \eta_\nu\, p_\rho\, k_\sigma\, 2F +$$

$$+ \epsilon_\mu \eta_\nu \left[g_{\mu\nu}(M_B^2 - M_{K^*}^2)\, G_1 - 2k_\mu q_\nu\, G_2\right], \tag{61}$$

such that

$$\Gamma(B \to K^* V) = \frac{\lambda^{3/2}}{16\pi M_B^3} \left\{ 2|F|^2 + \frac{(M_B^2 - M_{K^*}^2)^2}{\lambda} \cdot \left(2 + \frac{(M_B^2 - M_V^2 - M_{K^*}^2)^2}{4M_V^2 M_{K^*}^2}\right) |G_1|^2 + \right.$$

$$\left. + \frac{\lambda}{4M_V^2 M_{K^*}^2} |G_2|^2 - \frac{(M_B^2 - M_V^2 - M_{K^*}^2)(M_B^2 - M_{K^*}^2)}{2M_V^2 M_{K^*}^2} \mathrm{Re}\, G_1 G_2^* \right\}, \tag{62}$$

where $\lambda = (M_B^2 - M_V^2 - M_{K^*}^2)^2 - 4M_V^2 M_{K^*}^2$.

The analogues of eq.(56) now become:

$$F = G\, V(M_V^2)$$

$$G_i = G\, A_i(M_V^2) \tag{63}$$

with G given in (57). In this way the branching ratios for $B \to K^* \Psi(\Psi')$ are again a prediction. The results we obtain for these decays are reported in Tab. 2.

4. Concluding remarks

We have studied in this paper the short- and long-distance contributions to the exclusive rare decays $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$ in a hadronic model based on QCD sum rules embedded in a general Vector Meson Dominance framework.

For the $q^2 = 0$ normalization of the form factors of the short-distance amplitudes we find results which are consistent (within the uncertainties) with the Constituent Quark Model predictions of [9, 11].

The main difference with [9, 11] lies in the q^2 dependence of the form factors, as we try to account for deviation from single pole dominance. This is to expected a priori because of the large extrapolation involved in the given q^2 range. We have parametrized these corrections in terms of a meson radial excitation. The effect is a pronounced increase in the differential rate in the vicinity of the end point q^2_{max}, as shown in Figs. 4 and 6. In Fig. 7 we show the integrated rates as a function of the top quark mass for $B \to K(K^*) l^+ l^-$, with and without corrections to single pole dominance. The difference is roughly a factor of two, which then gauges the true theoretical uncertainty. It should be stressed that this is quite typical and acceptable from the phenomenological point of view. Indeed it reflects the systematic uncertainty affecting the current state of the art of form factor models. This model dependency would be substantially reduced as soon as experimental information on semileptonic form factors will become available.

For comparison we also show in Fig. 7 the inclusive rate for $b \to s l^+ l^-$, without l.d. contributions (the latter have been included in [25]).

The effect of the l.d. contributions from $B \to K(K^*)\Psi$ followed by $\Psi \to "\gamma" \to l^+ l^-$ (and same for Ψ'), is illustrated in Figs. 3 and 5. Except for the relative phase between short- and long-distance our results are again consistent with the quark model predictions of [12]. This phase difference is presumably due to our choice $(c_- + \frac{c_+}{3}) = -0.24$ in eq. (46). Anyway this is not particularly important as far as probing the purely short-distance amplitude is concerned. Indeed, for this purpose, measurements will have to be subject to cuts for $q^2 \lesssim M^2_{\Psi}$, and in this region the two approaches are consistent. This should result in a relatively model independent extraction of the s.d. part.

An important consistency check of the present framework is the prediction of the rates for $B \to K\Psi$, $B \to K^*\Psi$, $B \to K\Psi'$ and $B \to K^*\Psi'$, which do not involve additional parameters, and which are consistent with the present experimental data.

In conclusion, in spite of the long-distance effects which tend to obscure the short-distance contributions to $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$, thus requiring cuts in the spectrum and therefore reduction of the number of events, it should still be possible to test the one-loop electroweak theory from accurate measurements of these decays.

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Table 1

Values of the parameters used in obtaining Figs. 3-6. The definitions are found in the text.

$U_{cs} = 0.96$ $m_b = 5.1$ GeV $f_\psi = 1.2$ GeV^2	$U_{bc} = 0.045$ $m_t = 80$ GeV $f_{\psi'} = 1.02$ GeV^2	$C_- + \frac{C_+}{3} = -0.24$ $f_B = 1.3\, f_\pi$ $M_{B_s^*} = M_{B_s} = 5.32$ GeV
$F_{B_s^*}(0) = F_{B_s}(0) = 0.35$	$f_+(0) = 0.29$	$f_T(q^2) = f_+(q^2)/M_{B_s^*}$
$V = 0.066$ GeV^{-1} $F_1(0) = 0.35$	$A_1 = 0.068$ GeV^{-1} $F_2(q^2) = F_1(q^2)$	$A_2 = 0.092$ GeV^{-1} $F_3(q^2) = 0.3\, F_1(q^2)$

Table 2

Theoretical branching ratios of $B\to K\Psi(\Psi')$ and $B\to K^*\Psi(\Psi')$, and comparison with experimental data.

BRANCHING RATIOS (%)					
MODE	THIS PAPER	Ref. [9]	Ref. [15]	ARGUS	CLEO '87
$B\to K\Psi$	0.04	$0.06\left(\frac{U_{cb}}{0.05}\right)^2$	0.08 ± 0.02	0.07 ± 0.04	0.07 ± 0.02
$B\to K\Psi'$	0.02			0.22 ± 0.17	< 0.04
$B\to K^*\Psi$	0.16	$0.25\left(\frac{U_{cb}}{0.05}\right)^2$	0.17 ± 0.04	0.33 ± 0.18	0.12 ± 0.05
$B\to K^*\Psi'$	0.19			-	0.15 ± 0.09

Figure captions

Fig. 1: Vector meson dominance model for the form factors. J_μ represents the relevant current.

Fig. 2: Long-distance contributions in the vector meson dominance approach.

Fig. 3: Differential rate for $B \to K l^+ l^-$.

Fig.4: Differential rate for $B \to K l^+ l^-$: short-distance contribution only.
Curve (a): form factors as in eq. (25).
Curve (b): single-pole parametrization of the form factors.

Fig. 5: Differential rate for $B \to K^* l^+ l^-$.

Fig. 6: Differential rate for $B \to K^* l^+ l^-$: short distance contribution only.
Curve (a): form factors as in eqs. (36)-(44).
Curve (b): single-pole parametrization of the form factors.

Fig. 7: Integrated rates for $B \to K l^+ l^-$ and for $B \to K^* l^+ l^-$ as a function of the top quark mass. Dashed and solid lines correspond respectively to cases (a) and (b) of the preceding figures. Also shown is the rate of $b \to s\, l^+ l^-$.

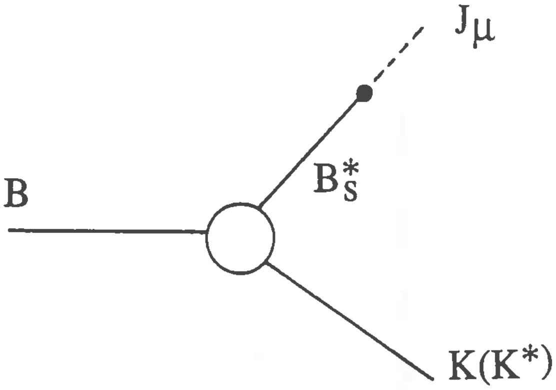


Fig. 1

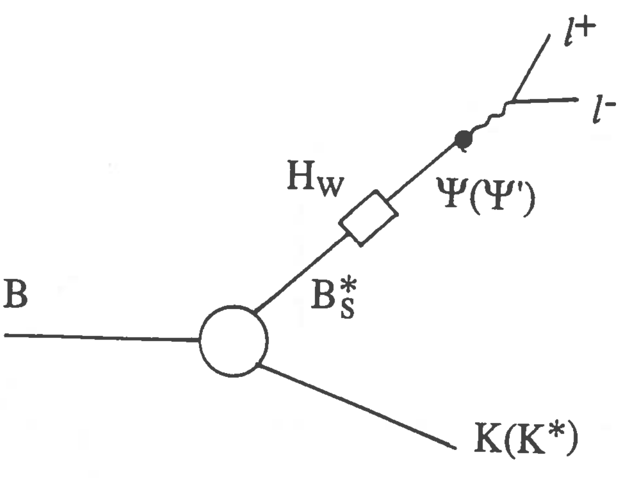


Fig. 2

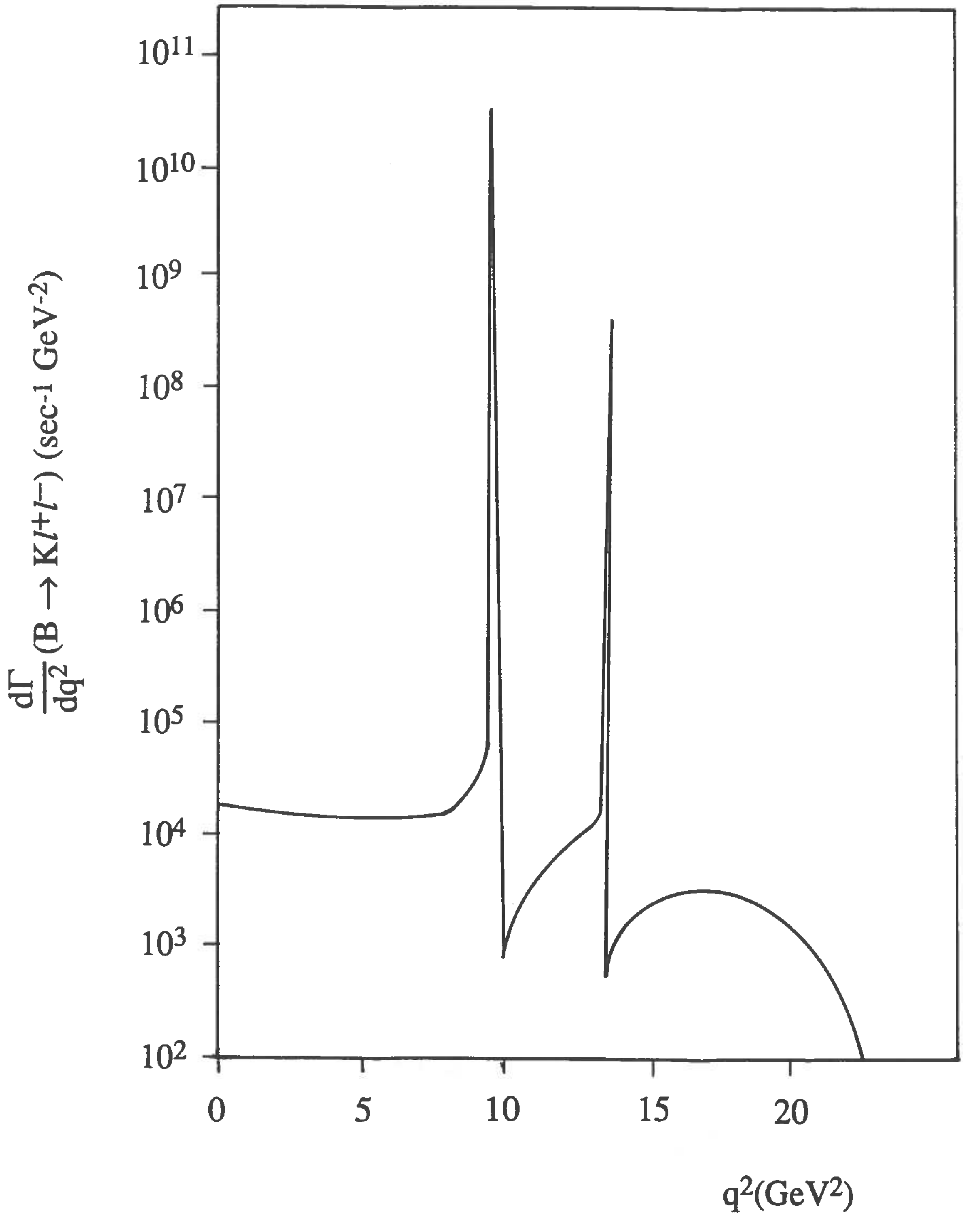

$\frac{d\Gamma}{dq^2}(B \to K l^+ l^-)$ (sec-1 GeV-2)
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
0
5
10
15
20
q^2(GeV2)

Fig. 3

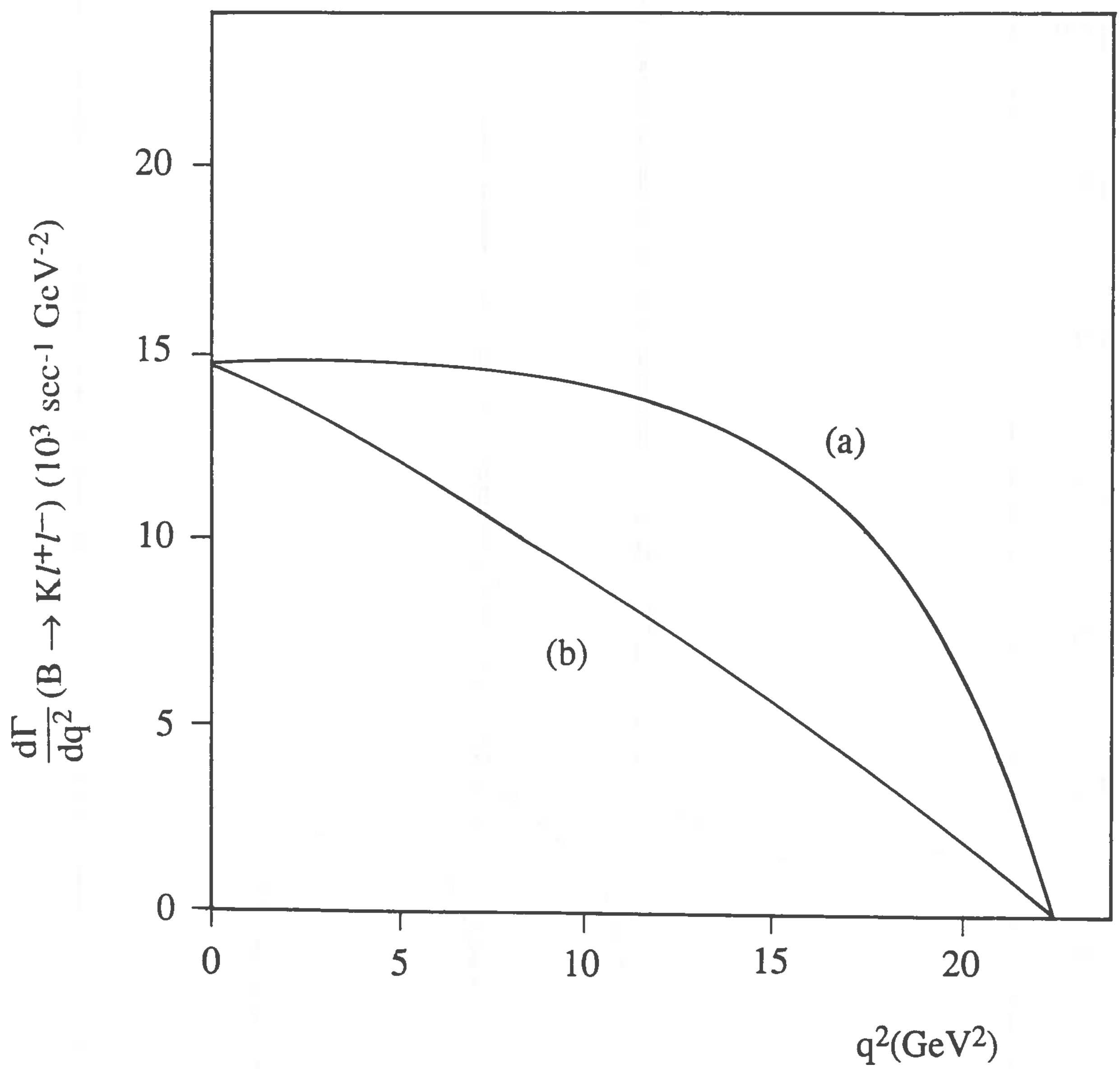
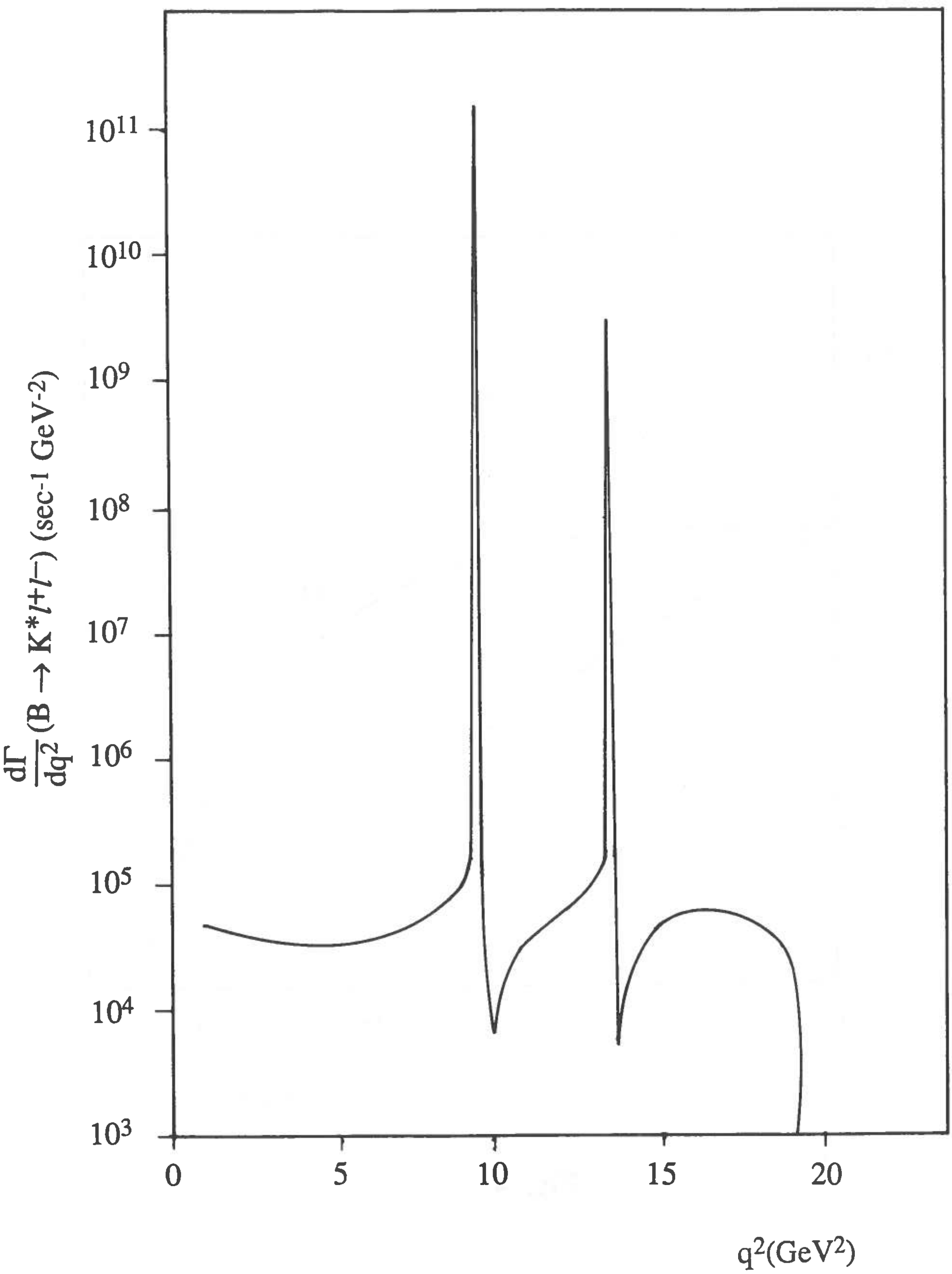


Fig. 4



dΓ/dq² (B → K*l+l−) (sec-1 GeV-2)
10^11
10^10
10^9
10^8
10^7
10^6
10^5
10^4
10^3
0
5
10
15
20
q²(GeV²)

Fig. 5

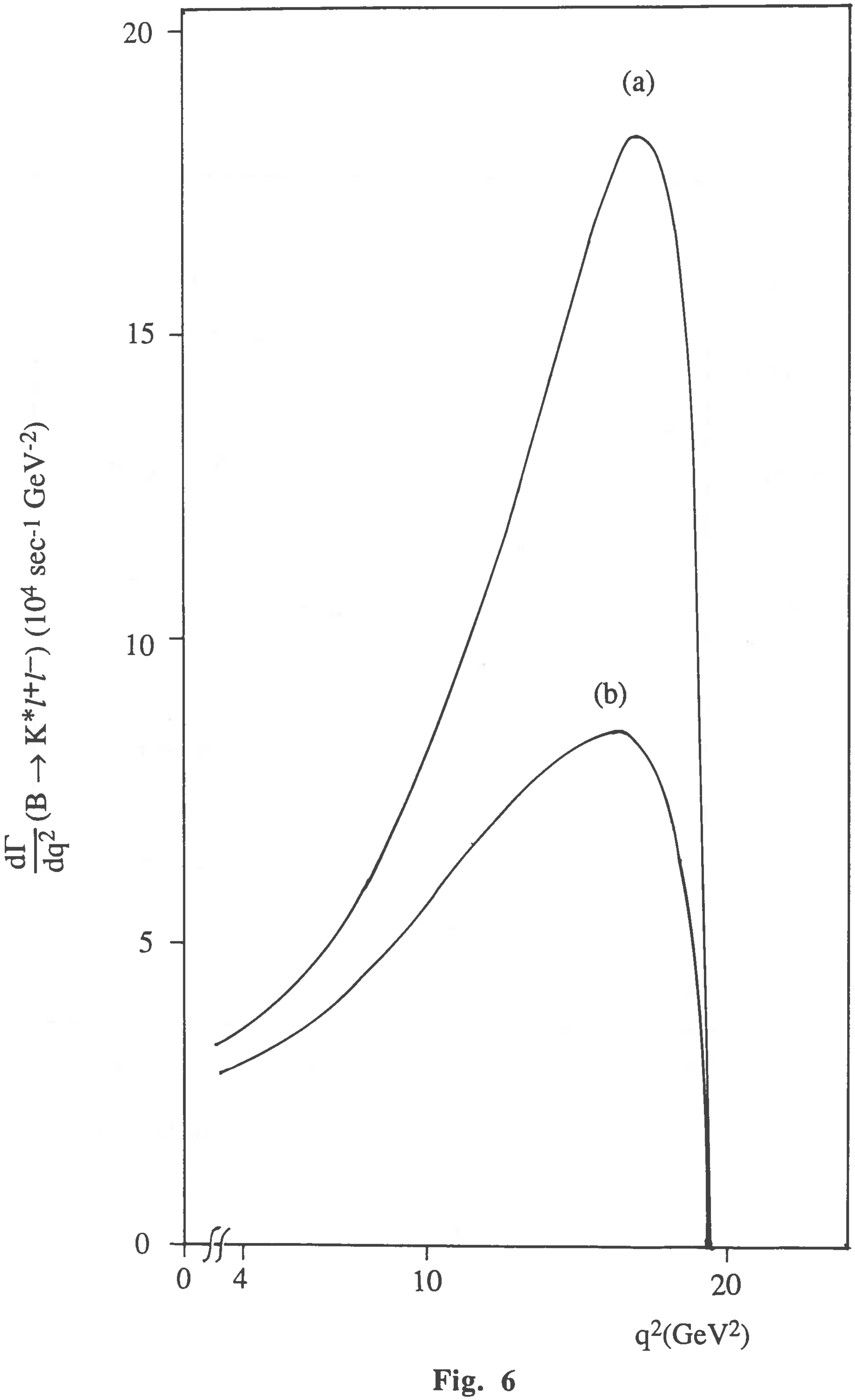


Fig. 6

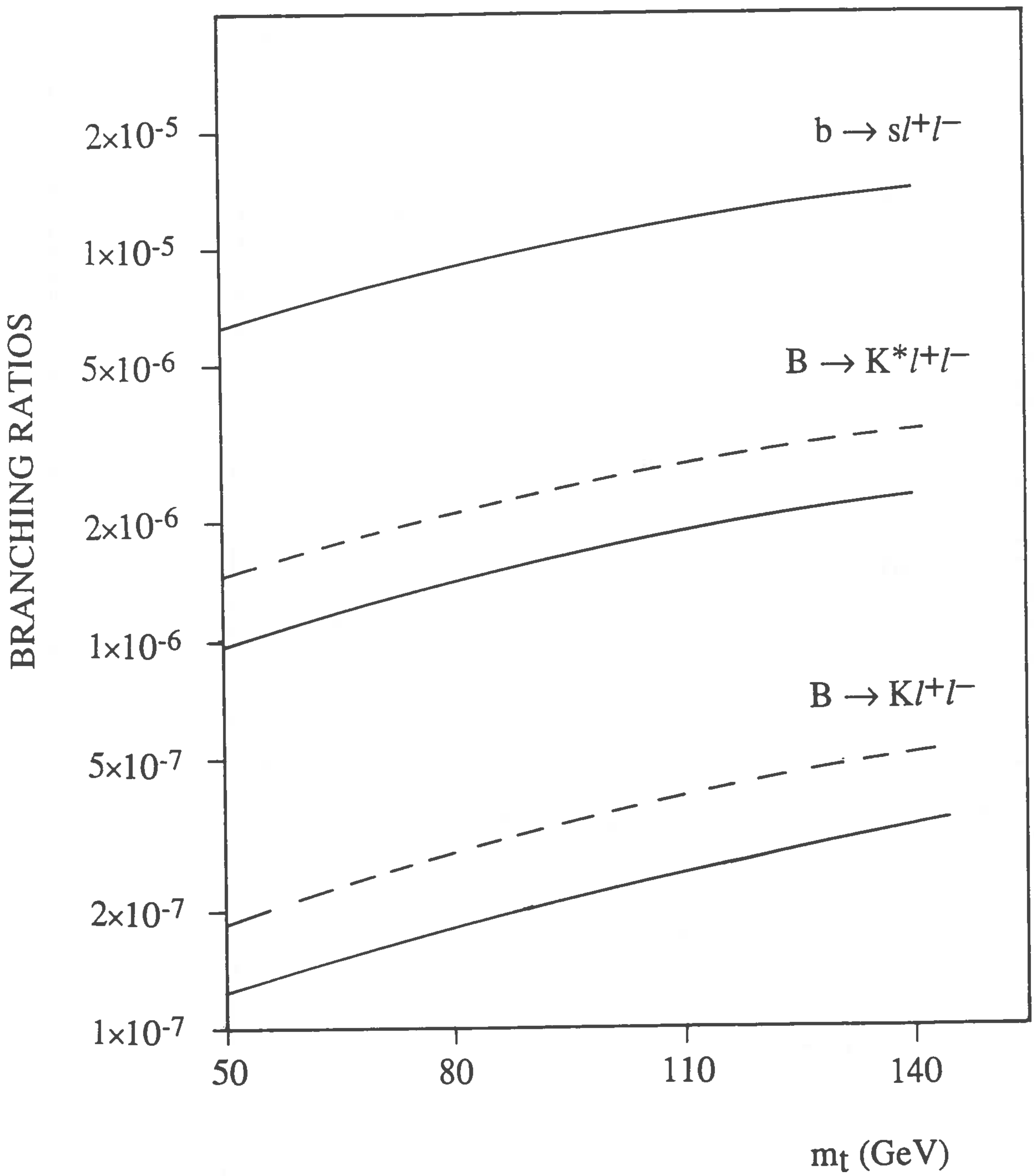

b → sl+l−
B → K*l+l−
B → Kl+l−
BRANCHING RATIOS
2×10-5
1×10-5
5×10-6
2×10-6
1×10-6
5×10-7
2×10-7
1×10-7
50
80
110
140
mt (GeV)

Fig. 7