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ROLE OF B-MESON SEMILEPTONIC FORM FACTORS IN B^0–$\overline{B^0}$ MIXING AND IN $b \to s$ TRANSITIONS *

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INTRODUCTION

Considerable effort has been devoted recently to theoretical estimates of exclusive B-meson semileptonic transitions [1]. These decays involve non perturbative matrix elements of the weak currents between the B meson and the possible final hadronic states. Different phenomenological models have been suggested in order to describe the relevant form factors, relying on the constituent quark model [2-6] and on QCD sum rules [7,8]. As well known, the particular interest of these form factors is that they play a key role in the experimental determination of the CKM mixing matrix elements of the b quark, where they are needed as a theoretical input. The other important aspect, which we wish to emphasize here, is that in principle they could also affect quite significantly the estimates of a number of non leptonic processes of the B meson, which also are expected to represent stringent tests of the Standard Model. In fact, most estimates of the hadronic matrix elements of the non leptonic weak Hamiltonian have relied so far on the vacuum insertion approximation, which in many cases involves the semileptonic form factors. In addition to that, and rather interestingly, these form factors can have a crucial role in trying to go beyond the framework of the vacuum saturation.

In the sequel we shall illustrate this last point by discussing, as examples, the cases of the $B^0 - \overline{B}^0$ mixing and of the long distance effects in radiative and in non leptonic $b \to s$ transitions.

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SINGLE PARTICLE CONTRIBUTIONS TO THE B PARAMETER

In the three generations Standard Model the $B^0 - \overline{B^0}$ mixing can be expressed as [9]:

$$x_d = \frac{\Delta m_B}{\Gamma_B} = \frac{\tau_B}{m_B}\left[\frac{G_F}{4\pi}V_{td}V_{tb}^*\right]^2 m_t^2\ f(y_t)\ \eta\ \mathcal{M}, \tag{1}$$

where $f(y_t)$ is a known function of $y_t = \frac{m_t^2}{M_W^2}$, and η is a perturbative QCD correction factor. Experimentally [10,11]:

$$x_d|_{EXP} = 0.70 \pm 0.13 \tag{2}$$

In eq.(1) $\mathcal{M}$ is the hadronic, non perturbative matrix element:

$$\mathcal{M} =< \overline{B^0}(p)|O_{\Delta B=2}|B^0(p) >=< \overline{B^0}(p)|:\bar{b}\gamma_\mu(1-\gamma_5)d\ \bar{b}\gamma^\mu(1-\gamma_5)d:|B^0(p)>, \tag{3}$$

which is usually written in the following notation:

$$\mathcal{M} = \frac{8}{3}m_B^2 f_B^2 B = \mathcal{M}_{VAC}B, \tag{4}$$

where f_B is the B-meson leptonic decay constant (normalized according to $f_\pi = 132\ MeV$), so that the relevant dimensionless parameter is B, such that $B = 1$ if strict vacuum saturation holds.

Dynamical estimates of B have appeared in the literature, based on lattice simulations [12,13] and on QCD sum rules [14,15]: within their intrinsic uncertainties they support, at least approximately, the validity of vacuum saturation. An alternative point of view, in this regard, could be to try going beyond the vacuum saturation approximation, by considering the latter as just the first step of a completeness insertion between the two currents in eq.(3). Thus there should be other contributions which add to the vacuum, and can potentially make $B = \frac{\mathcal{M}}{\mathcal{M}_{VAC}} \neq 1$. We shall describe here an attempt to account for the contribution of single particle intermediate states [16], which in this spirit should be considered as the next logical step following the vacuum saturation:

$$\mathcal{M}_{1Part} = \sum_n < \overline{B^0}|\bar{b}\gamma_\mu(1-\gamma_5)d|n>< n|\bar{b}\gamma^\mu(1-\gamma_5)d|B^0>, \tag{5}$$

where the sum runs over the lowest lying meson states $|n>= \pi^0, \eta, \eta', \rho$ and ω. The main features of our approach are the following:

Due to the precocity of the short distance behaviour one can take the currents in eq.(3) at a small spacetime separation x_0:

$$\mathcal{M} \simeq< \overline{B^0}(p)|T\left(J_\mu(x_0)J^\mu(0)\right)|B^0(p)>, \tag{6}$$

where $J_\mu = \bar{b}\gamma_\mu(1-\gamma_5)d$. Indeed the desired matrix element is identical to the leading light cone expansion of the two currents in eq.(6). Such a procedure is equivalent to introducing a cut-off of order $\mu \approx \frac{1}{|x_0|}$ in momentum space. As m_b is the natural subtraction point, one may take $\mu = m_B$. Thus we can write:

$$\mathcal{M} = \int \frac{d^4q}{(2\pi)^4}\theta(m_b^2 + q^2)T(q), \tag{7}$$

with

$$T(q) = g^{\mu\nu}T_{\mu\nu}(q) = g^{\mu\nu}\int d^4x \exp{(iq\cdot x)} < \overline{B^0}(p)|T\left(J_\mu(x)J_\nu(0)\right)|B^0(p) > . \tag{8}$$

Notice that $\mathcal{M}_{VAC}$ can be easily reobtained from eqs.(7) and (8) which admit however, in addition, the inclusion of more general hadronic intermediate states.

The second step is to decompose the tensor $T_{\mu\nu}$ into invariant amplitudes as follows:

$$\begin{aligned} T_{\mu\nu} =& (q_\mu q_\nu - g_{\mu\nu}q^2)f_1 + (p_\mu - q_\mu\frac{\nu}{q^2})(p_\nu - q_\nu\frac{\nu}{q^2})f_2 \\ &+ (-2\nu g_{\mu\nu} + p_\mu q_\nu + p_\nu q_\mu)\frac{f_3}{q^2} + g_{\mu\nu}\frac{f_4}{q^2}, \end{aligned} \tag{9}$$

where $f_i = f_i(q^2,\nu)$ with $\nu = (p\cdot q)$. The invariant amplitudes f_i should satisfy well-convergent dispersion relations, because their asymptotic behaviour in $|\nu|$ at fixed q^2 is dominated by exotic $\Delta B = 2$ Regge trajectories with negative intercept. Thus it makes sense to assume such dispersion relations to be well approximated by the inclusion of the lowest hadronic states in $Im\ T$.

At this is point, in order to estimate the single particle contributions of eq.(5), we need the matrix elements of the current J_μ between the B and the $0^-, 1^-$ meson states. To this purpose we adopt the model of Refs.[2,3], where relativistic meson wave functions and infinite momentum frame techniques are used to compute the form factors at $q^2 = 0$, and then a simple pole behavior in q^2 is assumed. The results are compatible with estimates based on QCD sum rules [7,8].

The q-integration in eq.(7) is most simply performed by rotating the contour in the q_0 plane, obtaining

$$\mathcal{M} = \mathcal{M}_{Cott} + \mathcal{M}_{Pole}, \tag{10}$$

where $\mathcal{M}_{Cott}$ is a Cottingham-like term:

$$\mathcal{M}_{Cott} = \frac{i}{(2\pi)^3}\int_0^{m_b^2} dQ^2 \int_{-\sqrt{Q^2}}^{\sqrt{Q^2}} dk_0\sqrt{Q^2 - k_0^2}\ T(Q^2, k_0) \tag{11}$$

($Q^2 = -q^2$, $k_0 = -iq_0$), and $\mathcal{M}_{Pole}$ is the contribution of the poles in the first and third quadrant that can be computed by the Cauchy theorem.

The numerical results for the various contributions to $\mathcal{M}$, as computed in our approach, are the following (in GeV^4 units):

$$\mathcal{M}(\pi) = -4.24; \quad \mathcal{M}(\eta) = -1.17; \quad \mathcal{M}(\eta') = -1.41; \quad \mathcal{M}(\rho) = \mathcal{M}(\omega) = +3.40 \quad (12)$$

The values in eq.(12) should be compared to $\mathcal{M}_{VAC} = (0.74 - 3.9)\ GeV^4$, which would be obtained from eq.(4) using the vacuum saturation value $B = 1$ and the range $f_B = (100 - 230)\ MeV$ which encompasses all the theoretical results for the leptonic B decay [12], [17-21]. The indication is that the single particle contributions can be non negligible with respect to the vacuum insertion result. This is contrary to the case of the $K^0 - \overline{K^0}$ mixing [22]. This fact can been understood by simple dimensional arguments: for an $M^0 - \overline{M^0}$ system $\mathcal{M}_{VAC}$ scales as $m_M^2 f_M^2$, whereas $\mathcal{M}_{1Part}$ varies as $m_M^4 F_M^2$ (with F_M the coupling of the form factor at $q^2 = 0$). Since $f_K \simeq f_B$ and $\frac{F_B}{F_K} \simeq (0.2 - 0.3)$, one can expect a larger relative importance of $\mathcal{M}_{1Part}$ for $B^0 - \overline{B^0}$ than for $K^0 - \overline{K^0}$.

On the other hand the values in eq.(12) indicate that the single particle contributions tend to compensate; actually one obtains

$$B = \frac{\mathcal{M}_{VAC} + \mathcal{M}_{1Part}}{\mathcal{M}_{VAC}} \simeq 1 \quad (13)$$

irrespective of the value of f_B. Thus, within the theoretical uncertainty which admittedly affects eqs.(12) and (13), and which we are not able to reliably assess at the moment, the approximate validity of the vacuum saturation follows, in the approach exposed here, from the cancellation of appreciable hadronic intermediate states. It should be interesting to try to clarify the mechanism underlying such a cancellation.

RARE $b \to s$ EXCLUSIVE DECAYS

The theoretical descrition of these decays is based on an effective weak Hamiltonian expressed in terms of quark and gluon fields, which is estimated in the Standard Model from higher order "penguin" diagrams. Correspondingly, these transitions are expected to have a particularly important role in order to test the Standard Model dynamics beyond the tree level approximation [23]. Crucial to this program is however the dominance of short-distance physics, implicitly assumed in the derivation of such an H_{eff}, so that the underlying quark and gluon dynamics is not obscured by long-distance effects. Thus in principle, for a meaningful comparison with experimental data, one should consider for each mode the complete phenomenological situation in which long-range effects are taken into account.

In this context, particular attention has been given recently to the radiative decay:

$$B(p) \to K^*(p', \eta) + \gamma(q, \epsilon). \quad (14)$$

The short-distance effective Hamiltonian for this transition is the penguin operator [24,25]:

$$H_{eff}(b \to s\gamma) = C\ m_b\ \bar{s}\epsilon^{\mu}\sigma_{\mu\nu}\frac{1}{2}(1+\gamma_5)q^{\nu}b, \tag{15}$$

where C depends, in addition to electroweak coupling constants and mixing angles, on the top quark mass m_t. Also, C contains virtual gluon QCD corrections, which greatly enhance the rate of $b \to s\gamma$ [26-28]; actually an additional significant effect is expected from SUSY particle exchanges [29]. All that justifies the great theoretical interest of the $b \to s\gamma$ transitions.

In general the amplitude for the process (14) has a parity conserving plus a parity violating part:

$$\begin{aligned}\mathcal{A}(B \to K^*\gamma) =& i\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu}\eta^{\nu}p^{\rho}q^{\sigma}F_1 \\ &+ \left[(\epsilon\cdot\eta)(m_B^2 - m_{K^*}^2) - (\epsilon\cdot(p+p'))(\eta\cdot q)\right]F_2.\end{aligned} \tag{16}$$

The short-distance (sd) contribution to eq.(16) is obtained by taking the matrix element of H_{eff} in eq.(15) between the hadronic B and K^* states. The result can be written as [30]:

$$F_1|_{sd} = 2F_2|_{sd} = 9.9\sqrt{\eta_{sd}}\left(\frac{m_t}{60\ GeV}\right)^{0.58} 10^{-9}GeV^{-1}, \tag{17}$$

where

$$\eta_{sd} = \frac{\Gamma(B \to K^*\gamma)|_{sd}}{\Gamma(b \to s\gamma)}. \tag{18}$$

The constant η_{sd} depends on the different non perturbative hadronization models; to compare with the long distance contributions we take the range

$$\eta_{sd} \cong 0.05 - 0.4, \tag{19}$$

which encompasses the various theoretical predictions [31-34]. Correspondingly the prediction for the branching ratio would be $BR(B \to K^*\gamma) \sim 10^{-5}$, while the present limit is $BR(B \to K^*\gamma)|_{EXP} < 2.4\ 10^{-4}$. Experiments are thus very close to make a decisive test of the predictions above.

Long range effects (ld) to $B \to K^*\gamma$ have been catalogued in [30]. They correspond to the diagrams in Fig.1, where K_n, B_n and V_n are intermediate hadronic states with appropriate quantum numbers, and the circles denote the mixing weak Hamiltonian H_W. Actually gauge invariance forbids Fig.1c for a real photon, so that only Figs.1a,b remain [35].

In [30] the K-pole of Fig.1a has been discussed as an example, by using the vacuum insertion approximation to estimate the matrix elements of the corresponding H_W. The result for this contribution is negligible compared to the (sd) amplitude in eq.(17). There exists however one more source of long range effects from Figs.1a,b, which potentially can affect the results. Indeed the needed mixing matrix elements $< K|H_W|B >$, $< K^*|H_W|B >$, etc. can be induced also by the weak non leptonic

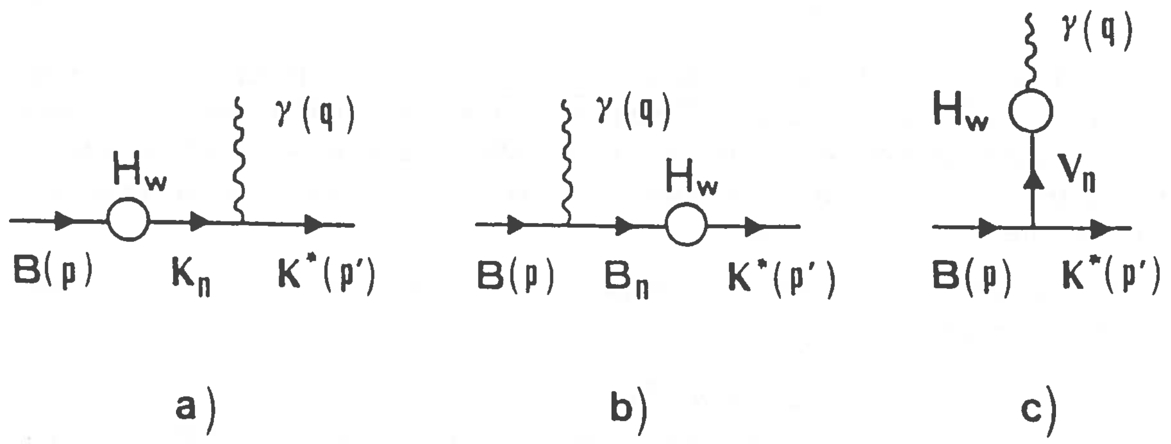

Fig.1 Long distance contributions to $B \to K^*\gamma$

Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}}\left(c_1 + \frac{c_2}{3}\right) V^*_{sc} V_{bc}\, \bar{b}\gamma_\mu(1-\gamma_5)c\, \bar{c}\gamma^\mu(1-\gamma_5)s, \tag{20}$$

where c_1 and c_2 are QCD coefficients (with $c_1 + \frac{c_2}{3} \simeq 1$). The H_W of eq.(20) cannot be vacuum saturated, but can give a non vanishing $< K|H_W|B >$ mixing via single particle D and D^* intermediate state saturation:

$$< K|H_W|B > \propto < K|\bar{c}\gamma_\mu(1-\gamma_5)s|D, D^* >< D, D^*|\bar{b}\gamma^\mu(1-\gamma_5)c|B > . \tag{21}$$

Also, and very importantly, eq.(20) has the enhancement $V_{bc}/V_{bu} \geq 10$ as compared to the Born contribution considered in [30].

To evaluate the mixing matrix elements in Figs.1a,b via the H_W of eq. (20), and thus to complete the estimate of the (ld) contributions to the transition (16), we can adopt the same formalism for the single particle saturation used in the previous Section [36]. The only (small) conceptual difference is that in the present case one has to account for the fact that the intermediate K_n and B_n states are off-shell. Thus for example, for the case of the B-K mixing matrix element in Fig.1a

$$\mathcal{A}_{BK}(p^2) =< K(p)|H_W|B(p) >, \tag{22}$$

the analogue of eq.(7) would be

$$\mathcal{A}(p^2) = \frac{G_F}{\sqrt{2}}\left(c_1 + \frac{c_2}{3}\right) V^*_{sc} V_{bc} \int \frac{d^4q}{(2\pi)^4}\theta(q^2+\mu^2)T(q,p^2) \tag{23}$$

where

$$T(q,p^2) = g^{\mu\nu}T_{\mu\nu}(q) = g^{\mu\nu}\int d^4x \exp{(iq\cdot x)} < K(p)|T\left(\tilde{J}_\mu(x)J_\nu(0)\right)|B(p)>, \quad (24)$$

with $\tilde{J}_\mu = \bar{c}\gamma_\mu(1-\gamma_5)s$ and $J_\mu = \bar{b}\gamma_\mu(1-\gamma_5)c$. In eq.(23) μ is the high-frequency cutoff, representing the onset of scaling behaviour; we can take

$$\mu = \sqrt{p^2}, \quad (25)$$

since this mass fixes the overall scale of the matrix element.

For the current matrix elements needed to evaluate eq.(21) we adopt again the parametrizations and the results of Refs.[2,3], generalized to the specific situation of eq.(22) where the kaon is off-shell ($p^2 \neq m_K^2$), and where in general in Figs.1a,b the intermediate mesons M have $p^2 \neq m_M^2$. More specifically we assume for the various form factors F_j:

$$F_j(q^2,p^2) = g_M(p^2)h_j(p^2)\Big/\left(1-\frac{q^2}{m_j^2}\right). \quad (26)$$

For the p^2 dependence we assume

$$g_M(p^2) = \frac{m_{M'}^2 - m_M^2}{m_{M'}^2 - p^2}, \quad (27)$$

where M' is the first radial excitation of M (in particular $m_{K'} = 1.4\,GeV$ and $m_{K^{*\prime}} = 1.7\ GeV$), as it is consistent with a M'-dominated dispersion relation in p^2. The dependence of h_j on p^2 can be argued by applying the LSZ formalism to the matrix element of eq.(22); to account for such an effect we can make the replacement $m_M^2 \to p^2$ in the explicit formulae giving the on-shell $h_j(m_M^2)$ in Ref. [2].

Eq.(22) can now be estimated, by performing the q-integration in eq.(23) similarly to the previous Section. One finds in analogy to eq.(10):

$$\mathcal{A}_{BK}(p^2) = \sum_{n=D,D^*}\left[\mathcal{A}_{Pole}^{(n)} + \mathcal{A}_{Cott}^{(n)}\right], \quad (28)$$

and the final numerical result is

$$\mathcal{A}_{BK}(m_B^2) = (3.7 + i\ 0.9)\ 10^{-7}\ GeV^2. \quad (29)$$

A similar calculation can be performed for the transition

$$< K^*(p,\eta)|H_W|B(p)> = i\ \mathcal{A}_{BK^*}(p^2)(p\cdot\eta), \quad (30)$$

with the result:

$$\mathcal{A}_{BK^*}(m_B^2) = (-0.4 + i\ 1.4)\ 10^{-8}\ GeV. \quad (31)$$

One can see by dimensional arguments that the contributions of the diagrams in Fig.1b should be much smaller than eqs.(29) and (31). Indeed, amplitudes are computed in Fig.1b at a much smaller value of μ (say $\mu \sim 1\ GeV$), so that the q-integration region is quite reduced. Thus it is a good approximation to retain, in the evaluation of the (ld) effects, only the contributions of Figs.1a.

By using these results we can now compute the (ld) contributions to $B \to K^*\gamma$ from Figs.1a,b. In particular, eq.(22) contributes to the parity conserving amplitude of eq.(16):

$$F_1|_{ld} = \frac{\mathcal{A}_{BK}(m_B^2)}{m_B^2 - m_K^2} g_{KK^*\gamma}(m_B^2) \tag{32}$$

where the off-shell $KK^*\gamma$ coupling constant can be written as

$$g_{KK^*\gamma}(p^2) = g_K(p^2) g_{KK^*\gamma} \tag{33}$$

with $g_K(p^2)$ given by eq.(27) and the on-shell $g_{KK^*\gamma}$ determined from the $K^* \to K\gamma$ partial width. Analogously, eq.(30) contributes to the parity violating amplitude in eq.(16):

$$F_2|_{ld} = -\frac{\mathcal{A}_{BK^*}(m_B^2)}{2m_{K^*}^2} g_{K^*K^*\gamma}(m_B^2), \tag{34}$$

where $g_{K^*K^*\gamma}$ is the normal component of the magnetic moment of the K^* which was estimated e.g. in Ref.[37].

The long distance contributions computed in this way are presented in Tab.1, and are compared to the short distance contributions given by eq.(17) for $m_t = 70\ GeV$ and η_{sd} as in eq.(19). From this Table we can see that the long distance amplitudes estimated in the present framework are appreciably larger than the results $O(10^{-11})$ obtained in Ref.[30] by the vacuum insertion approximation. On the other hand it is reassuring that they are still small enough as to not obscure the quark process underlying the short distance amplitude, in particular the QCD or the possible SUSY enhancement.

Decay mode	$F_1\|_{ld}$	$F_2\|_{ld}$	$F_1\|_{sd} = 2F_2\|_{sd}$
$\overline{B^0} \to \overline{K^{*0}}\ \gamma$	$4\ 10^{-10}\ GeV^{-1}$	$\simeq 0$	$(2.4 - 6.8)\ 10^{-10}\ GeV^{-1}$
$B^- \to K^{*-}\ \gamma$	$2\ 10^{-10}\ GeV^{-1}$	$2\ 10^{-10}\ GeV^{-1}$	"

Tab.1: Parity conserving and parity violating amplitudes for $B \to K^*\gamma$

Turning now to non leptonic exclusive $b \to s$ decays, such as e.g. the processes $B \to M\ V$, $B \to M\ M$ and $B \to V\ V$, with M and V a pseudoscalar and a vector meson respectively, the long distance effects in this case can be discussed quite similarly to $B \to K^*\gamma$. Indeed, while the short distance amplitudes are given by the effective penguin Hamiltonian [3,38-39], the long range effects arise from a similar class of diagrams as considered in Figs.1a,b,c, in which the photon and the K^* are replaced by the relevant V and M mesons. Also, by the same dimensional argument as above, the diagrams of the kind in Fig.1a should largely dominate these long range effects.

Thus in the example of $B \to M\ V$, which includes the channels $B \to K^*\pi$, $B \to K\rho$ and $B \to K\phi$, we would have the (ld) contribution

$$\mathcal{A}_K(B \to M\ V) = i\ \frac{\mathcal{A}_{BK}(m_B^2)}{m_B^2 - m_K^2} g_{KMV}(m_B^2)\ \epsilon^\mu(p+q)_\mu, \tag{35}$$

where analogously to eq.(33) the strong coupling constant $g_{KMV}(p^2)$ can be determined using eq.(27) and the measured $K^* \to K\pi$ width (and assuming SU(3) symmetry to obtain the other couplings). Also, it is easy to show that the contribution of $\mathcal{A}_{BK^*}$ in eq.(30) cannot contribute to this case.

The other $b \to s$ decays listed above can be discussed quite analogously. We show in Tab.2 the contributions of the long distance effects to the branching ratios for the different decay channels, together with the results obtained from the short distance penguin effective Hamiltonian.

Decay mode	*ld*	*sd*	*Exp. limit*
$\overline{B^0} \to \overline{K^0}\ \rho^0$	$7\ 10^{-5}$	$4\ 10^{-6}$ $^{(a)}$	$5.8\ 10^{-4}$ $^{(c)}$
$\overline{B^0} \to K^-\ \rho^+$	$1\ 10^{-4}$	$4\ 10^{-7}\ -\ 2\ 10^{-5}$ $^{(b)}$	
$\overline{B^0} \to K^{*-}\ \pi^+$	$1\ 10^{-4}$	$2\ 10^{-5}\ -\ 4\ 10^{-4}$ $^{(b)}$	$4.4\ 10^{-4}$ $^{(c)}$
$\overline{B^0} \to \overline{K^0}\ \phi$	$8\ 10^{-5}$	$5\ 10^{-5}$ $^{(a)}$	$4.9\ 10^{-4}$ $^{(c)}$
$\overline{B^0} \to K^-\ \pi^+$	$2\ 10^{-8}$	$7\ 10^{-6}\ -\ 9\ 10^{-6}$ $^{(b)}$	$0.9\ 10^{-4}$ $^{(c)}$
$\overline{B^0} \to \overline{K^{*0}}\ \rho^0$	$1\ 10^{-4}$		$5.0\ 10^{-4}$ $^{(d)}$
$\overline{B^0} \to \overline{K^{*0}}\ \phi$	$2\ 10^{-4}$	$5\ 10^{-5}$ $^{(a)}$	$4.4\ 10^{-4}$ $^{(c)}$
$B^- \to K^-\ \rho^0$	$7\ 10^{-5}$	$1\ 10^{-5}$ $^{(a)}$	$0.7\ 10^{-4}$ $^{(c)}$
$B^- \to \overline{K^{*0}}\ \pi^-$	$1\ 10^{-4}$	$5\ 10^{-5}$ $^{(a)}$	$1.3\ 10^{-4}$ $^{(c)}$
$B^- \to K^-\ \phi$	$7\ 10^{-5}$	$4\ 10^{-5}$ $^{(a)}$	$0.8\ 10^{-4}$ $^{(c)}$
$B^- \to \overline{K^0}\ \pi^-$	$2\ 10^{-8}$	$6\ 10^{-5}$ $^{(a)}$	$0.9\ 10^{-4}$ $^{(c)}$
$B^- \to K^{*-}\ \rho^0$	$1\ 10^{-4}$		$5.4\ 10^{-4}$ $^{(d)}$
$B^- \to K^{*-}\ \phi$	$2\ 10^{-4}$	$4\ 10^{-5}$ $^{(a)}$	$1.1\ 10^{-3}$ $^{(d)}$

Tab.2: Our estimated (ld) contributions to branching ratios for excusive $b \to s$ decays, compared to the (sd) contributions of Ref.[38](a) and of Ref.[39](b). Also shown are the experimental limits from Ref.[40](c) and from Ref.[41](d).

Due to the incomplete knowledge of the current matrix elements and of the form factors, the theoretical uncertainty affecting the predictions in Tab.2 is quite large, both for the (ld) and for the (sd) contributions. However we observe from Tab.2 that, differently from the $B \to K^*\gamma$ decay, the long distance amplitudes, which should be combined with the penguin matrix elements with a model dependent relative phase, cannot be safely neglected a priori.

CONCLUSIONS

As we have seen in the preceding Sections, the semileptonic form factors, in addition to be needed for better determinations of the CKM matrix elements, play a key role in a number of other physical processes, in which the B meson is involved. The two examples discussed here, the $B^0 - \overline{B^{0'}}$ mixing matrix element beyond the vacuum saturation approximation and the long distance effects in exclusive $b \to s$ radiative and nonleptonic decays, are particularly illustrative in this regard. Clearly the improved knowledge (both theoretical and experimental) of these form factors should be of great importance in order to refine the estimates presented above and to test the ideas which underly such a calculation.

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