ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Trieste

INFN/AE-89/4

14 luglio 1989

G. Calucci and D. Treleani

MINIJETS:
CROSS SECTION AND ENERGY DISTRIBUTION IN VERY HIGH ENERGY NUCLEAR COLLISIONS

Servizio Documentazione
dei Laboratori Nazionali di Frascati

MINIJETS: CROSS SECTION AND ENERGY DISTRIBUTION IN VERY HIGH ENERGY NUCLEAR COLLISIONS

G. Calucci and D. Treleani

Dipartimento di Fisica Teorica dell'Università di Trieste, Trieste, I 34014 Italy
INFN, Sezione di Trieste, Italy

ABSTRACT

The energy spectrum from semi-hard partonic interactions in nucleus nucleus collisions with c.m. energies of the order of one TeV per nucleon is discussed. The presence of a large number of nucleons induces incoherence among most of the partonic collisions while the large number of partonic interactions makes the unitarization of the cross section an essential tool for a meaningful description of the processes. This goal is achieved accounting for all semi-hard partonic scatterings, namely including both disconnected collisions and rescatterings. The characteristic feature of the interaction, resulting from this analysis, is that it is basically a geometrical one. As a consequence of the unitarization the energy distribution of the scattered partons turns out to be a regular function of the cut off p_t^{min} distinguishing semi-hard events from soft ones.

This divergent behaviour is, at first sight, unexpected since the divergence affects the differential cross section but not the integrated one.

The nature of this divergence can be traced back to the singular behaviour of average quantities, like the average energy produced by semi-hard partonic collisions, that are still given by the perturbative QCD parton model expression, and therefore are divergent when the cut off p_t^{min} goes to zero.

As an example the average energy produced by semi-hard partonic collisions is given by

$$\langle E\rangle\sigma_H = \int E\frac{d\sigma_H}{dE}dE \tag{5}$$

with

$$\frac{d\sigma_H}{dE} = \int d^2\beta \sum_{n=1}^{\infty} \frac{1}{n!}\frac{d\langle n(\beta)\rangle}{dE_1}\cdots\frac{d\langle n(\beta)\rangle}{dE_n}\delta(E-E_1-\ldots-E_n)e^{-\langle n(\beta)\rangle}dE_1\ldots dE_n \quad (6).$$

One has then

$$\begin{aligned}\langle E\rangle\sigma_H &= \int d^2\beta \sum_{n=1}^{\infty}\frac{1}{n!}(E_1+\ldots+E_n)\frac{d\langle n(\beta)\rangle}{dE_1}\cdots\frac{d\langle n(\beta)\rangle}{dE_n}e^{-\langle n(\beta)\rangle}dE_1\ldots dE_n \\ &= \int d^2\beta \sum_{n=1}^{\infty}\frac{1}{(n-1)!}\int E_1\frac{d\langle n(\beta)\rangle}{dE_1}dE_1\langle n(\beta)\rangle^{n-1}exp[-\langle n(\beta)\rangle]\end{aligned} \tag{7}$$

so that

$$\langle E\rangle\sigma_H = \int E\frac{d\sigma_{QCD}}{dE}dE \tag{8}$$

where $\frac{d\sigma_{QCD}}{dE}$ is the differential cross section to produce energy by means of semi-hard parton collisions as given by the single hard scattering expression provided by the QCD parton model.

When considering the case of a very large number of partonic collisions one may estimate the energy spectrum by means of the central limit theorem[7], getting

$$\frac{1}{\sigma_H}\frac{d\sigma_H}{dE} = \frac{1}{\sqrt{D\pi}}exp\left(-\frac{(E-\langle E\rangle)^2}{D}\right) \tag{9}$$

where D is defined as $D = \langle(E-\langle E\rangle)^2\rangle$ and can be estimated analogously to the average energy $\langle E\rangle$.

Altough the integral of Eq.(9) gives 1, the energy spectrum is itself meaningless for small values of the cut off.

The problem of the divergent behaviour of average quantities in the limit of $p_t^{min} \to 0$ will be discussed in the next paragraphs, where it will be shown how a solution can be obtained keeping properly into account all multiple parton collisions.

The energy spectrum from semi-hard partonic interactions will then be estimated.

II. Semi-hard cross section and average number of collisions

The divergent behaviour of average quantities like the average energy produced in semi-hard partonic collisions discussed in the previous paragraph has to be solved, quite in general, unitarizing the parton distributions[8] at small values of x.

Although that, in general terms, is the problem one has to solve, we observe that it can appear quite differently in particular cases. We think, in fact, that the problem will look much simpler when considering semi-hard nuclear, rather than hadronic, interactions. In that case, in fact, the unitarity problem will appear also at fixed x when increasing the atomic mass A. The advantage is that one can work discussing partonic distributions in ranges of x where they are well defined, or, more precisely, in a region where one does not need to take into account the unitarization of the parton distributions at the nucleon level. (One must point out, however, that, for a realistic calculation, in the relevant region of x, namely $x \simeq 10^{-3}$, the resummation of logs of $1/x$ has to be included[9]).

In such a case the problem with unitarity will look much simpler since the origin of the divergences can be traced back in a rather clean way and a solution can be found using, in practice, only geometrical arguments.

In fact when looking at the dependence on the atomic mass A of the average energy associated with semi-hard interactions, one will observe that, using Eq.(8), the growth with A is faster than that one of the total energy available:

If the atomic mass numbers of the two interacting nuclei are A and B respectively, then the total energy available is proportional to $A + B$, while the average number of partonic collisions is rather proportional to $A \times B$. The average energy produced by hard partonic collisions is given, in the QCD parton model, by the average number of partons in nucleus A multiplied by the average number of partons in nucleus B, the hard partonic cross section and the energy of the interacting partons; everything then is divided by the hard nucleus nucleus cross section.

When looking at semi-hard interactions one will notice that the semi-hard nucleus nucleus cross section, in the limit of a very large number of collisions and for $A \simeq B$, is behaving like $A^{2/3}$ so that one gets effectively, for the average energy, a behaviour of $A^{4/3}$ that will violate energy conservation for large values of A.

Although the problem does not seem to give a real contradiction at regimes that one can realistically forsee, at lest in a near future, still the observation seems to us an interesting one, since it poses a problem of consistency in a rather clear way.

In fact, the problem will be present even considering the simplest nuclear structure: two spherical nuclei with no Fermi motion, all nucleons carrying $1/A$ of the total momentum.

It is useful to remind that an experimental measure of the deviation of the partonic nuclear structure from this simplified description is provided by the EMC effect[10]. This effect however is small, being confined in corrections to the nuclear parton distribution of the order or less then 10%. We will then focus on this simplified case only, since we are more interested in the consistency problem rather than in a very detailed prediction; moreover the corrections due to Fermi motion and binding energy will not spoil the overall general picture.

Since the value of the initial state energy is finite, the origin of the trouble for the total energy produced by semi-hard interactions is to be looked for in an inconsistent treatment of the interaction. Actually, keeping into account disconnected semi-hard processes only, as it was done in ref.3 to estimate the semi-hard cross section, one might get the interaction probability, for a parton to interact with a nucleus, larger than one when the number of partonic interactions will become very large. As a consequence the value of the semi-hard cross section will not be much affected, but that will not be the case for other physical observables.

We will then start discussing the semi-hard cross section more in detail.

Let us assume a Poissonian for the small x component of the nuclear parton distribution.

The assumption of a Poissonian distribution for the nuclear partons is consistent with the assumption of an incoherent superposition of Poissonian distributions of partons at the nucleon level. The information that the nucleus is made up with nucleons is present, in this framework, when giving the kinematical limits for the fractional momenta x, that are scaled to the nucleon energy. The picture than consists of a nucleus made with A non interacting nucleons: when interacting with a high-energy parton only the partonic structure of the individual nucleons is seen, the information on the intermediate nucleonic structure being present only in the kinematical limits of the partons.(Further discussions on this point are presented in appendix C).

The probability density for having n partons of nucleus A (being A the nuclear mass) with fractional momenta $x_1, \dots, x_n$ and with transverse coordinates $\mathbf{b}_1, \dots, \mathbf{b}_n$ is then given by:

$$\frac{1}{n!}\Gamma_A^{f_1}(x_1, \mathbf{b}_1)\dots\Gamma_A^{f_n}(x_n, \mathbf{b}_n) \times exp[-\int\sum_f \Gamma_A^f(x, \mathbf{b})dxd^2b] \qquad (10)$$

where $\Gamma_A^f(x, \mathbf{b})$ is the average number of partons in the A nucleus with momentum fraction x (with respect to the nucleon momentum), transverse coordinate $\mathbf{b}$ and the index f is counting the various species of partons.

The normalization of $\Gamma_A^f(x, \mathbf{b})$ is A times that of the nucleon parton distributions and the integral in Eq.(10) is regularized with a cut off related to p_t^{min}.

One will notice that, in writing Eq.(10), the total energy of the nucleus is not fixed any more: one is in fact introducing a dispersion in the total energy proportional to A. The total energy can then vary by amounts proportional to $A^{1/2}$ around its average value (that is proportional to A). Since we will keep, in the following, only the leading terms in the atomic mass number we will disregard this problem.

When writing the semi-hard cross section we will make the simplifying assumption of complete incoherence between different partonic semi-hard collisions.

That amounts to assume incoherence not only between disconnected semi-hard collisions, but also between different rescatterings of the same parton. In case of collisions on different nucleons this hypothesis is consistent both with previous works on inelastic nucleus-nucleus collisions[11,12] and with a recent analysis on rescattering of partons on nuclear targets[13].

We will make the same assumption also if the multiple collisions will happen on the same nucleon, in such a way that all partons will be treated on the same footing.

Although incoherence is less obvious in this last case, we will keep this assumption for simplicity reasons and because multiple collisions on the same nucleon are a small subset of all multiple collisions, when A is large and the interaction is not very peripheral.

The semi-hard cross section will then be expressed as:

$$\begin{aligned}\sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \dots f_n} \frac{1}{n!} \Gamma_A^{f_1}(x_1, \mathbf{b}_1) \dots \Gamma_A^{f_n}(x_n, \mathbf{b}_n) e^{-\int \sum_f \Gamma_A^f(x,\mathbf{b}) dx d^2b} \times \\ \times \sum_{l=1}^{\infty} \sum_{f'_1 \dots f'_l} \frac{1}{l!} \Gamma_B^{f'_1}(x'_1, \mathbf{b}'_1 - \beta) \dots \Gamma_B^{f'_l}(x'_l, \mathbf{b}'_l - \beta) \times \\ \times e^{-\int \sum_{f'} \Gamma_B^{f'}(x',\mathbf{b}') dx' d^2b'} \times [1 - \prod_{i=1}^{n} \prod_{j=1}^{l} \prod_{f_i, f'_j} (1 - \hat{\sigma}_{ij}^{f_i f'_j})] \times \\ \times dx_1 d^2b_1 \dots dx_n d^2b_n dx'_1 d^2b'_1 \dots dx'_l d^2b'_l\end{aligned} \tag{11}$$

where $\hat{\sigma}_{ij}^{f_i f'_j} \equiv \hat{\sigma}_{ij}^{f_i f'_j}(\mathbf{b}_i - \mathbf{b}_j, x_i x_j)$ is the probability for the parton f_i from nucleus A to have a semi-hard interaction with parton f'_j from nucleus B, so it will depend on $x_i x_j$, on the difference of the transverse relative distance $\mathbf{b}_i - \mathbf{b}'_j$ and on the indices f_i, f'_j.

The square parenthesis in Eq.(11) will then represent the probability to have at least one semi-hard partonic interaction between nucleus A and nucleus B, and the cross section is constructed summing over all possible partonic configurations of the two nuclei and integrating on the nuclear impact parameter β.

One will notice that in Eq.(11) all possible interactions between partons of

I. Introduction

At very high c.m. energies the description of hadronic interactions by short distance dynamics as given by the QCD parton model faces a unitarity problem:

The QCD parton model provides the inclusive hard cross section expressed as:

$$\sigma_{incl} = \sum_{ff'} \int_{p_t^{min}} G_A^f(x) G_B^{f'}(x') \hat{\sigma}^{ff'}(xx') dx dx' \qquad (1)$$

with $G^f(x)$ the parton distributions, depending explicitly on the fractional momentum x and parton species f, $\hat{\sigma}^{ff'}$ the parton parton cross section (integrated on the parton parton c.m. polar scattering angle) and p_t^{min}, the cut off distinguishing large p_t events from soft ones, (providing a regularization of the divergences originated from the distributions G and also of the Rutherford singularity of $\hat{\sigma}$).

In fact one can realize that there are conditions in which the cut off is large enough to allow a perturbative approach to the interaction, and at the same time, expression (1) gets larger than the total cross section[1]. Actually, when keeping p_t^{min} fixed and increasing the c.m. energy squared s, the inclusive cross section is increasing faster than the total one.

The kinematical region where Eq.(1) gets comparable to the size of the total inelastic cross section is called region for semi-hard interactions.

Being the inclusive cross section to produce large p_t partons proportional to the average number of partons in the final state[2], ($\sigma_{incl} = \langle n \rangle \sigma_H$, with $\sigma_{inel} = \sigma_H + \sigma_{soft}$) there is no inconsistency in having σ_{incl} larger than σ_{tot} provided that the average number of partons with $p_t > p_t^{min}$ in the final state grows with energy faster than the total cross section.

The problem that one is facing is, however, the one of discussing the semi-hard cross section σ_H in the framework of perturbative QCD.

One will notice that, while the QCD parton model expression for σ_{incl} is not characterized by any intrinsic scale, so that expression (1) gets its size by the cut off p_t^{min}, the semi-hard cross section σ_H is, on the contrary, characterized by a natural scale because of the bound $\sigma_H < \sigma_{inel}$.

As discussed in ref.3 one will find a natural solution to the problem taking into account all disconnected semi-hard parton collisions (real and virtual): Increasing the number of interacting partons one will obviously increase the parton multiplicity in the final state, and multiple parton collisions will introduce a characteristic scale into the picture because the multiparton distributions are dimensional quantities. In fact multiparton distributions, essentially for geometrical reasons, are quantities whose size is given by the inverse of the typical transverse hadronic dimension[4].

Assuming a Poisson distribution for multiparton distributions one gets the semi-hard cross section σ_H expressed as:

$$\sigma_H = \int d^2\beta(1 - exp[-\langle n(\beta)\rangle]) = \sum_{n=1}^{\infty} \int d^2\beta \frac{1}{n!} \langle b(\beta)\rangle^n exp(-\langle n(\beta)\rangle) \qquad (2)$$

with

$$\langle n(\beta)\rangle = \sum_{ff'} \int_{p_t > p_t^{min}} \Gamma_A^f(x, \mathbf{b}) \Gamma_B^{f'}(x, \mathbf{b} - \beta) \hat{\sigma}^{ff'}(x, x') dx dx' d^2 b \qquad (3)$$

where Γ_A^f and $\Gamma_B^{f'}$ are the parton distributions of the two interacting hadrons A and B (depending explicitly on the parton transverse coordinate b and such that $\int \Gamma^f(x, \mathbf{b}) d^2 b = G^f(x)$), β is the hadronic impact parameter and $\langle n(\beta)\rangle$ the average number of semi-hard parton parton collisions at a given value of β.

Eq.(2) shows how the semi-hard cross section can be expressed as a sum of multiple semi-hard collisions of partons with Poissonian distribution at fixed impact parameter.

One can see, moreover, how the hadronic dimension enters in giving the size to the semi-hard cross section, so that this one has only a smooth dependence on the cut off p_t^{min} for $p_t^{min} \to 0$. One has, in fact, that for small values of the cut off $\langle n(\beta)\rangle$ becomes very large, so that the exponential in Eq.(2) is practically zero. When however β is larger than some typical hadronic radius R, for any value of the cut off, $\langle n(\beta)\rangle$ becomes zero because there is no overlap between the interacting hadrons. The size of σ_H is then bounded by πR^2.

When evaluating the average number of semi-hard partonic interactions one gets back the QCD parton model expression for the inclusive cross section:

$$\langle n\rangle\sigma_H = \sum_{n=1}^{\infty} \int d^2\beta \frac{n}{n!} \langle n(\beta)\rangle^n exp[-\langle n(\beta)\rangle] = \int d^2\beta \langle n(\beta)\rangle = \sigma_{incl} \qquad (4)$$

consistently with the cancellation rules for initial and final state interaction[5].

In fact, while in ref.3, one gets Eq.(2) for the semi-hard cross section discussing the partonic interaction in the perturbative regime, one can obtain a similar expression in the framework of eikonal models for high energy hadronic interaction[6].

Although the semi-hard cross section σ_H is a well defined quantity, still the situation is not satisfactory: one has in fact a lot of difficulties in defining other well behaved physical observables in the limit of small cut off values. This problem will appear not only when looking at cases like Eq.(4), where the origin of the divergent behaviour is in the nature of the cross section itself, that counts the parton multiplicity, but also when looking at quantities (like the energy spectrum produced by semi-hard partonic interactions) that, when integrated, will give the semi-hard cross section σ_H.

nucleus A and partons of nucleus B are taken into account, so that also all possible semi-hard rescatterings in nuclear matter are included.

The semi-hard cross section can be conveniently expressed introducing the probabilities for the parton f'_j from nucleus B to have at least one semi-hard interaction with a given configuration of n partons of nucleus A, defined as:

$$P^{f'_j}_{f_1\dots f_n} \equiv P^{f'_j}_{f_1\dots f_n}(x'_j, \mathbf{b}'_j; x_1, \mathbf{b}_1 \dots x_n, \mathbf{b}_n) \equiv 1 - \prod_{i=1}^{n} \prod_{f_i} (1 - \hat{\sigma}^{f_i f'_j}_{ij}) \tag{12}$$

One can then write:

$$\begin{aligned} \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1\dots f_n} \frac{1}{n!} \Gamma_A^{f_1}(x_1, \mathbf{b}_1) \dots \Gamma_A^{f_n}(x_n, \mathbf{b}_n) e^{-\int \sum_f \Gamma_A^f(x,\mathbf{b}) dx d^2b} \times \\ \times \sum_{l=1}^{\infty} \sum_{f'_1\dots f'_l} \frac{1}{l!} \Gamma_B^{f'_1}(x'_1, \mathbf{b}'_1 - \beta) \dots \Gamma_B^{f'_l}(x'_l, \mathbf{b}'_l - \beta) \times \\ \times e^{-\int \sum_{f'} \Gamma_B^{f'}(x',\mathbf{b}') dx' d^2b'} \times \\ \times [1 - \prod_{j=1}^{l} \prod_{f'_j} (1 - P^{f'_j}_{f_1\dots f_n}(x'_j, \mathbf{b}'_j; x_1, \mathbf{b}_1 \dots x_n, \mathbf{b}_n)] \times \\ \times dx_1 d^2b_1 \dots dx_n d^2b_n dx'_1 d^2b'_1 \dots dx'_l d^2b'_l \end{aligned} \tag{13}$$

The sums over l and f'_l can be performed giving:

$$\begin{aligned} \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1\dots f_n} \frac{1}{n!} \Gamma_A^{f_1}(x_1, \mathbf{b}_1) \dots \Gamma_A^{f_n}(x_n, \mathbf{b}_n) e^{-\int \sum_f \Gamma_A^f(x,\mathbf{b}) dx d^2b} \times \\ \times \left[1 - e^{(-\int \sum_{f'} \Gamma_B^{f'}(x',\mathbf{b}'-\beta) P^{f'}_{f_1\dots f_n} dx d^2b')}\right] \times dx_1 d^2b_1 \dots dx_n d^2b_n \end{aligned} \tag{14}$$

The number of partonic collisions can grow very rapidly with the atomic mass, but the number of partons involved in the interaction will grow much less. It is then convenient to introduce the concept of wounded parton (analogously to that one of wounded nucleon[11]) as a parton that has suffered at least one semi-hard interaction. Eq.(14) is a convenient expression to estimate the average number of wounded partons.

The reason to look for wounded partons is that they are directly related to measurable quantities. For example the total energy produced by semi-hard parton collisions is given by the number of wounded parton multiplied by the energy carried by each of them. One will notice that such a quantity is finite in the limit of small cut off values since the total energy carried by all partons is finite in the small x limit. The QCD parton model expression , on the other hand, being related to the average number of partonic collisions, will be badly divergent and will coincide with the correct expression only in the limit of a small number of hard interactions.

One may in fact expand Eq.(14) in terms of number of wounded partons of nucleus B:

$$\begin{aligned}\sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \dots f_n} \frac{1}{n!} \Gamma_A^{f_1}(x_1, \mathbf{b}_1) \dots \Gamma_A^{f_n}(x_n, \mathbf{b}_n) e^{-\int \sum_f \Gamma_A^f(x,\mathbf{b}) dx d^2 b} \times \\ \times \sum_{k=1}^{\infty} \frac{1}{k!} \Big[\int \sum_{f'} \Gamma_B^{f'}(x', \mathbf{b}' - \beta) P_{f_1 \dots f_n}^{f'} dx' d^2 b'\Big]^k \times \\ \times e^{-\int \sum_{f'} \Gamma_B^{f'}(x', \mathbf{b}' - \beta) P_{f_1 \dots f_n}^{f'} dx' d^2 b'} \times dx_1 d^2 b_1 \dots dx_n d^2 b_n\end{aligned} \tag{15}$$

so that one may write

$$\sigma_H^{AB} = \sum_{k=1}^{\infty} \sigma_{AB}^{(k)}. \tag{16}$$

The average number of wounded partons is then easily obtained:

$$< k > \sigma_H^{AB} = \sum_{k=1}^{\infty} k \sigma_{AB}^{(k)} \tag{17}$$

that can be immediately written as:

$$\begin{aligned}< k > \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \dots f_n} \frac{1}{n!} \Gamma_A^{f_1} \dots \Gamma_A^{f_n} e^{-\int \sum_f \Gamma_A^f(x,\mathbf{b}) dx d^2 b} \times \\ \times \sum_{f'} \Gamma_B^{f'}(x', \mathbf{b}' - \beta) P_{f_1 \dots f_n}^{f'} dx' d^2 b' dx_1 d^2 b_1 \dots dx_n d^2 b_n\end{aligned} \tag{18}$$

Using Eq.(12) one easily obtains:

$$\begin{aligned}< k > \sigma_H^{AB} = \int d^2\beta dx' d^2 b' \sum_{f'} \Gamma_B^{f'}(x', \mathbf{b}' - \beta) \times \\ \times \big[1 - exp(-\int \sum_f \Gamma_A^f(x, \mathbf{b}') \hat{\sigma}^{ff'}(xx') dx)\big].\end{aligned} \tag{19}$$

In Eq.(19) the probability of semi-hard interaction $\hat{\sigma}^{ff'}(xx', \mathbf{b}-\mathbf{b}')$ has been treated as a δ function in $\mathbf{b}-\mathbf{b}'$ in comparison with the much smoother $\mathbf{b}$ dependence of the average number of partons $\Gamma(x,\mathbf{b})$. The cross section $\hat{\sigma}^{ff'}(xx')$ is then the usual parton parton cross section integrated on the polar c.m. angle with the cut off provided by p_t^{min}. The integral in the exponent is also regularized with the same cut off.

Expression (19) has a transparent physical interpretation: the square parenthesis represents the probability for a parton of nucleus B to have at least one semi-hard interaction with nucleus A, so that the average number of wounded partons of B is given by the average number of partons of B multiplied by the interaction probability.

If one wanted rather to look at the average number of semi-hard partonic collisions $<\nu>$ then (as discussed in appendix A) one will have to replace the square parenthesis in Eq.(11) with $\sum_{p=1}^{nl}\hat{\sigma}_p$, so that

$$
\begin{aligned}
<\nu>\sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty}\sum_{f_1\dots f_n}\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\dots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)\times \\
\times\sum_{l=1}^{\infty}\sum_{f'_1\dots f'_l}\frac{1}{l!}\Gamma_B^{f'_1}(x'_1,\mathbf{b}'_1-\beta)\dots\Gamma_B^{f'_l}(x'_l,\mathbf{b}'_l-\beta)\times \\
\times e^{-\int\sum_f\Gamma_A^f(x,\mathbf{b})dxd^2b}\times e^{-\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}')dx'd^2b'}\times \\
\times\sum_{p=1}^{nl}\hat{\sigma}_p^{ff'}\times dx_1d^2b_1\dots dx_nd^2b_ndx'_1d^2b'_1\dots dx'_ld^2b'_l
\end{aligned}
\qquad (20)
$$

and since the sum over p in Eq.(20) can be replaced with $nl\times\hat{\sigma}_1^{ff'}$, being all partons identical for the present purpose, one may write

$$
<\nu>\sigma_H^{AB} = \sum_{ff'}\int d^2\beta\Gamma_A^f(x_1,\mathbf{b}_1)\Gamma_B^{f'}(x'_1,\mathbf{b}_1-\beta)\hat{\sigma}^{ff'}(x_1x'_1)dxdx'd^2b_1 \qquad (21)
$$

that is the single scattering expression given by the QCD parton model.

One has then checked that the approach is consistent with the cancellation involving the average number of collisions.

On the other hand, in such an extreme regime, this average quantity is of little interest. On the contrary one is interested in quantities that are more directly accessible experimentally and that, as it will be discussed in the next paragraph, are rather related to averages involving wounded partons.

III. Average energy and dispersion

The energy produced by semi-hard partonic interactions is the energy carried by the wounded partons (since a parton is wounded when it suffers at least one semi-hard interaction).

Also the dispersion in the energy produced will be related to various averages involving wounded partons.

We will then start discussing these averages.

Another way to obtain the average number of wounded partons of nucleus B (and that will be convenient later) is the following: one may sum over the partons of each given partonic configuration of B (the sum over the index s in the following expression) and then for each term in the sum one will ask for the probability that the parton of B taken into consideration, will have at least one semi-hard interaction with A (the square parenthesis):

$$
\begin{aligned}
< k > \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \dots f_n} & \frac{1}{n!} \Gamma_A^{f_1}(x_1, \mathrm{b}_1) \dots \Gamma_A^{f_n}(x_n, \mathrm{b}_n) \times \\
& \times \sum_{k=1}^{\infty} \sum_{f'_1 \dots f'_k} \frac{1}{k!} \sum_{s=1}^{k} \sum_{f'_s} \Gamma_B^{f'_1}(x'_1, \mathrm{b}'_1 - \beta) \dots \\
& \dots \Gamma_B^{f'_s}(x'_s, \mathrm{b}'_s - \beta) \dots \Gamma_B^{f'_k}(x'_k, \mathrm{b}'_k - \beta) \times \\
& \times e^{-\int \sum_f \Gamma_A^f(x,\mathrm{b}) dx d^2b} \times e^{-\int \sum_{f'} \Gamma_B^{f'}(x',\mathrm{b}') dx' d^2b'} \times \\
& \times [1 - \prod_{l=1}^{n} \prod_{f_l} (1 - \hat{\sigma}_{l_s}^{f_l f'_s})] \times \\
& \times dx_1 d^2b_1 \dots dx_n d^2b_n dx'_1 d^2b'_1 \dots dx'_s d^2b'_s \dots dx'_k d^2b'_k
\end{aligned}
\tag{22}
$$

With a little of algebra Eq.(22) will give back Eq.(19).

One may now easily obtain $< k^2 >$ and $< k \times n >$ (namely an average involving both nuclei A and B).

A natural extension of Eq.22 will lead to:

$$
\begin{aligned}
< kn > \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1\dots f_n} \frac{1}{n!} \sum_{j=1}^{n} \sum_{f_j} \Gamma_A^{f_1}(x_1, \mathbf{b}_1)\dots \\
\dots \Gamma_A^{f_j}(x_j, \mathbf{b}_j)\dots\Gamma_A^{f_n}(x_n, \mathbf{b}_n) \times e^{-\int \sum_f \Gamma_A^f(x,\mathbf{b})dxd^2b} \times \\
\times \sum_{k=1}^{\infty} \sum_{f'_1\dots f'_k} \frac{1}{k!} \sum_{s=1}^{k} \sum_{f'_s} \Gamma_B^{f'_1}(x'_1, \mathbf{b}'_1 - \beta)\dots \\
\dots \Gamma_B^{f'_s}(x'_s, \mathbf{b}'_s - \beta)\dots\Gamma_B^{f'_k}(x'_k, \mathbf{b}'_k - \beta) \times e^{-\int \sum_{f'} \Gamma_B^{f'} dx'd^2b'} \times \\
\times[1 - \prod_{l=1, l\neq j}^{n} \prod_{f_l} (1 - \hat{\sigma}_{ls}^{f_l f'_s})] \times [1 - \prod_{i=1, i\neq s}^{k} \prod_{f'_i} (1 - \hat{\sigma}_{ji}^{f_j f'_i})] \times \\
\times dx_1 d^2b_1 \dots dx_n d^2b_n dx'_1 d^2b'_1 \dots dx'_k d^2b'_k
\end{aligned} \tag{23}
$$

In Eq. (23) the simplification was made of neglecting the case $l = j$ and $i = s$ that will correspond to the possibility of interaction for the parton s of B with the parton j of A, being s and j the partons taken into consideration in evaluating the average. (The relevance and validity of this simplification is further discussed in appendix B).

Given the very large amount of partonic interactions at the regime considered here, this simplification will make only marginal corrections to an exact result. On the other hand the final expression obtained in this way has a very simple and transparent physical meaning.

With some manipulations one gets in fact:

$$
\begin{aligned}
< kn > \sigma_H^{AB} = \int d^2\beta dx' d^2b' dx d^2b \sum_{ff'} \Gamma_B^{f'}(x', \mathbf{b}' - \beta)\Gamma_A^f(x, \mathbf{b}) \times \\
\times[1 - exp(-\int \sum_{f_1} \Gamma_B^{f_1}(x_1, \mathbf{b} - \beta)\hat{\sigma}^{ff_1}(x_1 x)dx_1)] \times \\
\times[1 - exp(-\int \sum_{f_2} \Gamma_A^{f_2}(x_2, \mathbf{b}')\hat{\sigma}^{f_2 f'}(x_2 x')dx_2)]
\end{aligned} \tag{24}
$$

where the probability for a parton to have a semi- hard interaction with a nucleus is bounded by one also in the limit of p_t^{min} close to zero:

At a given transverse coordinate $\mathbf{b}$ one has, in fact, some average number of partons of kind f: $\Gamma_A^f(x, \mathbf{b})$ from nucleus A multiplied by the interaction probability with nucleus B (actually $[1 - exp(-\int \sum_{f'} \Gamma_B^{f'}(x_1, \mathbf{b} - \beta)\hat{\sigma}^{ff'}(x_1 x)dx_1)]$). The same operation is done with B and finally one has to sum over all possibilities.

The quantities that we are more interested in are, however, the averages involving the energy of the wounded partons.

The reason is that one can then estimate the semi-hard energy spectrum, or better its average value and the dispersion.

A part a trivial rescaling we are then interested in $< x_B >$, $< x_B^2 >$, $< x_A >$, $< x_A^2 >$ and $< x_A x_B >$.

Let us then look for $< x_B >$:

$$
\begin{aligned}
< x_B > \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \ldots f_n} & \frac{1}{n!} \Gamma_A^{f_1}(x_1, \mathbf{b}_1) \ldots \Gamma_A^{f_n}(x_n, \mathbf{b}_n) \times \\
& \times \sum_{k=1}^{\infty} \sum_{f'_1 \ldots f'_k} \frac{1}{k!} \Gamma_B^{f'_1}(x'_1, \mathbf{b}'_1 - \beta) \ldots \Gamma_B^{f'_k}(x'_k, \mathbf{b}'_k - \beta) \times \\
& \times (x'_1 P^{f'_1}_{f_1 \ldots f_n} + \ldots + x'_k P^{f'_k}_{f_1 \ldots f_n}) \times \\
& \times e^{-\int \sum_f \Gamma_A^f(x, \mathbf{b}) dx d^2 b} \times e^{-\int \sum_{f'} \Gamma_B^{f'}(x', \mathbf{b}') dx' d^2 b'} \times \\
& \times dx_1 d^2 b_1 \ldots dx_n d^2 b_n dx'_1 d^2 b'_1 \ldots dx'_k d^2 b'_k
\end{aligned}
\qquad (25)
$$

that will give

$$
\begin{aligned}
< x_B > \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \ldots f_n} & \frac{1}{n!} \Gamma_A^{f_1}(x_1, \mathbf{b}_1) \ldots \Gamma_A^{f_n}(x_n, \mathbf{b}_n) \times \\
& \times e^{-\int \sum_f \Gamma_A^f(x, \mathbf{b}) dx d^2 b} \times \sum_{f'} x' \Gamma_B^{f'}(x, \mathbf{b}' - \beta) P^{f'}_{f_1 \ldots f_n} \times \\
& \times dx_1 d^2 b_1 \ldots dx_n d^2 b_n dx' d^2 b'
\end{aligned}
\qquad (26)
$$

and finally

$$
\begin{aligned}
< x_B > \sigma_H^{AB} = \int d^2\beta dx' d^2 b' & \sum_{f'} x' \Gamma_B^{f'}(x', \mathbf{b}' - \beta) \times \\
& \times [1 - exp(-\int \sum_f \Gamma_A^f(x, \mathbf{b}') \hat{\sigma}^{ff'}(xx') dx)]
\end{aligned}
\qquad (27)
$$

that can be compared with Eq.(19).

One will notice that in the limit of small p_t^{min} values Eq.(27) is well defined, since the interaction probability is explicitly less than one and the average momentum carried by the partons of nucleus B is regular as a function of the cut off.

To estimate $< x_B^2 >$ one will rewrite Eq.(25) replacing the sum of x's with a sum squared.

One will then get two contributions:

$$(x'_1+\ldots+x'_k)^2=\sum_{i=1}^{k}(x'_i)^2+\sum_{i,j=1,i\neq j}^{k}x'_i x'_j.$$

The first one one will give:

$$\int d^2\beta dx' d^2b' \sum_{f'}(x')^2\Gamma_B^{f'}(x',\mathrm{b}'-\beta)[1-exp(-\int\sum_{f}\Gamma_A^f(x,\mathrm{b}')\hat{\sigma}^{ff'}(xx')dx)] \tag{28a}$$

and the second one:

$$\int d^2\beta\left[dx'd^2b'\sum_{f'}x'\Gamma_B^{f'}(x',\mathrm{b}'-\beta)(1-exp(-\int\sum_{f}\Gamma_A^f(x,\mathrm{b}')\hat{\sigma}^{ff'}(xx')dx))\right]^2 \tag{28b}$$

(after having neglected the possibility for the two partons of B carrying the fractional momentum x'_i and x'_j to interact with the same parton of A).

When looking at $< x_A x_B >$ one, analogously with $< kn >$, will get:

$$\begin{aligned}< x_A x_B > \sigma_H^{AB} = \int d^2\beta dx' d^2b' dx d^2b \sum_{ff'} x'\Gamma_B^{f'}(x',\mathrm{b}'-\beta)x\Gamma_A^f(x,\mathrm{b})\times \\ \times[1-exp(-\int\sum_{f_1}\Gamma_B^{f_1}(x_1,\mathrm{b}-\beta)\hat{\sigma}^{ff_1}(x_1x)dx_1)]\times \\ \times[1-exp(-\int\sum_{f_2}\Gamma_A^{f_2}(x_2,\mathrm{b}')\hat{\sigma}^{f_2f'}(x_2x')dx_2)].\end{aligned} \tag{29}$$

that, like in the previous case, is a regular function of the cut off.

Intoducing the probability $\eta_A^{f'}(x,\mathrm{b})$for a parton of kind f', with fractional momentum x and transverse coordinate b to have a semi-hard interaction with the nucleus A (and analogously with B) as

$$\eta_A^{f'}(x,\mathrm{b})\equiv 1-exp(-\int\sum_{f}\Gamma_A^f(x',\mathrm{b})\hat{\sigma}^{ff'}(xx')dx') \tag{30}$$

one will get for the dispersion at fixed impact parameter $D(\beta)$:

$$\begin{aligned}D(\beta) &\equiv < (x_A+x_B)^2 >_\beta - < x_A+x_B >_\beta^2 \\ &= \int\sum_{f}x^2\Gamma_A^f(x,\mathrm{b}-\beta)\eta_B^f(x,\mathrm{b})dxd^2b+ \\ &+\int\sum_{f'}x^2\Gamma_B^{f'}(x,\mathrm{b}-\beta)\eta_A^{f'}(x,\mathrm{b})dxd^2b.\end{aligned} \tag{31}$$

The energy spectrum is then easily written down with the help of the central limit theorem:

$$\frac{d\sigma_H}{dE} = \int d^2\beta[1 - exp(- < n(\beta) >)]\frac{1}{\sqrt{D(\beta)\pi}}exp\left(-\frac{(E - \langle E(\beta)\rangle)^2}{D(\beta)}\right) \qquad (32)$$

In Eq.(32) $< n(\beta) >$ is defined implicitly by Eq.(11) giving the semi-hard cross section σ_H^{AB}. For practical purposes, given the large interaction probabilities, one can approximate well $< n(\beta) >$ using Eq.(3).

The feature of Eq.(32) we want to stress is that it is a regular function in the limit of small values of the cut off p_t^{min}, since the average values entering in Eq.(32) are well defined also in that limiting case.

IV. Quantitative estimates and conclusions

The problem that has been discussed in the present paper has to be regarded as a contribution in the framework of the more ambitious project of describing large cross section physics by means of perturbation theory.

This project is justified when the C.M. energies become increasingly large since the scale for the coupling constant to be in the perturbative regime is fixed.

From a phenomenological point of view the argument is that, already at CERN Collider energies, the cross section for minijet production is a large fraction of the total inelastic one[14].

While, on general grounds, one will remark that such a project needs an infinite set of non perturbative inputs (represented by the multiparton distributions) our point of view is rather that one of trying to construct a consistent scheme with a minimal set of assumptions.

Our basic input is:

1) multiparton distributions take a Poissonian form (Eq.10),

2) the geometrical size of the nucleus is the scale factor giving the dimensions to the multiparton distributions,

3) different semi-hard partonic collisions are completely incoherent, so that the cross section is given by Eq.(11).

We think that these requirements, in particular the last one, are better satisfied when looking at nuclear, rather than hadronic, interactions.

Given this input we have addressed our attention to the calculation of the energy spectrum produced in semi-hard nuclear collisions, namely to the fraction of energy carried by the partons that are involved in semi-hard collisions.

In fact, given the assumptions above, the semi-hard energy spectrum can be computed in the QCD parton model. We mean that, making use of unitarity, we have obtained an expression that is regular as a function of the cut off p_t^{min} for $p_t^{min} \to 0$.

Such energy spectrum has the non trivial characteristic of being independent of the fragmentation of partons, so that it depends on the initial state and on the structure of the interaction only.

To be more precise it is independent of the fragmentation to the extent that one can neglect reinteraction of partons in the final state.

We point out that, in this respect, the transverse energy spectrum would depend, if to be compared with experiment, on further hypotheses on the fragmentation.

We mean that we expect that an energetic parton scattered with a momentum transfer of a few GeV will distribute its energy, through fragmentation, over a large rapidity interval.

Unfortunately, while the semi-hard energy spectrum is appealing from a theoretical point of view, the problem of its measurement is far from trivial. When looking also for fragments at small angles one will get a large amount of contributions from soft processes, that have to be estimated with a lot of precision to be able to extract the semi-hard contribution.

To have some qualitative feelings on these issues we find interesting to comment on the limiting situation of Eq.(32) for $A \to \infty$ and later to compare the limiting distribution to some more realistic case.

One may in fact notice that, at fixed impact parameter, the average energy produced $\langle E(\beta)\rangle$ and the dispersion $D(\beta)$ are linear in the atomic mass A (see Eq.(27) and Eq.(31)) in the same way as the total energy.

It is then convenient to introduce a new variable ϵ defined as the energy divided by the total energy available.

In the limit of $A \to \infty$ one will notice that the Gaussian energy distribution at fixed impact parameter β in Eq.(32) will became a delta function as a function of ϵ:

$$\frac{1}{\sqrt{D(\beta)\pi}} exp\left(-\frac{(E-\langle E(\beta)\rangle)^2}{D(\beta)}\right) \to \delta\left(\epsilon - \epsilon(\beta)\right)$$

For large values of A the dispersion in the semi-hard energy produced in the interaction is then negligible at given impact parameter.

The consequence is that a measure of the semi-hard energy is also a measure of the impact parameter: if A is very large the energy carried by the nucleons in the overlap region between the two interacting nuclei will be wholly released by semi-hard scatterings. Measuring that amount of energy will then give the amount of overlap and therefore the value of the impact parameter.

One will notice that, in such a situation, in order to measure the energy produced in the semi-hard scatterings, it would not be really necessary to measure all the energy carried by all the final state minijets. It would be enough to be able to measure the energy carried by the spectator nucleons (the ones that will not happen to be in the overlap region).

In order to have some more quantitative feeling on these remarks, and without any claim to perform detailed predictions, we have performed some numerical calculations.

Basically the output is that for values of p_t^{min} around $3GeV$ at nucleon-nucleon C.M. energies of $2TeV$ one, with heavy nuclei, is not far from the limiting case mentioned above.

The relevant kinematical region for the parton distributions is, as a consequence, the one of $x \simeq 10^{-3}$. For these values of x the usual evolution equation is not any more adequate[9] and one has to resum the logs of x. The behaviour of the parton distributions as a function of x in the small x limit gets therefore changed in a sizeable way: a behaviour of x^{-J} with J effectively of the order of 1.5 has been argued[15].

In performing our calculations we have used the set 3 parton distributions from ref.16 where the behaviour of the gluon distributions was obtained requiring the behaviour $x^{-1.5}$.

In the region of interest, both for x and Q^2, the values provided by these gluon distributions are not dramatically different from those obtained in the recent analysis of ref.17.

In the calculation we have assumed spherical nuclei with uniform nuclear density and the semi-hard cross section has been assumed to be the geometrical one, namely $\sigma_H^{AB} = 40 \times 4 \times A^{2/3} mb$. In the elementary partonic interaction a $k = 2$ factor has been assumed.

The results are presented in four figures:

In the first figure one is plotting the average fraction of energy released by means of semi-hard collisions as a function of the ratio β/R, with R the nuclear radius. The case taken into consideration is that of $A = B = 208$ and of $1TeV$ per nucleon c.m. energy. The dashed dotted curve refers to a cut off $p_t^{min} = 4GeV$, the dashed one to $p_t^{min} = 2.5GeV$ while the continuous curve is the limiting case discussed above.

In the second figure one is plotting $\frac{1}{\sigma_H^{AB}} \frac{d\sigma_H^{AB}}{d\epsilon}$ as a function of $\epsilon \equiv E/E_{tot}$ for Pb+Pb collisions with c.m energies of $1TeV$ per nucleon. The dashed-dotted curve corresponds to cut off $p_t^{min} = 4GeV$, the dashed curve to a cut off of $2.5GeV$ and the continuous curve to the limiting case where all the energy in the overlap region is released by semi-hard collisions.

In the third figure one is looking at the dependence on the atomic mass A, the cut off has been kept fixed equal to $2.5GeV$. The dashed-dotted curve is for $A+A$ with $A = 100$, the dashed one for $A = 208$ and the continuous is the limiting case. Although the calculation is not reliable any more for small atomic numbers, we have included in the figure also the case $A = 10$ (dotted curve).

The fourth figure shows the dependence on the c.m. energy at fixed atomic mass ($A = B = 100$) and cut off ($p_t^{min} = 2.5GeV$). The dashed-dotted curve is for $0.5TeV$ per nucleon collisions, the dashed case for $1TeV$ per nucleon collisions and the continuous curve refers to the limiting case.

The indication from the numerical calculation is that this limiting case is not as far as one could perhaps have expected. Very roughly, at these c.m. energies, the basic description of the semi-hard interaction between the two nuclei is represented by the simple geometrical picture were most of the constituents in the overlap region take part to the interaction all the others being spectators. The large variation of the output semi-hard energy distribution as a function of the cut off p_t^{min} shows that the input parton distributions at values of $x \simeq 10^{-3}$ or less are the really critical parameter.

A measurement of the semi-hard energy spectrum as a function of the cut off would then provide a sensitive measurement of the behaviour of the parton distributions at values of $x < 10^{-3}$.

While a precise numerical study is beyond the scope of the present paper, we point out some qualitative features that, in our opinion, are of more general nature.

The main point is that in the semi-hard region multiple partonic collisions are the most important feature of the interaction.

We can recognize, at this purpose two different regimes:

The first one is that where the disconnected semi-hard parton processes start, and it begins rather early: it will start when the inclusive cross section, as given by the QCD parton model, is comparable with the total inelastic one.

The second one starts when the rescattering probability, that can be estimated from Eq.(30), gets sizeable. That will happen when the average number of partons inside a cylinder of dimensions given by the semi-hard parton parton cross section multiplied by the nuclear diameter gets comparable with one.

This second regime will start quite later, or better at higher energies (or larger atomic masses) since it is linear with the parton distributions, while the first one is quadratic.

The unitarization of the interaction will then switch on different processes while moving from the hard to the semi-hard region: one will first start seeing multiple parallel parton scatterings and later one will see semi-hard rescatterings.

In the case of interactions among nuclei the unitarization of the semi-hard interaction can be achieved, as discussed here, accounting for all kinds of multiple collisions. When discussing semi-hard hadronic interactions the blacking of the hadron with respect to semi-hard partonic collisions is discussed in terms of unitarization of the parton distributions[8].

The argument for expecting two different regimes, the one of multiple parallel semi-hard partonic collisions and that one of the saturation of the parton distributions remains however valid[18].

Our remark is that at the regimes considered here both features are present and have to be taken into account.

The main claim of the present paper is that, when considering nuclear rather than hadronic minijet production, given the incoherence between the different partonic collisions, one can unitarize the interaction without need of further input, a part geometry, from the hadronic structure.

Having unitarized the cross section one is in the position to define physical quantities that are regular functions of the cut-off that makes the separation between soft and semi-hard processes. In the present case is the energy spectrum related to minijet production which has been discussed.

Appendix A

We want to show here how one can derive Eq.(20), representing the average number of partonic collisions, from the expression for the semi-hard cross section (Eq.11).

For simplicity we will indicate here all the degrees of freedom of the interaction probability $\hat{\sigma}_{ij}^{f_i f_j'}$ with only one index.

Since the factor

$$[1-\prod_{i=1}^{n}\prod_{j=1}^{l}\prod_{f_i,f_j'}(1-\hat{\sigma}_{ij}^{f_i f_j'})] \equiv [1-\prod_{\nu=1}^{Q}(1-\hat{\sigma}_\nu)] \tag{A.1}$$

in Eq.(11) is multiplied by a symmetric expression it is convenient to introduce the symmetrizing operator $\mathcal{S}$ defined as:

$$\mathcal{S}x_1x_2\dots x_k \equiv [x_1x_2\dots x_k + x_1x_2\dots x_{k+1}+\dots \\ \dots + x_1x_2\dots x_{k+3}+\dots + x_2x_3\dots x_{k+3}\dots]/\binom{n}{k} \tag{A.2}$$

so that one is choosing in all possible ways k elements in a set of n, summing the products and dividing by the number of combinations.

Let us first consider

$$\mathcal{S}\sum_{\nu=0}^{Q}\binom{Q}{\nu}(1-\hat{\sigma}_1)\dots(1-\hat{\sigma}_\nu)\hat{\sigma}_{\nu+1}\dots\hat{\sigma}_Q =$$

$$= \mathcal{S}\sum_{\nu=0}^{Q}\binom{Q}{\nu}\sum_{\mu=0}^{\nu}(-1)^\mu\hat{\sigma}_{j_1}\dots\hat{\sigma}_{j_\mu}\hat{\sigma}_{\nu+1}\dots\hat{\sigma}_Q\binom{\nu}{\mu} =$$

$$= \mathcal{S}\sum_{\nu=0}^{Q}\sum_{\mu=0}^{\nu}\frac{Q!}{(Q-\nu)!(\nu-\mu)!\mu!}(-1)^\mu\hat{\sigma}_{l_1}\dots\hat{\sigma}_{l_{Q-\nu+\mu}} =$$

$$= \mathcal{S}\sum_{\lambda=0}^{Q}\sum_{\mu=0}^{Q-\lambda}\frac{Q!(Q-\lambda)!}{(Q-\mu-\lambda)!\lambda!\mu!(Q-\lambda)!}(-1)^\mu\hat{\sigma}_{l_1}\dots\hat{\sigma}_{l_{Q-\lambda}} =$$

$$= \mathcal{S}\sum_{\lambda=0}^{A}\binom{Q}{\lambda}[1-1]^{Q-\lambda}\hat{\sigma}_{l_1}\dots\hat{\sigma}_{l_{Q-\lambda}} = 1 \tag{A.3}$$

One will then write the following relation:

$$S\sum_{\nu=0}^{Q-1}\binom{Q}{\nu}(1-\hat{\sigma}_1)\dots(1-\hat{\sigma}_\nu)\hat{\sigma}_{\nu+1}\dots\hat{\sigma}_Q = \qquad\qquad$$
$$= S\sum_{\nu=0}^{Q} -(\nu=Q) = 1-(1-\hat{\sigma}_1)\dots(1-\hat{\sigma}_Q) \qquad (A.4)$$

We can now evaluate the average number of collisions:

$$S\sum_{\nu=0}^{Q}\nu\binom{Q}{\nu}(1-\hat{\sigma}_1)\dots(1-\hat{\sigma}_\nu)\hat{\sigma}_{\nu+1}\dots\hat{\sigma}_Q =$$
$$=SQ\sum_{\rho=0}^{Q-1}\binom{Q-1}{\rho}(1-\hat{\sigma}_1)\dots(1-\hat{\sigma}_{Q-\rho-1})\hat{\sigma}_{Q-\rho}\dots\hat{\sigma}_Q = \qquad (A.5)$$
$$=SQ\hat{\sigma}_Q\sum_{\rho=0}^{Q-1}\binom{Q-1}{\rho}(1-\hat{\sigma}_1)\dots(1-\hat{\sigma}_\rho)\hat{\sigma}_{\rho+1}\dots\hat{\sigma}_{Q-1} =$$
$$=SQ\hat{\sigma}_Q = \hat{\sigma}_1+\hat{\sigma}_2+\dots\hat{\sigma}_Q$$

that is the relation we wanted to prove.

Appendix B

In this appendix we will show how to obtain the complete expression for $< x_A x_B > \sigma_H^{AB}$ and $< x_B^2 > \sigma_H^{AB}$, including the correction term that has been neglected when writing Eq.s(28,29) in the text.

To simplify the notation we do not write the indices identifying the various species of partons.

Following ref.11 we introduce the generating functions for the collisions between n partons from nucleus A and m partons from nucleus B:

$$F(x_1,\dots x_n;y_1,\dots y_m) = \prod_{i=1}^{n}\prod_{j=1}^{m}(1-\hat{\sigma}_{ij}+x_i y_j\hat{\sigma}_{ij}). \qquad (B.1)$$

The expansion of F in powers of x's and y's gives the probabilities for multiple collisions: e.g. the coefficient of the term containing $x_{i_1}^{n_1}x_{i_2}^{n_2}\dots y_{j_1}^{m_1}y_{j_2}^{m_2}\dots$ is the probability for i_1 to collide n_1 times, i_2 n_2 times etc. (One will notice that $\sum_r n_r = \sum_s m_s$ by construction).

We then introduce the notation:

$$\begin{aligned} F(1,1) &= F(1,\ldots 1;1\ldots 1) = 1 \\ F_i(0,1) &= F(1,\ldots x_i = 0\ldots 1;1\ldots 1) = \prod_{j=1}^{m}(1-\hat{\sigma}_{ij}) \\ F_j(1,0) &= F(1\ldots 1;1,\ldots y_j = 0\ldots 1) = \prod_{i=1}^{n}(1-\hat{\sigma}_{ij}) \\ F_{ij}(0,0) &= F(1\ldots x_i = 0\ldots 1;1,\ldots y_j = 0\ldots 1) \\ &= \prod_{l\neq j}(1-\hat{\sigma}_{il})\prod_{k\neq i}(1-\hat{\sigma}_{kj}) \times (1-\hat{\sigma}_{ij}). \end{aligned} \tag{B.2}$$

To get the probability for i to have at least one interaction one then writes:

$$\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{\partial}{\partial x_i}\right)^k F(x_1,\ldots x_A;y_1,\ldots y_B)|_{x=0,y=0} = F(1,1) - F_i(0,1). \tag{B.3}$$

The probability for x_i and y_j to have at least one interaction is analogously:

$$F(1,1) - F_i(0,1) - F_j(1,0) + F_{ij}(0,0). \tag{B.4}$$

One has then:

$$\begin{aligned} < x_A x_B > \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty}\frac{1}{n!}\Gamma_A(x_1,\mathrm{b}_1)\ldots\Gamma_A(x_n,\mathrm{b}_n) \times e^{-\int \Gamma_A(x,\mathrm{b})dxd^2b} \times \\ \times \sum_{k=1}^{\infty}\frac{1}{k!}\Gamma_B(x_1',\mathrm{b}_1'-\beta)\ldots\Gamma_B(x_k',\mathrm{b}_k'-\beta) \times e^{-\int \Gamma_B(x',\mathrm{b}')dx'd^2b'} \times \\ \times \sum_{i=1}^{n}\sum_{j=1}^{k} x_i x_k' \left[F(1,1) - F_i(0,1) - F_j(1,0) + F_{ij}(0,0)\right] \times \\ \times dx_1 d^2b_1 \ldots dx_n d^2b_n dx_1' d^2b_1' \ldots dx_k' d^2b_k'. \end{aligned} \tag{B.5}$$

The contribution from $F(1,1)$ is:

$$\int d^2\beta\Gamma_A(x_1,\mathrm{b}_1)\Gamma_B(x_1',\mathrm{b}_1'-\beta)x_1 dx_1 d^2b_1 x_1' dx_1' d^2b_1'. \tag{B.6}$$

The contribution from $F_i(0,1)$:

$$\begin{aligned} \int d^2\beta\Gamma_A(x_1,\mathrm{b}_1)[1-\hat{\sigma}_{11'}]\times \\ \times\Gamma_B(x_1',\mathrm{b}_1'-\beta)e^{-\int \Gamma_B(x_2',\mathrm{b}_2'-\beta)\hat{\sigma}_{12'}dx_2'd^2b_2'} x_1 dx_1 d^2b_1 x_1' dx_1' d^2b_1'. \end{aligned} \tag{B.7}$$

The contribution from $F_j(1,0)$ is obtained exchanging A and B in $F_i(0,1)$. The contribution from $F_{ij}(0,0)$ is:

$$\int d^2\beta x_1 dx_1 d^2b_1 x_1' dx_1' d^2b_1' \Gamma_A(x_1, \mathbf{b}_1)[1-\hat{\sigma}_{11'}]\Gamma_B(x_1', \mathbf{b}_1' - \beta)\times \\ \times e^{-\int \Gamma_B(x_2', \mathbf{b}_2' - \beta)\hat{\sigma}_{12'} dx_2' d^2b_2'} \times e^{-\int \Gamma_A(x_2, \mathbf{b}_2)\hat{\sigma}_{21'} dx_2 d^2b_2}. \tag{B.8}$$

All together one gets:

$$\int d^2\beta x_1 dx_1 d^2b_1 x_1' dx_1' d^2b_1' \times \\ \times \Gamma_A(x_1, \mathbf{b}_1)[1 - e^{-\int \Gamma_B(x_2', \mathbf{b}_2' - \beta)\hat{\sigma}_{12'} dx_2' d^2b_2'}]\times \\ \times \Gamma_B(x_1', \mathbf{b}_1' - \beta)[1 - e^{-\int \Gamma_A(x_2, \mathbf{b}_2)\hat{\sigma}_{21'} dx_2 d^2b_2}] \tag{B.9}$$

that is the expression given in Eq.(29), and the correction term:

$$\int d^2\beta x_1 dx_1 d^2b_1 x_1' dx_1' d^2b_1' \Gamma_A(x_1, \mathbf{b}_1)\hat{\sigma}_{11'}\Gamma_B(x_1', \mathbf{b}_1' - \beta)\times \\ \times [1 - (1 - e^{-\int \Gamma_B(x_2', \mathbf{b}_2' - \beta)\hat{\sigma}_{12'} dx_2' d^2b_2'})\times \\ \times (1 - e^{-\int \Gamma_A(x_2, \mathbf{b}_2)\hat{\sigma}_{21'} dx_2 d^2b_2})]. \tag{B.10}$$

One will notice that, in the limit of a large number of collisions, the term in the square brackets will get a negligible contribution from the overlap region between the two nuclei, unless $\mathbf{b}_1$ or $\mathbf{b}_1'$ are close to the nuclear border.

The correction term is then a term of order $A^{\frac{1}{3}}$ while the dominant term is of order $A^{\frac{2}{3}}$.

In order to evaluate the correction term to the squared energy emitted by the partons of nucleus B we make use twice of Eq.(B.3). We get:

$$< x_B^2 > \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \frac{1}{n!}\Gamma_A(x_1, \mathbf{b}_1)\ldots\Gamma_A(x_n, \mathbf{b}_n) \times e^{-\int \Gamma_A(x, \mathbf{b}) dx d^2b} \times \\ \times \sum_{k=1}^{\infty} \frac{1}{k!}\Gamma_B(x_1', \mathbf{b}_1' - \beta)\ldots\Gamma_B(x_k', \mathbf{b}_k' - \beta) \times e^{-\int \Gamma_B(x', \mathbf{b}') dx' d^2b'} \times \\ \times \Big(\sum_{j=1}^{B} x_j'^2 [F(1,1) - F_j(1,0)] + \sum_{j=1}^{B} \sum_{k\neq j}^{1\ldots B} x_j' x_k' [F(1,1) - F_j(1,0)]\cdot \\ \cdot [F(1,1) - F_k(1,0)]\Big) dx_1 d^2b_1 \ldots dx_k' d^2b_k'. \tag{B.11}$$

From the first term we get the expression ($28a$), the second term can be splitted into two pieces, the first one reproduces expression $28b$ while the second one is

$$\int d^2\beta x''\Gamma_B(x'',\mathbf{b}''-\beta)V(x',x'';\mathbf{b}',\mathbf{b}'')x'\Gamma_B(x',\mathbf{b}'-\beta)\times \\ \times e^{-\int\Gamma_A(x,\mathbf{b}')\hat{\sigma}(x,x')dx}e^{-\int\Gamma_A(x,\mathbf{b}'')\hat{\sigma}(x,x'')dx}dx'dx''d^2b'd^2b'' \tag{B.12}$$

where V is defined as

$$V = e^{\int\hat{\sigma}(x'',\mathbf{b}'';x,\mathbf{b})\Gamma_A(x,\mathbf{b})\hat{\sigma}(x,\mathbf{b};x',\mathbf{b}')dxd^2b} - 1.$$

One should notice that, since $\hat{\sigma}$ is a probability it will never exceed 1, as a consequence the positive exponential in V will be always compensated by the negative ones in expression (B.12). In the limit of a large number of partons one has then that Eq.(B.12), analogously to Eq.(B.10) gives a contribution of order $A^{\frac{1}{3}}$ to be compared with the dominant contributions that are rather of order $A^{\frac{2}{3}}$.

Appendix C

In this appendix origin and consequences of possible deviations from a strict Poissonian distribution for the nuclear parton population are discussed. We take into consideration two possible sources of deviation: the first one is the effect of the intermediate nucleon structure, which mainly enters into the game inducing correlations, in the second case the partonic distribution deviates from a Poissonian already at nucleonic level.

The treatment discussed till now relies on a partonic description of the whole nucleus (see Eq.10) and the only explicit remnant of the nuclear properties is in the index A appended to the functions $\Gamma^f_A(x_i,\mathbf{b}_i)$.

The effects of the intermediate nucleonic level could show up in different ways. We have chosen a well defined nuclear effect which is certainly present, it is the correlation between nucleons and the related fluctuation in the nuclear density.

In absence of density fluctuations, partonic Poissonian distributions from the single nucleons would result in a strictly Poissonian distribution of partons for the whole nucleus, at fixed b; if, on the contrary, there is a fluctuation in the nucleon number, at fixed impact parameter, then the resulting distribution cannot be Poissonian, as discussed, in general, in ref.11.

We will consider here the correlations raising from the Pauli principle. The nucleus will be represented by a Fermi gas (at zero temperature) in a rigid box as, e. g., in ref.19.

Starting from the normalized nuclear wave function one defines:

$$
\begin{aligned}
w_A(\mathbf{r}_1 \ldots \mathbf{r}_A) &= |\Phi(\mathbf{r}_1 \ldots \mathbf{r}_A)|^2 \\
w^{(1)}(\mathbf{r}_1) &= \int d\mathbf{r}_2 \ldots d\mathbf{r}_A w_A \\
w^{(2)}(\mathbf{r}_1, \mathbf{r}_2) &= \int d\mathbf{r}_3 \ldots d\mathbf{r}_A w_A .
\end{aligned}
\tag{C.1}
$$

Since, in this description, the surface effects are neglected, the one body distribution is constant: $w^{(1)} = 1/V$, with $V = \frac{4\pi}{3} R^3$ the volume.

We always work in impact parameter representation and therefore it turns out to be convenient to introduce the distributions:

$$
\begin{aligned}
F_A(\mathbf{s}_1 \ldots \mathbf{s}_A) &= \int dz_1 \ldots dz_A w_A \\
F^{(1)}(\mathbf{s}_1) &= \int dz_1 w^{(1)} \\
F^{(2)}(\mathbf{s}_1, \mathbf{s}_2) &= \int dz_1 dz_2 w^{(2)}
\end{aligned}
\tag{C.2}
$$

where $r^2 = s^2 + z^2$.
Following ref.19 we get:

$$
w^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{A}{(A-1)} \left[w(\mathbf{r}_1) w(\mathbf{r}_2) - c(\mathbf{r}_1, \mathbf{r}_2) \right].
$$

The correlation function c depends only on the relative coordinate $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. Actually:

$$
c(r) = \frac{1}{V} \cdot \frac{9}{u^4} \left(\frac{sinu}{u} - cosu \right)^2
$$

and

$$
\int d^3 r c(r) = \frac{1}{AV}
$$

where $u = r k_F$ and the Fermi momentum is taken equal both for neutrons and protons ($N = Z = \frac{1}{2} A$), $k_F^3 = \frac{3}{2} \pi^2 \frac{A}{V}$.

An important property of c is that it differs sizably from zero only in a range of r which is, in a heavy nucleus, quite smaller with respect to the nuclear radius; as a consequence the integration in r can be extended to infinity.

Going to the impact parameter representation we get:

$$
\begin{aligned}
F^{(1)}(\mathbf{s}) &= f(s) = \frac{2R}{V} \sqrt{1 - s^2/R^2} \theta(|R| - |s|) \\
F^{(2)}(\mathbf{s}_1, \mathbf{s}_2) &= f(s_1) f(s_2) + g(\mathbf{s}_1, \mathbf{s}_2)
\end{aligned}
\tag{C.3}
$$

with

$$g(\mathbf{s}_1,\mathbf{s}_2) = \frac{1}{A-1} f(s_1) f(s_2) - \frac{A}{A-1} \int dz_1 dz_2 c(\mathbf{r}_1,\mathbf{r}_2)$$

and

$$\int g(\mathbf{s}_1,\mathbf{s}_2) d\mathbf{s}_1 = \int g(\mathbf{s}_1,\mathbf{s}_2) d\mathbf{s}_2 = 0. \qquad (C.4)$$

The analytical expression for g is rather cumbersome; in the case $\mathbf{s}_1 \simeq \mathbf{s}_2$ (that is also the case where the effect of the Pauli principle is larger) on gets however a simple expression.

In this case $F^{(2)}$ is given by:

$$F^{(2)}\Big(\mathbf{S} - \frac{1}{2}\mathbf{s}, \mathbf{S} + \frac{1}{2}\mathbf{s}\Big) = \frac{A}{A-1}\Big[f^2(S)\Big(1 - \frac{s}{4}\frac{R^2+S^2}{(R^2-S^2)^2}\Big) - \\ -f(S)\frac{6\pi}{5}\frac{1}{Vk_F}\Big(1 - \frac{11}{63}(k_F s)^2\Big)\Big]. \qquad (C.5)$$

The first part is from the factorized part (ff), the second from the correction (g).

The main observation is that, while for $\mathbf{r}_1 = \mathbf{r}_2$ $w^{(2)}$ is zero, for $\mathbf{s}_1 = \mathbf{s}_2$ $F^{(2)}$ is not zero. The negative correcting term is roughly 0.2 for $A \simeq 200$, the effect then is not a very relevant one unless $\mathbf{s} \to \mathbf{r}$, where, on the other hand, the surface effects are important*.

As expected the $z-$integration is softening the effect of the Pauli principle in rather efficient way, as a consequence, whenever an impact parameter description is allowed, the effect, on large nuclei, can be treated perturbatively.

We will then work out a definite example (neglecting for simplicity flavour indices).

The semi-hard cross section (see Eq.11) takes then the form:

---

* *If* $\mathbf{s}$ *is very close to* $\mathbf{R}$ *the expression for* $F^{(2)}$ *cannot be trusted any more since the integration over* z *cannot be done over all the real axis in that case.*

$$
\begin{aligned}
\sigma_H^{AB} = \int d^2\beta \prod ds F_A(\mathrm{s}_1 \dots \mathrm{s}_A) \prod ds' F_B(\mathrm{s}'_1 \dots \mathrm{s}'_B) \times \\
\times \sum_{\{n_Z\}} \frac{1}{n_Z!} \Gamma(x_{Z,1}, \mathrm{b}_{Z,1} - \mathrm{s}_Z) \dots \Gamma(x_{Z,n_Z}, \mathrm{b}_{Z,n_Z} - \mathrm{s}_Z) \times \\
\times \sum_{\{l_Y\}} \frac{1}{l_Y!} \Gamma(x'_{Y,1}, \mathrm{b}'_{Y,1} - \mathrm{s}'_Y) \dots \Gamma(x'_{Y,l_Y}, \mathrm{b}'_{Y,l_Y} - \mathrm{s}'_y) \times \\
\times e^{-A \int \Gamma(x,\mathrm{b}) dx d^2 b} \times e^{-B \int \Gamma(x',\mathrm{b}') dx' d^2 b'} \times \\
\times [1 - \prod_{Z=1}^{A} \prod_{i=1}^{n_Z} \prod_{Y=1}^{B} \prod_{j=1}^{l_Y} (1 - \hat{\sigma}_{Z,i;Y,j})] \prod dx dx' \prod d^2 b d^2 b'
\end{aligned}
\qquad (C.6)
$$

where $Z = 1 \dots A, Y = 1 \dots B$ refer to the nucleons in A and B, Γ are the parton distributions of the single nucleon and $\{n_Z\}$ denotes the indices $n_1 \dots n_A$ that have to be summed.

The mean energy is easily obtained:

$$
\begin{aligned}
< x_B > \sigma_H^{AB} = \int d^2\beta \prod ds F_A \prod ds' F_B \sum_Y x' dx' \times \\
\times \Gamma(x', \mathrm{b}' - \mathrm{s}_Y)[1 - \prod_Z (1 - \eta(x', \mathrm{b}' - \mathrm{s}_Z)].
\end{aligned}
\qquad (C.7)
$$

η has been defined in Eq.(30), in this case however Γ refers to a single nucleon.

The function F_B is symmetric in its arguments so the sum $\sum_Y$ gives simply a factor B, moreover F_B can be integrated over $B - 1$ arguments giving $f_B(s')$; on the contrary all the arguments of F_A are present also in $1 - \eta$ and therefore a more detailed treatment is needed in this case.

Quite in general one may write:

$$
\begin{aligned}
F_A(\mathrm{s}) = f_A(s_1) \dots f_A(s_A) + f_A(s_1) \dots g_A(\mathrm{s}_j \mathrm{s}_k) \dots f_A(s_A) + \\
+ f_A(s_1) \dots h_A(\mathrm{s}_j \mathrm{s}_k \mathrm{s}_l) \dots f_A(s_A) + \\
+ f_A(s_1) \dots g_A(\mathrm{s}_j \mathrm{s}_k) \dots g_A(\mathrm{s}_m \mathrm{s}_n) \dots f_A(s_A) + \dots
\end{aligned}
\qquad (C.8)
$$

Since the effect of the correlation is not large we keep (besides the factorized term) only the first order term in g. Introducing this truncated expression in Eq.(C.7) and using the approximation $(1 - y)^A \simeq e^{-Ay}$ we get:

$$
\begin{aligned}
< x_B > \sigma_H^{AB} = & \int d^2\beta B f_B(s')\Gamma(x',\mathbf{b}'-\mathbf{s}')x' ds' \times \\
& \times \left[1 - e^{-A\int f_A(s)\eta(x',\mathbf{b}'-\mathbf{s})ds} - \frac{A^2}{2}\int g(\bar{s},\bar{\bar{s}})\eta(x',\mathbf{b}'-\bar{\mathbf{s}}) \times \right. \qquad (C.9) \\
& \left. \times \eta(x',\mathbf{b}'-\bar{\bar{\mathbf{s}}}) e^{-(A-2)\int f_A(s)\eta(x',\mathbf{b}'-\mathbf{s})ds} d\bar{\mathbf{s}} d\bar{\bar{\mathbf{s}}} + \dots \right]
\end{aligned}
$$

The nuclear parton distributions are obtained convoluting f with Γ. More precisely:

$$\Gamma_B(x',\mathbf{b}'-\beta) = B\int f_0(s')\Gamma(x',\mathbf{b}'-\beta-\mathbf{s}')ds'$$

where, keeping into account that f_B is centered around β, one has defined f_0 as: $f_0(|\mathbf{s}'-\beta|) \equiv f_B(s')$.

The comparison with Eq.(27) shows two differences: the first is the term in g, the second is the appearance of η at the exponent. One will notice that if η is "small" then $\eta \simeq \Gamma\hat{\sigma}$ and defining $\Gamma_A(x,\mathbf{b}_\nu) = A\int f_A(s_\nu)\Gamma(x,\mathbf{b}-\mathbf{s}_\nu)ds_\nu$ one gets the same exponent as in Eq.(27). The correction term in $(C.9)$ looks proportional to A^2 and thus potentially large, it may, however, be recasted in the form $\frac{1}{2}\int \Gamma_A\hat{\sigma}\Gamma_A\hat{\sigma}(g/ff)$ and, according to the previous discussion the term in parenthesis is small.

The same procedure can be applied also in computing $< x_B^2 >$ and $< x_A x_B >$, the details being rather complicated we will only sketch the calculation.

The term $< x_B^2 >$ has three different kinds of contributions:

one of kind x'^2_{jY} (energy squared of one parton),

one of kind $x'_{jY}x'_{iY}$ (energies of two partons of the same nucleon)

and one of kind $x'_{jY}x'_{iW}$ (energies of two partons of different nucleons).

The first two contributions do not involve the correlation function for the nucleus B, the third does.

The term $< x_A x_B >$ finally will depend on the correlation functions of both nuclei that will act between the terms linear in Γ and the exponential terms.

In conclusion we see that a systematic way to treat the density fluctuations of the nucleus is available and that this kind of perturbation does not play an important role in the problems discussed in the present paper.

There is a question about the parton distribution which is complementary to the problem analyzed above, namely one can ask how much the Poissonian distribution is fundamental for the conclusions which have been drawn. The question in this form is too general, we will then look to a much more specialized case, where one keeps the distribution still factorized, but not Poissonian. The alternative chosen is the negative binomial distribution, which, although in a different context, has been suggested as relevant in high energy multiplicity phenomena[20].

We start, therefore, by a distribution

$$\binom{-\alpha}{n}(-)^n\Lambda(x_1,\mathbf{b}_1)\ldots\Lambda(x_n,\mathbf{b}_n)\Big[1-\int\Lambda(x,\mathbf{b})dxd^2b\Big]^{\alpha}.$$

In this case the exclusive distribution Λ does not coincide with the partonic density which is:

$$\mathcal{D}(x,\mathbf{b})=\alpha\frac{\Lambda(x,\mathbf{b})}{1-\int\Lambda(x,\mathbf{b})dxd^2b}.$$

One can use the previously given formulation in order to calculate physical observables. As an example we find:

$$< x'_B > \sigma_H = \int d^2\beta dx'd^2b'x'\mathcal{D}(x',\mathbf{b}'-\beta)\Big(1-[1+\frac{1}{\alpha}\int\mathcal{D}(x,\mathbf{b}')\hat{\sigma}(xx')dx]^{-\alpha}\Big). \qquad (C.10)$$

The geometric limit corresponds to $\int\mathcal{D}\hat{\sigma}\rightarrow\infty$ at fixed α. As well known one can obtain the Poisson distribution and the related result in a limiting case: $\Lambda=\Gamma/\alpha$, $\alpha\rightarrow\infty$, in fact in this case one gets back, from Eq.(C.10), Eq.(27).

This kind of distribution can well be taken for the single nucleon, then inserted in the nuclear structure as in (C.7) (which remains valid provided an obvious redefinition of η is made) and, in the same way, Eq.(C.9) is obtained. When $\mathcal{D}$ is small, the approximation $\eta\simeq\mathcal{D}\hat{\sigma}$ holds and the parameter α disappears. We are induced to conclude, in this example, that all the features of the Poisson distribution are reproduced, not because of the original partonic distribution, but because of the large A effect.

REFERENCES

1) G. Pancheri and Y.N. Srivastava, Phys. Lett. **159B** (1985) 69, Phys. Lett. **182B** (1986) 199; T.K. Gaisser and F. Halzen, Phys. Rev. Lett. **54** (1985) 1754; T. Sjöstrand and M. van Zijl, Phys. Rev. **D36** (1987) 2019.
2) M. Jacob and P.V. Landshoff, Mod. Phys. Lett. **A1** (1986) 657.
3) Ll. Ametller and D. Treleani, Int. J. Mod. Phys. **A3** (1988) 521.
4) N. Paver and D. Treleani, Nuovo Cimento **A70**(1982)215; Zeit. Phys.**C28** (1985)187.
5) V.A. Abramovskii, V.N. Gribov and O.V. Kancheli, Yad. Fiz. **18** (1973) 595, (Sov. J. Nucl. Phys. **18** (1974) 308); J.L. Cardy and G.A. Winbow, Phys. Lett. **52B**(1974)95; C.E. De Tar, S.D. Ellis and P.V. Landshoff, Nucl. Phys. **B87** (1975) 176.
6) L. Durand and H. Pi, Phys. Rev. Lett. **58**(1987)303; A. Capella, J. Tran Thanh Van and J. Kwieciński, Phys. Rev. Lett. **58**(1987)2015; B. Margolis, P. Valin, M.M. Block, F. Halzen and R.S. Fletcher, Phys. Lett. **B213** (1988) 221.
7) K. Kajantie, P.V. Landshoff and J. Lindfors, Phys. Rev. Lett. **59**(1987) 2527.
8) L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Reports **100** (1983) 1.
9) Ya.Ya. Balitzkij and L.N. Lipatov, Yad. Fiz. **28**(1978)1597; T. Jaroszewicz, Acta Phys. Polon. **B11**(1980)965; N.L. Lipatov, Sov. Phys. JEPT **63** (1986) 904; J.C. Collins and J. Kwieciński, Nucl. Phys. **B316** (1989)307
10) EM Coll.,J.J. Aubert et al., Phys. Lett. **B123**(1983)275.
11) A. Białas, M. Bleszyński and W. Czyż, Nucl. Phys. **B111**(1976) 461.
12) C. Pajares and A.V. Ramallo, Phys. Rev. **D31**(1985)2800.
13) K. Kastella, Phys. Rev. **D36**(1987)2734; K. Kastella, G. Sterman and J. Milana, Phys. Rev. **D39**(1989)2586.
14) F. Ceradini et al., Study of minimum-bias-trigger events at $\sqrt{s} = 0.2-0.9TeV$ with magnetic and calorimetric analysis CERN-EP/85-196, November 1985, published in Bari Europhys. High Energy (1985) p. 363.
15) J.C. Collins, Semihard processes in QCD, *in* Physics simulations at high energies, ed. V. Barger, T. Gottschalk and F. Halzen (World Scientific, Singapore, 1987).
16) A.D. Martin, R.G. Roberts, W.J. Stirling, Phys. Rev. **37**(1988) 1161.
17) S. Lomatch, F.I. Olness and J.C. Collins, Nucl. Phys. **B317** (1989)617.
18) R.D. Carlitz, preprint PITT-88-05 (October 1988).
19) A.G. Sitenko and V.K. Tartakovskij, Lectures on the theory of nucleus (cap. 2), Pergamon Press (1975).
20) A. Giovannini and L.Van Hove, Zeit. Phys.**C30**(1986)391.

Figure Captions

1) Average fraction of energy released by means of semi-hard collisions as a function of the ratio β/R for $A = B = 208$ and C.M. energy of $1TeV$ per nucleon . The dashed dotted curve refers to a cut off $p_t^{min} = 4GeV$, the dashed one to $p_t^{min} = 2.5GeV$ while the continuous curve is the limiting case discussed in the text.

2) Differential cross section as a function of $\epsilon \equiv E/E_{tot}$ for Pb+Pb collisions with C.M. energies of $1TeV$ per nucleon and for $p_t^{min} = 2.5GeV$. The different curves are as in Fig. 1.

3) Dependence on the atomic mass A in $A + A$ collisions for $p_t^{min} = 2.5GeV$ and $1TeV$ per nucleon C.M. energy. Dotted curve $A = 10$, dashed-dotted $A = 100$, dashed $A = 208$ and the continuous curve limiting case.

4) Dependence on the CM energy at fixed atomic mass ($A = B = 100$) and cut off ($p_t^{min} = 2.5GeV$). Dashed-dotted curve $0.5TeV$ per nucleon collisions, dashed curve $1TeV$ per nucleon and continuous curve limiting case.

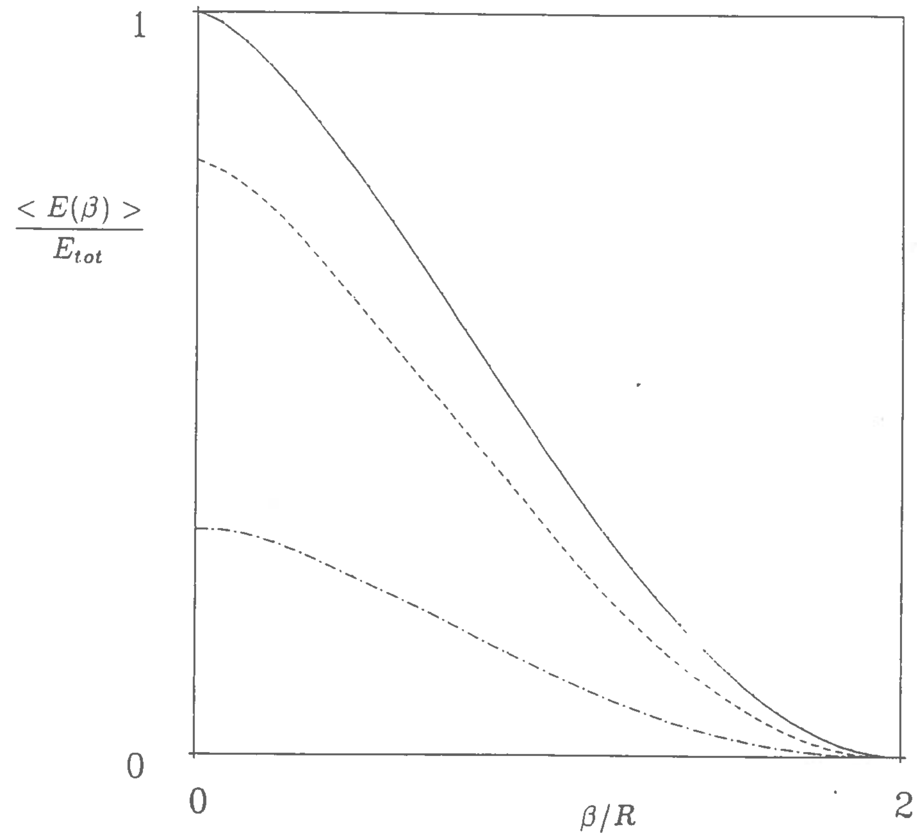
1
$\frac{< E(\beta) >}{E_{tot}}$
0
0
β/R
2

Fig. 1

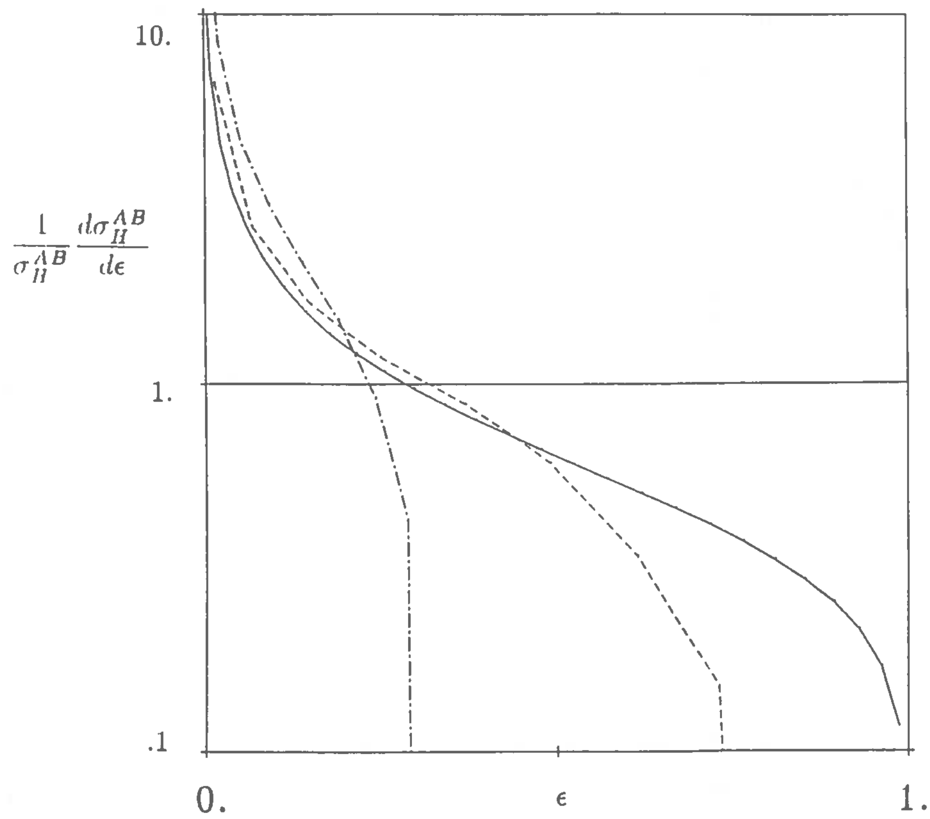
10.
$\frac{1}{\sigma_H^{AB}} \frac{d\sigma_H^{AB}}{d\epsilon}$
1.
.1
0.
ϵ
1.

Fig. 2

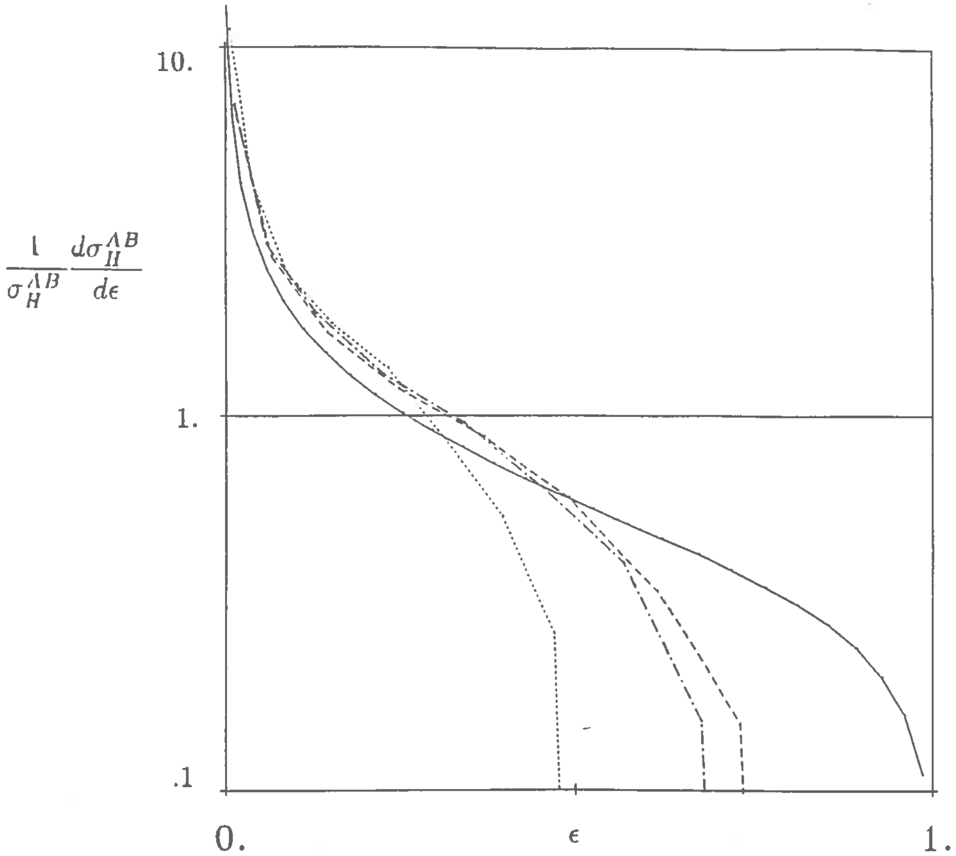
10.
1.
.1
$\frac{1}{\sigma_H^{AB}} \frac{d\sigma_H^{AB}}{d\epsilon}$
0.
ϵ
1.

Fig. 3

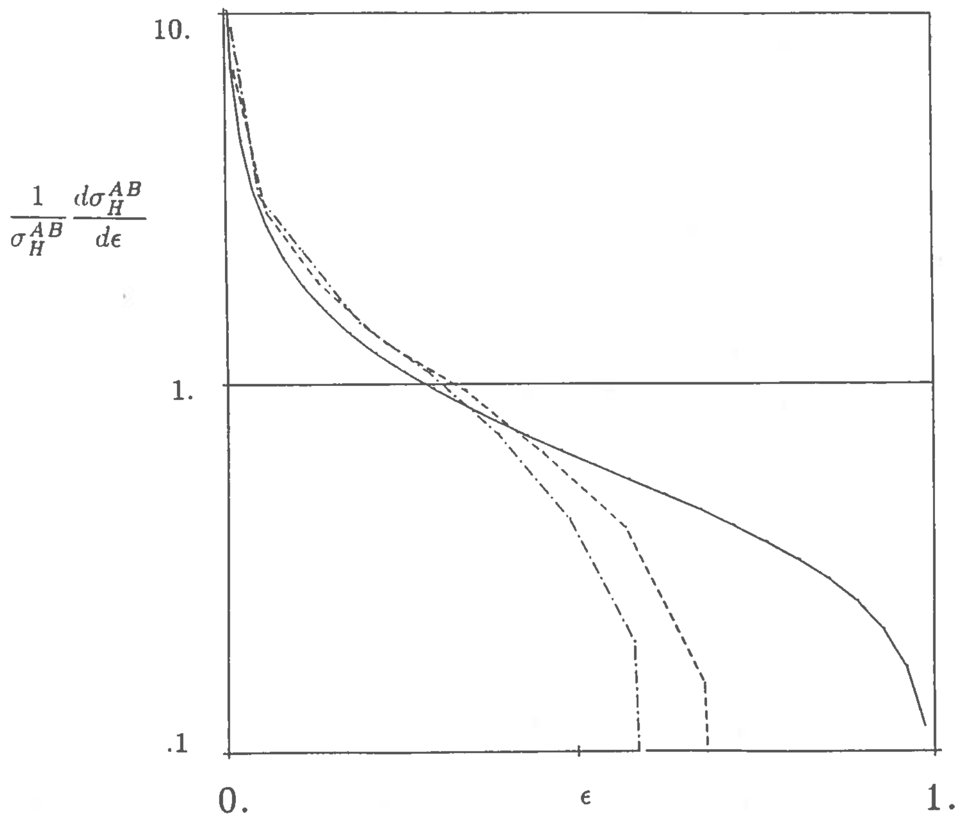
10.
1.
.1
$\frac{1}{\sigma_H^{AB}} \frac{d\sigma_H^{AB}}{d\epsilon}$
0.
ϵ
1.

Fig. 4