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DOES THERMODYNAMICS REQUIRE A NEW EXPANSION AFTER THE "BIG CRUNCH" OF OUR COSMOS?(+)

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ABSTRACT: Recently, a unified geometrical approach to gravitational and strong interactions was proposed, based on the methods of General Relativity. According to it, hadrons can be regarded as "black-hole type" solutions of new field equations describing *two* tensorial metric-fields (the ordinary gravitational, and the "strong" one). By extending the Bekenstein-Hawking thermodynamics to those "strong black-holes" (SBH), we show: (i) that SBH thermodynamics seems to require a new expansion of our cosmos after its "Big Crunch" (this thermodynamical indication being rather unique, up to now, in showing that a recontraction of our cosmos has to be followed by a new "creation"); (ii) that a collapsing star with mass $2M_{\odot} \leq M \leq 15M_{\odot}$, once overtaken the neutron-star phase, must re-explode reaching a diameter of at least a few light-days, thus failing to reach the black-hole state.

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"Matter possesses a coesion that keeps it together and against which the surrounding vacuum is impotent. Actually, the material world is supported by an immense force, and alternately contracts and expands following its physical transmutations, now consumed by fire, now giving on the contrary a new rise to the cosmos creation".

PLUTARCUS (ca. 135 ÷ 51 B.C.).

INTRODUCTION. Inspired by the Weyl-Eddington-Dirac "large numbers coincidences", recently we proposed[1-4] a unifield theory of strong and gravitational interactions, that (even if geometrical and classical, i.e. based on the methods of General Relativity) yields results similar to the "strong gravity" ones by Salam *et al.*[5].

This geometrical approach is twofold. On one side, it tries to describe the strong field inside a hadron just by means of "scaled down" Einstein equations (with attractive cosmological term).[1,2,4]. One thus naturally gets the confinement of the hadron constituents, and is allowed e.g. to calculate the hadron masses (for the meson mass-spectra, see the last one of refs.[4] and work in progress).

On the other hand, one is led to describe the strong field surrounding a hadron as a new tensorial field, to be "added" to the gravitational one; so that a *bi-scale* theory arises [1,3], where the total metric $g_{\mu\nu}$ is tentatively regarded as the sum of the ordinary gravitational metric-field $g^{(G)}_{\mu\nu} \equiv e_{\mu\nu}$ and of the "strong" metric-field $s_{\mu\nu}$, at least in a first approximation. In the surroundings of a hadron (and in suitable coordinates):

$$g_{\mu\nu} \simeq e_{\mu\nu} + s_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu} , \qquad (1)$$

where $\eta_{\mu\nu} \equiv \mathrm{diag}(+1, -1, -1, -1)$, and the new field-equations are[1,3]

$$R_{\mu\nu} + \lambda s_{\mu\nu} \simeq - \frac{8\pi}{c^4} (S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\rho}{}_{\rho}) , \ [\lambda > 0] \qquad (2)$$

with

$$\lambda \equiv \rho^{-2}\Lambda; \quad S_{\mu\nu} \equiv NT_{\mu\nu}; \quad N \equiv \rho^{-1}G, \tag{3}$$

where: G is the ordinary universal gravitation constant; Λ is the ordinary cosmological constant (with $|\Lambda| \simeq 10^{-56}\ \mathrm{cm}^{-2}$); and $\rho^{-1} \simeq 10^{41}$ is the ratio between the cosmos Hubble-radius $R(U) \simeq 10^{26}$ m and the typical hadron radius $r(h) \simeq 10^{-15}$ m. Quantity ρ^{-1}, by "coincidence", is equal to the ratio between the *strong* interaction and the *gravitational* interaction strength:

$$\rho^{-1} \equiv \frac{R(U)}{r(h)} \simeq \frac{S(s)}{S(g)} \approx 10^{41}.$$

The strong metric-tensor $s_{\mu\nu}$, entering eqs.(2), is of course defined by eq.(1): $s_{\mu\nu} \equiv g_{\mu\nu} - e_{\mu\nu} \simeq g_{\mu\nu} - \eta_{\mu\nu}$. Quantity Λ can be called the strong or hadronic "cosmological constant". All tensorial indices are raised (lowered) by the total metric-tensor $g^{\mu\nu}(g_{\mu\nu})$.

In this paper we shall refer ourselves only to the latter "side" of our theoretical approach. Let us recall that in the present bi-scale approach two kinds of objects are taken into consideration: (a) the ones — carrying ordinary gravitational mass, i.e. with intrinsic scale-factor $d = 1$ — that are subjected only to the gravitational metric $e_{\mu\nu}$, and: (b) the ones — carrying strong charge or "strong mass", i.e. with intrinsic scale-factor $d = \rho$ — subjected also to the strong metric $s_{\mu\nu}$. The latter are of course the hadronic objects.

On the field equations in the surroundings of a hadron. Our new field equations (2), to be valid in the surroundings of a hadron, are acceptable only if all the components $s_{\mu\nu}$ vanish for $r \gg 1$ fm. Actually, by linearizing eqs.(2) with respect to the flat metric, in the weak-field approximation, we get[3] at the static limit (when only $s_{oo} \neq 0$) for the *scalar* potential $V \equiv \frac{1}{2} c^2 s_{oo}$ the solution ($\lambda > 0$):

$$V = -\frac{g}{r}\exp(-r\sqrt{2\lambda})\,, \tag{4}$$

holding for a pointlike particle at rest in the origin and endowed with "strong mass" (or strong charge) g. By identifying $\sqrt{2\lambda} \equiv m_s c/\hbar$ we find [3] for the field mass the value $m_s = \hbar\sqrt{2\lambda}/c \simeq m_\pi$, and therefore it is varified that V (i.e., s_{oo}) is exponentially damped, and just in the Yukawian way.

Let us now recall that eqs.(2) can be easily written also as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R^{\rho}{}_{\rho} - \lambda(g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g_{\mu\nu}g^{\alpha\beta}\eta_{\alpha\beta}) \simeq -\frac{8\pi}{c^4} S_{\mu\nu} \tag{2'}$$

and therefore, *formally*, as:

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R^{\rho}{}_{\rho} \simeq -\frac{8\pi}{c^4}(S_{\mu\nu} + t_{\mu\nu})\,; & (5\underline{a}) \\ t_{\mu\nu} \equiv -\frac{c^4\lambda}{8\pi}(g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g_{\mu\nu}g^{\alpha\beta}\eta_{\alpha\beta})\,. & (5\underline{b}) \end{cases}$$

Incidentally, the last addendum in the r.h.s. of eq.(5b) can be considered as an "interference term" between the two metrics $s_{\mu\nu}$ and $e_{\mu\nu} \simeq \eta_{\mu\nu}$. We can geometrize also the strong field since the *inertia* m_I of the test-hadron in the surroundings of the source-hadron, when subjected to the strong field, coincides[1-3] with its *strong-mass* g', and *not* with its gravitational mass m', so that: $m_I \equiv g'$; and, in the special relativistic limit, it is for instance $g' = g'_o/\sqrt{1-v^2}$, quantity g'_o being the *rest* strong-mass. Notice in eqs.(5) that also $t_{\mu\nu}$ is a tensor, since $\eta_{\mu\nu}$ is only an approximation for $e_{\mu\nu}$. Moreover, let the *total* metric (acting on the test-hadron, in the surrounding of the considered source-hadron) be:

$$ds^2 = e^{a(r)}c^2dt^2 - e^{b(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta \cdot d\varphi^2)\,, \tag{6}$$

where a(r) and b(r) are functions to be determined. Then we find, e.g., that:

$$\begin{cases} t_{oo} = L(1 - e^{a-b}); \quad t_{11} = L(e^{b-a} - 1) \\ \\ t_{22} = L(e^{-a} + e^{-b} - 2); \; t_{33} = t_{22}\sin^2\theta, \end{cases} \qquad [L \equiv -\frac{c^4\lambda}{16\pi}]$$

so that the conditions $e^a, e^b \to 1$ for $r \gg 1$ fm are equivalent to the conditions $t_{\mu\nu} \to 0$ for $r \gg 1$ fm. Notice incidentally that, for a spherically-symmetric, static metric in the vacuum, eqs.(2') take on the simple form (valid, at least, where eq.(1) holds):

$$\begin{cases} B' + B(\frac{2}{r} + \frac{A'}{A}) = \frac{2}{r}; \\ \\ B' + B(Hr - \frac{A'}{A}) = \frac{Hr}{A}, \end{cases} \tag{7}$$

where the functions $A(r)$, $B(r)$ are defined as: $A \equiv e^a$; $B^{-1} \equiv e^b$. Work is still in progress (in collaboration with J. M. Martínez and W. A. Rodrigues), as far as the solution of eqs.(7) is concerned. Let us here anticipate only that, in units such that $c = \hbar = 1$ and under the approximation in which (see eq.(4)) $s_{oo} \simeq 2V$; $s^o_o \simeq s_{oo}$; and $g_{oo} \equiv A \equiv e^a \simeq 1 + s_{oo} \simeq 1 - 2g\exp(-m_\pi r)/r$, then our field equations *yield* that also $g_{11} \equiv -B^{-1} \equiv e^b$ tends exponentially to 1 for $r \gg 1$ fm:

$$B \equiv e^{-b} \equiv -(g_{11})^{-1} \simeq 1 + \lambda V(r + m_\pi^{-1})/m_\pi \, .$$

This implies that the space-time associated witn eqs.(2 ÷ 7) is *asymptotically flat* (cf. eq.(6)); and therefore that all the components $t_{\mu\nu}$ do actually go to zero for $r \gg 1$ fm.

The point relevant to us in this paper, *however*, is that hadrons can actually be associated (by solving a Schwarzschild-like problem) with the spherically-symmetric solutions of our field equations (2),(2'),(5). Namely, it was suggested [1,3] that hadrons be regarded as the "black-hole *type*" solutions of our field equations, related to the presence of the *strong* - gravity field $s_{\mu\nu}$ (besides the gravitational field $e_{\mu\nu}$). The Schwarzschild-type radii of our strong black-holes (SBH) are, e.g., of the order of 1 fm ([1,3]). Remember, moreover, the eq.(4). Such SBHs will *confine*, of course, only objects carrying a "strong charge", i.e. only hadronic particles.

On the thermodynamics of "strong black-holes" (SBH). In the second items of refs.([1]) and ([3]), as well as in ref.([6]), one of us already mentioned that fashinating consequences would follow by assuming the Bekenstein-Hawking ordinary black-hole (BH) thermodynamics to hold good also for our SBHs. Let us approach this question more closely.

In the surroundings of a hadron, the cosmological constant Λ plays no role. In connection with eqs(2 ÷ 5), therefore, we can assume our space-time manifold V to be: (i) asymptotically flat; and moreover([7]): (ii) time-orientable and Lorentzian (= pseudo-Riemannian), with a metric $g_{\mu\nu}$ of Lorentz signature - 2 and satisfying the two postulates: (iii) of local causality, and: (iv) of *local* conservation of energy-momentum.

The space-time property (i) implies the positivity of energy.([8])

As to our field equations, we firstly notice that our approach implies gravity and strong-gravity to be attractive for positive matter and strong-matter densities, respectively. Second, we explicitly assume([7,9]) that: (v) quantity $R_{\mu\nu}K^{\mu}K^{\nu}$ is non-negative for any non-spacelike Killing vector K^{μ}. Notice, incidentally, that all the above assumptions (i)÷(v) do agree with our physical model (the only problem still partially open at this point being whether the form actually chosen for the field equations is *fully* compatible with them).

All the ordinary theorems on singularities([7]), then, hold good also for our manifold V. Let us deal, in particular, with the Penrose theorem([10]) and the Second Law of BH thermodynamics([11÷13]), in order to be able to draw briefly the most relevant consequences.

Let us consider, e.g., a SBH solution of our field equations in the static case ($\partial g_{ik} / \partial x_o = 0$; $g_{ok} = 0$), with its strong-horizon(1÷3,7). Always with reference to objects endowed with strong-mass (= strong charge), such solutions will present at least one asymptotically flat region, or "external universe", possessing a timelike (strong) future infinity I_s^+ and past infinity I_s^-, a spacelike (strong) infinity I_s^o, and a null (strong) future infinity J_s^+ and past infinity J_s^-. The strong--horizon divides of course the external universe (which can send hadronic particles —"strong-signals"— out to J_s^+), from the SBH interior (which cannot). Let us define the union of the future SBH horizons as the *boundary* $B \equiv \dot{J}_s^-(J_s^+)$ of the causal past (via strong-signals) of the future null strong-infinity (i.e., of the domain $J_s^-(J_s^+)$ that can send future-directed causal strong-curves to future null strong-infinity).

From assumptions (i)÷(iv) above, it is then straightforward to generalize the Penrose theorem(10) on the structure of the future strong-horizons; i.e., to conclude that B is generated by null strong-geodesics which do not have future end-points.(*) Let us clarify that the strong-geodesics are the ones followed by the hadronic test-particles, i.e. by particles affected by the strong metric $s_{\mu\nu}$ besides the gravitational metric $e_{\mu\nu}$ (negligible in the vicinity of the source-hadron). The *null* strong--geodesics, therefore, are geometrically well defined, even if

(*) Let us recall that, more specifically, Penrose's theorem states that(9): (a) the "generators" of B are defined as null strong-geodesics that lie in B, at least for some finite interval of the affine parameter; (b) a generator, followed into the past, may leave B entering $J_s^-(J_s^+)$; (c) once a generator, followed into the future, enters the boundary B of $J_s^-(J_s^+)$ at point C (a "caustic" of B), it will never be able thereafter to leave B, nor to intersect another generator (the generators can intersect each other only at the caustics); (d) only *one* generator (apart from a normalization of the geodesic affine-parameter) can pass through a non-caustic event.

test-hadrons would not exist travelling exactly at the speed of light.(**) But in the naïve version of our approach there should exist, for instance, "strong gravitions".

At this point, let us consider our asymptotically-flat space--time, endowed with gravitational and strong charges, i.e. not only "gravitationally" but also "strongly" (locally) *curved;* and the union $\mathcal{B}$ of all its future strong-horizons. Let us divide[9] the null strong-geodesics, which generate $\mathcal{B}$, into a large number of infinitesimal bundles, each one identified by a number α. Consider along a generic bundle α_o of strong - generators (all lying in a surface of constant phase), a particular event P and various observers (endowed with different velocities) all measuring the cross-section $a(\alpha_o,P)$ at P of α_o. Then, from assumptions (i) ÷ (v) above and following e.g. ref.[9] — taking into account, in particular, that the geometric optics in curved space-times exploited therein holds good also in our approach—, one can show that the area $a(\alpha_o,P)$ does *not* depend on the observer velocity; and that, in general, $a(\alpha,P)$ depends only on the localization of P along the bundle α, according to the "focusing theorem": If condition (v) holds for any one of the observers measuring $a(\alpha,P)$, then for any point P along α:

$$\frac{d^2\sqrt{a(\alpha,P)}}{dk_\alpha^2} \leq 0, \quad \forall P \in \alpha,$$

where $k_\alpha \equiv k(\alpha,P)$ is the affine parameter along the bundle α. Let us discuss, in connection with eq.(8), the sign of the first derivative $da_\alpha^{1/2}/dk_\alpha$, where $a_\alpha \equiv a(\alpha,P)$.

If $da_\alpha^{1/2}/dk_\alpha$ were negative at a certain event P along α, then it would thereafter go on being negative, and fatally $a^{1/2}$ would reach the value zero, after a finite interval of the

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(**) Remember that the global space-time dilations, considered by us[2], do *not* affect the speeds, in particular the value of the light-speed.

affine parameter. At the point where $a^{1/2}$ vanishes, all the (adjacent) null strong-geodesics belonging to α cross each other, thus violating the Penrose theorem; *unless* those null geodesics hit a real strong-singularity, and disappear, before intersecting each other. But the last hypothesis implies the existence of a naked singularity. We wish, on the contrary, to associate hadrons with SBHs, excluding the existence —just as it is generally done— of naked point-like singularities [a hadron associated with a naked singularity would appear to be point-like, whilst experimentally —as well as in our approach— they are *extended* objects]. We may therefore conclude that

$$\frac{da_\alpha^{1/2}}{dk_\alpha} \geq 0. \qquad (9)$$

Since new bundles can be born, but noone destroyed, when moving towards the future the total cross-section of $\mathcal{B}$ cannot decrease(11,9).

Therefore, we can believe the Second Law of the ordinary BH thermodynamics, by Bekenstein(12) and Hawking(11), to hold good also for our SBHs. This conclusion is confirmed by the fact that the purely thermodynamical considerations elegantly developed by Hawking(13) (by which he generalized the results above) seem to be easily translatable into the SBH language.

In particular, if two SBHs, forming an isolated system, *merge together* so to originate a unique new SBH, the final strong Schwarzschild are A_s must be larger than the sum of the initial strong Schwarzschild areas:

$$A_s \geq A'_s + A''_s \,. \qquad (10)$$

Consequences and applications for collapsing stars and for the universe "Big Crunch". Let us, first, consider an ordinary star S, collapsing towards its gravitational Schwarzschild radius $r_{BH}=2GM/c^2$. We are interested in the case in which S passes through the neutron-star phase *before* collapsing beyond its horizon; i.e., in which its BH-density $\rho_{BH} = 3c^6/(32\pi G^3 M^2)$ is *high-*

er than the nuclear-matter density $\rho_n \simeq 10^{14}$ g/cm^3. Under the simplifying hypothesis of a homogeneous collapse, we get: $M \leq 15M_\odot$.

If M is smaller than two solar masses, the star S stops collapsing and transforms into a neutron star. If, however,

$$2M_\odot \leq M \leq 15M_\odot \,, \qquad (11)$$

it is usually expected that S go on collapsing *beyond* the neutron-star state.(***) But the star neutrons, if regarded as SBHs, when melting together must comply with eq.(10), so that they should thermodynamically tend to give origin to a cosmological "hadronic object" with area(°) $A > Na_n$, quantify N being the number of neutrons corresponding to the star mass M, and a_n being the neutron SBH area (in refs.[1÷4] we found for the neutron $r_{SBH} \simeq 0.8$ fm, agreeing with the experimental nucleon-radius). This means that, for $2M_\odot \leq M \leq 15M_\odot$, *the collapsing body* S *will not become a gravitational black-hole*; on the contrary, it is predicted by Thermodynamics to re-*explode* tending to become an (evaporating) SBH with horizon-radius of a few light-days:

$$R > \bar{R}, \quad \text{with} \quad 1 \leq \bar{R} \leq 5 \text{ light-days.} \qquad (12)$$

Of course, such an explosion will be complicated by the fact that the "super-neutrons" formed during the process will evaporate[13,14], just as expected for *generic* SBHs[1÷4].

The most interesting case is the one corresponding to the collapse of a mass as big as the one of our cosmos[1÷4]: $M \approx 10^{54}$ kg. In this case, however, the Schwarzschild radius is enormously large, $r_{BH} \simeq 1.5 \times 10^{27}$ m, and we have to regard ourselves as *inside* such a cosmic horizon; i.e. to regard the considered col-

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(***) Notice that, within the mass limits of eq.(11), the collapsing body S will never stabilize itself at the level of a "quark star".

(°) Just as in the case of ordinary hadrons, we assume the horizon-radius to be representative of the *effective* (or average) radius of the "hadronic object".[1÷4] apse as taking place in the interior of such a "cosmic black--hole".

Equation (10) requires that, if during the contraction phase our cosmos U reduces at a certain moment to a neutron super-star of $\sim 10^{80} \div 10^{81}$ neutrons, then it will re-explode tending to form a new object with radius(°°)

$$R_U \geqslant 2 \times 10^{25} \text{ m}, \qquad (13)$$

which is not far from the presently accepted "Hubble radius" of our cosmos. (We may have, then, sucessive expansion/contraction cycles, all taking probably place inside the cosmic horizon).

That indication of Thermodynamics, anticipated by one of us in refs.(15), is the only (thermodynamical) hint till now known to us that our world, after the recontraction, has to ≪newly give origin to the creation of the cosmos≫, to use Posidonius' words (~ 100 B.C.). But one should not be surprised that the laws of physics seem to allow us to make predictions beyond the "Big Crunch": in fact, in our approach, no final (nor initial) singularity exists in the cosmos history, so that the physical laws do not break down in the temporal vicinity of such a *non--singular* Big Crunch.

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(°°) In a sense, at the end of every cycle our cosmos can be regarded as jumping to a higher step in the considered field strength hierarchy; but this would not be perceivable inside the cosmos, due to the scale covariance of the fundamental equations(1,2). The important point, once more, is that the horizon-radius is not far from the *average* radius (both our cosmos and hadrons are expected, in our approach, to pulsate between a minimum radius and their respective horizon-radius).

REFERENCES

(1) For an extended summary of it, see e.g. E.RECAMI: in *Quarks and the Nucleus: vol.8 of Progress in Particle and Nuclear Physics*, ed. by D. Wilkinson (Pergamon Press; Oxford, 1982), p.401. See also E. RECAMI: in *Old and New Questions in Physics, Cosmology...: Essays in Honour of W. Yourgrau*, ed. by A. van der Merwe (Plenum; New York, 1982), chapt.24, p.377.

(2) E. RECAMI and P. CASTORINA: Lett.Nuovo Cimento 15,357(1976); P. CALDIROLA, M. PAVŠIČ and E. RECAMI: Nuovo Cimento B48,205-271 (1978); Phys.Letters A66,9(1978); E.RECAMI:Found.Phys.13,341(1983); E.RECAMI: in *Problemy Statisticheskoi i Kvantovoi Fiziki*, ed.by Yu. I.Zaparovannyi and Yu.P.Rybakov (Peoples' Friendship Univ.Press; Moscow,1984), pp.3-22.

(3) P.CALDIROLA and E.RECAMI: Lett.Nuovo Cimento 24,565(1979); P.AMMIRAJU, E.RECAMI and W.A.RODRIGUES: Nuovo Cimento A78,172(1983);

(4) E.RECAMI: Found.Phys.13,341(1983); A.ITALIANO and E.RECAMI: Lett.Nuovo Cimento 40,140(1984); A.ITALIANO, M.LATTUADA, G.D.MACCARONE, E.RECAMI, F.RIGGI and D.VINCIGUERRA: Hadronic J.7,1321(1984).

(5) See e.g. A.SALAM and J.STRATHDEE: Phys.Rev.D18, 4596(1978), and refs. therein; A.SALAM: in *Proceedings of the 19th Internat. Conf. on High-Energy Physics (Tokyo,1978)*, p.937.

(6) E.RECAMI: in *Relativity, Quanta and Cosmology in the development of the scientific thought of A.Einstein*, ed.by F. de Finis and M.Pantaleo (Johnson Rep. Co.; New York,1979), vol.2, pp.537-597.

(7) See e.g. S.W.HAWKING and G.F.R.ELLIS : *The Large-Scale Structure of Space-Time* (Cambridge Univ.Press; Cambridge,1973).

(8) See e.g. Y.CHOQUET-BRUHAT: in *Relativity, Groups and Topology - II: Proceedings of the 40th Les Houches Summer School in Theoretical Physics*, ed. by B.De Witt and R.Stora (North-Holland; Amsterdam, 1984), pp.739-785, and refs. therein.

(9) See e.g. C.W.MISNER, K.S.THORNE and J.A.WHEELER: *Gravitation* (Freeman; San Francisco,1973).

(10) R.PENROSE:"Structure of space-time", in *Battelle Rencontres:*

1967 Lectures in Mathematics and Physics, ed. by C.De Witt and J.A. Wheeler (Benjamin; New York, 1968).

([11]) S.W.HAWKING: Comm.Math.Phys.25,152(1972); Phys.Rev.Lett.26, 344(1971); and in *Black-holes*, ed. by B.De Witt and C.De Witt (Gordon & Breach; New York,1973).

([12]) J.D.BEKENSTEIN: Phys.Rev.D7,2333(1973); D9,3292(1974).

([13]) S.W.HAWKING: Phys.Rev.D13,191(1976).

([14]) See, e.g., S.W.HAWKING: Nature 248,30(1974); Comm. Math.Phys. 43,199(1975); B.S.De Witt: Phys.Rep. C19,295.

([15]) See, e.g.: the second item of ref.([1]); the last three items of refs.([2]), and ref.([6]).