ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Catania

AE

INFN/AE-89/13

8 Novembre 1989

E. Recami, V. Tonin-Zanchin:

Unusual Black-Holes: About Some Stable (Non-Evaporating) Extremal Solutions of Einstein Equations

Servizio Documentazione
dei Laboratori Nazionali di Frascati

INFN - Istituto Nazionale di Fisica Nucleare

Sezione di Catania

INFN/AE-89/13

8 Novembre 1989

UNUSUAL BLACK–HOLES: ABOUT SOME STABLE (NON–EVAPORATING) EXTREMAL SOLUTIONS OF EINSTEIN EQUATIONS.(*)

Vilson TONIN–ZANCHIN[a,b] and Erasmo RECAMI[a,c]

[a] *Dept. of Applied Mathematics, State University at Campinas, S.P., Brazil.*

[b] *"Gleb Wataghin" Inst. of Physics, State Univ. at Campinas, S.P., Brazil.*

[c] *Dipartimento di Fisica, Università Statale di Catania, Catania, Italy and I.N.F.N., Sezione di Catania, Catania, Italy.*

ABSTRACT – Within a purely classical formulation of "strong gravity", we associated hadron constituents (and even hadrons themselves) with suitable stationary, axisymmetric solutions of certain new Einstein–type equations supposed to describe the strong field inside hadrons. Such equations are nothing but Einstein equations – with cosmological term – suitably scaled down. As a consequence, the cosmological constant Λ and the masses M result in our theory to be scaled up, and transformed into a "hadronic constant" and into "strong masses", respectively. Due to the unusual range of Λ and M values considered, we met a series of solutions of the Kerr–Newman–de Sitter (KNdS) type with so uncommon horizon properties (e.g., completely impermeable horizons), that it is worth studing them also in the case of ordinary gravity. This is the aim of the present work.

The requirement that those solutions be stable, i.e., that their temperature (or surface gravity) be *vanishingly small*, implies the coincidence of at least two of their (in general, three) horizons.

(*) Work partially supported by CAPES, FAPESP, and by INFN, M.P.I. and CNR.

In the case of ordinary Einstein equations and for stable black holes of the KNdS type, we get Regge–like relations among mass M, angular momentum J, charge q and J cosmological constant Λ. For instance, with the standard definitions $Q^2 \equiv Gq^2/(4\pi\varepsilon_0 c^4))$; $a \equiv J/(Mc)$; $m \equiv GM/c^2$, in the case $\Lambda = 0$ in which $m^2 = a^2 + Q^2$ and q is negligible we find $M^2 = J$, where $c = G = 1$. When considering, for simplicity, $\Lambda > 0$ and $J = 0$ (and q still negligible), then we obtain $m^2 = 1/(9\Lambda)$. In the most general case, the condition, for instance, of "triple coincidence" among the three horizons yields for $|\Lambda a^2| << 1$ the couple of independent relations $m^2 = 2/(9\Lambda)$; $m^2 = 8(a^2 + Q^2)/9$.

One of the interesting points is that – with few exceptions – all such relations (among M, J, q, Λ) lead to solutions that can be regarded as (stable) cosmological models. Worth of notice are those representing isolated worlds, bounded by a two–way impermeable horizon.

1. INTRODUCTION.

Within a purely classical approach to "strong gravity", that is to say, within our geometric approach to hadron structure[1], we came to associate hadron constituents with suitable, stationary, axisymmetric solutions of certain new Einstein–type equations, supposed to describe the strong field inside hadrons.

Such Einstein–type equations are nothing but the ordinary Einstein equations (with cosmological term) suitably scaled down[2]. As a consequence, the cosmological constant Λ and the masses M result to be scaled up, in such a theory, and transformed into a "hadronic constant" and into "strong masses", respectively.[1,2].

Due to the unusual range of values therefore assumed by Λ and M, we met a series of solutions of the Kerr–Newman–de Sitter type, which had not received attention in the previous literature. Moreover, the requirement that those "(strong) black–hole" solutions be stable (i.e., that their surface temperature, or surface gravitity[3], be *vanishingly small*), implies the coincidence of at least *two* of their (three, a priori) horizons. This fact makes such black–hole solutions so interesting and with so uncommon horizon–properties, that it is worthwhile studying them also in the case of ordinary gravity, that is to say of ordinary Einstein equations.

2. THE FOUR SPACE–REGIONS ASSOCIATED WITH A CENTRAL, STATIONARY BODY. HORIZONS AND THEIR MAIN PROPERTIES.

Let us consider Einstein equations with cosmological term

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\rho}_{\rho} + \Lambda g_{\mu\nu} = -kT_{\mu\nu} \; ; \quad [k \equiv \frac{8\pi G}{c^4}], \tag{1}$$

choose (whenever convenient) units such that $G = 1$; $c = 1$, and look for the vacuum solutions describing the stationary axisymmetric field created by a rotating charged source. This solutions is the Kerr–Newman–de Sitter (KNdS) space–time, whose metric in Boyer–Lindquist– type[4] coordinates (t, r, θ, φ) writes [5] (with the signature -2):

$$\begin{aligned} ds^2 \;=\; & -\rho^2[dr^2/B + d\theta^2/D] - \rho^{-2}A^{-2}[(adt - (r^2+a^2)d\varphi]^2 \sin^2\theta + \\ & + BA^{-2}\rho^{-2}[dt - a\sin^2\theta \; d\varphi]^2, \end{aligned} \tag{2}$$

with $m \equiv GM/c^2$; $a \equiv J/Mc$; $\rho^2 \equiv r^2 + a^2\cos^2\theta$; $A \equiv 1 + \Lambda a^2/3$; $B \equiv B(r) \equiv (r^2+a^2)(1-\Lambda r^2/3) - 2mr + Q^2$; $D \equiv D(\theta) \equiv 1 + (\Lambda a^2\cos^2\theta)/3$; $Q^2 \equiv (G/4\pi\varepsilon_0 c^4)q^2$; quantities M, J and q being mass, angular momentum and electric charge of the source, respectively. For simplicity, let us here analyze only the case $\Lambda > 0$.

One meets the event horizons of the space (2) in correspondence with the divergence of the coefficient g_{rr}, i.e., when $B(r) = 0$. This equation,

$$(r^2+a^2)(1-\frac{\Lambda r^2}{3}) - 2mr + Q^2 = 0, \tag{3}$$

admits four roots, one of which, r_0, is always real and negative. The interesting case is when eq. (3) has four real solutions; in that case we shall have *three* positive roots: let us call them r_1, r_2, r_3, with $r_3 \geq r_2 \geq r_1$. [The case in which $\Lambda < 0$ is less interesting, since it yields at most two real positive roots]. We shall see that at $r = r_3$ we have a cosmological horizon[6], while at $r = r_2$ and $r = r_1$ we meet two black–hole horizons analogous to the two wellknown $r = r_+$ and $r = r_-$ horizons of the Kerr metric.

The three horizons 1, 2, 3, in the general case when they are all real, divide the space in the four parts I, II, III and IV (see Fig.1). On each horizon, quantity $g_{rr} \equiv g_{11}$ diverges, i.e. $g^{rr} = 0$. Quantity g^{rr} does change sign when passing from any region to the adjacent ones.

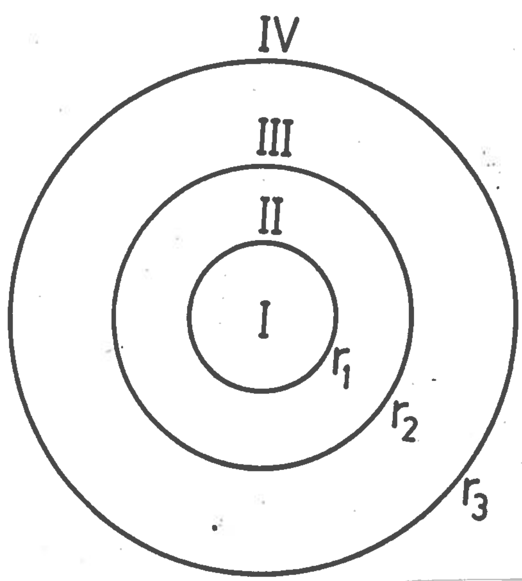


Figure 1: Given an (almost) pointlike stationary body, generating, when $\Lambda \neq 0$, a Kerr–Newman–de Sitter space–time, it will in general possess *three* horizons 1,2,3, which divide the associated space into the four regions I, II, II, IV. On the horizons g_{rr} diverges, i.e., $g^{rr} = 0$. Quantity g^{rr} does change sign when passing from any region to the adjacent ones. Surface 3 is the cosmological horizon and surfaces 2,1 are the outer and inner black–hole horizons, respectively.

For instance, in regions III and I it is always $g^{rr} < 0$, as expected in the case of an ordinary Kerr black–hole; on the contrary, in regions II and IV it is always $g^{rr} > 0$. Actually, it is possible to define a Killing vector K^μ which is simultaneously time–like in regions III and I (but not in regions II and IV too). Therefore one can have stationary observers ($r =$ const.) only in regions III and I, in the sense that only there the $r =$ const. trajectories are time-like.

Let us call *time-like* the (ordinary type) *regions* III and I; and *space-like* the other two *regions* II and IV. Let us explicitly mention also that if the (time-like) Killing vector K^μ is future-pointing in region III, then it will be past-pointing in region I. An analogous sign-change occurs when passing from region II to region IV (in which, however, K^μ is space-like), so that – if the geometry of region IV is expanding – then that of region II is collapsing[7].

From a more formal point of view, let us represent the properties of such regions, and of their horizons, by depicting the behaviour of their various radial null geodesics.

For simplicity's sake, let us confine ourselves to the static case (Reissner–Nordström–de Sitter metric), which is not qualitatively different. From eq. (2), by putting $a = 0$, we find for those geodesics:

$$ds^2 = Fdt^2 - F^{-1}dr^2 = 0 \quad ; \quad F \equiv B/r^2 \tag{4}$$

By integration of eq. (4), after some algebra one gets $[m = 0, 1, 2, 3]$:

$$t = \mp 3\Lambda^{-1} \sum_{m=0}^{3} \alpha_m r_m^2 \log |\frac{r}{r_m} - 1| + C_\mp \tag{4'}$$

where $C_\mp$ are integration constants, and α_m are "constants" whose value depends on the values of the four roots r_0, r_1, r_2, r_3 of eq. (3). The behaviour of the radial null geodesics $t = t(r)$ is given in Fig.2 for the four regions. In eq. (4') the upper (lower) sign corresponds to outgoing (ingoing) geodesics.

As confirmed by Figs. 1, 2, our four horizons are semi-permeable surfaces. With our choice of the time direction, both horizons 1 and 2 can be crossed by causal (time-like or null) curves only in the direction of decreasing r; whilst the opposite holds for horizon 3. For instance, causal particles can only be emitted by (but not enter) the cosmological, $r = r_3$, horizon. In Fig.3 we indicate by arrows the permeability of the various horizons to causal particles in the *general* case of Fig.1, i.e. of the metric (2). Notice that the horizon permeability properties are in this general (stationary) case the same as in the particular (static) case of Fig.2, so that Fig.2 can be chosen (without loss of generality) to illustrate the horizon permeability properties in the general case.

We are particularly interested, however, in the case when two or more of our horizons coincide.

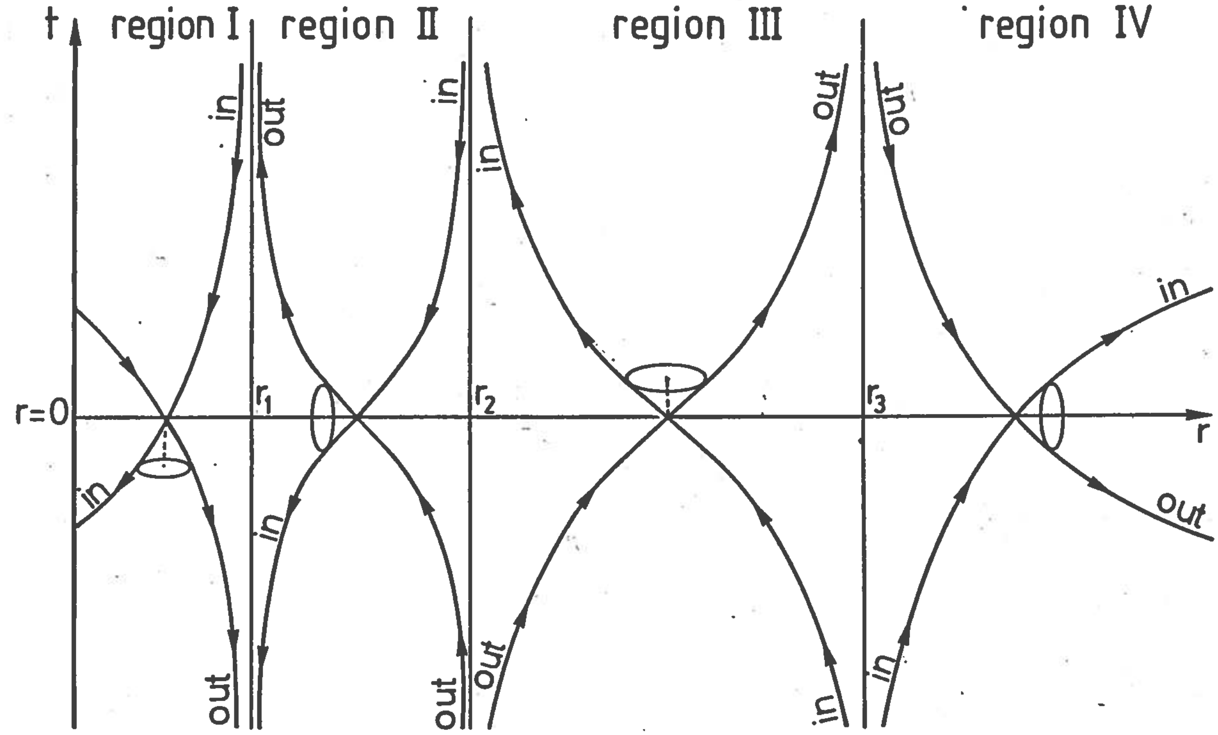


Figure 2: The behaviour of the outgoing and ingoing radial null geodesics in the different (four) regions of the Reissner–Nordström–de Sitter geometry (static case). The case here depicted corresponds in particular to $r_2 = 2r_1$; $r_3 = 3r_1$ (with $r_0 = -r_1 - r_2 - r_3$). The semi–cones appearing in this figure point towards the future.

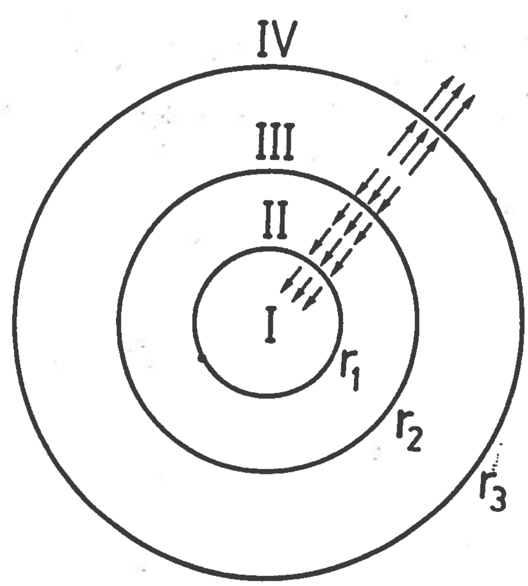


Figure 3: We indicate by arrows the semi–permeability to causal particles of the various horizons in the general case of Fig.1 (metric (2)). Let us recall that, a priori, the opposite choice is also allowed ("white–hole" solution) by time reversal invariance.

3. ON THE HORIZON TEMPERATURES (SURFACE GRAVITIES).

In the general (stationary, i.e. KNdS) case of metric (2), the Bekenstein–Hawking temperature[9] T_n of each horizon in Figs. 1, 3 is known to be proportional to the horizon surface gravity as follows:

$$T_n = \varepsilon \gamma_n \quad ; \qquad \varepsilon \equiv \hbar/(2\pi k_B c) \tag{5}$$

where k_B is the Boltzman constant and $n = 1, 2, 3$. On any null-surface (in particular on every horizon) the surface gravity[10,11] can be defined by the equation $\partial_\mu (K_\nu K^\nu) = -2\gamma K_\mu$, symbols ∂_μ representing the covariant derivatives.

To evaluate the surface gravities γ_n, let us then recall that our metric (2) does admit two Killing vector K_t^μ, K_φ^μ, the former being related to time-translation invariance and the latter to rotational invariance of the space-time. By linear combination of K_t^μ and K_φ^μ one can construct Killing vectors $K_n^\mu = K_t^\mu + \omega_n K_\varphi^\mu$ which vanish on the n-th horizon; i.e., which there satisfy the relation $(K_n)^\mu (K_n)_\mu = 0$. One finds $\omega_n = g_{t\varphi}/g_{\varphi\varphi}$ (evaluated at $r = r_n$). We finally get for the horizon temperatures (and for $\Lambda \neq 0$) the expressions $T_n = \varepsilon \gamma_n$ with

$$T_n = \frac{\varepsilon \Lambda}{6A(r_n^2 + a^2)} \cdot \left| \Pi_{\ell \neq n}^{0,3} (r_n - r_e) \right| \quad , \quad [\ell = 0, 1, 2, 3] \tag{6}$$

Eq. (6) yields the result that the horizon temperature can be vanishing small only when two (or more) horizons tend to coincide; i.e., when two (or more) roots r_i of eq. (3) tend to coincide. This result is important since it leads to the conditions for a BH (black–hole) to be stable, i.e. it implies some *relations* among mass, radius, charge, angular momentum (and Λ) of a stable BH.

In the particular (Kerr–Newman) case when $\Lambda = 0$, one gets only two (or no) horizons, corresponding to $r_\pm = m \pm \sqrt{m^2 - a^2 - Q^2}$, and eq. (6) has to be replaced by $T_\pm = \varepsilon (r_+ - r_-)/(r_\pm^2 + a^2)$.

We get a stable ($T = 0$) black–hole solution when

$$r_+ = r_- = m \quad , \tag{7a}$$

that is to say, when the Regge–like condition does hold:

$$m^2 = a^2 = Q^2 \; . \tag{7b}$$

Incidentally, let us notice that in this particular case the stable black–hole is *still* bounded by a semi-permeable horizon [in fact, r_-, r_+ behave as r_1, r_2 of Figs. 2, 3]. However, since in this case region II disapeared, then the whole BH–interior is time–like!, so as the external region III (with the difference that the light–cones reverse when passing from region III to region I, i.e. the time–flow direction in region I is opposite to that of region III). Let us call a solution of this type a semipermeable *"time–like black–hole"* (at variance with the ordinary "space–like BHs").

4. THE STABLE SCHWARZSCHILD–DE SITTER BLACK–HOLE: A FIRST EXAMPLE OF NON–PERMEABLE HORIZON.

A more interesting case is that of the Schwarzschild–de Sitter metric, in which $Q^2 = a^2 = 0$, so that $B = -\Lambda r^4/3 + r^2 - 2mr$ and two horizons only (with radii $r_- \equiv r_{\rm B}$, $r_+ \equiv r_{\rm C}$, respectively) are met, whose surface temperatures result to be

$$T_\pm = \frac{\varepsilon\Lambda}{3r_\pm^2}(\frac{3m}{\Lambda} - r_\pm^2) \tag{8}$$

Once more, the requirement $T = 0$ implies that $r_{\rm B} = r_{\rm C} \equiv r$ and that $r = (3m/\Lambda)^{1/3}$. The last equation can be read as

$$r = \Lambda^{-1/2} = 3m \quad , \quad [r_{\rm B} = r_{\rm C} \equiv r] \tag{9}$$

since those two radii coincide (only) when

$$9\Lambda m^2 = 1 \tag{10}$$

For completeness' sake, let us mention that the two horizons radii can be written as $2r_\pm = (\beta_1 + \beta_2) \pm \sqrt{-3}(\beta_1 - \beta_2)$, with $\beta_{1,2} \equiv [\beta \pm \sqrt{\beta^2 - \Lambda^{-3}}]^{1/3}$; $\beta \equiv 3m/\Lambda$.

Let us observe that the condition for a BH to be stable yields, besides the BH radius (as a function of m and Λ), a further relation between m and Λ.

More interesting, here, is the observation that r_- and r_+ behave so as r_2 and r_3, respectively, of Figs. 2 and 3. For this reason we called $r_- \equiv r_{\rm B}$; $r_+ = r_{\rm C}$ (B $\equiv$ black–hole horizon; C $\equiv$ cosmological horizon). When $r_{\rm B}$ tends to coincide with $r_{\rm C}$, the (time–like) region III disappears, so that we are left only with regions II and IV, and the BH tends to occupy the whole space inside the cosmological horizon (roughly speaking, the BH itself can be regarded as a model for a cosmos). See Fig.4(a). As a consequence, the $r = 3m$ horizon of the stable BH becames a *totally impermeable surface* in either ways, i.e. a (two–way) *non–permeable* membrane, since it is the superposition of two membranes semi–permeable in antiparallel directions. It is worthwhile mentioning that, by choosing for Λ the value $|\Lambda| \approx 10^{-52}$ m^{-2} ordinarily assumed for our cosmos, the condition (10) yields $m \simeq \frac{1}{2} \times 10^{53}$ kg, which is close to the estimated mass of our own cosmos. Incidentally, when passing to the "strong BH" case [1,2], with Λ replaced by $\lambda \simeq (10^{40})^2\Lambda$, one would get $m = m_\pi$.

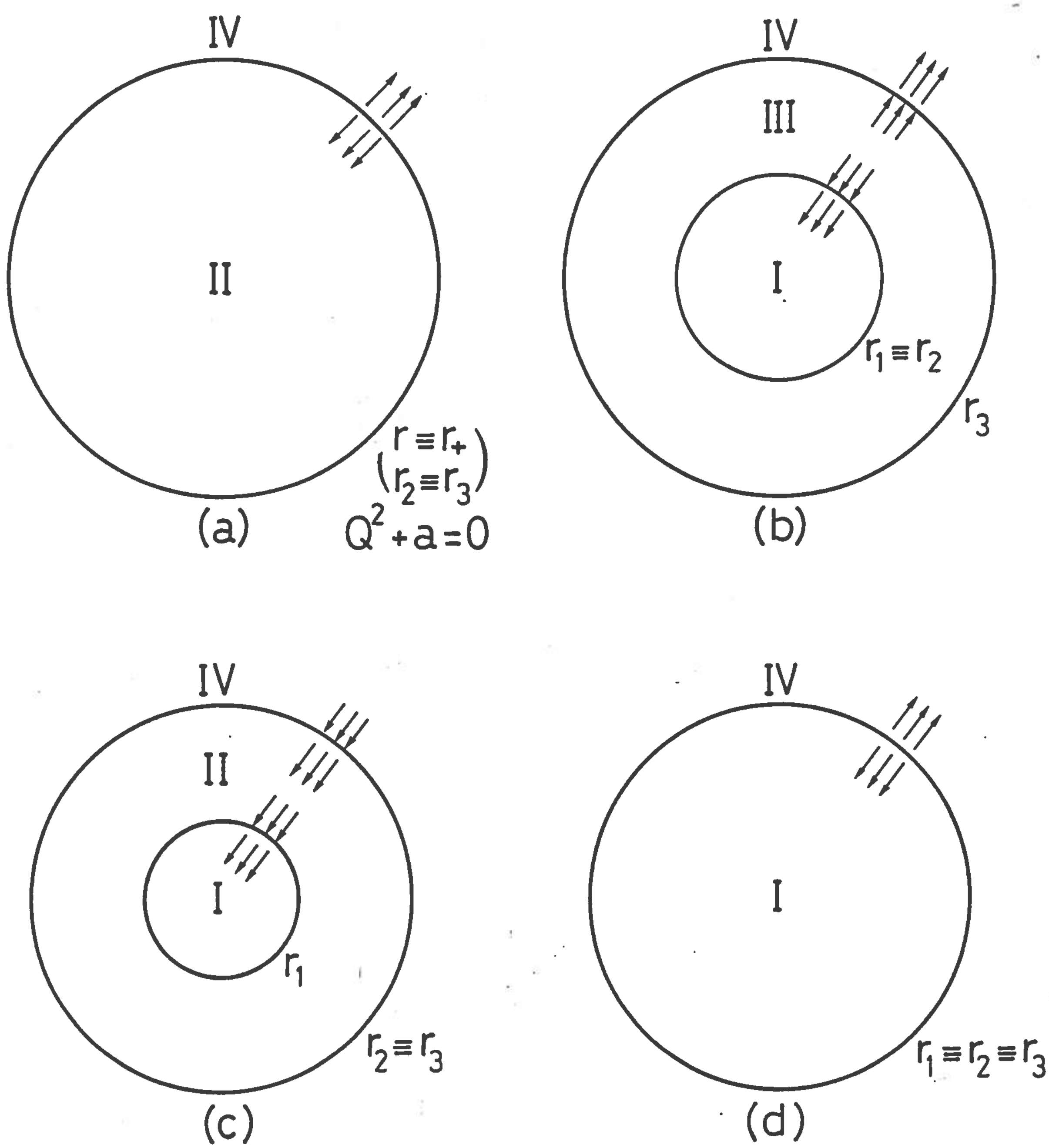


Figure 4: These figures depict the horizons and the available regions for the *stable* BH solutions (and/or cosmological models) considered in this paper. Namely: 4(a) refers to the Schwarzschild– de Sitter solution with vanishing surface gravity (i.e., for $r_- = r_+$); 4(b) and 4(c) refer to the general (Kerr–Newman–de Sitter) solutions for $r_1 = r_2$ and $r_2 = r_3$, respectively. In 4(b), the non–evaporating horizon is the internal, while in (c) is the external one. Fig.4(d) does still refer to the general stationary case, but for $r_1 \equiv r_2 \equiv r_3$. We meet semi–permeables horizons only in Fig.4(b), and (as far the internal horizon is concerned) in Fig.4(c). All the other horizons in Figs. (a), (b), (c), (d) are totally non–permeable. See the text.

5. CHARACTERISTICS OF STABLE BH (= BLACK–HOLES) AND THEIR HORIZONS.

Let us now consider the characteristics of stable BHs in the general (KNdS) case when the source is endowed also with angular momentum (stationary case) and charge. We have at our disposal two equations: (i) the equation $B(r) = 0$ which yields, as before, the radii r_n corresponding to the horizons; (ii) the equation $T = 0$, implying the coincidence of two, or more, (*) radii r_n, which guarantees the horizon stability. Those two equations yield the system:

$$\left.\begin{aligned} -\frac{\Lambda r^4}{3} + (1 - \frac{\Lambda a^2}{3})r^2 - 2mr + a^2 + Q^2 = 0 \\ \\ -2\frac{\Lambda r^3}{3} + (1 - \frac{\Lambda a^2}{3})r - m = 0 \end{aligned}\right\} \tag{11}$$

whose second equation requires the vanishing of the derivative $B'(r)$ in correspondence with the values r_n which satisfy the first equation $[B(r) = 0]$. Such second equation, therefore, ensures the solutions of the system to be double (or triple) "roots" of eq. $B(r) = 0$.

After some algebra, we get explicitly – besides a first equation, yielding the stable BH radii – a second equation providing us with a link among the various parameters m, Λ, a, Q:

$$\left.\begin{aligned} r = \frac{3m\sigma}{E} \\ \\ 9m^2\sigma(\delta\sigma - E) + 2\eta E^2 = 0 \end{aligned}\right\} \tag{12}$$

with $E \equiv 3\delta^2 + 4\Lambda\delta\eta - 18m^2\Lambda$; $\delta \equiv 1 - \Lambda a^2/3$; $\eta \equiv a^2 + Q^2$; $\sigma \equiv \delta^2 - 4\Lambda\eta$.

It is easy to verify that: (i) for $\Lambda = 0$, eqs. (12) reduce to eqs. (7a), (7b); and that (ii) for $\eta \equiv a^2 + Q^2 = 0$, eqs. (12) reduce to eqs. (9), (10).

Eqs. (12) do yield, of course, *both* the stable BH solut dence of r_1, r_2, *and* those resulting from the coincidence of r_2, r_3. second of eqs. (12) can be written as

$$\frac{3m\sigma}{E} = \frac{3m}{2\delta} \pm \sqrt{\frac{9m^2}{4\delta^2} - \frac{2\eta}{\delta}}, \tag{13}$$

from which one can of course construct two independent systems (yielding r_+ and r_-, respectively, as a function of three out of the four remaining parameters m, Λ, a, Q).

Let us consider the two cases separately:

(*) Let us recall that the real positive radii (the roots of eq. (3)) can be at most three; actually they can be one or three. The latter case is of course the only interesting one: and we shall imagine in the following that three out of those (four) roots have actually positive real values, even if our formulas have general validity.

(i) When $r_1 \equiv r_2 = r_-$, region II of Figs. 1 – 3 does disapear and we obtain a *stable* semi–permeable Kerr–Newman–de Sitter BH, similar to the *stable* BH encountered at the end of section **3**, in the particular Kerr–Newman case. In that case, however, the stable BH was surrounded by an asymptotically flat region III; whilst in the present case our stable BH is surrounded by *two* regions (since we are still in presence of a cosmological $r = r_3$ horizon): region III, and a (space–like) region IV, asymptotically de Sitter.

In other words, both the external (III) and the internal (I) regions of the present stable BH are time–like regions, separated just by a semi–permeable membrane. See Fig.4(b). In such regions, however, time flows in opposite directions, in the sense that the light–cones get reversed when corssing the horizon (cf. Fig.2 when eliminating region II, by gluing r_1 to r_2). For the interpretation of this fact see e.g. refs. [8,7].

Let us emphasize that the internal region (I) of our stable BH is not collapsing; any causal observer O_c can live therein without falling into the singularity $r = 0$: that, incidentally, will appear to O_c as a *naked singularity.*

At last, region IV is analogous to the exterior of a de Sitter (cosmological) horizon.

(ii) When $r_2 \equiv r_3 = r_+$, region III of Figs. 1 – 3 does disappear and we obtain a *stable* BH bounded by a *non–permeable* horizon originating from the fusion of a BH–type (r_2) surface and a cosmological–type (r_3) horizon. The stable $r_2 \equiv r_3$ null–surface can be regarded, therefore, both as a BH–membrane and as a cosmological horizon. Outside such a surface, we meet region IV, asymptotically de Sitter. See Fig.4(c). No causal particle can cross the (impermeable) $r_2 \equiv r_3$ surface, either coming from region II, or coming from region IV.

Both the internal (II) and the external (IV) black–hole regions are space–like, since the time–like region III (where causal observers usually live) disappeared. In regions II and IV the light–cones, now, point in the direction of decreasing and increasing r, respectively; no stationary observers can exist therein: region II is collapsing while region IV is expanding.

Inside the $r_2 \equiv r_3$ surface we moreover have at $r = r_1$ a null surface that can be considered the internal BH boundary, so as in the Kerr–Newman (or Kerr) case. In other words, the r_1 (semi–permeable) horizon separates a space–like region II from a time–like region I, so as it occurs in the *interior* of an ordinary Kerr–Newman BH.

6. TOTALLY IMPERMEABLE BHs AND "COSMOSES": THE PARTICULAR CASE OF THE TRIPLE COINCIDENCES.

In the very special case when all the three positive roots of eq. (3) do coincide, i.e. when $r_1 = r_2 = r_3$, we shall meet a stable BH with a single horizon, whose radius takes on a simple analytical expression. Let us write eq. (13), more conveniently, as:

$$r = \frac{3m}{2\delta} \pm \sqrt{\frac{9m^2}{4\delta^2} - \frac{2\eta}{\delta}}$$

and observe that the conditions of triple coincidence (which implies the existence of a single positive solution) requires the vanishing of the square root, i.e. yields the solution:

$$r = \frac{3m}{2\delta} \ , \tag{14}$$

with the two simultaneous Regge–like constraints [$\eta \equiv a^2 + Q^2$]:

$$\begin{cases} m^2 = \dfrac{8}{9}\delta(a^2 + Q^2) \ ; & \text{(14a)} \\ & [\delta \equiv 1 - \dfrac{\Lambda a^2}{3}] \\ m^2 = \dfrac{2}{9}\dfrac{\delta^3}{\Lambda} \ . & \text{(14b)} \end{cases}$$

Eq. (14b) comes from inserting eqs. (14), (14a) in either of eqs. (11).

In the present case, both regions II and III did disapear, and the horizon of our stable BH becames a *totally impermeable* membrane separating a space–like external region (IV) from a time–like internal region (I). Such solution is therefore a (non–permeable) *"time–like black–hole"*. Region IV is asymptotically de Sitter. See Fig.4(d).

Such a BH solution is more conveniently interpretable (like in the cases in sections **4** and **5**(ii)) as a cosmological model: namely, as a model of a totally impermeable (stable) cosmos.

7. A FEW COMMENTS.

Let us stress, first of all, that for stable BHs we got "Regge–like" relations among their mass and angular momentum and/or charge and/or the cosmological constant. For instance, in the case $\Lambda = 0$ we got eq. (7b):

$$m^2 = a^2 + Q^2 \ , \tag{7b}$$

which – when q is negligible – can just be written $M^2 = cJ/G$, that is to say [with $c = G = 1$]:

$$M^2 = J. \tag{7'}$$

On the contrary, when $J = 0$ and q is still negligible, then we meet eq. (10), which can read $M^2 = (c^4/9G^2)\Lambda^{-1}$, or [with $c = G = 1$]:

$$M^2 = \frac{1}{9}\Lambda^{-1}. \tag{10'}$$

In the most general case, the considered relation (among M, J, q, Λ) is involute, and was given by the second one of eqs. (12). In the (simpler) case of section **6**, i.e. of the "triple coincidence", we obtained two such relations, namely eqs. (14a), (14b), which are still complicated. However, if $|\Lambda a^2| << 1$, eqs. (14) yield both

$$m^2 \simeq \frac{8}{9}(a^2 + Q^2) \ , \tag{14'a}$$

to be compared with eqs. (7b), (7'), and [with $c = G = 1$]:

$$M^2 \simeq \frac{2}{9}\Lambda^{-1} \quad , \tag{14'b}$$

to be compared with eq. (10').

The most interesting point is that – with the exception of eqs. (7b), (7') – all such "Regge–like" relations can be attributed to our (stable) cosmological models, i.e., to the stable *"cosmoses"*.

Still with reference to our stable "cosmological objects", particularly worth of notice are those characterized by a totally (two–way) impermeable cosmological horizon: cf. sections **4**, **5**(ii) and **6**. Such solutions may well represent (semi–classically, at least) isolated worlds; whilst the ordinary cosmological models are usually bounded by semi–permeable membranes!

Finally, let us mention that elsewhere we shall apply (and interpret) the results presented in this paper to the case of "strong gravity" theories and "strong BHs": i.e., to the case of hadronic physics.

Acknowledgements: Useful discussions are acknowledged with P.S. Letelier, W.A. Rodrigues Jr., Q.A.G. de Souza, L.A.B. Annes and J.A. Roversi, as well as the collaboration of M. Lourdes S. Silva.

References

[1] See e.g. P. Caldirola, M. Pavšič and E. Recami: Nuovo Cimento *B48* (1978) 205; Phys. Lett. *A66* (1978) 9; Lett. Nuovo Cimento *24* (1979) 565. See also E. Recami and P. Castorina: ibidem, *15* (1976) 357; A. Italiano and E. Recami: ibidem, *40* (1984) 140; E. Recami: Found. Phys. *13* (1983) 341; A. Italiano *et al:* Hadronic Journal *7* (1984) 1321.

[2] E. Recami: Prog. Part. Nucl. Phys. *8* (1982) 401. See also E. Recami in *"Old and New Questions in Physics, Cosmology ..."*, ed. by A. van der Merwe (Plenum; N.Y., 1982), p. 377.

[3] E. Recami, J.M. Martínez and V. Tonin–Zanchin: Prog. Part. Nucl. Phys. *17* (1986) 143; V. Tonin–Zanchin: M. Sc. thesis (UNICAMP, 1987). See also E. Recami and V. Tonin–Zanchin: Phys. Lett. *B177* (1986) 304; *B181* (1986) 416.

[4] R.H. Boyer and R.W. Lindquist: Jour. Math. Phys. *8* (1967) 265.

[5] B. Carter: Comm. Math. Phys. *17* (1970) 233; in *"Les Astres Occlus"* (Gordon and Breach; N.Y., 1973).

[6] See e.g. S.W. Hawking and G.F.R. Ellis: *"The Large Scale Structure of Space–Time"* (Cambridge Univ. Press, 1973), and references therein.

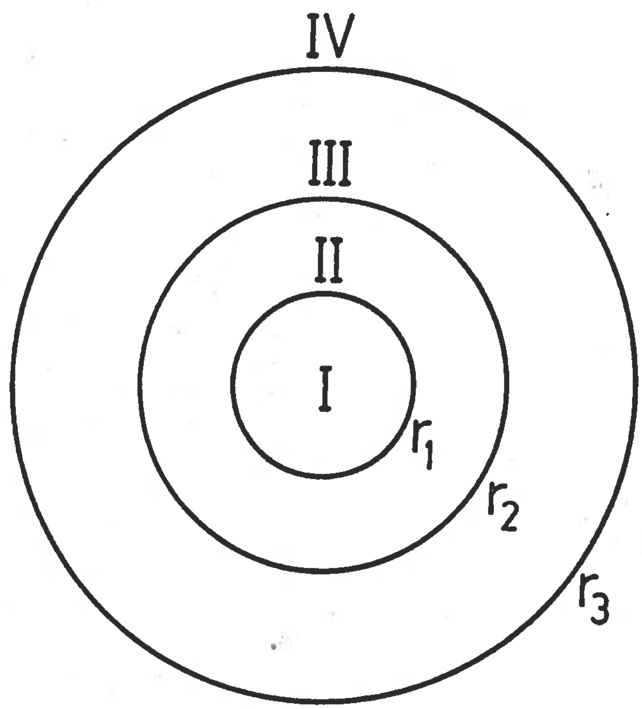
[7] Cf. also A.P. Trofimenko and V.S. Gurin: Pramãna *28* (1987) 379; E. Recami and K.T. Shah: Lett. Nuovo Cimento *24* (1979) 115; E. Recami: Riv. Nuovo Cimento *9* (1986), issue no. 6.

[8] See e.g. E. Recami: Found Phys. *8* (1978) 329; M. Pavšič and E. Recami: Lett. Nuovo Cim. *34* (1982) 357; E. Recami and W.A. Rodrigues Jr.: Found. Phys. *12* (1982) 709; A. Italiano: Hadronic J. *9* (1986) 9.

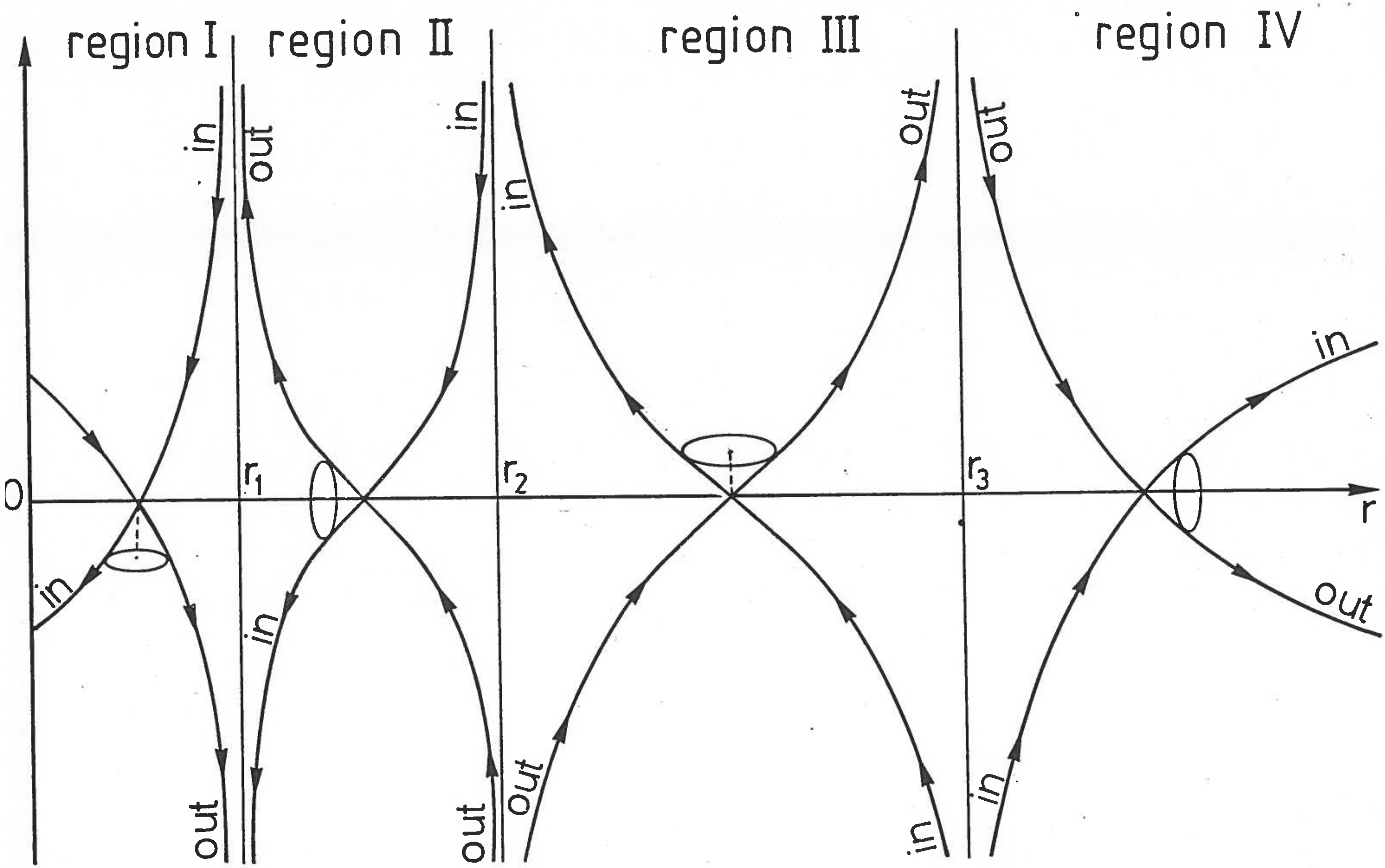
[9] See e.g. J. D. Bekenstein, Phys. Rev. *D9* (1974) 3292; S.W. Hawking; Comm. Math. Phys. *43* (1975) 199.

[10] J.M. Bardeen, B. Carter and S.W. Hawking: Comm. Math. Phys. *31* (1973) 1161.

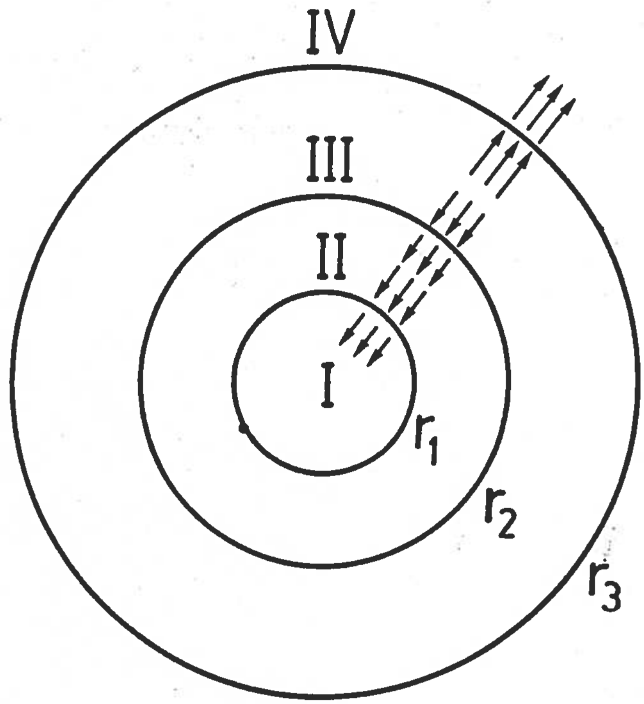
[11] Z. Zheng and G. Yuanxing: *Proc. 3rd Marcel Grossmann Meeting on Gen. Relat.*, ed. by H. Ning (North–Holland; Amsterdam, 1983), p. 1177.



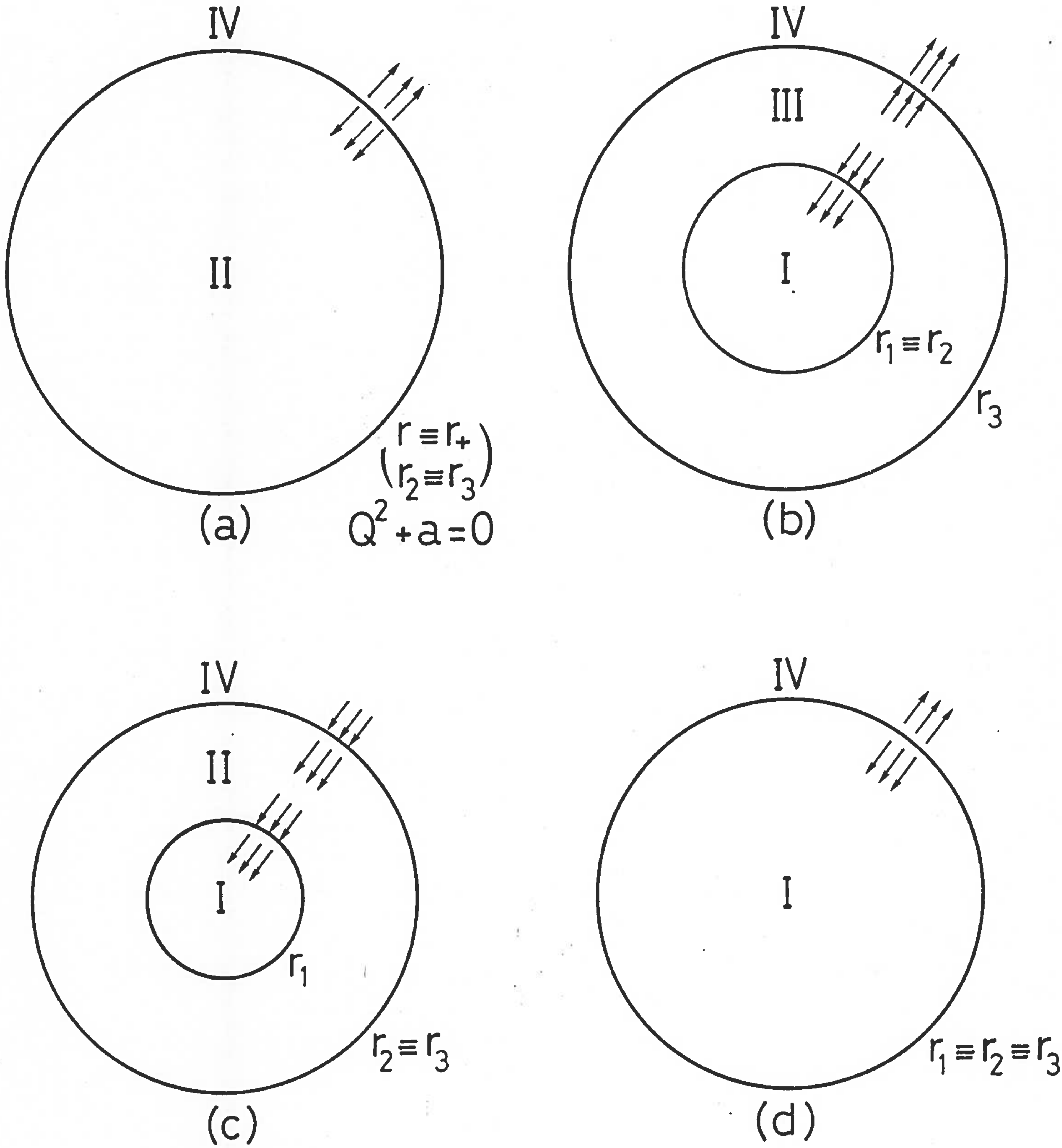
– Fig. 1 –


region I
region II
region III
region IV
in
out
in
out
in
out
in
out
in
out
in
out
in
out
in
out
0
r_1
r_2
r_3
r

- Fig. 2 -



– Fig. 3 –


IV
II
r ≡ r+
(r2 ≡ r3)
(a)
Q2 + a = 0
IV
III
I
r1 ≡ r2
r3
(b)
IV
II
I
r1
r2 ≡ r3
(c)
IV
I
r1 ≡ r2 ≡ r3
(d)

– Figs. 4 –