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POSSIBLE COSMOLOGICAL CONSEQUENCES OF THERMODYNAMICS IN A UNIFIED APPROACH TO GRAVITATIONAL AND STRONG INTERACTIONS (*)

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<<Matter possesses a coesion that keeps it together and against which the vacuum is impotent. Actually, the material world is supported by an immense force, and alternately it contracts and expands following its physical trasmutations, now consumed by fire, now giving rise to a new cosmos creation>>.

Plutarcus (~135 ÷ 51 B.C.).

ABSTRACT

A unified geometrical approach to strong and gravitational interactions has been recently proposed, based on the classical methods of General Relativity. According to it, hadrons can be regarded as "black-hole type" solutions of new field equations describing *two* tensorial metric -fields (the ordinary gravitational field, and the "strong" one). In this paper, we first seize the opportunity for an improved exposition of some elements of the theory relevant to our present scope. Secondly, by extending the Bekenstein-Hawking thermodynamics to the abovementioned "strong black-holes" (SBH), we show: (i) that SBH thermodynamics seems to require a new expansion of our cosmos after its *Big Crunch* (i.e., that a recontraction of our cosmos has to be followed by a new "creation"); (ii) that a collapsing star with mass M approximately in the range 3 to 5 solar masses, once reached the neutron-star density, could re-explode *tending* to form a (radiating) object with a diameter of the order of 1 light-day: thus failing to create a gavitational black-hole.

KEYWORDS

Unified theories; classical "strong gravity"; strong interactions; gravitational interactions; General Relativity; black-hole thermodynamics; "strong black-holes"; Big Bang; collapsing stars; neutron stars; cosmology; "Big Crunch".

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1. INTRODUCTION

Inspired by the Weyl-Klein-Eddington-Dirac "large numbers coincidences", we proposed a few years ago a unified approach to strong and gravitational interactions (for an extended summary of it see Recami, 1982a,b), that happens to be *formally* analogous to N.Rosen's (1979, 1980a,b) *bi-metric General Relativity theory*, and that —even if geometrical, i.e. based on the classical methods of General Relativity (GR)— yields results similar to the "*strong gravity*" ones by Salam and coworkers (Salam, 1978; Salam & Strathdee, 1978).

Our geometrical approach is twofold:

(i) on one side, it tries to describe the strong field inside a hadron just by means of "scaled down" Einstein equations , with attractive cosmological term (Recami & Castorina, 1976; Caldirola, Pavsic & Recami, 1978a,b; Recami, 1983, 1984). One thus naturally gets the *confinement* of the hadron constituents, and e.g. is allowed to calculate the meson mass-spectra (Recami, 1983; Italiano & Recami, 1984; Italiano, Lattuada, Maccarrone, Recami, Riggi & Vinciguerra, 1984 and work in progress in coll. also with L.A.B.Annes, P.Ammiraju, J.S. Roversi and W.A.Rodrigues);

(ii) on the other hand, one is led to describe the strong field surrounding a hadron as a new tensorial field, to be "added" to the gravitational one: so that a *bi-scale theory* arises (Caldirola & Recami, 1979; Ammiraju, Recami & Rodrigues, 1983), in which the total metric $g_{\mu\nu}$ is the superposition of the ordinary gravitational metric-field $g_{\mu\nu}(\text{Grav.}) \equiv e_{\mu\nu}$ and of the "strong" metric-field $s_{\mu\nu}$.

Here we shall stick to question (ii) only.

2. THEORETICAL FRAMEWORK

1) The strong field surrounding a hadron is a (second) tensorial field $s_{\mu\nu}$ such that $s_{\mu\nu} \to 0$ for $r \gg 1$ fm. All tensorial indices, incidentally, are raised (lowered) by the *total* metric-tensor $g^{\mu\nu}(g_{\mu\nu})$.

2) Hadrons possess, besides the ordinary (gravitational) mass m, a "strong mass" g (or *strong charge*) which *directly* affects (deforms) space-time via the constant $N \equiv \rho^{-1}G$, where ρ^{-1} is the ratio of the strong interaction strength S to the gravitational interaction strength s:

3) $\rho^{-1} \equiv S/s \equiv (Ng^2/\hbar c)\cdot(Gm^2/\hbar c)^{-1} \simeq 10^{41}$,
and where, by "coincidence", ρ is approximately equal to the ratio of the typical hadron radius r(h) to the Hubble radius R(U) of our cosmos. If, e.g., we put conventionally $g = m$, then $N \simeq 10^{41}G \simeq hc/m_\pi^2$.

4) *Far* from a source-hadron, $g_{\mu\nu} = e_{\mu\nu}$, and both hadrons and non-hadrons will feel only $e_{\mu\nu}$: So that our approach does correctly yield the theory of GR.

5) Near a source-hadron, the test-hadrons will feel *both* $e_{\mu\nu}$ and $s_{\mu\nu}$. For a test-hadron endowed with mass m' and strong-mass g' in the surroundings of a source-hadron, the simplest field equations would be

$$R_{\mu\nu} + \Lambda g_{\mu\nu} + \lambda s_{\mu\nu} = -8\pi c^{-4}\,(GT_{\mu\nu} + S_{\mu\nu} - \tfrac{1}{2}Gg_{\mu\nu}T^{\rho}_{\ \rho} - \tfrac{1}{2}g_{\mu\nu}S^{\rho}_{\ \rho}).$$

Disregarding the terms negligible in the neibourhood of the source-hadron, we end up with the *field equations* $[\mu,\nu = 0,1,2,3]$:

$$R_{\mu\nu} + \lambda s_{\mu\nu} \simeq -\,8\pi c^{-4}\,(S_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}S^{\rho}_{\ \rho}) \qquad (1)$$

with

$$\lambda \equiv \rho^{-2}\Lambda; \quad S_{\mu\nu} \equiv NT_{\mu\nu}; \quad N \equiv \rho^{-1}G, \qquad (1')$$

where: $S_{\mu\nu}$ is the "strong-matter tensor", specifying the strong-matter discribution associated with the source-hadron (just as $T_{\mu\nu}$ is the "ordinary-matter tensor"); G is the ordinary gravitation universal constant; Λ is the ordinary cosmological constant ($|\Lambda| \simeq 10^{-56}\text{cm}^{-2}$); and —as already mentioned— by "coincidence" $\rho \equiv s/S \simeq r(h)\,/\,R(U) \simeq 10^{-15}/10^{26} = 10^{-41}$. Quantity λ can be called the "strong (or hadronic) cosmological constant". By assuming spherical symmetry (and a static source, i.e. $\partial g_{ik}/\partial x_o = 0$; $g_{ok} = 0$; $[i,k = 1,2,3]$), the *total* metric acting on the test-hadron —in the surroundings of the source-hadron— writes in polar coordinates, as usual,

$$ds^2 = g_{oo}dt^2 - g_{11}dr^2 - r^2(d\theta^2 + \sin^2\theta \cdot d\phi^2) .$$

5) *Very* near the source-hadron, we may assume that $s_{\mu\nu} \to g_{\mu\nu}$. This condition can be useful when looking for explicit solutions of our new field-equations.

6) On the contrary, sufficiently far from the source hadron (when our field equations can be linearized), we may write

$$g_{\mu\nu} \simeq e_{\mu\nu} + s_{\mu\nu} , \qquad (2)$$

that is to say, in suitable coordinates,

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu} ; \qquad [\eta_{\mu\nu} \equiv \mathrm{diag}(+1,-1,-1,-1)] . \qquad (2')$$

7) At last, let us recall that —as we saw under point 4)— very far from the source-hadron our field equations (1) must imply that $g_{\mu\nu} \to \eta_{\mu\nu}$ when $r \to \infty$.

3. WEAK FIELD APPROXIMATION (Linear Approximation)

In the "weak-field approximation", we can assume eqs.(2') to hold: $g_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu}$. Then, if $\partial_\alpha\partial^\alpha \equiv \Box \equiv {}_{,\rho\sigma}\eta^{\rho\sigma}$ is the D'Alambertian operator, where the *comma* represents the ordinary (partial) derivative, it follows that

$$R_{\mu\nu} \simeq \tfrac{1}{2}\partial_\alpha\partial^\alpha s_{\mu\nu}$$

$$\partial_\alpha\partial^\alpha s_{\mu\nu} \equiv s_{\mu\nu,\rho\sigma}\eta^{\rho\sigma} = [(\partial^2/\partial t^2) - \nabla^2]\, s_{\mu\nu}$$

with the conditions $(s_\mu{}^\nu - \tfrac{1}{2}s^\rho{}_\rho\delta_\mu{}^\nu)_{,\nu} = 0$; *so that our field equations* (1) *yield*

$$\partial_\alpha\partial^\alpha s_{\mu\nu} + 2\lambda s_{\mu\nu} = -16\pi c^{-4}\,(S_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}S^\rho{}_\rho) . \qquad (3)$$

For a source-hadron at rest, at the origin of the reference frame, in the *static* case (with zero pressure-terms), —i.e., when $\partial_\alpha\partial^\alpha s_{\mu\nu} = -\nabla^2 s_{\mu\nu}$—, in polar coordinates it is $S_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}S^\rho{}_\rho =$ $= \mathrm{diag}\,\tfrac{1}{2}c^2\delta_S(c^2,1,r^2,r^2\sin^2\theta)$; and in Cartesian coordinates:

$$S_{\mu\nu} = \mathrm{diag}\,(c^4\delta_S,0,0,0) ;$$

$$S_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}S^\rho{}_\rho = \mathrm{diag}\,\tfrac{1}{2}c^2\delta_S(c^2,1,1,1) ,$$

with $\delta_S = N\gamma$ being the strong-matter density. So that eq.(3) reduces to

$$\nabla s_{oo} - 2\lambda s_{oo} = 8\pi N\gamma . \qquad (4)$$

For a point-like *nucleon* (for instance) at the origin, s_{oo} must be a function only of $r =$ $= (x^2 + y^2 + z^2)^{\frac{1}{2}}$, and moreover $\gamma = m\delta(r)$; $\delta_S = Mn\delta(r) \equiv g\delta(r)$; notice that $\delta(r)$ is nothing but the Dirac delta; we get, therefore,

$$s_{oo} = 2Nm(\exp\{-\sqrt{2\lambda}\,r\})/r = 2g\,\frac{\exp\{-\sqrt{2\lambda}\,r\}}{r} . \qquad [g \equiv Nm]$$

By identifying $\sqrt{2\lambda} \equiv m_S c/\hbar$, we can *calculate*

$$m_S = \hbar\sqrt{2\lambda}/c \simeq m_\pi . \qquad (5)$$

In conclusion, by linearizing our field eqs.(1), at the static limit (when the *tensorial* strong potential goes into a *scalar* strong potential) one obtains the CORRECT YUKAWIAN BEHAVIOUR:

$$V \equiv \tfrac{1}{2}(g_{00} - 1) = -g \frac{e^{-m_\pi rc/\hbar}}{r} , \tag{6}$$

where we had to remember that (for weak fields and not too high speeds) $g_{00} \equiv 1 + 2V$.

Still in the weak field approximation, when eq.(2') holds: $g_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu}$, but relaxing the condition $v \ll c$, it is easy to verify that (in Cartesian coordinates) *all* the components $s_{\mu\nu} \to 0$ for $r \gg 1$ fm. And in polar coordinates we simply get that $s_{11} = s_{00}/c^2$, so that *both* s_{00} and s_{11} are *exponentially damped*.

4. THE SCHWARZSCHILD-TYPE (spherically symmetric) PROBLEM FOR THE NEW FIELD-EQUATIONS

Let us eventually *abandon* the weak-field approximation; but still consider the source-hadron to be a *spherically symmetric distribution* of strong-matter. In polar coordinates, then, let us explicitly put $ds^2 = \exp\{\nu(r)\}dt^2 - \exp\{-\lambda(r)\}dr^2 - d\Omega$, that is to say

$$g_{\mu\nu} = (e^{\nu(r)}, -e^{\lambda(r)}, -r^2, -r^2\sin^2\theta) , \tag{7}$$

where $\nu=\nu(r)$ and $\lambda=\lambda(r)$ are functions still *to be determined*.

We *cannot* make recourse any longer to approximations (2), and finding out exact solutions of our field eqs.(1) is a difficult task.

We shall therefore use the trick of following the procedures usually adopted (in connection with the ordinary Einstein equations) when in presence of an electric charge *and field*, besides a mass. We purport to follow an iterative procedure; and, as the starting point, we take from what precedes the model of a static strong-field source, whose field at the zero-order approximation is Yukawian:

$$V = -g \frac{e^{-m_0 r}}{r} . \qquad \left[m_0 \equiv m_\pi c/\hbar\right]$$

Let us define an energy-density $u=u(r)$ for the Yukawa potential (by following, however, our philosophy):

$$W = \tfrac{1}{2}\iint \delta(x) \frac{\delta(x')}{|x-x'|} e^{-m_0|x-x'|} d^3x \, d^3x' = \tfrac{1}{2}\int \delta(x)\, V(x)\, d^3x , \tag{8}$$

where $V(x) = \int \delta(x') \cdot \exp\left[-m_0|x-x'|\right] \cdot |x-x'|^{-1} d^3x'$ is the Yukawa potential. Therefore:

$$\nabla^2 V(x) = m_0^2\, V(x) - 4\pi\delta(x) ,$$

and eq.(8) becomes

$$W = \tfrac{1}{2}\int \left[-\nabla^2 V(x) + m_0^2\, V(x)\right] \cdot \frac{V(x)}{4\pi} d^3x = \frac{1}{8\pi}\int \left[|\nabla V|^2 + m_0^2\, |V|^2\right] d^3x ,$$

so that

$$u(r) = \frac{1}{8\pi}\left[|\nabla V|^2 + m_0^2\, |V|^2\right] = \frac{1}{8\pi} g^2 \frac{e^{-m_0 r}}{r} (r^{-2} + 2m_0 r^{-1} + 2m_0^2) . \tag{9}$$

Formally, we are dealing with a *strong*-energy tensor $t_{\mu\nu} \equiv \mathrm{diag}(u(r),0,0,0)$, and with the "Einstein" equations:

$$R_\mu^{\ \nu} - \tfrac{1}{2}\delta_\mu^{\ \nu} R^\rho_{\ \rho} = -\frac{8\pi}{c^4} t_\mu^{\ \nu} , \qquad \text{(in the vacuum)},$$

which become

$$\begin{cases} -8\pi c^{-4}\, u(r) = -\dfrac{e^{-\lambda}}{r}\lambda' + \dfrac{e^{-\lambda}}{r^2} - \dfrac{1}{r^2} ; \\[2ex] 0 = \dfrac{e^{-\lambda}}{r}\nu' + \dfrac{e^{-\lambda}}{r^2} - \dfrac{1}{r^2} . \end{cases} \qquad \left[\,' \equiv \frac{d}{dr}\right] \tag{10}$$

The *first one* of eqs.(10) is exactly solvable: if we set $\exp\{-\lambda\} \equiv y$, it writes

$$y'r + y = 1 - \alpha^2 e^{-2m_o r}(r^{-2} + 2m_o r^{-1} + 2m_o^2)\,, \qquad [\alpha^2 \equiv g^2/c^4]$$

whose solution is

$$e^{-\lambda(r)} \equiv y = 1 - \frac{2\ell}{r} + \frac{m_o}{r}\alpha^2 e^{-2m_o r} + \frac{\alpha^2}{r^2} e^{-2m_o r}\,, \tag{11}$$

ℓ being an integration constant related to the value of the strong-charge g of the considered source-hadron.

The *second one* of eqs.(10) can be written:

$$\nu' = \frac{1}{r}\left(\frac{1}{y} - 1\right)\,. \tag{12}$$

By recalling eq.(7), in the present "strong Schwarzschild problem" the *strong Schwarzschild radii* will be given by the conditions

$$\exp\{-\lambda(r)\} = 0; \quad \exp\{\nu(r)\} = 0\,. \tag{13}$$

The first one of eqs.(13), *i.e.* $\exp\{\lambda(r)\} = \infty$, when associated with eq.(11), does yield *two* "strong Schwarzschild radii", r_1 and r_2. In the case when the source-hadron is a nucleon, we get

$$r_1 \simeq 1 \text{ fm}\,; \quad r_2 \simeq 10^{-2} \text{ fm}\,. \tag{14}$$

In general:

$$r_1 \simeq 2\ell\,; \quad r_2 \ll r_1\,.$$

From eq.(12), not exactly solvable, we can for instance *verify* that for $r = r_1$

$$\exp\{\nu(r_1)\} \simeq \exp\{-\lambda(r_1)\}\,.$$

5. APPLICATIONS

For a possible application to the E > 100 TeV cosmic-ray phenomenology, we refer to Ammiraju, Recami & Rodrigues (1983). That phenomenology seems to require the existence of a few "fireballs", whose decay-modes —called *Centeuros, Geminions, Chirons,...*— have been tentatively associated by the abovementioned authors with the "evaporation" of some exotic "strong black-holes".

Here let us only recall the following. Just as black bodies (defined as objects absorbing all electromagnetic radiation impinging on them) have been discovered to emit a characteristic, thermodinamical spectrum of photons, so the gravitational *black-holes* (BH) —which are objects absorbing all kinds of particles impinging on them— are expected to emit characteristic, Planck-type spectra of every kind of particles (Bekenstein, 1973,1974; Hawking, 1976). Analogously, our *strong black-holes* (SBH) —which will absorb and confine only particles sensitive to the strong field— are expected to emit characteristic spectra of hadrons only (Recami 1979,1982b).

6. OUR SPACE-TIME MANIFOLD AND STRONG BLACK-HOLE (SBH) THERMODYNAMICS

The results met in Sect.3 and Sect.4 allow us to associate hadrons with the "SBH" solutions of our (new) field equations (1); solutions related to the presence of the strong-gravity field $s_{\mu\nu}$ (besides $e_{\mu\nu}$). Such SBHs will *confine* — let us repeat it— only objects carrying a strong charge: i.e., only hadronic particles.

Let us recall, for instance, that on the basis of our eqs.(1) we derived: (a) that $s_{\mu\nu}$ vanish for r larger than a few fm ; (b) the correct Yukawa behaviour (under the suitable hypotheses) of the *scalar* strong potential; (c) the fact that the external "strong Schwarzschild radius" for ordinary particles is of the order of 1 fm. Moreover, we can geometrize also the strong field since the *inertia* m_I of the test-hadron in the neighbourhood of the source-hadron, when it is subjected to the strong field, coincides (cf. e.g. Caldirola et al.,1978a) with its *strong*

-*mass* g', and *not* with its gravitational mass m', so that $m_I \equiv g'$; and, in the special relativistic limit, it is for example $g' = g'_o/\sqrt{1-v^2}$, quantity g'_o being the *rest* strong-mass. At last, let us remind the reader that in the present bi-scale approach two kinds of particles are taken into consideration: the ones —carrying ordinary gravitational mass, i.e. with intrinsic scale-factor $\kappa=1$ — that are subjected (only) to the gravitational metric $e_{\mu\nu}$; and the ones —carrying a strong-charge or "strong mass", i.e. with intrinsics scale-factor $\kappa=\rho\simeq 10^{-41}$— that are subjected also to the strong metric $s_{\mu\nu}$ (the latter being of course the hadrons).

We already mentioned elsewhere (Caldirola et al.,1978a; Recami, 1979,1982b) that a fashinating consequence would follow by assuming the Bekenstein-Hawking ordinary BH thermodynamics to hold good also for our SBHs. Let us approach the question more closely.

Since in the surroundings of a hadron the cosmological constant Λ plays no role, and g_{oo}, g_{11} tend to 1, -1, respectively, for $r \gg 1$ fm, we can assume the space-time manifold V —associated with our field equations (1)— to be: (i) *asymptotically flat*, such a property implying also the *positivity* of energy (see e.g. Choquet-Bruhat, 1984); (ii) and moreover (see e.g. Hawking & Ellis, 1973) time-orientable and Lorentzian (= pseudo-Riemannian), with a metric $g_{\mu\nu}$ of Lorentz signature -2 and satisfying the two ordinary postulates of: (iii) local causality, and (iv) *local* conservation of energy-momentum.

As to our field eqs.(1), we first notice that the present approach implies gravity and strong-gravity to be attractive for positive matter and strong-matter densities, respectively. Secondly, we explicitly assume (see e.g. Misner, Thorn & Wheeler, 1973) that: (v) quantity $R_{\mu\nu}K^{\mu}K^{\nu}$ is non-negative for any non-spacelike Killing vector K^{μ}. Notice, incidentally, that all the above "assumptions" (i)÷(v) do agree with our physical model (even if the question, whether the form actually chosen for our field equations is *fully* compatible with them, is still *partially* open).

All the ordinary theorems on singularities (Hawking & Ellis,1973), then, hold good also for our manifold V. Let us deal, in particular, with the Penrose theorem (1968) and the Second Law of BH Thermodynamics (Hawking, 1971,1972,1973; Bekenstein, 1973,1974), in order to be able to draw briefly the most relevant consequences.

Let us consider, e.g., a SBH solution of our field equations in the static case, with its strong horizon (Ammiraju et al.,1983, and references therein). Always with reference to objects endowed with strong-mass, such solutions will present at least one asymptotically flat region, or "external universe", possessing a timelike (strong) future infinity I^+_s and past infinity I^-_s, a spacelike (strong) infinity I^o_s, and a null (strong) future infinity $\mathcal{J}^+_s$ and past infinity $\mathcal{J}^-_s$. The strong-horizon divides of course the external universe (which can send hadronic particles —"strong signals"— out to $\mathcal{J}^+_s$) from the SBH interior (which cannot). Let us define the union of the future SBH horizons as the *boundary* $B \equiv \dot{J}^-_s(\mathcal{J}^+_s)$ of the causal past —via strong-signals— of the future null strong-infinity (i.e., of the domain $J^-_s(\mathcal{J}^+_s)$ that can send future-directed causal strong-curves to future null strong-infinity).

From assumptions (i)÷(iv) above, it is then straightforward to generalize the Penrose (1968) theorem on the structure of the future strong-horizons; i.e., to conclude that B is generated by null strong-geodesics which do not have future end-points.(1) Let us clarify that the strong-geodesics are are the ones followed by the hadronic test-particles, i.e. by particles affected

(1) Let us recall more specifically that Penrose's theorem states that: (a) the "generators" of B are defined as null strong-geodesics that lie in B, at least for some finite interval of the affine parameter; (b) a generator, followed into the past, may leave B entering $J^-_s(\mathcal{J}^+_s)$; (c) once a generator, followed into the future, enters the boundary B of $J^-_s(\mathcal{J}^+_s)$ at point ^sC (a "caustic" of B), it'll never be able thereafter to leave B, nor to intersect another generator (the generators can intersect each other only at the caustics); (d) only *one* generator (apart from a normalization of the geodesic affine-parameter) can pass through a non-caustic event.

(2) Remember that the global space-time dilations, considered by us (cf. e.g. Caldirola et al., 1978a), do *not* affect the speeds, in particular do not affect the value of the light-speed c.

by the strong metric $s_{\mu\nu}$ besides the gravitational metric $e_{\mu\nu}$ (negligible in the vicinity of the source hadron). The *null* strong-geodesics, therefore, are geometrically well defined, even if test-hadrons would not exist travelling exactly at the speed of light.(2) But in the naïve version of our approach there should exist, for instance, "strong gravitons".

At this point, let us consider our asymptotically-flat space-time, endowed with gravitational and strong charges, i.e. not only "gravitationally" but also "strongly" (locally) *curved*; and the union $\mathcal{B}$ of all its future strong-horizons. Let us divide the null strong-geodesics, which generate $\mathcal{B}$, into a large number of infinitesimal bundles, each one identified by a number α. Consider along a generic bundle α_o of strong-generators (all lying in a surface of constant phase), a particular event P and various observers (endowed with different velocities) all measuring the cross-section $a(\alpha_o,P)$ at P of α_o. Then, from assumptions (i)÷(v) above and following e.g. Misner et al.(1973) —taking into account in particular that the geometric optics in curved space-times exploited therein holds good also in our approach—, one can show that the area $a(\alpha_o,P)$ does *not* depend on the observer velocity; and that, in general, $a(\alpha,P)$ depends only on the localization of P along the bundle α, according to the "focusing theorem": If condition (v) holds for any one of the observers measuring $a(\alpha,P)$, then for any point P along α:

$$d^2\sqrt{a(\alpha,P)}\,/\,dk_\alpha^2 \leq 0\,, \qquad \forall\, P \in \alpha\,, \tag{15}$$

where $k_\alpha \equiv k(\alpha,P)$ is the affine parameter along the bundle α. Let us discuss, in connection with eq.(15), the sign of the *first derivative* $d\sqrt{a_\alpha}/dk_\alpha$, where $a_\alpha \equiv a(\alpha,P)$.

If $d\sqrt{a_\alpha}/dk_\alpha$ were negative at a certain event P along α, then it would thereafter go on being negative, and fatally $\sqrt{a}$ would reach the value zero, after a finite interval of the affine parameter. At the point where $\sqrt{a}$ vanishes, all the (adjacent) null strong-geodesics belonging to α cross each other, thus violating the Penrose theorem; unless those null geodesics hit a real strong-singularity, and disappear, before intersecting each other. But the last hypothesis implies the existence of a naked singularity. We wish, on the contrary, to associate hadrons with SBHs, excluding the existence —just as it is generally done in GR— of naked point-like singularities [a hadron associated with a naked singularity would appear to be pointlike, whilst experimentally —as well as in our theoretical approach— they are *extended-type* objects]. We may therefore conclude that

$$d\sqrt{a_\alpha}\,/\,dk_\alpha \geq 0\,. \tag{16}$$

Since now bundles can be born, but noone destroyed, when moving towards the future the total cross-section of $\mathcal{B}$ cannot decrease.

Therefore, we can believe the Second Law of the ordinary BH Thermodynamics, by Bekenstein (1973, 1974) and Hawking (1971,1972,1973), to hold good also for our SBHs. This conclusion is confirmed by the fact that the purely thermodynamical considerations elegantly developed by Hawking in 1976 (by which he generalized the results above) seem to be easily translatable into the SBH language.

In particular, if two SBHs, forming an isolated system, *merge together* so to originate a single new SBH, the final strong Schwarzschild area A_s must be larger than the sum of the initial strong Schwarzschild areas:

$$A_s \geq A'_s + A''_s\,. \tag{17}$$

7. CONSEQUENCES OF SBH THERMODYNAMICS FOR THE "BIG CRUNCH" OF OUR UNIVERSE

Let us first consider an ordinary star S, collapsing towards its gravitational Schwarzschild radius $r_{BH} = 2GM/c^2$. We are interested in the case in which S —or rather its core— reaches the nuclear-matter density *before* collapsing beyond its gravitational horizon; i.e., in which its BH-density $\rho_{BH} = 3c^6/(32\pi G^3M^2)$ is *higher* than the density $\rho_n \simeq 10^{15}$ g/cm^3 of nuclear matter. Here, and in what follows, M should represent the S core mass. However, under the simplifying hypothesis of a homogeneous collapse (when M is just the mass of S), we get: $M \lesssim 5M_\odot$.

If M is smaller than about three solar masses, the star S stops collapsing and transforms (as well-known) into a neutron star. If, however,

$$3M_\Theta \lesssim M \lesssim 5M_\Theta \,, \tag{18}$$

it is usually expected that S goes on collapsing *beyond* the neutron-star phase.(3) But the star neutrons, if regarded as SBHs, when melting together must comply with eq.(17); so that they should thermodynamically tend to originate a "hadronic object" with area(4) $A > Na_n$, quantity N being the number of neutrons corresponding to the star mass M, and a_n being the neutron SBH area (by our approach, we found for the nucleon a strong Schwarzschild radius $r_{SBH} \simeq 0.8$ fm, in agreement with the experimental nucleon-radius). This would mean that, for $3M_\Theta \lesssim M \lesssim 5M_\Theta$, the collapsing body S should not become a gravitational black-hole; on the contrary, Thermodynamics seems to predict that it ought to re-explode *tending* to become an (evaporating!) SBH with a horizon-radius of the order of 1 light-day:

$$R \gtrsim \bar{R} \,, \qquad \text{with} \quad 1 \lesssim \bar{R} \lesssim 3 \text{ light-days.}$$

Of course, such an explosion will be complicated by the fact that the "super-neutrons" formed during the process do evaporate (Hawking, 1974,1975; de Witt, 1975), just as expected for *generic* SBHs. Even more, the interval represented by eq.(18) should not be taken too seriously, since it actually depends on the *critical* density ρ_c of nuclear-matter *beyond* which the neutrons start to merge together. And ρ_c may depend a priori on M. For example, if (instead of choosing $\rho_c \simeq 10^{15}$g/cm^3) we identify ρ_c with the neutron-star core-density d given by the standard formula $d \simeq 0.823\ (M/M_\Theta)^2 \times 10^{15}$g/cm^3, then the "allowed" range in eq.(18) does even disappear, since the condition $\rho_{BH} > d$ yields the constraint $M < 2.17M_\Theta$ (even if one ought to not forget that M should represent the core-mass, rather tha the star-mass). Clearly, the question whether the phenomenon above described does take place in nature, or not, is at this point still an open problem, since it crucially depends on the chosen equation of state.

Let us pass, therefore, to the more interesting case corresponding to the collapse of a mass as big as the one of our cosmos: $M \simeq 10^{54}$kg. In this case the difficulties met above do not appear, since the Schwarzschild radius is so large, $r_{BH} \simeq 1.5 \times 10^{27}$ m, that we have to regard ourselves as *inside* such an enormous cosmic horizon: i.e., to regard the considered collapse as taking place in the interior of this "cosmic black-hole".

Equation (17) requires that, if during the contraction phase our cosmos U reduces at a certain moment to a neutron super-star of $\sim 10^{80} \div 10^{81}$ neutrons, then it will re-explode tending to form a new object with radius(5)

$$R_U >> 2 \times 10^{25} \text{ m} \,, \tag{19}$$

which is not far from the presently accepted "Hubble radius" of our cosmos. (We may have, then, successive expansion/contraction cycles, all taking possibly place inside the cosmic horizon).

This indication of Thermodynamics (anticipated e.g. in Caldirola et al.,1978a; Recami, 1979, 1982b,1983,1984) is the only (thermodynamical) hint known to us till now that our world, after

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(3) Notice that, within the mass limits of eq.(18), the collapsing body S is *not* expected to stabilize itself at the level of a "quark star".

(4) Just as in the case of ordinary hadrons, we assume the horizon-radius to be representative of the *effective* (or average) radius of the "hadronic object".

(5) In a sense, at the end of every cycle our cosmos can be regarded as jumping to a higher step in the considered field-strength hierarchy; but this would not be perceivable inside the cosmos, due to the scale covariance of the fundamental equations (Recami, 1982a; Caldirola et al.,1978). The important point, once more, is that the horizon-radius is not far from the *average* radius (both our cosmos and hadrons are expected, in our approach —as well as in other similar models—, to pulsate between a minimum radius and their respective horizon-radius).

its contraction, has to <<give rise to a new cosmos creation>>, —to use Posidonius' words (about 100 B.C.). But one should not be surprised that the laws of physics seem to allow us to make predictions beyond the "Big Crunch" : in fact, in our approach, no final (nor initial) singularity exists in the cosmos history (cf. also Rosen, 1979, 1980a,b), so that the physical laws do not break down in the temporal vicinity of such a *non-singular* Big Crunch.

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