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FIFTH FORCE, SIXTH FORCE, AND ALL THAT: A THEORETICAL (CLASSICAL) COMMENT. (*)

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ABSTRACT

In the recent literature, a few claims appeared about possible deviations from the ordinary gravitational laws (both at the terrestrial and at the galactic level). The experimental evidence does not seem to be conclusive; nor it is clear if new forces are showing up, or if we have to accept actual deviations from Newton or Einstein gravitation (in the latter case, the validity of the very Equivalence Principle might be on the stage). In such a situation, the attempts by various authors at explaining the "new effects" just *on the basis* of the ordinary theory of General Relativity (for instance, in terms of quantum gravity) can be regarded as logically questionable. In this pedagogically oriented paper, we approach the problem within the classical realm, by exploring whether the possible new effects can be accounted for through *minimal* modifications of the standard formulation of General Relativity: in particular, through exploitation and extension of the rôle of the cosmological constant.

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Introduction – A merit of the seminal paper by Fischbach *et al.* [1] is having triggered a series of delicate experiments [2] aimed – purposedly – at testing the exact validity of Newton and Einstein gravitational theories at the ordinary macroscopic scale. [3]

The experimental results, however, are still rather contradictory and confusing [4], and - for instance - there is no conclusive evidence about the existence of a fifth [1] or a sixth [5] force.

Even more, if new forces are really showing up, it is not clear [6] whether they are to be admitted into the restricted club of the fundamental forces. A priori, as the ordinary forces are associated with the strong, electric, weak and gravitational "charges", respectively, so new force-fields can correspond to the other known additive charges: baryonic charge, leptonic charge, strong flavor charges (strangeness, charm, etc.). But the new possible experimental effects might be due to deviations from the ordinary gravitational laws. In such a case, the very validity of the Equivalence Principle could be jeopardized.

In such an unclear situation [6], it appears to be dangerous trying to explain the "new effects" *on the basis* of the *ordinary* theory of General Relativity (for instance, in terms of quantum gravity). [7]

In this pedagogically oriented paper, we want just to explore whether, and how, one can account (at least a priori) for the possible new experimental evidence without abandoning the classical realm: and, namely, by modifying as little as possible the standard formulation of General Relativity (GR). The most natural path is exploiting, and extending, the rôle of the cosmological constant Λ which enters Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\rho}{}_{\rho} + \Lambda g_{\mu\nu} = -K T_{\mu\nu} \; ; \quad K \equiv \frac{8\pi G}{c^4} \tag{1}$$

In so doing, we shall not forget that possible deviations from the Newton law have been already invoked also at the galactic (and hyper-galactic) scale [8]; even if the experimental data, at those scales, are presently even less conclusive.

Exploiting the rôle of the cosmological term: a discrete-valued Λ - Let us repeat that our approach, being largely didactic in its aim, does not call for being taken too seriously.

In eqs.(1) the ordinary value Λ_0 attributed to Λ (i.e., its value at the cosmological scale) is $|\Lambda_0| \simeq (10^{26}\text{m})^{-2} \equiv 10^{-52}\text{m}^{-2}$. Let us recall that, e.g., in a de Sitter cosmological model, $\Lambda_0 = 3/R^2$, quantity R being the cosmos radius.

Such a tiny value plays an actual rôle only for very large (cosmological) distances. It cannot influence, therefore, the physics at the galactic, at the terrestrial or at the ordinary macroscopic scale.

Within the hierarchical-type theories [9], however, Λ can be regarded (since $1/\sqrt{\Lambda}$ does assentially constitute a "fundamental length") as assuming different values at the different physical scales. It is already known, for instance, that at the level of hadrons and strong interactions [10] it is $\Lambda_h \simeq (10^{41})^2 \Lambda_0 \simeq 10^{30}\text{m}^{-2} \simeq (1\text{ fm})^{-2}$. In fact - by making

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recourse at this point to Mandelbrot's language [(11)] and to his general equation for the self-similar structures - we may regard our cosmos and hadrons as constituting systems of scale n and $n-1$, respectively, with fractal dimension $D = 2$(quantity D being the self-similarity exponent, which does characterize the hierarchy). This led, incidentally, to the construction of a unified geometrical theory of gravitational and strong interactions.[(12)]

Following e.g. Oldershaw [(13)], let us assume that Λ can take on, in our cosmos, a series of discrete values Λ_n, with $|\Lambda_n| \simeq (1/R_n^2)$, quantity R_n being the fundamental length of the physics considered. On the contrary, we assume quantity G to be everywhere constant inside our cosmos.[*1] At the n-th level, in the simple case of a static body with mass M and of Schwarzschild-type coordinates, eqs. (1) yield in the vacuum the metric coefficient

$$g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda_n r^2}{3}\,, \tag{2}$$

which in the weak field approximation corresponds to the gravitational potential $V \equiv \frac{c^2}{2}(g_{00} - 1) = -\frac{GM}{r} - \frac{\Lambda_n c^2 r^2}{6}$ and therefore to the gravitational force

$$F = -\frac{GmM}{r^2} + \frac{c^2 \Lambda_n m}{3} r. \tag{2'}$$

For $\Lambda_n > 0$ $(\Lambda_n < 0)$ this force does represent – besides the Newton term – a repulsive (attractive) force.

Alternatively, without assuming the field to be weak (but still assuming small velocities, $|v| << c$), from the geodesic equation one derives

$$F = -\frac{GmM}{r^2} - \frac{GmM\Lambda_n}{3} + \frac{c^2 m \Lambda_n}{3} r - \frac{c^2 m {\Lambda_n}^2}{9} r^3. \tag{3}$$

Eqs. (2'), (3) could be interesting at the galactic or hyper-galactic level. For instance, eq.(2') with $\Lambda_n < 0$ would cause a galaxy to rotate almost rigidly, since its last term leads to a constant angular frequency ω. In the case of spiral galaxies, for example, eqs. (2'),(3) could explain the stability in time of the spirals; without any odd assumption about dark matter distribution over the whole galaxy.

Those equations, however, do not seem to be suited to represent intermediate-range $(r \approx 10^2 \div 10^3\text{m})$ forces, since the terms added to Newton's increase with the distance (unless unprobable cancellations take place in the r.h.s. of eq. (3)). A way out can be found – however – by a local redefinition of the vacuum. In fact, let us linearize eq.(1), by putting $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$:

$$\frac{1}{2}\Box h_{\mu\nu} - \Lambda_n h_{\mu\nu} = -K(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\rho}{}_{\rho}) + \Lambda_n \eta_{\mu\nu}. \tag{1'}$$

In the vacuum, we can assume the whole r.h.s. of eq.(1') to vanish; this corresponds to assuming (via a Higgs-type mechanism) that in the vacuum the whole tensor $\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} - \Lambda_n \eta_{\mu\nu}/K$ (and not $T_{\mu\nu}$) does vanish. Under such a hypothesis we obtain, for distances of the order of $r \approx 10^2 \div 10^3\text{m}$ and in the static limit $(|v| << c)$, the equation $\nabla^2 h_{00} + 2\Lambda_n h_{00} = 0$, which yields a potential of the Yukawa-Nernst type:

$$V \equiv \frac{c^2}{2} h_{00} \simeq -\frac{GM}{r} exp[-r/r_n] \; ; \quad r_n^{\;-1} \equiv \sqrt{-2\Lambda_n} \; , \tag{4}$$

which is real when $\Lambda_n < 0$. Eq.(4) corresponds to the gravitational force

$$F \simeq -\frac{GmM}{r}\left(\frac{1}{r} + \frac{1}{r_n}\right) exp[-r/r_n], \tag{4'}$$

which, by expansion

$$F \simeq -\frac{GmM}{r^2}[1 - \frac{r^2}{2r_n^{\;2}} + \frac{r^3}{3r_n^{\;3}} - \ldots] \; , \tag{5}$$

can a priori account for both a repulsive correction ("fifth force") and an attractive correction ("sixth force") to the Newton force. Let us repeat that eq.(5) is expected to hold only for $r \approx 10^2 \div 10^3$m and $r < r_n$. To fit the experimental data, it is needed for r_n a value of the order of $r_n \approx 10$ km, which does correspond to the value $|\Lambda_n| \simeq (10^{22})^2 \Lambda_0 \simeq 10^{-8}\text{m}^{-2}$.

Of course, when passing to the solar-system level, Λ_n has to take on a much smaller value, compatible with the well-verified validity - therein - of the Newton law.

May Λ vary continuously with the distance? - A possible weak point of the Oldershaw-type hierarchical theories (in which Λ can assume, inside our cosmos, a set of discrete values Λ_n) is that the range Δr_n over which the value Λ_n applies is not well defined. It is tempting, therefore, to check the consequence of assuming Λ to vary continuously with r. To preserve the general covariance, Λ has to be a scalar function of the coordinates. It must be noticed, however, that (if G is constant, and Λ is a scalar function of r) a non-constant Λ implies that $-KT^{\mu\nu}{}_{;\nu} = g^{\mu\nu}\Lambda_{,\nu} \neq 0$, where ";" and "," represent the covariant derivative and the ordinary derivative, respectively. On the contrary, if G too (as well as Λ) is allowed to be a scalar function of the coordinates [10,12], then we get $g^{\mu\nu}\Lambda_{,\nu} = -kG_{,\nu}T^{\mu\nu} - kGT^{\mu\nu}{}_{;\nu}$, where $k \equiv 8\pi/c^4$, in which case it may well be $KT^{\mu\nu}{}_{;\nu} = 0$.

The most natural [12,10] choice would be $\Lambda = \Lambda(r) = \pm C/r^2$, with C a dimensionless, positive constant of the order of unity (e.g., $C = 3$). However, such a choice does not lead to an interesting potential. More interesting it seems to be the case $\Lambda(r) = C\dfrac{exp[-\alpha r]}{r^2}$, with α a positive (dimensional) constant. In fact, in this case one finds $g_{00} = 1 - 2GM/(c^2 r) + C\; exp[-\alpha r] \;/\; (\alpha r)$, which corresponds to the potential

$$V = -\frac{GM}{r}(1 + A\, e^{-\alpha r}) \; ; \quad A \equiv -\frac{c^2 C}{2\alpha GM} \; , \tag{6}$$

that, incidentally, has the same form of the Fischback formula [1]. The present choice does obviously guarantees the validity of the Newton law at the solar system scale, as well as for larger systems.

There look interesting also the choices $\Lambda(r) = D/r$ and $\Lambda(r) = (D/r) \cdot exp[-\alpha r]$, which yield $g_{00} = 1 - 2GM/(c^2 r) - Dr/2$ and $g_{00} = 1 - 2GM/(c^2 r) + \frac{D}{\alpha}(exp[-\alpha r] + \dfrac{exp[-\alpha r]}{\alpha r})$, respectively.

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Footnote

*1 We are assuming, therefore, that G varies with the scale-size (even if less drastically than Λ; i.e. $G_n = G_0R_0/R_n$, so as predicted by the hierarchical-type theories [9–13]), *only* when passing from a (gravitational) cosmos to a (hadronic) micro-cosmos [10,12]. In conclusion, we discuss in this paper only the possible effects of a varying Λ.

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