ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Catania

INFN/AE-85/6
4 Luglio 1985

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TACHYON KINEMATICS AND CAUSALITY
(A SYSTEMATIC, THOROUGH ANALYSIS)

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TACHYON KINEMATICS AND CAUSALITY(o)
(A SYSTEMATIC,THOROUGH ANALYSIS)

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ABSTRACT - The chronological order of the events along a space-like path is not invariant under Lorentz transformations, as wellknown. This led to an early conviction that tachyons would give rise to causal anomalies. A relativistic version of the Stückelberg-Feynman "switching procedure" (SWP) has been invoked as the suitable tool to eliminate those anomalies. The application of the "SWP" does eliminate the motions backwards in time, but interchanges the roles of *source* and *detector*. This fact triggered the proposal of a host of causal "paradoxes".

Till now, however, it has not been recognized that such paradoxes can be sensibly discussed (and completely solved, at least "in microphysics") *only* after having properly developed the tachyon relativistic mechanics.

We start by showing how to apply the "SWP", both in the case of *ordinary* Special Relativity, and in the case with tachyons. Then, we carefully exploit the kinematics of the tachyon-exchange between to (ordinary) bodies.

Being finally able to tackle the tachyon-causality problem, we successively solve the paradoxes: (i) by Tolman-Regge; (ii) by Pirani; (iii) by Edmonds; (iv) by Bell. At last, we discuss a further, new paradox associated with the transmission of signals by modulated tachyon beams.

I - INTRODUCTION

It is well-known that the chronological order of the events along a space-like path is not invariant under Lorentz transformations.

This led to an early conviction that tachyons —if they existed— would give rise to causal anomalies; Tolman's proposal of the "anti-telephone paradox" (1), e.g., goes back to 1917.

A relativistic version of the Stückelberg-Feynman "Switching Procedure" (also known as "reinterpretation principle") was invoked in 1962 by Sudarshan and coworkers(2) as the suitable tool to solve those anomalies for tachyons. The application of the "Switching" does eliminate the motions backwards in time,(3,4) but has the consequence that the roles of *source* and *detector* get interchanged (i.e., they become in their turn frame-dependent). This fact triggered the proposal of a host of causal paradoxes, and a lot of discussions.

(o) Work partially supported by INFN-Sez.di Catania, by CNR, and by CIME/IILA.

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Till now, however, it has not been widely recognized —nor sufficiently, emphasized— that the causal paradoxes can be sensibly discussed (and, in our opinion, completely clarified and solved, at least "in microphysics") *only* after having properly developed the relativistic mechanics of tachyons. We think it to be the time, therefore, for a renewed view over the whole subject, paying any attention to clarify in a more convincing way the many aspects of the question.

To prepare the ground, in Sect.2 we shall show merely how to apply the Switching Procedure ("SWP") in *ordinary* Special Relativity, i.e. forgetting about tachyons. On the contrary, in Sect. 4 we shall extend, in a preliminary form, the "SWP" to the case with tachyons. In Sect.5, then, we shall carefully exploit the kinematics of the tachyon-exchange between two (ordinary) bodies.

Finally, we shall be able to tackle —in Sect.7— the tachyon-causality issue.

2 - ORDINARY SPECIAL RELATIVITY WITH ORTHO- AND ANTI-CHRONOUS LORENTZ TRANSFORMATIONS

In this Part I we shall *forget* about Tachyons.

From the ordinary postulates of Special Relativity (SR) it follows that in such a theory —which refers to the class of Mechanical and Electromagnetic Phenomena— the class of reference-frames equivalent to a given inertial frame is obtained by means of transformations L (Lorentz Transformations, LT) which satisfy the following sufficient requirements: (i) to be linear

$$x'^{\mu} = L^{\mu}_{\ \nu}\, x^{\nu}; \qquad (1)$$

(ii) to preserve space-isotropy (with respect to electromagnetic and mechanical phenomena); (iii) to form a group; (iv) to leave the quadratic form invariant:

$$\eta_{\mu\nu}dx^{\mu}dx^{\nu} = \eta'_{\alpha\beta}dx'^{\alpha}dx'^{\beta}. \qquad (2)$$

From condition (i), if we confine ourselves to sub-luminal speeds, .t follows that in eq.(2):

$$\eta'_{\alpha\beta} = \mathrm{diag}(+1,-1,-1,-1) = \eta_{\mu\nu}. \qquad (3)$$

Eqs.(1)-(3) imply that $\det \underline{L}^2 = 1$; $(\underline{L}^\circ{}_\circ)^2 \geq 1$. The set of all subluminal (Lorentz) transformations satisfying all our conditions consists —as is well-known— of four pieces, which form a noncompact, nonconnected group (the Full Lorentz Group). Wishing to confine ourselves to space-time "rotations" only, i.e. to the case $\det \underline{L} = +1$, we are left which the two pieces

$$\{L_+^\uparrow\}:\ L^\circ{}_\circ \geq +1;\ \det L = +1; \tag{4a}$$

$$\{L_+^\downarrow\}:\ L^\circ{}_\circ \leq -1;\ \det L = -1, \tag{4b}$$

which give origin to the group of the proper (orthochronous *and* antichronous) transformations

$$\mathcal{L}_+ \equiv \mathcal{L}_+^\uparrow \cup \mathcal{L}_+^\downarrow \equiv \{L_+^\uparrow\} \cup \{L_+^\downarrow\} \tag{5}$$

and to the subgroup of the (ordinary) proper ortochronous transformations

$$\mathcal{L}_+^\uparrow \equiv \{L_+^\uparrow\}, \tag{6}$$

both of which being, incidentally, invariant subgroups of the Full Lorentz Group. For reasons to be seen later on, let us rewrite $\mathcal{L}_+$ as follows

$$\mathcal{L}_+ = \mathcal{L}_+^\uparrow \otimes \mathbb{Z}(2);\ \mathbb{Z}(2) \equiv \{\sqrt[2]{+1}\} \equiv \{+1,-1\}. \tag{5'}$$

We shall skip in the following, for simplicity's sake, the subscript + in the transformations $\underline{L}_+^\uparrow, \underline{L}_+^\downarrow$. Given a transformation $\bar{\underline{L}}^\downarrow$, another transformation $\bar{\underline{L}}^\uparrow \in \mathcal{L}_+^\uparrow$ always exists such that

$$\bar{L}^\downarrow = (-\mathbb{1}) \cdot \bar{L}^\uparrow, \forall \bar{L}^\downarrow \in \mathcal{L}_+^\downarrow, \tag{7}$$

and vice-versa. Such a one-to-one correspondence allows us to write formaly

$$\mathcal{L}_+^\downarrow = -\mathcal{L}_+^\uparrow. \tag{7'}$$

It follows in particular that the central elements of $\mathcal{L}_+$ are:

$\mathbb{C} \equiv (+\mathbb{1}, -\mathbb{1})$.

Usually, even the piece (4*b*) is discarded. Our present aim is to show —on the contrary— that a physical meaning can be attributed also to the transformations (4*b*). Confining ourselves here to the *active* point of view (cf. ref.(74) in the following and references therein), we wish precisely to show that the theory of SR, once based on the *whole* proper Lorentz group (5) and not only on its orthochronous part, will describe a Minkowski space-time populated by both matter and antimatter.

2.1. The Stückelberg-Feynman "switching principle" in SR

Besides the usual postulates of SR (Principle of Relativity, and Light-Speed Invariance), let us assume —as commonly admitted, e.g. for the reasons in Garuccio *et al* (5), Mignani and Recami (6)— the following:

Assumption - <<negative-energy objects travelling forward in time do *not* exist>>. We shall give this Assumption, later on, the status of a fundamental postulate.

Let us therefore start from a positive-energy particle P travelling forward in time. As wellknown, any orthochronous LT (4*a*) transforms it into another particle still endowed with positive energy and motion forward in time. On the contrary, any antichronous (= non-orthochronous) LT (4*b*) will change sign —among the others— to the time-components of *all the four-vectors* associated with P. Any $L^{\downarrow}$ will transform P into a particle P' endowed in particular with negative energy *and* motion backwards in time (Fig.1).

In other words, SR together with the natural Assumption above *implies* that a particle going backwards in time (Gödel(7)) (Fig. 1) corresponds in the four-momentum space, Fig. 2, to a particle carrying negative energy; and, vice-versa, that changing the energy sign in one space corresponds to changing the sign of time in the dual space. It is then easy to see that these two paradoxical occurrences ("negative energy" and "motion backwards in time") give rise to a phenomenon that any observer will describe in a quite *orthodox* way, when they are —as they actually are— simultaneous (Recami(3),(8) and refs. therein).

Notice, namely, that: (i) every observer (a macro-object) explores space-time, Fig. 1, in the positive *t*-direction, so that we shall meet *B* as the first and *A* as the last event; (ii) emission of positive quantity is equivalent to absorption of nega-

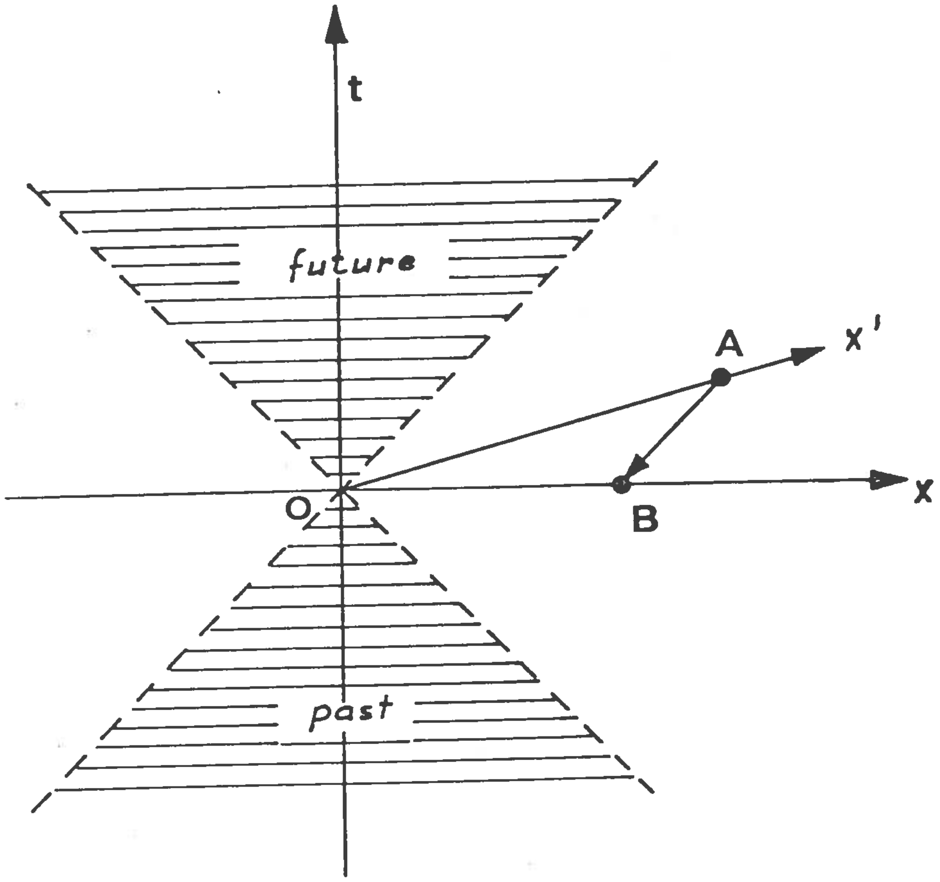

Fig.1

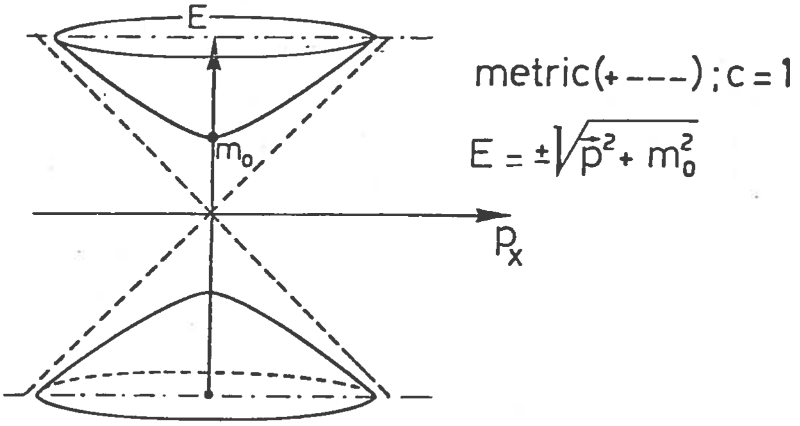

Fig.2

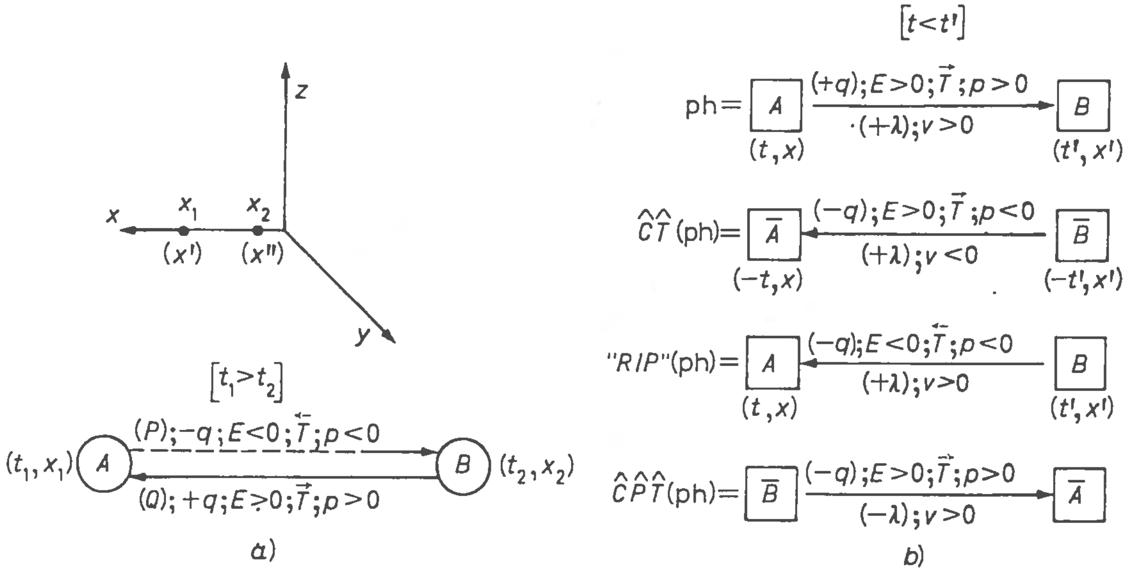

Fig.3

tive quantity, as $(-)\cdot(-) = (+)\cdot(+)$; and so on.

Let us now suppose (Fig.3) that a particle P' with negative energy (and e.g. charge $-e$) moving backwards in time is emitted by A at time t, and absorbed by B at time $t_2 < t_1$. Then, it follows that at time t_1 the object A "looses" negative energy and charge, i.e. *gains* positive energy and charge. And that at time $t_2 < t_1$ the object B "gains" negative energy and charge, i.e. *looses* positive energy and charge. The physical phenomenon here described is nothing but the exchange *from* B *to* A of a particle Q with *positive* energy, charge $+e$, and going *forward* in time. Notice that Q has, however, charges *opposite* to P'; this means that in a sense the present "switching procedure" (previously called "RIP") effects a "charge conjugation" C, among the others. Notice also that "charge", here and in the following, means *any* additive charge; so that our definitions of charge conjugation, etc., are more general than the ordinary ones (Review I(9), Recami (10)). Incidentally, such a switching procedure has been shown to be equivalent to applying the chirality operation γ_5 (Recami and Ziino (11)).

2.2. Matter and Antimatter from SR

A close inspection shows the application of any antichronous transformation $L^{\downarrow}$, together with the switching procedure, to trans form P into an object

$$Q \equiv \bar{P}$$

which is indeed the *antiparticle* of P. We are saying that *the concept of anti-matter is a purely relativistic one*, and that, on the basis of the double sign in [c = 1]

$$E = \pm \sqrt{p^2 + m_o^2} \tag{9}$$

the existence of antiparticles could have been predicted from 1905, exactly with the properties they actually exibited when later discovered, provided that recourse to the "switching procedure" had been made. We therefore maintain that the points of the lower hyperboloid sheet in Fig.2 —since they correspond not only to negative energy but also to motion backwards in time— represent the kinematical states of *antiparticle* $\bar{P}$ (of the particle P represented by the upper hyperboloid sheet). Let us explicitly observe that the switching procedure exchanges the roles of *source* and *detector*, so that (Fig.1) any observer will describe B to be the source and A the detector of the antiparticle $\bar{P}$.

Let us stress that the switching procedure not only can, but *must* be performed, since any observer can do nothing but explore space-time along the positive time-direction. That procedure is merely the translation into a purely relativistic language of the Stückelberg (12) - Feynman (13) "Switching principle". Together with our Assumption above, it can take the form of a "Third Postulate": <<Negative-energy objects travelling forward in time do not exist; any negative-energy object P travelling backwards in time can and must be described as its anti-object $\bar{P}$ going the opposite way in space (but endowed with positive energy and motion forward in time)>>. Cf. e.g. Caldirola and Recami (4), Recami (8) and references therein.

2.3. Further remarks

a) Let us go back to Fig.1. In SR, when based only on the two ordinary postulates, nothing prevents a priori the event A from influencing the event B. Just to forbid such a possibility we introduced our Assumption together with the Stückelberg-Feynman "Switching procedure". As a consequence, not only we eliminate any particle-motion backwards in time, but we also "predict" and naturally explain within SR the existence of antimatter.

b) The Third Postulate, moreover, helps solving the paradoxes connected with the fact that all relativistic equations admit, besides standard "retarded" solutions, also "advanced" solutions : The latter will simply represent antiparticles travelling the opposite way (Mignani and Recami (14)). For instance, if Maxwell equations admit solutions in terms of *outgoing* (polarized) photons of helicity $\lambda = +1$, then they will admit also solutions in terms of *incoming* (polarized) photons of helicity $\lambda = -1$; the actual intervention of one or the other solution in a physical problem depending only on the initial conditions.

c) Eqs. (7),(8) tell us that, in the case considered, any $\underline{\bar{L}}^{\downarrow}$ has the same kinematical effect than its "dual" transformation $\underline{L}^{\uparrow}$, just defined through eq.(7), except for the fact that it moreover transforms $\underline{P}$ into its antiparticle $\underline{\bar{P}}$. Eqs.(7),(7') then lead (Mignani and Recami (15÷17)) to write

$$-\mathbb{1} \equiv \bar{P}\,\bar{T} = CPT\,, \tag{10}$$

where the symmetry operations $\bar{P}, \bar{T}$ are to be understood in the "strong sense": For instance, $\bar{T}$ = reversal of the time-components of all fourvectors associated with the considered phenomenon (namely, inversion of the time *and* energy axes). We shall come back to this point. The discrete operations P,T have the ordinary meaning. When the particle $\underline{P}$ considered in the beginning can be regarded as an extended object, Pavšič and Recami (18) have shown the "strong" operations $\bar{P}, \bar{T}$ to be equivalent to the space, time reflections acting on the space-time *both* external *and* internal to the particle world-tube.

Once accepted eq.(10), then eq.(7') can be written

$$\mathcal{L}_{+}^{\downarrow} = (\bar{P}\,\bar{T})\,\mathcal{L}_{+}^{\uparrow} \equiv (CPT)\mathcal{L}_{+}^{\uparrow}. \tag{7"}$$

In particular, the total-inversion $\underline{\bar{L}}^{\downarrow} = -\mathbb{1}$ transforms the process $\underline{a} + \underline{b} \to \underline{c} + \underline{d}$ into the process $\underline{\bar{d}} + \underline{\bar{c}} \to \underline{\bar{b}} + \underline{\bar{a}}$ without any change in the velocities.

d) All the ordinary relativistic laws (of Mechanics and Electromagnetism) are actually already covariant under the *whole* proper group $\mathcal{L}_{+}$, eq.(5), since they are CPT-symmetric besides being covariant under $\mathcal{L}_{+}^{\uparrow}$.

e) A few quantities, that happened (cf. Sect.4.4 in the following) to be Lorentz-invariant under $\underline{L}^{\uparrow} \in \mathcal{L}_{+}^{\uparrow}$, are no more invariant under the transformations $\underline{L} \in \mathcal{L}_{+}$. We have already seen this to be true for the *sign* of the additive charges, e.g. for the sign of the eletric charge $\underline{e}$ of a particle $\underline{P}$. The ordinary derivation of the eletric-charge invariance is obtained by evaluating the integral flux of a current through a surface which, under $\underline{L}^{\uparrow}$, moves, changing the angle formed with the current. Under $\underline{L}^{\downarrow} \in \mathcal{L}_{+}^{\downarrow}$ the surface "rotates" so much with respect to the current, that the current enters it through the opposite face; as a consequence, the integrated flux (i.e. the charge) changes sign.

3 - ON THE POSTULATES OF SPECIAL RELATIVITY

3.1. Foreword

Let us now take on the issue of Tachyons.

As wellknown, Superluminal particles have been given the name "*Tachyons*"(T) by Feinberg (19) from the Greek word ταχῦς = = fast. << *Une particule qui a un nom possède déjà un début d'existence*>> (A particle bearing a name has already taken on some existence) was later commented on by Arzeliès (20). We shall call "*Luxons*"(ℓ), following Bilaniuk *et al.*(2), the objects travelling exactly at the speed of light, like photons. At last, we shall call "*Bradyons*"(B) the ordinary *subluminal* ($\underline{v}^2 < \underline{c}^2$) objects, from the Greek word βραδῦς = slow, as it was independently proposed by Cawley(21), Barnard and Sallin (22), and Recami(23); see also Baldo *et al* (24).

Tachyons, or space-like particles, are already known to exist as *internal, intermediate states* or exchanged objects (25). Can they also exist as "asymptotically free" objects?

We shall see that the particular —and unreplaceable— role in SR of the light-speed $\underline{c}$ in vacuum is due to its *invariance* (namely, to the experimental fact that $\underline{c}$ does not depend on the velocity of the source), and *not* to its being or not the maximal speed (see e.g. Recami and Modica (26)

Since a priori we know nothing about Ts, the safest way to build up a theory for them is trying to generalize the ordinary theories (starting with the classical relativistic one, only later on passing to the quantum field theory) through "minimal extensions", i.e. by performing modifications as small as possible. Only after possessing a theoretical model we shall be able to start experiments: Let us remember that, not only good experiments are required before getting sensible ideas (Galilei (27, 28)), but also a good theoretical background is required before sensible experiments can be performed.

The first step consists therefore in facing the problem of extending SR to Tachyons. In so doing, some authors limited themselves to consider objects both subluminal and Superluminal, always referred however to subluminal observers ("weak approach"). Other authors attempted on the contrary to generalize SR by introducing both subluminal observers (s) and Superluminal observers (S), and then by extending the Principle of Relativity ("strong approach"). This second approach is theoretically more worth of consideration (tachyons, e.g., get *real* proper-masses), but it meets of course the greatest obstacles (25) [in fact, the extension of the Relativity Principle to Superluminal inertial frames seems to be *straightforward* only in the pseudo-Euclidean space-times M(n,n) having the same number n of space-axes and of time axes (29)].

Let us first revisit, however, the postulates of the ordinary SR.

3.2. The Postulates of SR Revisited

Let us adhere to the ordinary postulates of SR. A suitable choice of Postulates is the following one (Review I; Maccarrone and Recami (30) and refs. therein):

1) First Postulate - *Principle of Relativity*: <<The physical laws of Electromagnetism and Mechanics are covariant (=invariant in form) when going from an inential frame f to another frame moving with constant velocity u relative to f.>>

2) Second Postulate - "Space and time are homogeneous and space is isotropic". For future convenience, let us give this Postulate the form: <<The space-time accessible to any inertial observer is four-dimensional. To each inertial observer the 3-dimensional Space appears as *homogeneous and isotropic*, and the 1-dimensional Time appears as *homogeneous*>>.

3) Third Postulate - *Principle of Retarded Causality*: << Positive-energy objects travelling backwards in time *do not exist*; and any negative-energy particle P travelling backwards in time can and must be described as its antiparticle $\overline{P}$, endowed with positive energy and motion forward in time (but going the opposite way in space)>>. See Sects.2.1,2.2.

The *First Postulate* is inspired to the consideration that all inertial frames should be *equivalent* for a careful definition of "equivalence" see e.g. Recami (8)); notice that this Postulate does *not* impose any constraint on the relative speed $u \equiv |\mathbf{u}|$ of the two inertial observers, so that a priori $-\infty < u < +\infty$. The *Second Postulate* is justified by the fact that from it the conservation laws of energy, momentum and angular-momentum follow, which are well verified by experience (at least in our "local" space-time region); let us add the following comments: (i) The words homogeneous, isotropic refer to space-time properties assumed —as always— with respect to the electromagnetic and mechanical phenomena; (ii) Such properties of space-time are supposed by this Postulate to be covariant within the class of the inertial frames; this means that SR assumes the vacuum (i.e. space) to be "at rest" with respect to *every* inertial frame. The *Third Postulate* is inspired to the requirement that *for each observer* the "causes" chonologically precede their own "effects" (for the definition of causes and effects see e.g. Caldirola and Recami (4)). Let us recall that in Sect.2 the initial statement of the Third Postulate has been shown to be equivalent —as it follows from Postulates 1) and 2)— to the more natural Assumption that <<negative-energy objects travelling forward in time do *not* exist>>.

3.3. Existence of an Invariant Speed

Let us initially *skip* the Third Postulate.

Since 1910 it has been shown (Ignatowski (31), Frank and Rothe (32), Hahn (33), Lalan (34), Severi (35), Agodi (36), Di Jorio (37)) that the postulate of the light-speed invariance is not strictly necessary, in the sense that our Postulates 1) and 2) *imply* the existence of an invariant speed (*not* of a maximal speed, however). In fact, from the first two Postulates it follows (Rindler (38), Berzi and Gorini (39), Gorini and Zecca (40) and refs. therein, Lugiato and Gorini (41)) that one *and only one* quantity w^2 —having the physical dimensions of the square of a speed— must exist, which has the same value according to all inertial frames:

$$w^2 = \text{invariant.} \tag{11}$$

If one assumes $\underline{w} = \infty$, as done in Galilean Relativity, then one would get Galilei-Newton physics; in such a case the invariant speed is the infinite one: $\infty \oplus \underline{v} = \infty$, where we symbolically indicated by $\oplus$ the operation of speed composition.

If one assumes the invariant speed to be finite and real, then one gets immediately Einstein's Relativity and physics. Experience has actually shown us the speed $\underline{c}$ of light in vacuum to be the (finite) invariant speed: $\underline{c} \oplus \underline{v} = \underline{c}$. In this case, of course, the infinite speed is no more invariant: $\infty \oplus \underline{v} = \underline{V} \neq \infty$. It means that in SR the operation $\oplus$ is not the operation + of arithmetics.

Let us notice once more that the unique role in SR of the light-speed $\underline{c}$ in vacuum rests on its being invariant and not the maximal one (see e.g. Shankara (42), Recami and Modica (26)); if tachyons —in particular infinite-speed tachyons— exist, they could *not* take over the role of light in SR (i.e. they could not be used by different observers to compare the sizes of their space and time units, etc.), just in the same way as bradyons cannot replace photons. The speed $\underline{c}$ turns out to be a limiting speed; but any limit can possess *a priori* two sides (Fig. 4).

Of course one can substitute the light-speed invariance Postulate for the assumption of space-time homogeneity and space isotropy (see the Second Postulate).

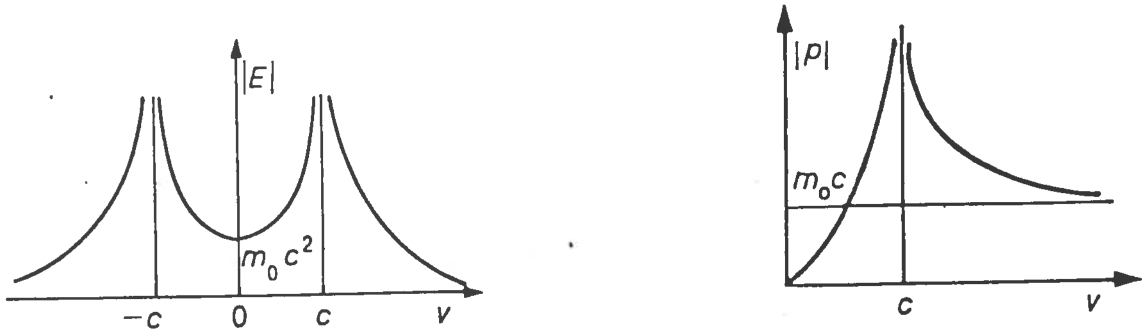

Fig. (a) Fig. (b)

Fig.4

4 - THE "SWITCHING PROCEDURE" (SWP) FOR TACHYONS

4.1 Introduction

Till now we have not taken account of tachyons. Let us finally take them into consideration, starting from a *model-theory*, i.e. from "Extended Relativity" (ER) (Maccarrone and Recami (30, 29), Maccarrone *et al* (43), Barut *et al* (44), Review I) in two dimensions.

Since tachyons are just usual particles w.r.t. their own rest-frames *f*, where the *f*s are Superluminal w.r.t. us (see Recami and Mignani (45), Leiter (46), Parker (47)), they too will possess *real* rest-masses m_o. We shall however have (25, 29) (cf. Fig. 5):

$$p_\mu p^\mu = \begin{array}{lll} \nearrow & +\, m_o^2 > 0 & \text{for bradyons (time-like case)} \\ \rightarrow & 0 & \text{for luxons (light-like case)} \\ \searrow & -\, m_o^2 < 0 & \text{for tachyons (space-like case).} \end{array}$$

Notice that in the present case (eqs.(12)) it is $\mu = 0,1$. Notice also that tachyons slow down when their energy increases and accelerate when their energy decreases. In particular, divergent energies are needed to slow down the tachyons' speed towards its (lower) limit c. On the contrary, when the tachyons' speed tends to infinity, their energy tends to zero; in ER, therefore, energy can be transmitted only at *finite* velocity. From figs. 5a,c it is apparent that a bradyon may have zero momentum (and minimal energy $m_o c^2$); however Bs cannot exist at zero energy, and tachyons cannot exist at zero momentum (w.r.t. the observers to whom they appear as tachyons!). Incidentally, since *transcendent* (=infinite-speed) tachyons do *not* transport energy but do transport momentum ($m_o c$), they allow getting the *rigid body* behaviour even in SR (Bilaniuk and Sudarshan (48), Review I, Castorina and Reca

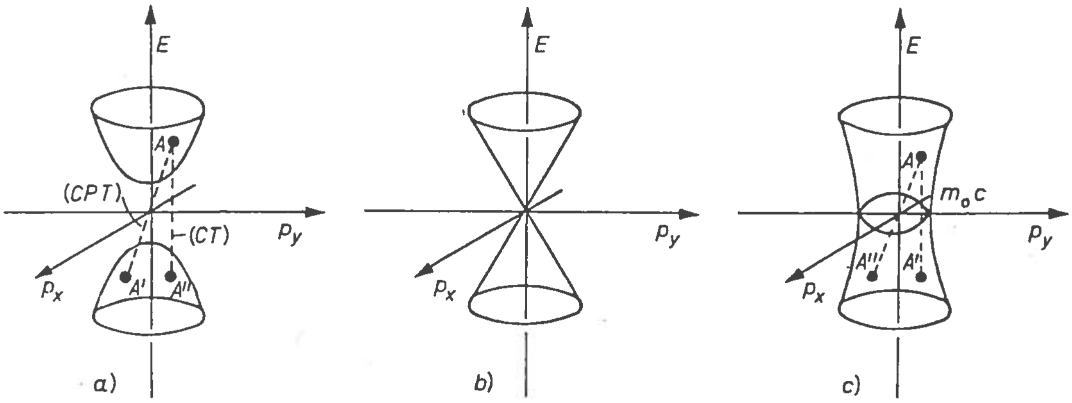

Fig.5

mi (49)). In particular, in elementary particle physics —see also Sect. 5.7 in the following— they might a priori be useful for interpreting in the suitable reference frames the diffractive scatterings, elastic scatterings, etc. (Maccarrone and Recami (50) and refs. therein).

From eq. (12$\underline{c}$) we derive that for tachyons (25,29)

$$E = \pm\sqrt{\vec{p}^{\,2} - m_o^2}\,. \qquad [\underline{V}^2 > 1;\ \underline{m}_o \text{ real}] \qquad (13)$$

And, more in general (cf. Review I):

$$m = \pm \frac{m_o}{\sqrt{|1 - v^2|}}\,, \qquad [-\infty < v < +\infty] \qquad (14)$$

so as anticipated in Fig. 4$\underline{a}$.

The problem of the double sign in eq. (14) has been already taken care of in Sect. 2 for the case of bradyons. Let us pass now from eq. (9) to eq. (13).

Inspection of Fig. 5$\underline{c}$ shows that, in the case of tachyons, it is enough a (suitable) ordinary subluminal orthochronous Lorentz transformation $\underline{L}^{\uparrow}$ to transform a positive-energy tachyon T into a negative-energy tachyon T'. For simplicity let us here confine ourselves, therefore, to transformations $\underline{L} \equiv \underline{L}^{\uparrow} \in \mathcal{L}_+^{\uparrow}$, acting on free tachyons.

On the other hand, it is wellknown in SR that the chronological order along a space-like-path is not $\mathcal{L}_+^{\uparrow}$-invariant.

However, in the case of Ts it is even clearer than in the bradyon case that the same transformation $\underline{L}$ which inverts the energy-sign will also reverse the motion-direction in time (Review I, Recami (51,52,8), Caldirola and Recami (53); see also Garuccio et al (5)). In fact, from Fig. 6 we can see that for going from a positive-energy state $\underline{T}_i$ to a negative-energy state $\underline{T}'_f$ it is necessary to bypass the "transcendent" state $\underline{T}_\infty$ (with $\underline{V} = \infty$). From Fig. 7$\underline{a}$ we see moreover that, given in the initial frame s_o a tachyon T travelling e.g. along the positive $\underline{x}$-axis with speed $\underline{V}_o$, the "critical observer" (i.e. the ordinary subluminal observer $s_c \equiv (\underline{t}_c, \underline{x}_c)$ seeing T with *infinite* speed) is simply the one whose space-axis $\underline{x}$ is superimposed to the world-line OT; its speed $\underline{u}_c$ w.r.t. s_o, along the positive $\underline{x}$-axis, is evidently

$$u_c = c^2/V_o\,; \quad u_c V_o = c^2, \quad [\text{"critical frame"}] \qquad (15)$$

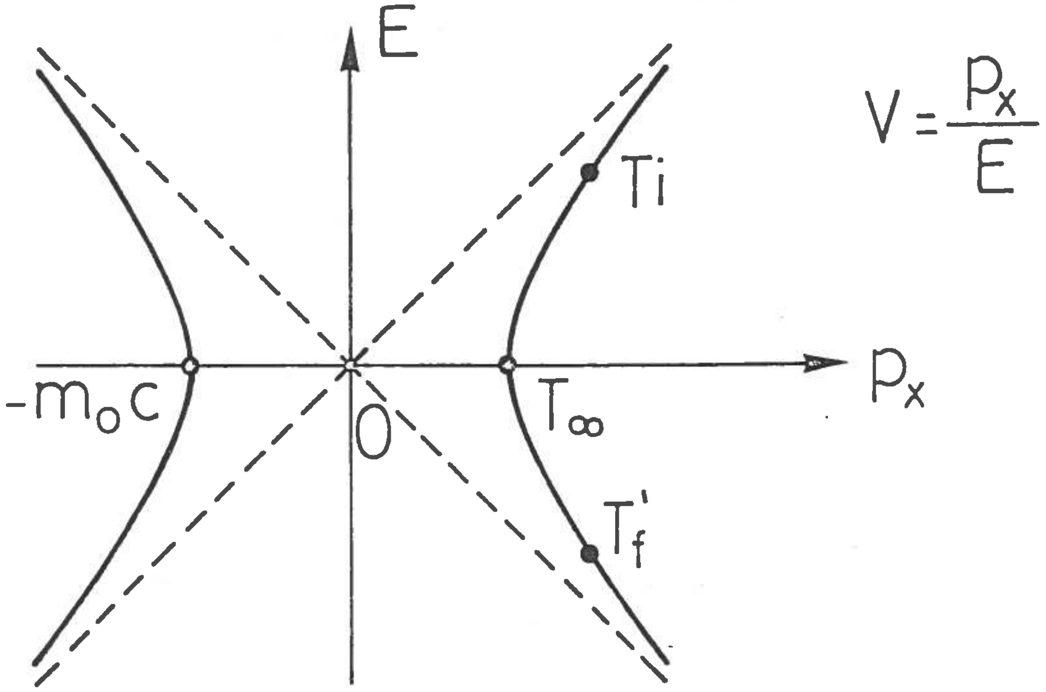

Fig.6

dual to the tachyon speed $\underline{V}_o$. Finally, from Fig. 6 and Fig.7b we conclude that any "trans-critical" observer $s' \equiv (\underline{t}', \underline{x}')$ such that $\underline{u}'\underline{V}_o > c^2$ will see the tachyon T not only endowed with negative energy, but also travelling backwards in time. Notice, incidentally, that nothing of this kind happens when $\underline{u}\,\underline{V}_o < 0$, i.e. when the final frame moves in the direction opposite to the tachyon's.

Therefore Ts display negative energies in the same frames in which they would appear as "going backwards in time", and vice-versa. As a consequence, we can —and must— apply also to tachyons the Stückelberg-Feynman "switching procedure" exploited in Sects.2.1-2.3. As a result, point $\underline{A}'$ (Fig. 5c) or point $\underline{T}'_f$ (Fig. 6) do *not* refer to a "negative-energy tachyon moving backwards in time", but rather to an *antitachyon* $\bar{T}$ moving the opposite way (in space), forward in time, and with positive energy. Let us repeat that the "switching" never comes into the play when the sign of $\underline{u}$ is opposite to the sign of $\underline{V}_o$. (Review I, Recami (3), Caldirola and Recami (4)).

The "Switching Principle" has been first applied to tachyons by Sudarshan and coworkers (Bilaniuk *et al* (4); see also Gregory (54,55)).

Recently Shwartz (56) gave the switching procedure an interesting formalization, in which —in a sense— it becomes "automatic".

4.2 Sources and Detectors. Causality

After the considerations in the previous Sect. 4.1, i.e. when we apply our Third Postulate (Sect 3.2) also to tachyons, we are left with *no* negative

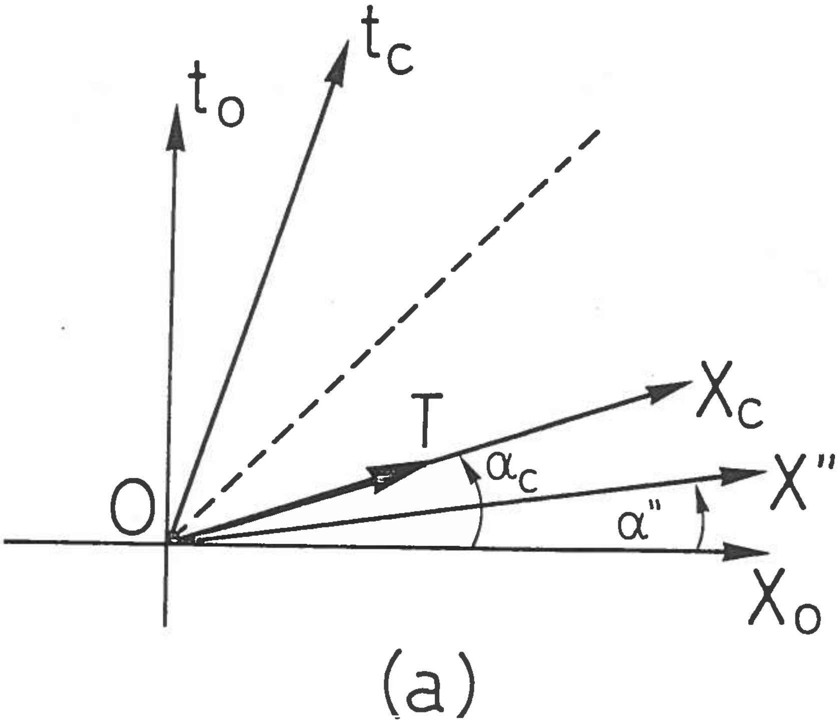

Fig.7(a)

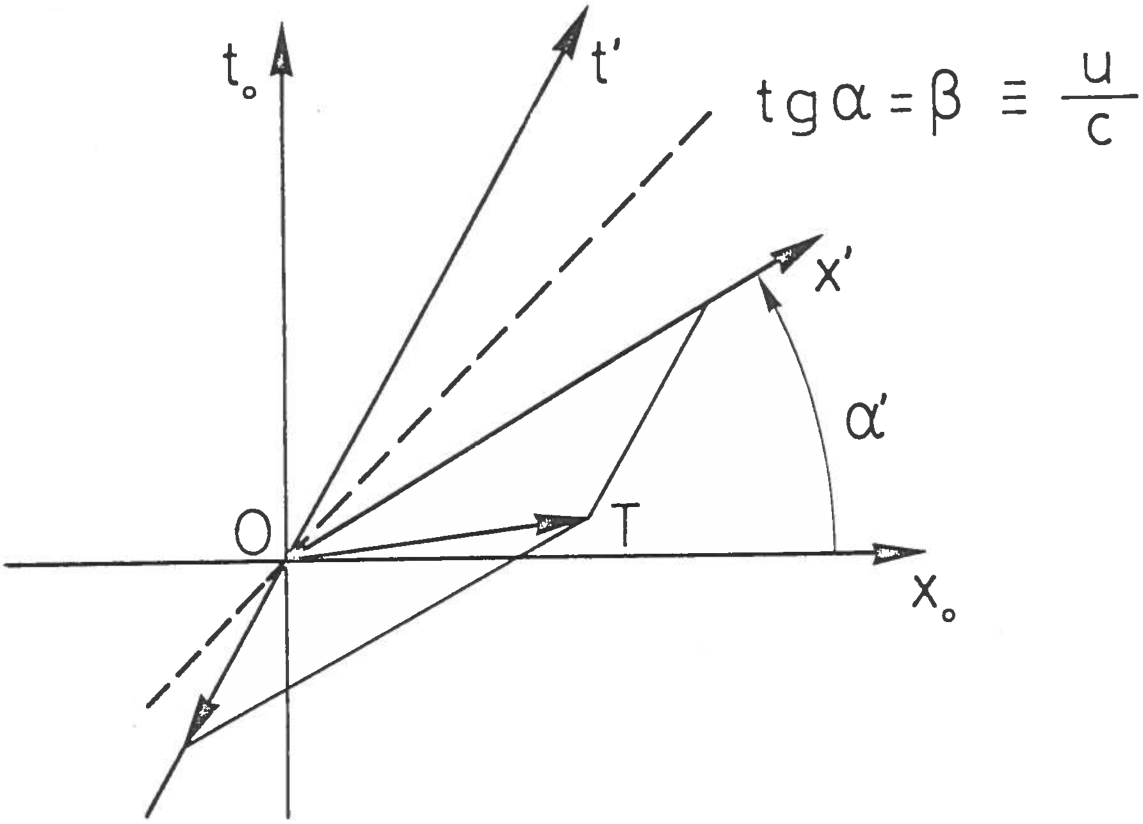

Fig.7(b)

energies (Recami and Mignani [57]) and with *no* motions backward in time (Maccarrone and Recami [58,50]), and refs. therein).

Let us remind, however, that a tachyon T can be transformed into an antitachyon $\overline{T}$ "going the opposite way in space" even by (suitable) ordinary subluminal Lorentz transformations $\underline{L} \in \mathcal{L}_+^\uparrow$. It is always essential, therefore, when dealing with a tachyon T, to take into proper consideration also its *source* and *detector*, or at least to refer T to an "interaction-region". Precisely, when a tachyon overcomes the divergent speed, it passes from appearing e.g. as a tachyon T *entering* (leaving) a certain interaction-region to appearing as the antitachyon $\overline{T}$ *leaving* (entering) that interaction-region (Arons and Sudarshan [59], Dhar and Sudarshan [60], Glück [61], Baldo et al [24], Camenzind [62]). More in general, the "trans-critical" transformations $\underline{\overline{L}} \in \mathcal{L}_+^\uparrow$ (Fig. 7b) lead from a T emitted by A and absorbed by B to its $\overline{T}$ emitted by B and absorbed by A (see Figs. 1 and 3b, and Review I).

The already mentioned fact (Sect. 2.2) that the Stückelberg - Feynman-Sudarshan "switching" exchanges the roles of source and detector (or, if you want, of "cause" and "effect") led to a series of apparent "causal paradoxes" (see e.g. Thoules (63), Rolnick (64,65) et al (66), Strnad (67), Strnad and Kodre (67)) which —even if easily solvable, at least in microphysics (Caldirola and Recami (4) and refs. therein, Maccarrone and Recami (50,58); see also Recami (3,10,51) and refs. therein, Trefil (68), Recami and Modica (26), Csonka (69), Baldo et al (24), Sudarshan (70), Bilaniuk and Sudarshan (71), Feinberg (19), Bilaniuk et al (2))— gave rise to much perplexity in the literature.

We shall deal with the causal problem in due time (see Sect. 7), since various points should rather be discussed about tachyon mechanics, shape and behaviour, before being ready to propose and face the causal "paradoxes". Let us here anticipate that, —even if in ER the judgement about which is the "cause" and which is the "effect", and even more the very existence of a "causal connection", is *relative* to the observer—, nevertheless in microphysics the law of "retarded causality" (see our Third Postulate) remains covariant, since any observers will always see the cause to precede its effect.

Actually, a sensible procedure to introduce Ts in Relativity is postu-

build up an ER in which the validity of both postulates is enforced. Till now we have seen that such an attitude —which extends the procedure in Sect.2 to the case of tachyons— has already produced, among the others, the description within Relativity of both matter and antimatter (Ts and $\overline{\text{T}}$s, and Bs and $\overline{\text{B}}$s).

4.3 Bradyons and Tachyons. Particles and Antiparticles

In two dimensions it is possible to extend straightforwardly SR to Superluminal frames, introducing in a rigorous may,a onq, the other, the Superluminal Lorentz transformations (SLT): see e.g. ref. (25,29) and refs. therein.

Fig. 8 shows, in the energy-momentum space, the existence of *two* different "symmetries", which have nothing to do one with the other.

The symmetry particle/antiparticle is the mirror symmetry w.r.t. the axis $\underline{E} = 0$ (or, in more dimensions, to the hyperplane $\underline{E} = 0$).

The symmetry bradyon/tachyon is the mirror symmetry w.r.t. the bisectors, i.e. to the two-dimensional "light-cone".

In particular, when we confine ourselves to the proper orthochronous subluminal transformation $L^{\uparrow} \in \mathcal{L}^{\uparrow}_{+}$, the "matter" or "antimatter" character is invariant for bradyons (but not for tachyons).

We want at this point to put forth explicitly the following simple but important argumentation. Let us consider the two "most typical" generalized frames: the frame at rest, $s_o \equiv (\underline{t},\underline{x})$ and its *dual* Superluminal frame (cf. Fig.9), i.e. the frame $S'_\infty \equiv (\underline{t}',\underline{x}')$ endowed with infinite speed w.r.t. s_o. The world-line of S'_∞ will be of course superimposed to the <u>x</u>-axis. With reference to Fig. 10<u>b</u>, observer S'_∞ will consider as time-axis <u>t</u>' our <u>x</u>-axis and as space-axis <u>x</u>' our <u>t</u>-axis; and vice-versa for s_o w.r.t. S'_∞. Due to the "extended principle of relativity", observers s_o, S'_∞ have moreover to be equivalent.

In space-time (Fig.1) we shall have bradyons and tachyons going both forward and backwards in time (even if for each observer —e.g. for s_o— the particles travelling into the past have to bear negative energy, as required by our Third Postulate). The observer s_o will of course interpret all —sub- and Super-luminal— particles moving *backwards* in *his* time <u>t</u> as antiparticles; and he

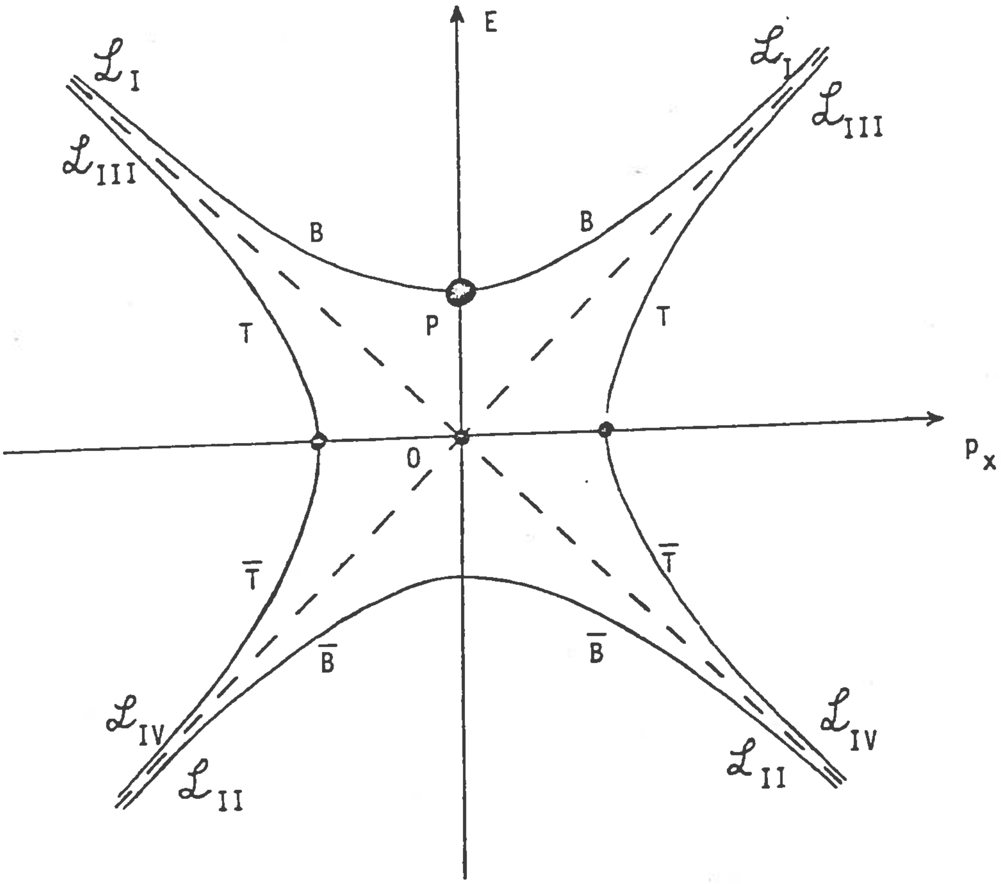

Fig.8

will be left only with objects going forward in time.

Just the same will be done, in his own frame, by obsever S'_∞ , since to him all —sub- or Super-luminal— particles travelling *backwards* in *his* time t' (i.e. moving along the negative x-direction, according to us) will appear endowed with negative energy. To see this, it is enough to remember that "the transcendent transformation" S does exchange the values of energy and momentum (see refs.(25),(29) and Review I). The same set of bradyons and tachyons will be therefore described by S'_∞ in terms of particles and antiparticles all moving along its positive time-axis t'.

But, even if axes t' and x coincide, the observer s_o will see bradyons and tachyons moving (of course) along both the positive and the negative x-axis! In other words, we have seen the following: The fact that S'_∞ sees only particles and antiparticles moving along its positive t'- axis *does not mean* at all that s_o sees only bradyons and tachyons travelling along his *positive* x-axis! This erroneous belief entered, in connection with tachyons, in the (otherwise interesting) two-dimensional approach by Antippa (72), and later on contributed to lead Antippa and Everett (73) to violate space-isotropy by conceiving that even in four dimensions tachyons had to move just along a unique, privileged direction —or "*tachyon corridor*"— .

Let us add that the *dual couples* of objects (or frames; see Fig. 9)—i.e. the couples of two objects having divergent *relative* speed— are such that their speeds v, V in a third frame correspond to each-other in a one-to-one way:

$$v \leftrightarrow V \equiv c^2/v \quad . \tag{16}$$

In such a particular conformal mapping (inversion) the speed c is the "united" one, and the speeds zero, infinite correspond to each other.

Even in more dimensions we shall call "dual" two objects (or frames) moving *along the same line* with speeds satisfying the equation

$$vV = c^2, \tag{16'}$$

i.e. with infinite relative speed. Let us notice that, if p_μ and p^μ are the energy-momentum vectors of the two objects, then the condition of infinite relative speed writes in invariant way as:

$$p_\mu p^\mu = 0 \quad . \tag{16''}$$

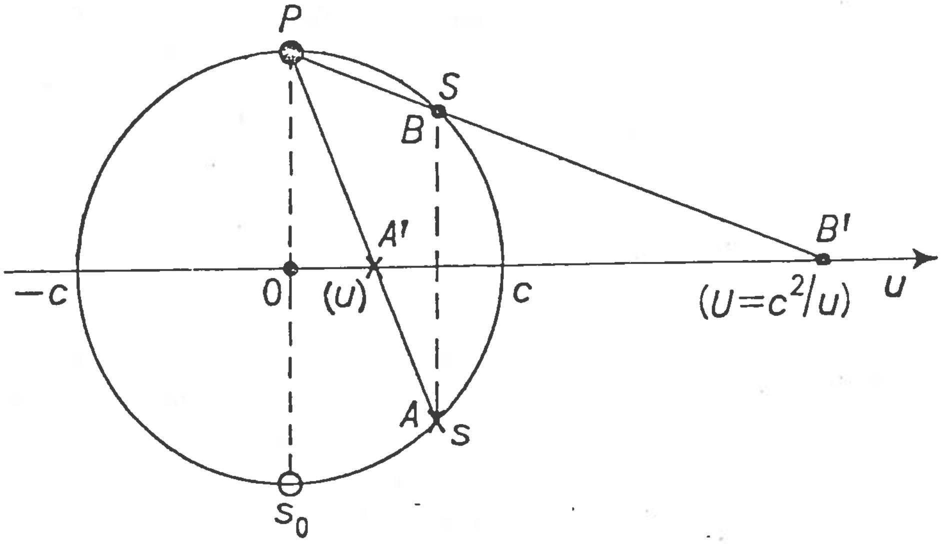

Fig.9

4.4 Laws and Descriptions. Interactions and Objects

Given a certain phenomenon ph, the principle of relativity (First Postulate) requires two different inertial observers $0_1, 0_2$ to find that ph is ruled by the same physical laws, but it does *not* require at all $0_1, 0_2$ to give the same description of ph (cf. e.g. Review I; p.555 in Recami (^{8}); p.715, Appendix, in Recami and Rodrigues (^{74})).

We have already seen in ER that, whilst the "Retarded Causality" is a *law* (corollary of our Third Postulate), the assignment of the "cause" and "effect" labels is *relative* to the observer (Camenzind (^{62})); and is to be considered a *description-detail* (so as, for instance, the observed colour of an object).

In ER one has to become acquainted with the fact that many description-details, which by chance were Lorentz-invariant in ordinary SR, are no more invariant under the GLTs. For example, what already said (see Sect.2.3, point e) with regard to the possible non-invariance of the sign of the additive charges under the transformations $\underline{L} \in \mathcal{L}_+$ holds *a fortiori* under the GLTs, i.e. in ER. Nevertheless, the total charge of an isolated system will have of course to be constant during the time-evolution of the system —i.e. to be conserved— as seen by *any* observer.

Let us refer to the explicit example in Fig. 11 (Feinberg (19), Baldo et al (24)), where the pictures (a), (b) are the different descriptions of the same interaction given by two different (generalized) observers. For instance, (a) and (b) can be regarded as the descriptions, from two *ordinary* subluminal frames $0_1, 0_2$, of one and the same process involving the tachyons a, b (c can be a photon, e.g.). It is apparent that, before the interaction, 0_1 sees one

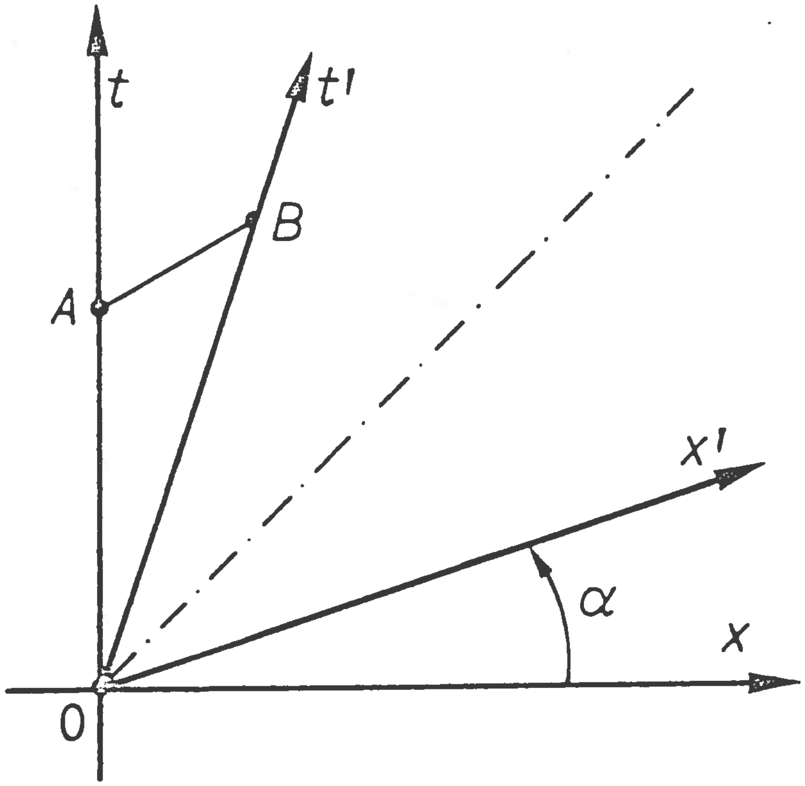

FIG. 10a

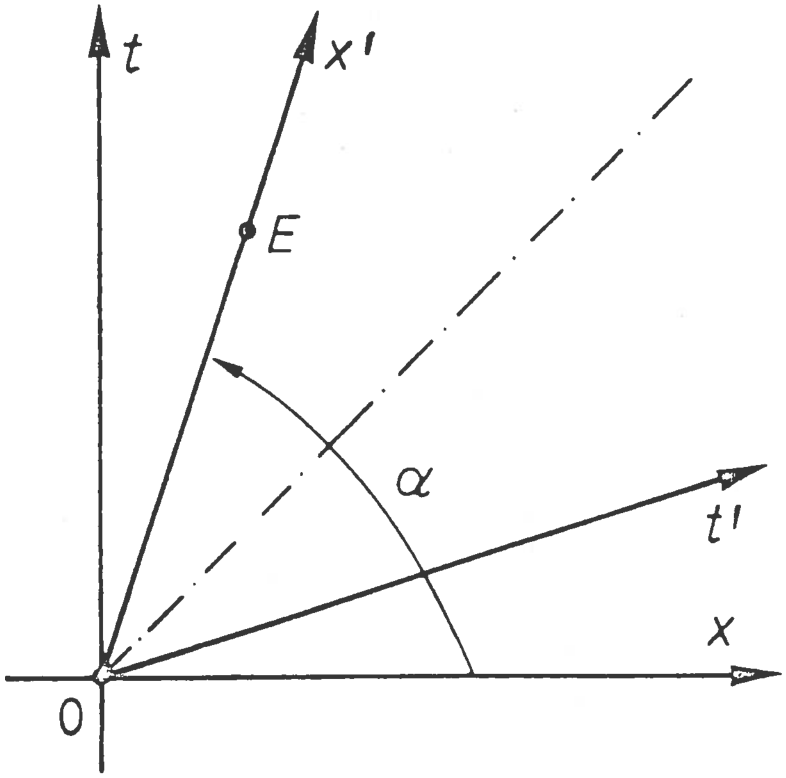

FIG. 10b

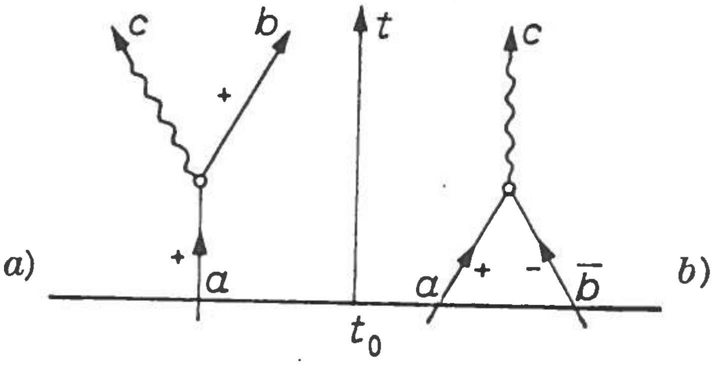

Fig.11

tachyon while 0_2 sees two tachyons. Therefore, the very number of particles —e.g. of tachyons, if we consider only subluminal frames and LTs— observed at a certain time-instant is not Lorentz-invariant. However, the *total* number of particles partecipating in the reaction either in the initial or in the final state *is* Lorentz-invariant (due to our initial three Postulates). In a sense, ER prompts us to deal in physics with interactions rather than with objects (in quantum-mechanical language, with "amplitudes" rather with "states"); (cf. e. g. Gluck (61), Baldo and Recami (75)).

Long ago Baldo *et al* (24) introduced however a vector-space $H = \mathcal{H} \otimes \bar{\mathcal{H}}$, direct procuct of two vector-spaces $\mathcal{H}$ and $\bar{\mathcal{H}}$, in such a way that any Lorentz transformation was unitary in the H - space even in presence of tachyons. The spaces $\mathcal{H}$ ($\bar{\mathcal{H}}$) were defined as the vector-spaces spanned by the states representing particles and antiparticles only in the initial (final) state.

5 - TACHYONS IN FOUR DIMENSIONS

5.1 Introduction

In four dimensions, we can start as a first step by studying the behaviour of tachyons within the "weak approach" (Sect.3.1), i.e. confining preliminarily the observers to be all subluminal. Therefore, we shall only assume the existence of sub- and Super-luminal (observed) objects. Tachyons are the space-like ones, for which in four dimensions, it is $\underline{ds}^2 \equiv \underline{dt}^2 - d\vec{x}^2 < 0$ and [$\underline{m}_o$ real] :

$$p_\mu p^\mu \equiv E^2 - \vec{p}^2 = - m_o^2 < 0. \qquad [\mu = 0,1,2,3] \qquad (17)$$

It is important to notice (25) that the results in Sect. 4 remain valid in four dimensions (see Sects. 4.1 and 2.1). provided that one takes into account that the *relevant* speed is now the component $\underline{V}_x$ of the tachyon velocity $\underline{V}$ along the (subluminal) boost-direction (Review I, Maccarrone *et al* (43) p.108, Maccarrone and Recami (29) Sect.8). Namely, if $\vec{u}$ is the (subluminal) boost - velocity, then the new observer s' will see instead of the initial tachyon T an antitachyon $\bar{T}$ travelling the opposite way ("switching principle") if and only if (Maccarrone and Recami (50)):

$$\vec{u} \cdot \vec{V} > c^2. \qquad (15')$$

Remember once more that if $\vec{u} \cdot \vec{V}$ is negative the "switching" does never come into the play.

Notice that, of course, the following results that do *not* depend on the very existence of SLTs. As an example, let us consider some tachyon kinematics.

5.2 On Tachyon Kinematics

Let us first explore the *unusual* and *unexpected* kinematical consequences of the mere fact that in the case of tachyons (see eq. (^{16})) it holds

$$|E| = +\sqrt{\vec{p}^{\,2} - m_o^2}\,, \qquad [\; m_o \text{ real; } V^2 > 1 \;], \qquad (13')$$

as partially depicted in Fig. 4.

To begin with, let us recall (Feinberg (^{19}), Dhar and Sudarshan (^{60}), Review I) that a bradyon at rest —for instance a proton p—, when absorbing a tachyon or antitachyon t, may *transform into itself*: p + t → p. This can be easily verified (see the following) in the rest-frame of the initial proton. It can be similarly verified that, in the same frame, the proton cannot decay into itself plus a tachyon. However, if we pass from that initial frame to another subluminal frame moving e.g. along the x-axis with positive speed $u \equiv u_x > 1/V_x$ (where V_x, assumed to be positive too, is the velocity x-component of t or $\bar{t}$), we know from Sect.4 that in the new frame the tachyon t *entering* the above reaction will appear as an *outgoing* antitachyon: p → p + $\bar{t}$. In other words, a proton in flight (but not at rest!) may a priori be seen to decay into itself plus a tachyon (or antitachyon).

Let us examine the *tachyon kinematics* with any care, due to its *essential role* in the proper discussion of the *causality problems*.

5.3 "Intrinsic emission" of a tachyon

Firtly, let us describe (Maccarrone and Recami (^{50,58}), and refs. therein) the phenomenon of "intrinsic emission" of a tachyon, as seen in the rest-frame of the emitting body, and in generic frames as well. Namely, let us first consider *in its rest-frame* a bradyonic body C, with initial rest-mass M, which emits towards a second bradyonic body D a tachyon (or antitachyon) T, endowed with (real) rest-mass m and 4-momentum $p \equiv (E_T, \vec{p})$ and travelling with speed V in the x - direction. Let M' be the final rest-mass of the body C. The 4-momentum conservation requires

$$M = \sqrt{\vec{p}^{\,2} - m^2} + \sqrt{\vec{p}^{\,2} + M'^2} \qquad \text{(rest-frame)} \qquad (18)$$

that is to say

$$2M\,|\vec{p}\,| = \sqrt{[m^2 + (M'^2 - M^2)]^2 + 4m^2M^2}\,, \qquad (18')$$

wherefrom it follows that a body (or particle) C cannot emit in its rest-frame any tachyon T (whatever it rest-mass m be), unless the rest-mass M of C "*jumps*" *classically* to a *lower* value $\underline{M}'$ such that $\left|E_T \equiv +\sqrt{\vec{p}^2 - m^2}\right|$:

$$\Delta \equiv M'^2 - M^2 = - m^2 - 2ME_T, \qquad \text{(emission)} \qquad (19)$$

so that

$$- M^2 < \Delta \leq - \vec{p}^2 \leq - m^2. \qquad \text{(emission)} \qquad (20)$$

Eq.(18') can read

$$V = \sqrt{1 + 4m^2M^2/(m^2+\Delta)^2}. \qquad (18'')$$

In particular, since infinite-speed Ts carry zero energy but non-zero impulse, $|p|=m_oc$, then C cannot emit any transcendent tachyon without lowering its rest-mass; in fact, in the case of infinite-speed T emission, i.e. when $E_T=0$ (in the rest-frame or C), eq.(19) yields

$$\Delta = - m^2. \qquad |V = \infty; \quad E_T=0| \qquad (21)$$

Since emission of transcendent tachyon (antitachyons) is equivalent to absorption of transcendent antitachyons (tachyons), we shall get again eq.(21) also as a limiting case of tachyon absorption (cf.eq.(27)).

It is essential to notice that Δ is, of course, an <u>invariant</u> quantity; in fact, in a generic frame $\underline{f}$ eq.(19) can be read

$$\Delta = - m^2 - 2p_\mu P^\mu, \qquad (22)$$

where P^μ is now the initial 4-momentum of body C w.r.t. the generic frame $\underline{f}$. It is still apparent that $-M^2<\Delta<-m^2$. If we recall (cf.eq.(16")) that two objects having infinite relative speed possess orthogonal 4-momenta

$$p_\mu P^\mu = 0 \qquad (16'')$$

we get again eq.(21) for the case in which T is transcendent w.r.t. body C.

<u>5.4</u>. <u>Warnings</u>

The word "emission" in eq.(20) aims at indicating —let us repeat— an <u>intrinsic</u>, "<u>proper</u>" behaviour, in the sense that if refers to emission (as seen) in the rest-frame of the emitting or particle. In suitably moving frames $\underline{f}$ such an <<emission>> can even appear as an absorption.

Conversely, other (suitably moving) frames f' can observe a T-emission from C (in flight), which does *not* satisfy inequalities (20) since it corresponds in the rest-frame of C to an (intrinsic) *absorption*.

However, if —in the moving frame f— inequalities (20) appear to be satisfied, this implies that in the C-rest-frame the process under exam is a tachyon emission, both when f observes an actual emission and when f observes on the contrary an absorption. We can state the following theorem:

Theorem 1: <<Necessary and sufficient condition for a process, observed either as the emission or as the absorption of a tachyon T by a bradyon C, to be a tachyon-emission in the C-rest-frame —i.e. to be an *''intrinsic emission''*— is that during the process C *lowers* its rest-mass (invariant statement!) in such a way that: $-M^2 < \Delta \leq -m^2$ >>, where M, m, Δ are defined above.

Let us anticipate that, in the case of "intrinsic absorption", relation (25') will hold instead of relation (20); and let us observe the following. Since the (invariant) quantity Δ in the relation (25') *can* assume also *positive* values (contrary to the case of eqs. (19)-(20)), if an observer f sees body C to *increase* its rest-mass in the process, then the "proper description" of the process can be nothing but an intrinsic absorption.

Let us stress once again that the body C, when in flight, can appear to emit suitable tachyons without lowering (or even changing) its rest-mass: In particular, a particle *in flight* can a priori emit a suitable tachyon t transforming into itself. But in such cases, if we pass in the rest frame of the initial particle, the "emitted" tachyon appears then as an *absorbed* antitachyon $\bar{t}$.

At last, when Δ in eqs. (19)÷(22) can assume only known discrete values (so as in elementary particle physics), then —once M is fixed— eq. (19) impose a link between m and E_T, i.e. between m and $|\vec{p}|$.

5.5 "Intrinsic absorption" of a tachyon

Secondly, let us consider (Maccarrone and Recami (50, 58)) our bradyon C, with rest-mass M, absorbing now *in its rest-frame* a tachyon (or antitachyon) T' endowed with (real) rest-mass m, 4 momentum $p \equiv (E_T, \vec{p})$, emitted by a second bradyon D, and travelling with speed V (e.g. along the x-direction).

The 4-momentum conservation requires that

$$M + \sqrt{\vec{p}^{\,2} - m^2} = \sqrt{\vec{p}^{\,2} + M'^2}\,, \qquad \text{(rest-frame)} \qquad (23)$$

wherefrom it follows that a body (or particle) C at rest can *a priori* absorb (suitable) tachyons both when increasing or lowering its rest-mass, and when conserving it. Precisely, eq. (23) gives

$$|\vec{p}| = \frac{1}{2M}\sqrt{(m^2+\Delta)^2 + 4m^2M^2} \qquad \text{(rest frame)} \qquad (24)$$

which corresponds to

$$\Delta = -m^2 + 2ME_T \qquad (25)$$

so that

$$-m^2 \leq \Delta < \infty. \qquad \text{(absorption)} \qquad (25')$$

Eq. (24) tells us that body C in its rest-frame can absorb T' only when the tachyon speed is

$$V = \sqrt{1 + 4m^2M^2/(m^2+\Delta)^2}. \qquad (26)$$

Notice that eq.(25) differs from eq.(19), such a difference being in agreement with the fact that, if bradyon C *moves* w.r.t. tachyon T', then —in the C-rest-frame— eq. (23) *can* transform into eq.(18): Cf. Sects. 4.1 ÷ 4.3. Eqs.(24), (26) formally coincide, on the contrary, with eqs.(18'),(18''), respectively; *but* they refer to different domains of Δ: In eq. (18'') we have $\Delta < -m^2$, while in eq. (26) we have $\Delta > -m^2$.

In particular eq.(26) yields that C can absorb (in its rest-frame) infinite-speed tachyons only when $m^2 + \Delta = 0$, i.e.

$$V = \infty \Leftrightarrow \Delta = -m^2 \qquad \text{(rest-frame)} \qquad (27)$$

in agreement with eq.(21), as expected.

Quantity Δ, of course, is again *invariant*. In a generic frame f eq. (25) can be written

$$\Delta = -m^2 + 2p_\mu P^\mu \qquad (28)$$

P^μ being now the initial C-fourmomentum in f. Still $\Delta \geq -m^2$. Notice also here that the word absorption in eq. (25') mean "intrinsic absorption", since it refers to <<absorption (as seen) in the rest-frame of the absorbing body or particle>>. This means that, if a moving observer f sees relation (25) being satisfied, the "intrinsic" description of the process, in the C-rest-frame, is a tachyon absorption, both when f observes an actual absorption and when f observes on the contrary an emission. Let us state the following theorem:

Theorem 2: <<Necessary and sufficient condition for a process, observed either as the emission or as the absorption of a tachyon T' by a bradyon C, to be a tachyon-absorption in the C-rest-frame —i.e. to be an "*intrinsic absorption*"— is that $-m^2 \le \Delta + \infty$.>> In the particular case $\Delta = 0$, one simply gets

$$2ME_T = m^2. \qquad (M' = M)$$

When Δ in eqs. (24)÷(28) can assume only known discrete values (so as in elementary particle physics) then —once M is fixed— eqs. (24)÷(28) provide a link between m and E_T (or $|p|$, or V).

5.6 Remarks

We shall now describe the *tachyon-exchange* between two bradyonic bodies (or particles) A and B, because of its importance not only for causality but possibly also for particle physics. We have to write down the implications of the 4-momentum conservation *at* A *and at* B; in order to do so we need choosing a unique frame wherefrom to describe the processes both at A and at B. Let us choose the rest-frame of A.

However, before going on, let us explicitly remark the important fact that, when bodies A and B exchange *one* tachyon T, the unusual tachyon kinematics is such that the "*intrinsic descriptions*" of the processes at A and at B (in which the process at A is described form the rest-frame of A and the process at B is now described from the rest-frame of B) can a priori be of the following *four* types (Maccarrone and Recami (50,58)):

(i) emission - absorption;
(ii) absorption - emission; (29)
(iii) emission - emission;
(iv) absorption - absorption.

Notice that the possible cases are *not* only (i) and (ii). Case (iii) can take place only when the tachyon-exchange happens in the receding phase (i. e. while A,B are receding from each other); case (iv) can take place only when the tachyon-exchange happens in the approaching phase (i.e. when A,B are approaching to each other).

Let us repeat that the descriptions (i)÷(iv) above do *not* refer to one and the same observer, but on the contrary *add together* the "local" descriptions of observers A and B.

5.7 A preliminary application

For instance, let us consider an *elastic* scattering between two (different) particles a,b. In the c.m.s., as wellknown, a and b exchange momentum but no energy. While no bradyons can be the realistic carriers of such an interaction, an infinite-speed tachyon T can be on the contrary a suitable interaction-carrier (notice that T will appear as a finite-speed tachyon in the a, b rest-frames). However, if a, b have to retain their rest-mass during the process, then the tachyon-exchange can describe that elastic process only when "*intrinsic absorptions*" take place both at a and at b (and this can happen only when a,b are approaching to each other).

5.8 Tachyon exchange when $\vec{u}\cdot\vec{V} < c^2$. Case of "intrinsic emission" at A

Let $\vec{V},\vec{u}$ be the velocities of the tachyon T and the bradyonic body B, respectively, in the rest-frame of A. And let us consider A,B to exchange a tachyon (or antitachyon) T when $\vec{u}\cdot\vec{V} < c^2$. In the rest-frame of A we can have either intrinsic emission or intrinsic absorption from the bradyonic body A. Incidentally, the case $\vec{u}\cdot\vec{V} < c^2$ includes both tachyon exchanges in the "approaching phase" (for intrinsic T emission at A), and in the "recession phase" (for intrinsic T absorption at A).

Let us first confine ourselves to the case when one observes in the A-rest-frame an (intrinsic) tachyon *emission* from A. In such a case *both* A *and* B will see the exchanged tachyon to be emitted by A and absorbed by B. In fact, the observer B would see an antitachyon $\overline{T}$ (travelling the opposite way in space w.r.t. tachyon T, according to the "switching principle") *only when* $\vec{u}\cdot\vec{V} > c^2$, whilst in the present case $\vec{u}\cdot\vec{V} < c^2$.

Imposing the 4-momentum conservation at A, we get in the A-rest-frame all the equations (18)÷(22), where for future clarity a subscript A should be introduced to identify the quantities ($M_A, M'_A, \Delta_A, P^{\mu}_A$) pertaining to A.

Let us remain in the rest-frame of A, and study now the kinematical conditions under which the tachyon T emitted by A *can* be absorbed by the second body B.

Let M_B and $P_B \equiv (M_B, \vec{P}_B)$ be rest-mass and 4-momentum of body B, respectively. Then:

$$\sqrt{\vec{P}_B^{\,2} + M_B^2} + \sqrt{\vec{p}^{\,2} - m^2} = \sqrt{(\vec{P}_B + \vec{p})^2 + M_B'^2}\,, \tag{30}$$

where M'_B is the B final mass. Let us define $\Delta_B \equiv M'^2_B - M^2_B$, which reads:

$\Delta_B = -\underline{m}^2 + 2\underline{\tilde{m}}\,\underline{\tilde{M}}_B(1-\underline{u}\,\underline{V}\cos\alpha)$, where $\underline{\tilde{m}} \equiv \underline{E}_T$, $\underline{\tilde{M}}_B \equiv \underline{E}_B \equiv \sqrt{\vec{P}_B^2 + \underline{M}_B^2}$ are the relativistic masses of T and B, and $\alpha = \widehat{\vec{u}\vec{V}}$. The invariant quantity Δ_B in a generic frame $\underline{f}$ would be written

$$\Delta_B = -m^2 + 2\,p_\mu P_B^\mu \qquad (31)$$

with p_μ, P_B^μ the T and B fourmomenta in $\underline{f}$. At variance with the process at A (intrinsic emission: eq.(19)), now Δ_B can a priori be both negative and positive or null:

$$-m^2 \leq \Delta_B < +\infty\,. \qquad \text{(intrinsic absorption)} \qquad (32)$$

Notice that, if relation (32) is verified, then the process at B will appear in the B-rest-frame as an (intrinsic) absorption, whatever the description of the process given by $\underline{f}$ may be. Of course the kinematics associated with the eq.(30) is such that Δ_B can even be smaller than $-\underline{m}^2$; but such a case $[\underline{u}\,\underline{V}\cos\alpha > 1]$ would correspond to intrinsic emission at B (and no more to intrinsic absorption).

In conclusion, the tachyon exchange here considered is allowed when in the A-rest-frame the following equations are simultaneously satisfied:

$$\left\{\begin{array}{l} \Delta_A = -m^2 - 2\,M_A E_T \\ \\ \Delta_B = -m^2 + 2\,E_T E_B(1-\vec{u}\,.\,\vec{V})\,, \end{array}\right. \qquad (33)$$

with

$$-M_A^2 < \Delta_A < -m^2\,;\ \Delta_B > -m^2. \qquad (33')$$

When B is at rest w.r.t. A we recover Sect. 5.5.

Differently from Δ_A, quantity Δ_B can even vanish; in this case the second of eqs.(33) simplifies into $2\underline{E}_T\underline{E}_B\,(1-\vec{u}.\vec{V}) = \underline{m}^2$. In the very particular case when both P_B and Δ_B are null, we get $\underline{V} = \sqrt{1+4\underline{M}_B^2/\underline{m}^2}$. Further details can be found in Maccarrone and Recami (50), which constitutes a basis also for Sects.5.9 ÷ 5.12.

5.9 The case of "intrinsic absorption" at A (when $\vec{u}.\vec{V} \leq c^2$)

Let us consider tachyon exchanges such that the process at A appears, in the A rest-frame, as an (intrinsic) absorption. The condition $\vec{u}.\vec{V} \leq c^2$ then implies body B to appear as *emitting* the tachyon T both in the A-rest-frame and in its own rest-frame.

The present case, therefore, is just the symmetrical of the previous one (Sect.5.8); the only difference being that we are now in the rest-frame of the *absorbing* body A. In conclusion, this tachyon-exchange is allowed when eqs.(33) are simultaneously satisfied, but with

$$\Delta_A \geq -m^2; \quad -M_B^2 < \Delta_B \leq -M^2. \tag{34}$$

In the particular case in which B moves along the same motion-line than T (along the x-axis, let us say), so that $\vec{P}_B // (\pm\vec{p})$, then

$$2M_B^2 |\vec{p}| = E_B \sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} \mp (m^2 + \Delta_B) |\vec{P}_B|; \; [\vec{P}_B // (\pm \vec{p})] \tag{35}$$

whilst for the analogous situation of the case in Sect.6.8 we would have obtained owing to evident symmetry reason (35) with opposite signs in its r.h.s. Moreover, when B is at rest w.r.t. body A, so that $\vec{P}_B = 0$, we recover *mutatis mutandis* eq.(18'), still with $-M_B^2 < \Delta_B < -m^2$.

5.10 Tachyon exchange with $\vec{u}.\vec{V} \geq c^2$. Case of "intrinsic emission" at A

Still in the A-rest-frame, let us now consider A,B to exchange a tachyon T when $\vec{u}.\vec{V} \geq c^2$. Again we can have either "intrinsic emission" or "intrinsic absorption" at A. The present cases differ from the previous ones (Sects. 5.8, 5.9) in the fact that now —due to the "switching procedure" (cf. the Third Postulate)— any process described by A as T emission at A and a T absorption at B is described in the B-rest-frame as a $\overline{T}$ absorption at A and a $\overline{T}$ emission at B, respectively.

Let us analyse the case of "intrinsic emission" by body A. Due to the condition $\vec{u}.\vec{V} > c^2$ (cf. eq.(15')) and to the consequent "switching", in the rest-frame of B one then observes an antitachyon $\overline{T}$ absorbed by A. *Necessary condition* for this case to take place is that A,B be *receding* one from the other (i.e., be in the "recession phase").

In any case, for the process at A (in the A-rest-frame) we get the same kinematics already expounded in Sects.5.8 and 5.3.

As to the process at B, in the A rest-frame the body B is observed to absorb a tachyon T, so that eq.(30) holds. In the B rest-frame, however, one observes an (intrinsic) $\bar{T}$ emission, so that Theorem 1 is here in order: Namely, $-M_B^2 < \Delta_B \le -m^2$. Notice that, when passing from the A to the B rest-frame (and applying the switching procedure), in eq.(30) one has: i) that quantity E_T changes sign, so that quantity $\sqrt{\vec{p}^2 - m^2}$ appears added to the r.h.s., and no longer to the l.h.s.; ii) that the tachyon 3-momentum $\vec{p}$ changes sign as well (we go in fact from a tachyon T with impulse $\vec{p}$ to its antitachyon $\bar{T}$ with impulse $-\vec{p}$).

In conclusion, the tachyon exchange is kinematically allowed when the two eqs.(30) are simultaneously verified, but now with

$$-M_A^2 < \Delta_A \le -m^2; \qquad -M_B < \Delta_B \le -m^2. \tag{36}$$

In the particular case when $\vec{P}_B$ and $\vec{p}$ are collinear (we can have only $\vec{P}_B // \vec{p}$: recession phase), we get

$$2M_B^2 |\vec{p}| = E_B \sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} + (m^2 + \Delta_B)|\vec{p}|; \qquad [\vec{P}_B // \vec{p}] \tag{37}$$

with Δ_B in the range given by eq.(36).

5.11 The case of "intrinsic absorption" at A (when $\vec{u}.\vec{V} \ge c^2$)

Due to present condition $\vec{u}.\vec{V} \ge c^2$, and to the consequent "switching", if we observe the body A in its own rest-frame to absorb (intrinsically) a tachyon T, then in the B-rest-frame we shall observe an antitachyon $\bar{T}$ emitted by A. *Necessary condition* for this case to take place is that A,B be *approaching* to each other (i.e., be in the "approaching phase").

In any case, for the process at A in the A-rest-frame we obtain the same kinematics as expounded in Sects.5.9 and 5.5. As to the process at B, in the A-rest-frame the body B is observed to emit a tachyon T:

$$\sqrt{\vec{P}_B^2 + M_B^2} = \sqrt{\vec{p}^2 - m^2} + \sqrt{(\vec{P}_B - \vec{p})^2 + M_B'^2}; \tag{38}$$

in the B-rest-frame, however, one would observe an (intrinsic) $\bar{T}$ absorption, so that it must be $\Delta_B \ge -m^2$.

In conclusion, the present tachyon exchange is kinematically allowed when eqs.(33) are satisfied, but now with

$$\Delta_A \geqslant -m^2; \; \Delta_B \geqslant -m^2. \tag{39}$$

In the particular case in which $\vec{P}_B$ and $\vec{p}$ are collinear, we can have only $(-\vec{P}_B)//\vec{p}$ (approaching phase), and we get

$$2M_B^2\,|\vec{p}| = E_B\,\sqrt{(m^2+\Delta_B)^2 + 4m^2M_B^2} - (m^2+\Delta_B)\,|\vec{P}_B|, \qquad [\vec{P}_B//(-\vec{p})] \quad (40)$$

with $\Delta_B \geqslant -\underline{m}^2$.

Finally, let us recall that in the present case ("intrinsic absorptions" at B and at A) both quantities Δ_A, Δ_B *can* vanish. When $\Delta_A = 0$, we simply get $2\underline{M}_A\underline{E}_T = \underline{m}^2$. In the particular case when $\Delta_B = 0$ one gets: $2\underline{E}_T\underline{E}_B\,(\vec{u}\cdot\vec{V} - 1) = \underline{m}^2$, and then: $|\vec{p}| = (\underline{m}/2\underline{M}_B^2)\,[\underline{E}_B(m^2 + 4\underline{M}_B^2)^{1/2} - \underline{m}\,|\vec{P}_B|]$.

5.12 Conclusions about the tachyon exchange

With regard to the process at B, the kinematical results of Sects. 5.8 ÷ 5.11 yield what follows:

$$\vec{u}\cdot\vec{V} \leq c^2: \qquad \Delta_B = \begin{cases} \nearrow\; -m^2 + 2\,p_\mu P_B^\mu \geq -m^2; \\ \searrow\; -m^2 - 2\,p_\mu P^\mu \leq -m^2; \end{cases} \qquad (41\underline{a})$$

$$\vec{u}\cdot\vec{V} \geq c^2: \qquad \Delta_B = \begin{cases} \nearrow\; -m^2 + 2\,p_\mu P^\mu \leq -m^2; \\ \searrow\; -m^2 - 2\,p_\mu P^\mu \geq -m^2. \end{cases} \qquad (41\underline{b})$$

More in general, the kinematical *conditions* for a tachyon to be *exchangeable* between A and B can be summarized as follows (notice that the case $\vec{u}\cdot\vec{V} < \underline{c}^2$ *includes* of course the case $\vec{u}\cdot\vec{V} < 0$):

a) in the case of "intrinsic emission" at A:

$$\begin{cases} \vec{u}\cdot\vec{V} < c^2 \Rightarrow \Delta_B > -m^2 \Rightarrow \text{intrinsic absorption at B;} \\ \vec{u}\cdot\vec{V} > c^2 \Rightarrow \Delta_B < -m^2 \Rightarrow \text{intrinsic emission at B;} \end{cases} \qquad (42)$$

b) in the case of "intrinsic absorption" at A:

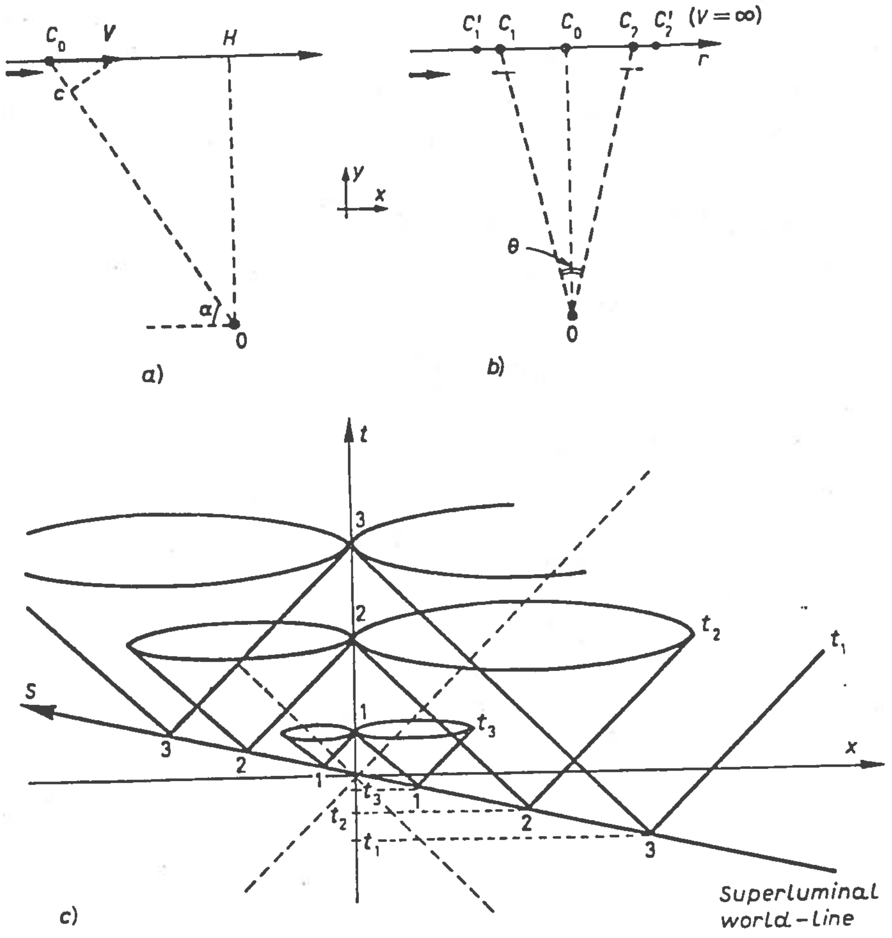

Fig.12

$$\begin{cases} \vec{u}\cdot\vec{V} < c^2 \Rightarrow \Delta_B < -m^2 \Rightarrow \text{intrinsic emission at B;} \\ \vec{u}\cdot\vec{V} > c^2 \Rightarrow \Delta_B > -m^2 \Rightarrow \text{intrinsic absorption at B.} \end{cases} \qquad (43)$$

5.13 On radiating tachyons

Many other results, actually independent of the very existence of SLTs, will appear in the following Sections.

Here, as a further example, let us report the fact that a tachyon —when seen by means of its electromagnetic emissions (see the following, and Review I, Baldo et al (24)— will appear in general as occupying *two positions* at the same time (Recami (76,77,10,8), Barut et al (44); see also Grøn (78)). Let us start by considering a macro-object C emitting spherical electromagnetic waves (Fig.12c). When we see it travelling at constant Superluminal velocity $\vec{V}$, because of the distortion due to the large relative speed $|\vec{V}| > \underline{c}$, we shall observe the electromagnetic waves to be internally tangent to an enveloping cone Γ having as its axis the motion-line of C (Recami and Mignani (45), Review I); even if this cone has nothing to do with Cherenkov's (Mignani and Recami (79)). This is analogous to what happens with an airplane moving at a constant supersonic speed in the air. A first observation is the following: as we hear a sonic boom when the sonic contact with the supersonic airplane does start (Bondi (80)), so we shall analogously *see* an "optic boom" when we first enter in radio-contact with the body C, i.e. when we meet the Γ-cone surface. In fact, when C is seen by us under the angle (Fig.12a)

$$V \cos \alpha = c \qquad [V \equiv |\vec{V}|] \tag{44}$$

all the radiations emitted by C in a certain time-interval around its position C_o reach us simultaneously. Soon after, we shall receive at the same time the light emitted from suitabel *couples of points*, one on the left and one on the right of C_o. We shall thus see the initial body C, at C_o, split in two luminous objects C_1, C_2 which will then be observed to recede from each other with the Superluminal "transverse" relative speed $\underline{W}$ (Recami *et al* (81), Barut *et al.* (44)):

$$W = 2b \, \frac{1 + d/bt}{[1 + 2d/bt]^{1/2}} ; \quad b \equiv \frac{V}{\sqrt{V^2 - 1}} , \quad [V^2 > 1] \tag{45}$$

where $d \equiv \overline{OH}$, and $t = 0$ is just the time-instant when the observer enters in radiocontact with C, or rather sees C at C_o. In the simple case in which C moves with almost infinite speed along $\underline{r}$ (Fig.12b), the apparent *relative* speed of C_1 and C_2 varies in the initial stage as $\underline{W} \simeq (2c\underline{d}/\underline{t})^{1/2}$, where now $\overline{OH} = \overline{OC}_o$ while $t = 0$ is still the instant at which the observer sees $C_1 \equiv C_2 \equiv C_o$.

Let us add the observation that the radiation associated with one of the images of C (namely, the radiation emitted by C while approaching us, in the simple case depicted in Fig.12c) will be received by us in the *reversed* chronological order; cf. Mignani and Recami (82), Recami (77).

It may be interesting to quote that the circumstance, that the image of a tachyon suddenly appears at a certain position C_o and then splits into two images, was already met by Bacry (83) and Bacry *et al.* (84) while exploiting a group-theoretical definition of the motion of a charged particle in a homogeneous field; definition which was valid for all kind of particles (bradyons, luxons, tachyons). Analogous solutions, simulating a pair-production, have been later on found even in the *subluminal* case by Barut (85), when exploring *non-linear* evolution equations, and by Sala (86), by merely taking account of the finite speed of the light which carries the image of a moving subluminal object. Sala (86) did even rediscover —also in subluminal cases— that one of the two images can display a time-reversed evolution.

At this point, we might deal with the problem of *causality* for tachyons (since the most relevant aspects of that problem do arise w.r.t. the class of the *subluminal observers*). We shift such a question, however, to Sect.7,because we want preliminarily to touch the problem of tachyon localization.

6. ON TACHYON LOCALIZATION

We have already noticed that a tachyon —observed by means of its light-signals— will generally appear as occupying two positions at the same time (Sect. 5.14, and Figs.12).

Still at a preliminary level, let us moreover recall that free bradyons always admit a particular class of subluminal reference-frames (their rest-frames) wherefrom they appear —in Minkowski space-time— as "points" in space extended in time along a line. On the contrary, free tachyons always admit a particular class of subluminal (w.r.t. us) reference-frames —the critical frames— wherefrom they appear with divergent speed $\underline{V} = \infty$, i.e. as "points" in time extended in space along a line (cf. Figs. 7, 10). Considerations of this kind correspond to the fact that the "localization" groups (little groups) of the timelike and spacelike representations of the Poincaré group are SO(3) and SO(2,1), respectively (see e.g. Barut (87)), so that tachyons are not expected to be localizable in our ordinary *space* (cf. also Peres (88), Souček (89)).

It is therefore necessary to study the *shape* of tachyons in detail. This has been already done, however, in Barut *et al.* (44), and in Recami and Maccarrone (90), and will not be repeated here. We simply refer to the results in Refs. (44,90).

We want ,however, to face the causality problem for tachyons in Relativity only after having at least made clear that tachyons are not trivially localizable in the ordinary space. Actually a tachyon T is moresimilar to a field than to a particle (25). There are reasons however to believe that, in general, most of the tachyon mass be concentrated near the center C of T [see Ref. (44, 90)], so that in the following we shall regard tachyons as "almost localized" in space. In what follows, therefore , we shall essentially make recourse only to the results in Sects 4.1÷4.3 (which, incidentally, have been seen to hold also in four dimensions) and to our results about tachyon kinematics (Sect. 5). As mentioned above, we shall confine ourselves only to the subluminal observers (in presence , of course, of both bradyons and tachyons) and, for simplicity, to the orthochronous Lorentz transformations only.

The results in Sects.4.1÷4.3, in particular, showed us that each observer will *always* see only tachyons (and antitachyons) moving with positive energy *forward in time*. As expounded in Sects.4.2 and 4.4, however, this success is obtained at the price of releasing the old conviction that judgement about what is "cause" and what is "effect" is independent of the observer; in Sect.4.4 we concluded that the assignment of the "source" and "detector" labels is to be regarded as a *description-detail*. As anticipated in Sect.4.2, this fact le to the proposal of a series of seeming "causal paradoxes", that we are going to discuss and (at least "in microphysics") to solve.

7. THE CAUSAL PROBLEM

As mentioned at the end of Sect.5.1, the discussion that will follow in this Sect. 7 is *independent of the very existence of the* SLTs, since the most relevant causal problems arise when describing tachyons (and bradyons) from the ordinary subluminal frames.

7.1 Solution of the Tolman-Regge Paradox

The oldest paradox is the "anti-telephone" one, originally proposed by Tolman(1) (see also Bohm (91)) and then reproposed by many authors.

Let us refer to its most recent formulation (Regge (92)), and spend some care in solving it since it is the kernel of many other paradoxes.

7.1.1. The paradox - In Figs.13 the axes t and t' are the world-lines of two devices A and B respectively, able to exchange tachyons and moving with constant relative speed u, ($u^2 < 1$). According to the terms of the paradox (Fig. 13a), A sends tachyon 1 to B (in other words, tachyon 1 is supposed to move forward in time w.r.t. A). The apparatus B is constructed so to send back a tachyon 2 to A as soon as it receives a tachyon 1 from A. If B has to emit (in its rest-frame) tachyon 2, then 2 must move forward in time w.r.t. B, that is to say its world-line BA_2 must have a slope smaller than the x-axis slope BA' (where $BA' // x'$); this means that A_2 must stay above A'. If the speed of tachyon 2 is such that A_2 falls between A' and A_1, it seems that 2 reaches back A (event A_2) *before* the emission of 1 (event A_1). This appears to realize an *anti-telephone*.

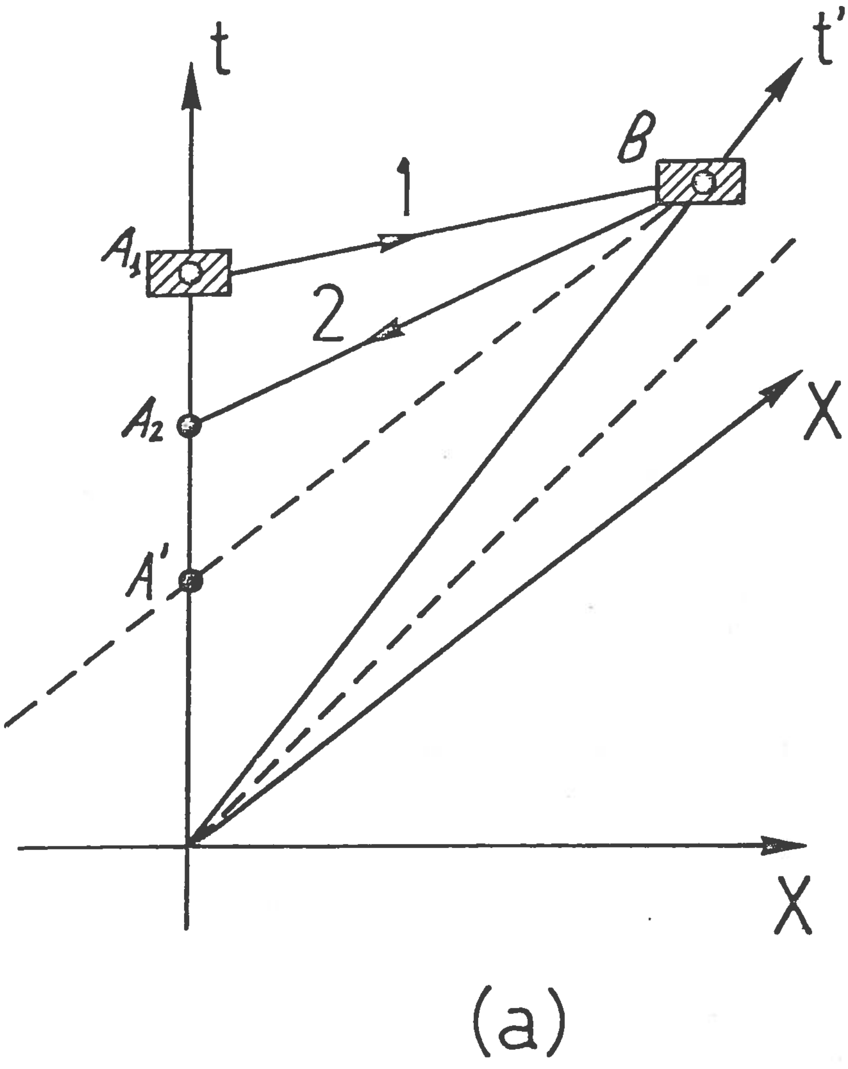

Fig.13a

7.1.2. The solution - First of all, since tachyon 2 moves backwards in time w.r.t. A, the event A_2 will appear to A as the emission of an antitachyon $\bar{2}$. The observer "t" will see his apparatus A (able to exchange tachyons) emit successively towards B the antitachyon $\bar{2}$ and the tachyon 1.

At this point, some supporters of the paradox (overlooking tachyon kinematics, as well as relations (29)) would say that, well, the description forwarded by observer "t" can be orthodox, but then the device B is no more working according to the premises, because B is no more emitting a tachyon 2 on receipt of tachyon 1. Such a statement would be wrong, however, since the fact that "*t*" sees an "intrinsic emission" at A_2 *does not mean* that "*t'*" will see an "intrinsic absorption" at B. On the contrary, we are just in the case of Sect.5.10: intrinsic emission by A, at A_2, with $\vec{u}\cdot\vec{V}_{\bar{2}} > c^2$, where $\vec{u}$ and $\vec{V}_{\bar{2}}$ are the velocities of B and $\bar{2}$ w.r.t. A, respectively; so that *both* A *and* B suffer an intrinsic *emission* (of tachyon 2 or of antitachyon $\bar{2}$) in their own rest-frames.

But the terms of the paradox were cheating us even more, and *ab initio*. In fact Fig.13*a* makes clear that, if $\vec{u}\cdot\vec{V}_{\bar{2}} > c^2$, then for tachyon 1 *a fortiori* $\vec{u}\cdot\vec{V}_1 > c^2$, where $\vec{u}$ and $\vec{V}_1$ are the velocities of B and 1 w.r.t. A. Due to Sect. 5.10, therefore, observer "*t'*" will see B intrinsically *emit* also tachyon 1 (or, rather, antitachyon $\bar{1}$). In conclusion the proposed chain of events does not include any tachyon absorption by B.

For body B to *absorb* tachyon 1 (in its own rest-frame), the world-line of 1 ought to have a slope larger than the *x*'-axis slope (see Fig.13*b*). Moreover, for body B to *emit* ("intrinsically") tachyon 2, the slope of 2 should be smaller than *x*'-axis'. In other words, when the body B, programmed to emit 2 as soon as it receives 1, does actually do so, the event A_2 does regularly happen *after* A_1 (cf. Fig. 13*b*).

7.1.3. The moral - The moral of the story is twofold: (i) one should never mix together the descriptions of one phenomenon yielded by different observers, otherwise —even in ordinary physics— one would immediately meet contradictions: in Fig. 13*a*, e.g., the motion-direction of 1 is assigned by A and the motion-direction of 2 is assigned by B; this is illegal; (ii) when proposing a problem about tachyons, one must comply (Caldirola and Recami (4)) with the rule of tachyon kinematics (Maccarrone and Recami (50)), just as when formulating the text of an ordinary problem one must comply with the laws of ordinary physics (otherwise the problem in itself is "wrong").

Most of the paradoxes proposed in the literature suffered the shortcomings above.

Notice that, in the case of Fig. 13 *a*, neither A nor B regard event A_1 as the cause of event A_2 (or vice-versa). In the case of Fig. 13 *b*, on the contrary, both A and B consider event A_1 to be the cause of event A_2: but in this case A_1 does chronologically *precede* A_2 for both observers, in agreement with the relativistic covariance of the Law of Retarded Causality. We shall come back to such considerations.

7.2 Solution of the Pirani Paradox

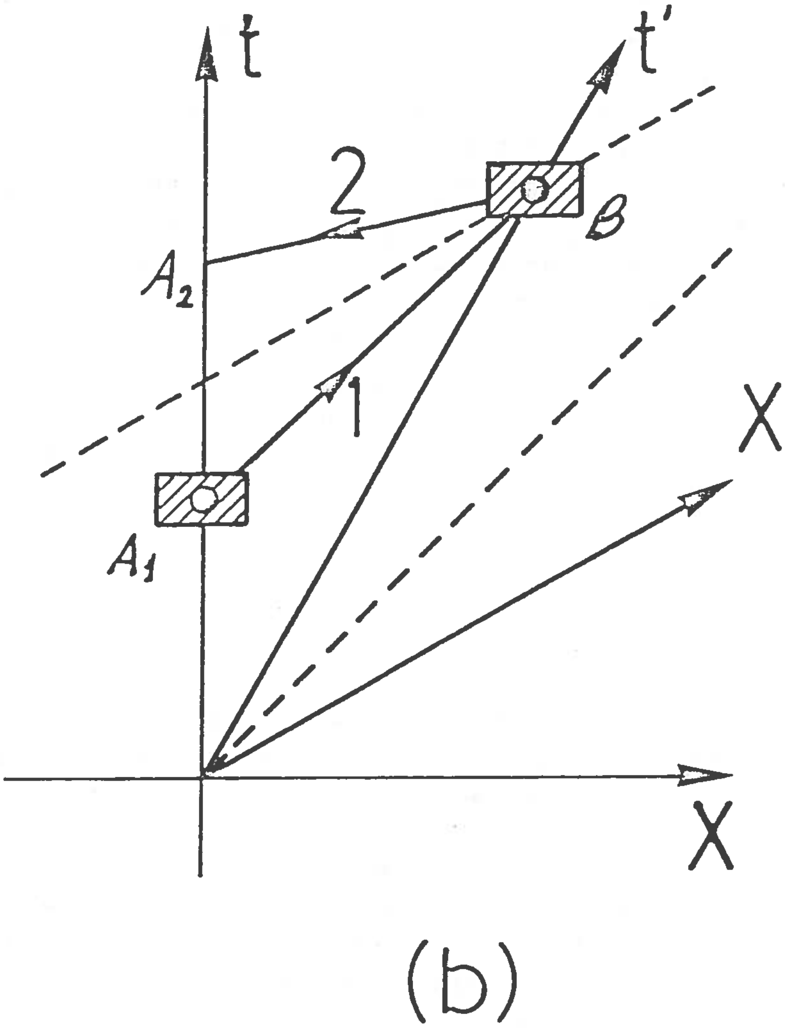

Fig.13b

A more sophisticated paradox was proposed, as wellknown, by Pirani (93) . It was substancially solved by Parmentola and Yee (44), on the basis of the ideas initially expressed by Sudarshan (70,95), Bilaniuk and Sudarshan (71) , Csonka (69), etc.

7.2.1. The paradox - Let us consider four observers A,B,C,D having given velocities in the plane ($\underline{x},\underline{y}$) w.r.t. a fifth observer s_o. Let us imagine that the four observers are given in advance the instruction to emit a tachyon as soon as they receive a tachyon from another observer, so that the following chain of events (Fig.14)

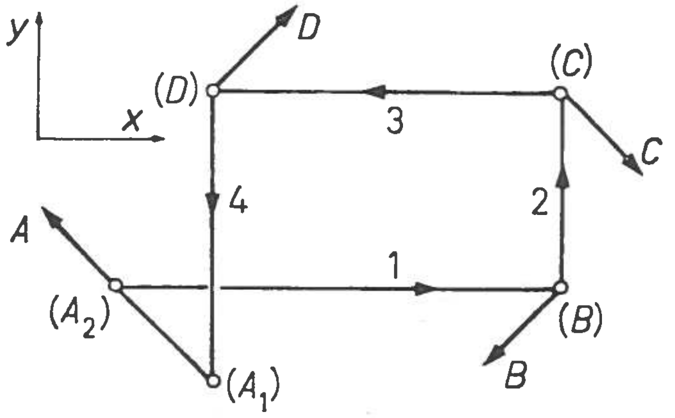

Fig.14

takes place. Observer A initiates the experiment by sending tachyon 1 to B; observer B immediately emits tachyon 2 towards C; observer C sends tachyon 3 to D; and observer D sends tachyon 4 back to A, with the result —according to the paradox— that A receives tachyon 4 (event $\underline{A}_1$) *before* having initiated the experiment by emitting tachyon 1 (event $\underline{A}_2$). The sketch of this "Gedankenexperiment" is in Fig. 14, where oblique vectors represent the observer velocities w.r.t. s_o and lines parallel to the Cartesian axes represent the tachyon paths.

7.2.2. The solution - The above paradoxical situation arises once more by mixing together observations by four different observers. In fact, the arrow of each tachyon line simply denotes its motion direction w.r.t. the observer which emitted it. Following the previous Sect. 7.1, it is easy to check that Fig. 14 does not represent the actual description of the process by any observer. It is necessary to investigate, on the contrary, how each observer describe the event chain.

Let us pass, to this end, to the Minkowski space-time and study the description given e.g. by observer A. The other observers can be replaced by objects (nuclei, let us say) able to absorb and emit tachyons. Fig. 15 shows that the absorption of 4 happens *before* the emission of 1; it *might seem* that one can send signals into the past of A. However (cf. Sects. 4.1 ÷ 4.3 and Sect. 5,

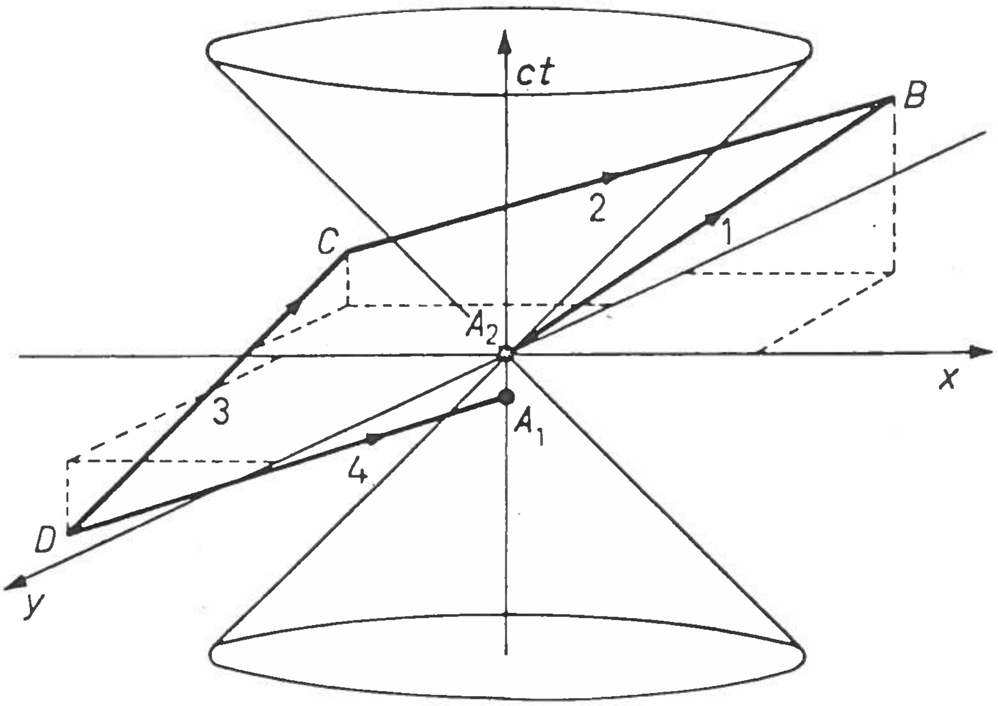

FIG. 15

as well as Recami (51,10), observer A will actually see the sequence of events in the following way: The event at $\underline{D}$ consists in the creation of the pair $\bar{3}$ and 4 by the object D; tachyon 4 is then absorbed at $\underline{A}_1$, while $\bar{3}$ is scattered at $\underline{C}$ (transforming into tachyon $\bar{2}$); the event $\underline{A}_2$ is the emission, by A itself, of tachyon 1 which annihilates at $\underline{B}$ with tachyon $\bar{2}$. Therefore, according to A, one has an initial pair-creation at $\underline{D}$, and a final pair-annihilation at $\underline{B}$, and tachyons 1, 4 (as well as events $\underline{A}_1$, $\underline{A}_2$) do *not* appear causally correlated at all. In other words, according to A, the emission of 1 does not initiate any chain of events that brings to the absorption of 4, and we are *not* in the presence of any "effect" preceding its own "cause".

Analogous, orthodox descriptions would be forwarded by the other observers. For instance, the tachyons and observers velocities chosen by Pirani (93) are such that all tachyons will actually appear to observer s_o as moving in directions *opposite* to the ones shown in Fig. 14.

7.2.3. Comments - The comments are the same as in the previous Sect.7.1. Notice that the "ingredients" that allow us to give the paradox a solution are always the "switching principle" (Sect 4.1 ; see also Schwartz (56)) and the tachyon relativistic kinematics (Sect. 5).

7.2.4. "Strong version" and its solution - Let us formulate Pirani's paradox in its strong version. Let us suppose that tachyon 4, when absorbed by A at $\underline{A}_1$, blows up the whole lab of A, eliminating even the physical possibility that tachyon 1 (believed to be the sequence starter) is subsequently emitted (at $\underline{A}_2$). Following Root and Trefil (68) we can see on the contrary how, e.g., observers s_o and A will really describe the phenomenon.

Observer s_o will see the lab of A blow up after emission (at $\underline{A}_1$) of the antitachyon $\bar{4}$ towards D. According to s_o, therefore, the antitachyon $\bar{1}$ emitted by B will proceed beyond A (since it is not absorbed at $\underline{A}_2$) and will eventually be absorbed at some remote sink-point $\underline{U}$ of the universe. By means of a LT, starting from the description by s_o, we can obtain (Caldirola and Recami (4)) the description given by A.

Observer A, after having absorbed at $\underline{A}_1$ the tachyon 4 (emitted at $\underline{D}$ together with $\bar{3}$), will record the explosion of his own laboratory. At $\underline{A}_2$, however, A will cross the flight of a tachyonic "cosmic ray" 1 (coming from the remote source $\underline{U}$), which will annihilate at $\underline{B}$ with the antitachyon $\bar{3}$ scattered at $\underline{C}$, i.e. with the antitachyon $\bar{2}$.

7.3 Solution of the Edmonds Paradox

The seeming paradoxes arising from the relativity of the judgment about "cause" and "effect" have been evidenced by Edmonds (96) in a clear (and amusing) way, with reference to the simplest tachyon process: the exchange of tachyons between two ordinary objects, at rest one w.r.t. the other.

7.3.1. The paradox - We build a long rocket-sled with a "tachyon-laser" at the left end and a "target-flower" at the right end. A short lever sticks out of the side of the "laser". If we trip the lever, the tachyon laser emits a very sharp, intense burst of tachyons for which we measure the speed of, let's say, $\underline{V}$. These tachyons then hit the "flower" and blast in into pieces. The flower absorbs all the tachyons in the pulse as it explodes, so that the tachyons disappear.

Now we accelerate the sled (with "charged" tachyon-laser and flower attached to it) up to an incoming speed of $-\underline{v} \equiv -\underline{v}_x$ relative to our frame, and then turn off its rocket engines. Moreover, we form a long line of "astronauts" floating in space along the $\underline{x}$-axis (i.e. along the rocket-sled motion-line). Each astornaut has a "roulette wheel" in his one hand, and keeps spinning his gambling wheel untill he gets, say, the number 13. When he happens to do so, he quickly put out a stick in front of him which *could* beat the trigger on the moving laser. No one in our frame knows when a given astronaut will get 13 to come up. Some astronauts may get 13, but too far down the line, or find the trigger has already passed them when they get it. But, finally, someone gets the right number, puts out his stick , finds that the lever is almost at his position and he triggers the laser.

Once the laser fires, the observer travelling with the sled sees — so as before— a burst of tachyons actually travelling from the laser to the flower. If the sled is moving slowly enough ($\underline{vV} < c^2$), then we also —together with the "astronauts"— see the flower blow up at a time *later* then the time at which the laser fires. However, if the sled is fast enough ($\underline{vV} > \underline{c}^2$), we see a pulse of antitachyons going from the flower to the laser: Namely, we *would see* the flower to blow up before the laser fires. Therefore, the astronaut who triggers the laser sees the laser immediately "swallowing" a pulse of antitachyons coming from the flower. In other words, the lucky astronaut will conclude that the flower had to *know in advance* who was going to get 13 (so that it can blow up and create the antitachyon-pulse just at the right time, in order for the beam to arrive at the lucky astronaut as he gets the number 13 to come up for him).

7.3.2. The solution - Since "source" and "detector" are supposed by Edmonds to be at rest one w.r.t. the other, according to both laser and flower — i.e. in the lab — there are *no* problems about the flight-direction of the tachyons. However, if we choose *other* observers (as the astronauts) they will in reality see the laser absorb antitachyons $\bar{T}$ coming from the flower (and *not* to fire tachyons T towards the flower). We have simply to accept it, since we learned (cf. e.g. Sect. 4.4) that only the "principle of retarded causality" (Third Postulate) is a *law*, and therefore has to be valid for each observer; whilst the assignement of the labels "source" and "detector" is a *description*-detail, which does not have to be relativistically invariant.

Then, to answer Edmonds (see Recami (97)), let us show by an example that *seeming* paradoxes as the one above arise also in ordinary Special Relativity (due to the Lorentz non-invariance of the descriptions). Let us therefore forget about tachyons in the following example.

Let us suppose we are informed about a cosmic fight taking place between two different kinds of extraterrestrial beings, each one driving his own rocket, where the rocket colors are *violet* for the first and *green* for the second species. Let us suppose moreover that we know the "green men" to possess an inviolable natural instinct that makes them peaceful; on the contrary the "violet men" possess an aggressive, warrior instinct. When we observe the interplanetary battle by our telescope, it *can* well happen —due to the Doppler effect, i.e. due to the "observation distorsions" caused by the relative motions— that, when a "violet man" fires his gun and strikes a green rocket, the violet color appears to us as green, and vice-versa, because of the rocket motions. Then, according to the spirit of Edmonds paradox, we should deduce that an inviolable law of nature has been badly violated (the instinctive law of those extraterrestrial beings). Within SR, however, we already know how to clarify the whole story: We "observe" at first a seeming violation of natural laws; but, if we know the relevant physics (i.e. SR and the rocket velocities), we can determine the "intrinsic (proper) colors" of the rockets in their own rest-frames, and solve any doubts.

In other words, any observer is capable of understanding the physical world in terms of his own observations only, *provided that* he is equipped with a suitable theory (he uses his knowledge of SR, in this case).

Going back to the tachyon "paradox", we conclude that the lucky astronaut, when knowing tachyon mechanics (i.e. the ER), can calculate the tachyons direction in the flower rest-frame and find out the "intrinsic behaviour" of the flower. The astronaut will find that in the flower-frame the tachyons are not emitted, but absorbed by the flower; even if the relative speed produces a high "distorsion" of the observed phenomenon. In analogy with our example, it is not important that the flower *seem* to the astronauts to precognize the future, but that the flower "intrinsically" *does not*.

The discussion of this paradox reminded us that: (i) one can scientifically observe (or *observe*, tout court) the natural world only if he is endowed with *theoretical* instruments, besides experimental and sensorial instruments: (ii) the "intrinsic properties" (so as the color) of a body appear to a moving observer distorted by the relative motion; if high relative speeds are involved, that distortion can be large as well.

Let us add a further comment.

7.3.3. Comment - In the case of a bradyon exchange, in which the roles of "source" and "detector" are independent of the observer, the emitter and receiver are well represented by a male and a female object, respectively. Such a habit is however misleading in the case of a tachyon exchange, in which the same object can now appear as the emitter, now as the receiver, depending on the observer. Devices such as "guns" and "lasers" ought to be avoided in the "Gedankenexperimente" regarding the exchange of tachyons. A round-shaped device, such as a sphere, should be the right one for representing objects able to emit/absorb tachyons.

7.4 Causality "in Micro-" and "in Macro-physics"

Let us *go on* investigating the paradoxes arising when two bradyonic objects A, B exchange tachyons T, since there we meet in nuce all the problems than one encounters in the more complicated processes.

Let us consider, namely, the situation in which "laser"(A) and "flower" (B) are no more at rest one w.r.t. the other.

Such a situation is much more problematic. Nevertheless, no real problems are actually present (cf. Sect.5) as far as the tachyon production is supposed to be a "spontaneous", uncontrollable phenomenon, just as particle production in elementary particle physics. By convention, let us refer to this as the case "of microphysics".

Problems arise, however, when the tachyon production is a priori regarded as controllable (we shall refer to this latter as the case "of macrophysics"). We are going to analyse such problems by means of two paradoxes.

The first one was proposed by Bell (98).

7.5 The Bell Paradox and its solution

7.5.1 The paradox - By firing tachyons you can commit a "perfec murder". Suppose that A purposes killing B, without risking prosecution. When he happens to see B together with a "witness" C, he aims his tachyon-pistol at the head of B, untill B and C (realizing the danger) start running away with speed, say, u. Then, A chooses to fire tachyonic projectiles T having a speed V such that $uV > c^2$. In the A rest-frame, tachyons T reach B soon and are absorbed by B's head, making him die. Due to the fact that $uV > c^2$ (and to Sects. 4.1 and 5), however, the witness C —when questioned by the police— will have to declare that actually he only saw antitachyons $\bar{T}$ come out of B's head and be finally absorbed by A's pistol. The same would be confirmed *by* B *himself*, were he still able to give testimony.

7.5.2. The solution; and comments - Let us preliminarly notice that B and C (when knowing tachyon mechanics) *could* at least revenge themselves on A by making A surely liable to prosecution: they should simply run *towards* A ! (cf. Sects. 4.1 and 5).

But let us analyse our paradox, as above expounded. Its main object is emphasizing that, when A and B are *moving* one w.r.t. the other, *both* A *and* B can observe "intrinsic emissions" in their respective rest-frames (Sect. 5.10).

It follows that it seems impossible in such cases to decide who is actually the beginner of the process; i.e., who is the cause of the tachyon exchange. There are no grounds, in fact, for privileging A or B.

In a pictoresque way —as Bell put it— it seems that, when A aims his pistol at B (which is running away) and decides to fire suitable tachyons T, then B is "obliged" to emit antitachyons $\bar{T}$ from his head and die.

To approach the solution, let us first rephrase the paradox (following the last lines of Sect.7.3) by substituing two spherical objects for A's pistol and B's head. About the properties of the emitters/absorbers of tachyons we know a priori only the results got in Sect.5; but, since this paradox simply exploits a particular aspect of the two-body interactions via tachyon exchange, we have just to refer to those results. Their teaching may be interpreted as follows, if we recall that we are assuming tachyon-production to be controllable (otherwise the paradox vanishes). The tachyon exchange takes place only when A, B possess suitable "tachyonic aptitudes", so as an electric discharge takes place between A and B only if A, B possess electrical charges (or, rather, are at different potential levels). In a sense, the couple of "spherical objects" A, B can be regarded as resembling a Van-de-Graaff generator. The tachyon-spark is exchanged between A and B, therefore, only when observer A gives his sphere (the "pistol") a suitable "tachyonic charge", or raise it to a suitable "tachyonic potential". The person responsible for the tachyon discharge between A and B (which may cause B to die) is therefore the one who intentionally prepares or modifies the "tachyonic properties" of his sphere: i.e., in the case above, it is A. In the same way, if one raises a conducting sphere A to a positive (electrostatic) potential high enough w.r.t. the earth to provoke a thunderbolt between A and a pedestrian B, he shall be the guilty murderer, even if the thunderbolt-electrons actually start form B and end at A.

Notice that we have been always considering tachyons emissions and absorptions, but never tachyon scatterings, since —while we know the tachyon mechanics for the former, simple processes— we do not know yet how tachyons *interact* with the (ordinary) matter.

7.6 Signals by modulated tachyon beams: Discussion of a Paradox

7.6.1. The paradox - Still "in macrophysics", let us tackle at last a more sophisticated paradox, proposed by ourselves (Caldirola and Recami (4)), which can be used to illustrate the most subtle hints contained in the "causality" literature (cf. e.g. Fox et al (99,100)).

Let us consider two ordinary inertial frames $s = (t, x)$ and $s' = (t', x')$ moving one w.r.t. the other along the x-direction with speed $u < c$, and let us suppose that s sends —in its own frame— a signal along the positive x-direction to s' by means of a modulated tachyon beam having speed $V > c^2/u$ (see Fig. 16). According to s' the tachyon-beam will actually appear as an antitachyon-beam emitted by s' itself towards s. We can imagine that observer s, when meeting s' at 0, hands him a sealed letter and tells him the following: << By means of my "tachyon-radio" A and starting at time t, I will transmit to your "tachyon-radio" B a multi-figured number. The number is written inside the envelope, to be opened only after the transmission >> .

Notice that the "free-will" of s' is *not* jeopardized nor under question, since s' can well decide to *not* switch on his tachyon-radio B. In such a case we would be back to the situation in Sect.7.3. In fact, s would see his tachyons T bypass s' without being absorbed and proceed further into the space ; s', on the contrary, would see antitachyons $\bar{T}$ coming from the space and reaching A. If s' knows extended relativity, he can transform his description of the phenomenon into the "intrinsic description" given by s, and find out that s is "intrinsically" emitting a signal by tachyons T. He can check that the signal carried by those tachyons T corresponds just to the number written in advance by s.

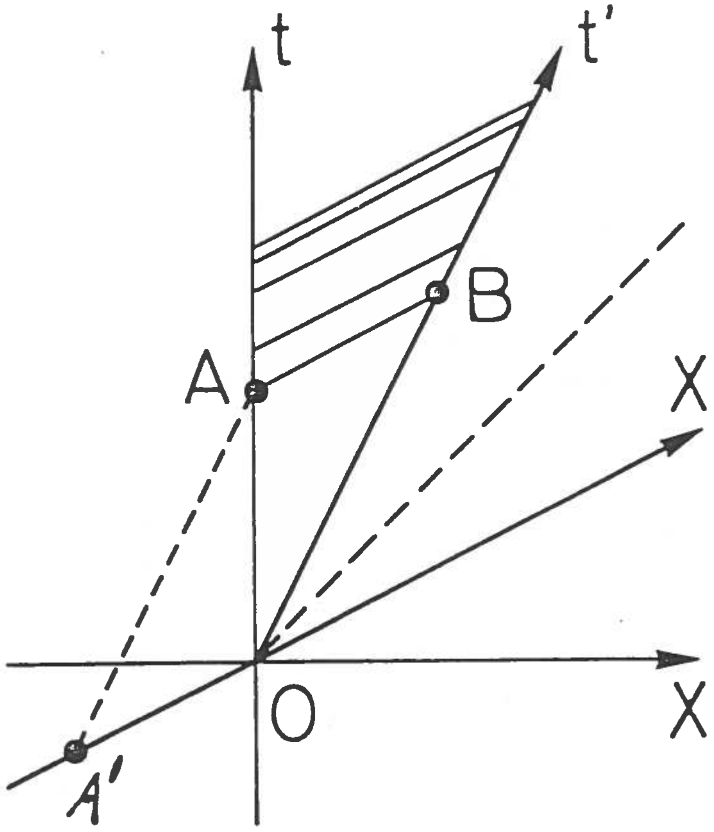

Fig.16

The paradox is actually met when s' does decide to switch on his tachyon-radio B. In fact (if $\underline{t}'$ is the Lorentz-transformed value of $\underline{t}$, and $\Delta\underline{t}'=\overline{A'O}/\underline{V}'$) the observer s' at time $\underline{t}'-\Delta\underline{t}'$ would see his radio not only broadcast the foretold multi-figured number (exactly the one written in the sealed letter, as s' can check straight after), but also emit simultaneously antitachyons $\overline{T}$ towards s: That is to say, transmit the same number to s by means of antitachyons. To make the paradox more evident, we can imagine s to transmit by the modulated tachyon-beam one of Beethovens's symphonies (whose number is shut up in advance into the envelope) instead of a plain number.

Further related paradoxes were discussed by Pavšič and Recami (101).

7.6.2. Discussion - Let us stress that s' would see the antitachyons $\bar{T}$ emitted by his radio B travel forward in time, endowed with positive energy. The problematic situation above arises only when (the tachyon-emission being supposed to be controllable) a well-defined pattern of correlated tachyons is used by s as a signal. In such a case, s' would observe his tachyon-radio B behave very strangely and unexpectedly, i.e. to transmit (by antitachyons $\bar{T}$) just the signal specified in advance by s in the sealed letter. He should conclude the intentional design of the tachyon exchange to stay on the side of s; we should not be in the presence of a real causality violation, however, since s' would *not* conclude that s is sending signals backward in time to him. We would be, on the contrary, in a condition similar to the one studied in Sect.7.5.2. The paradox has actually to do with the unconventional behaviour of the sources/detectors of tachyons, rather than with causality; namely s", observing his apparatus B, finds himself in a situation analogous to the one (Fig. 17) in which we possessed a series of objects b and saw them slip out sucked and "aspired" by A (or in which we possessed a series of metallic pellets and saw them slip out attracted by a variable, controllable electromagnet A).

From the behaviour of tachyon-radios in the above Gedankenexperiment it seems to follow that we are in need of a theory-formalization similar to Wheeler and Feynman's (102,103) (see also Gott III (104)). In particular, no tachyons can be emitted if detectors do not yet exist in the universe that will be able sooner or later to absorb them. This philosophy, as we already saw many times, is a must in ER, since tachyon physics cannot be developed without taking always into account the proper sources and detectors (whose roles can be inverted by a LT); it is not without meaning that the same philosophy was shown (102 103) to be adoptable in the limiting case of photons. Let us recall that —according to suitable observers— the two devices A,B are just exchanging infinite-speed tachyons (or antitachyons: an infinite-speed tachyon T going from A to B is exactly equivalent to an infinite-speed antitachyon $\bar{T}$ travelling from B to A). Any couple of bodies which exchange tachyons are thus realizing —according to those suitable observers— an instantaneous, mutual, *symmetrical* interaction. Thus tachyons can play an essential role at least as "internal lines" in bradyonic particle interactions (and vice-versa, passing to a Superluminal frame, bradyons would have a role as internal lines of tachyonic particle interactions).

This suggests that A and B can exchange that Beethovens's simphony by means of tachyons only if the inner structure of both A,B is "already" suited to such an exchange; this again is similar to what discussed in Sect.7.5.2, even if the situation is here more sophisticated.

Of course, all problems are authomatically (and simply) solved if we adopt the conservative attitude of assuming the tachyon exchanges between two

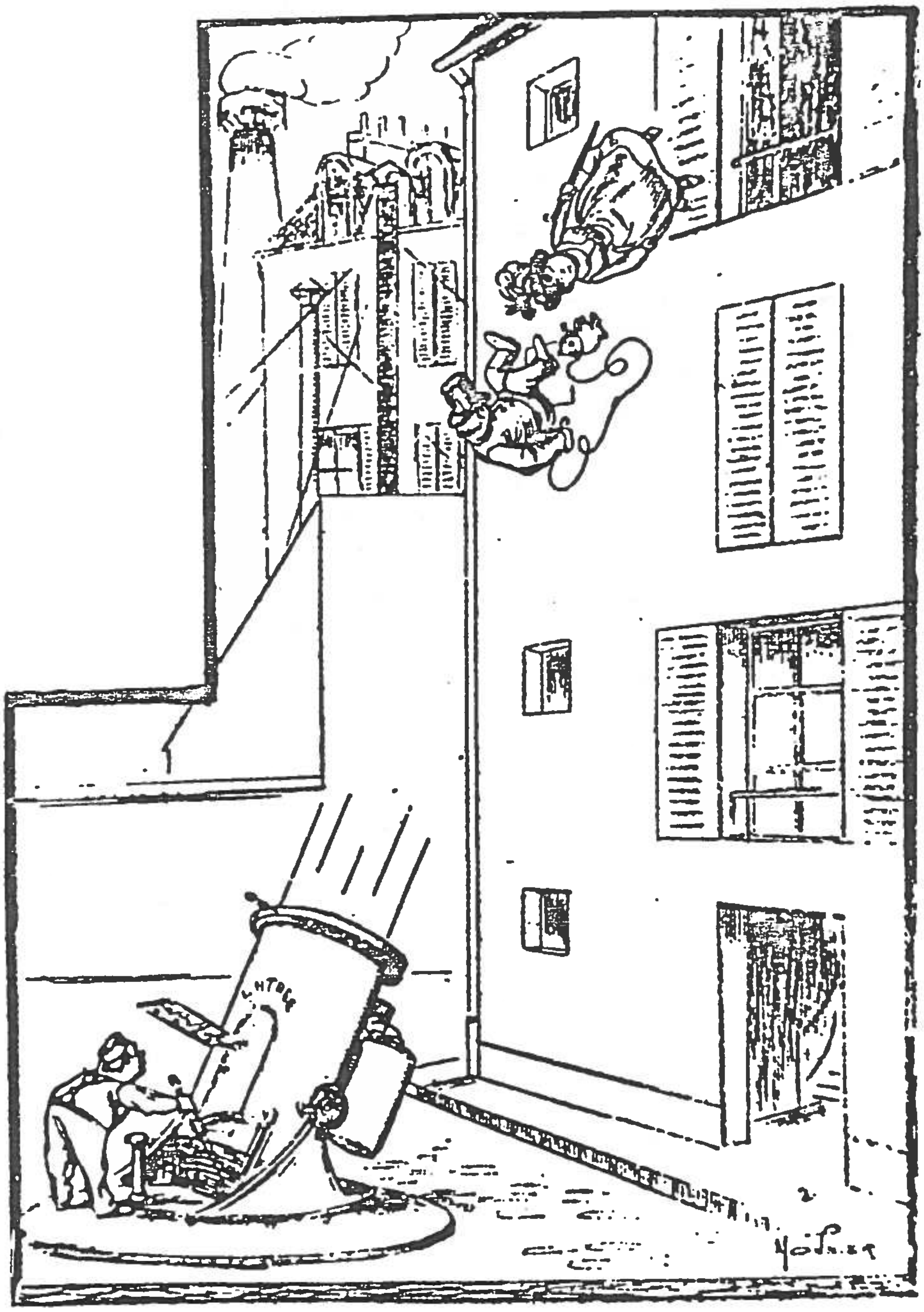

FIG. 17

bradyonic bodies A,B to be spontaneous and uncontrollable. For simplicity's sake, such a restrictive attitute might be actually adopted, even if unnecessary.

7.6.3. Further comments - When the signal does not consist of a well-defined pattern of tachyons, but is constituted by a few tachyons only —— typically by a unique tachyon——, we saw that no paradoxes survive. If on the contrary claims as the one put forth by Newton(105) were true, then one could send signals into the past even by ordinary antiparticles (which is *not* true, of course; cf. Recami and Modica(25), Recami(23)).

Moreover, to clarify further the terms of the paradox in Sects.7.6.1, 7.6.2 above, let us explicitly recall that: (i) the chronological order of the events can be reversed by an ordinary LT along a space-like path only; therefore the order of the events along the A,B world-lines cannot change; (ii) also the *proper-energies* (rest-masses) of A,B are Lorentz invariant, together with their "jumps"; (iii) while s see the *total energy* of A decrease, s' may see it increase (description details...); (iv) the paradox in Sects.7.6.1÷2 is connected with the question whether the entropy-variations and information-exchanges are to be associated with the changes in the proper energies: in this case, in fact, they would *not* necessarily behave as the "total energies" (see Caldirola and Recami (4) and Pavšič and Recami (101), where the paradoxical situations arising when one deals with macro-tachyons are furthemore discussed).

We mentioned in the previous discussion (Sect.7.6.2) that the behaviour of tachyon sources/detectors might appear paradoxical to us for the mere fact that we are not accustomed to it. To shed some light on the possible nature of such difficulties, let us report at last the following anecdote (Csonka(69)), which does not involve contemporary prejudices. For ancient Egyptians, who knew only the Nile and its tributaries, which all flow South to North, the meaning of the word "south" coincided with the one of "up-stream", and the meaning of the word "north" coincided with the one of "down-stream". When Egyptians discovered the Euphrates, which unfortunately happens to flow North to South, they passed through such a *crisis* that it is mentioned in the stele of Tuthmosis I, which tells us about *that inverted water that goes down-stream* (i. e. towards the North) *in going up-stream*.

7.7 A Further remark

Let us add the following remark. Let us consider (Fig.18) two bodies A and B which exchange (w.r.t. a frame s_o) a transcendent tachyon T_∞ moving along the $\underline{x}$-axis. From Fig. 3 and Sect. 5 we have seen that for transcendent particles the motion direction along AB is not defined. In such a limiting case, we can consider T_∞ either as a tachyon $T(\underline{v} = +\infty)$ going from A to B, or equivalently as an antitachyon $\overline{T}(\underline{v} = -\infty)$ going from B to A (cf. also Figs. 3). In QM language, we could write (Pavšič and Recami (101)):

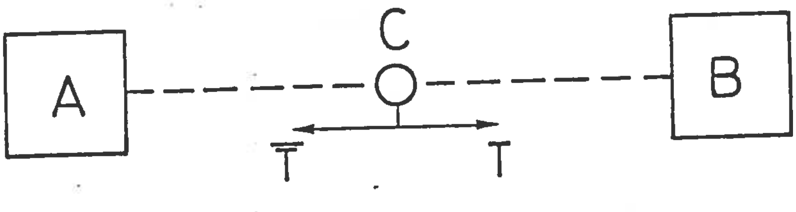

Fig.18

$$|T_\infty\rangle = a\,|T\,(v=+\infty)\rangle + b\,|\bar{T}\,(v=-\infty)\rangle\,; \quad a^2+b^2 = 1. \qquad (46)$$

Alternatively, it will be immediately realized that s_o can interpret his observations also as due to a *pair-creation* of infinite-speed tachyons T and $\bar{T}$ (travelling along x) at any point P of the x-axis between A and B (Mignani and Recami (6), Edmonds (106), Caldirola and Recami (4)): for instance, as the creation of a transcendent tachyon T travelling towards (and absorbed by) B and of a transcendent antitachyon $\bar{T}$ travelling towards (and absorbed by) A. Actually, for each observer the vacuum can become classically unstable *only* by emitting two (or more) *infinite-speed* tachyons, in such a way that the total 3-momentum of the emitted set is zero (the total energy emitted would be automatically zero: Figs. 4, 5 and 8).

It is interesting to check —cf. eq. (15) of Sect. 4.1— that any (subluminal) observer s_1, moving along x w.r.t. s_o in the direction A to B, will just see a unique (finite-speed) antitachyon $\bar{T}_1$ emitted by B, passing through point P without any interaction, and finally absorbed by A. On the contrary, any observer s_2 moving along x w.r.t. s_o in the direction B to A, will just see a unique (finite-speed) tachyon T_2 emitted by A, freely travelling from A to B (without any interaction at P) , and finally absorbed by B.

In what precedes we may consider the masses of A and B so large that the kinematical constraints, met in Sect.5, gets simplified. In such a case, s_o,s_1 and s_2 will all see an elastic scattering of A and B.

As we have seen above, any observer s_o can describe the particular process ph under examination in terms either of a vacuum decay or of a suitable tachyon emission by one of the two nearby bodies A,B. One can alternatively adopt one of those two languages. More generally, the probability of such decays must be related to the transcendent-tachyon emission-power (or absorption-power) of matter.

Furthermore, if A and B can exchange tachyons even when they are very far from each other, any observer s' (like s_1 and s_2) moving w.r.t. s_o will describe *ph* in terms either of an incoming, suitable tachyonic cosmic ray or of the emission of a suitable, finite-speed tachyon by a material object. One of the consequences, in brief, is that the tachyon cosmic flux is expected to have for consistency a Lorentz-invariant 4-momentum distribution, just as depicted in Figs.6 and 5*c*. The large majority of "cosmic" tachyons ought then appear to any observer as endowed with speed very near to the light-speed *c*. On this respect, it may be interesting to recall that an evaluation of the possible cosmic flux of tachyons yielded – even if very rough – a flux quite close to the neutrinos' one (Mignani and Recami (6)).

As an elementary illustration of other possible considerations, let us at last add the following. If s_o observes the process

$$\underline{a} \rightarrow \underline{b} + \underline{\bar{t}} \tag{47a}$$

where $\underline{\bar{t}}$ is an antitachyon, then —after a suitable LT— the new observer s' can describe the same process as

$$\underline{a} + \underline{t} \rightarrow \underline{b}\,. \tag{47b}$$

If, in eq. (47*a*), the emitted $\underline{\bar{t}}$ had travelled till absorbed by a (near or *far*) detector U, then in eq.(47*b*) *t* must of course be regarded as emitted by a (near or *far*) source U.

If $\Delta\tau$ is the mean-life of particle *a* for the decay (47*a*), measured by s_o, it will be the Lorentz transform of the average time $\Delta\tau'$ that particle *a* must spend according to s' before absorbing a "cosmic" tachyon *t* and transforming into *b*.

8. AGAIN ON THE "SWP" AND CHARGE CONJUGATION IN SR

We have seen that the "switching procedure" (Sects. 2 and 4.1) *has* to be applied —also in four dimensions— for both bradyons and tachyons. Let us therefore reconsider it on a more formal ground.

Notice that this Sect. 8 does *not* depend on the existence of tachyons, but depends on Sect.2 only.

8.1 Again about the "Switching procedure"

We indicate by "SWP" the *switching* procedure (previously often called "RIP"). Let us also call strong conjugation $\underline{\bar{C}}$ the discrete operation

$$\bar{C} \equiv C M_o \qquad (48)$$

where $\underline{C}$ is the conjugation of all additive charges and $\underline{M}_o$ the rest-mass conjugation (i.e. the reversal of the rest-mass sign). Recami and Ziino (11) showed that formally (cf. Fig. 3b)

$$\text{"SWP"} = \bar{C} \, . \qquad (49)$$

Then, by considering m_o as a *fifth* coordinate, besides the ordinary four (Einstein and Bergmann (107)), and shifting to the language of quantum mechanics, they recognized that $\underline{P}_5 \equiv \underline{\bar{C}}$, quantity $\underline{P}_5$ being the chirality operation, so that

$$\text{"SWP"} \equiv P_5 \, ; \qquad (50)$$

in fact, when dealing as usual with states with definite parity, one may write

$$\underline{\bar{C}}^{-1} \psi \, \underline{\bar{C}} \equiv \gamma^5 \psi \equiv \underline{P}_5^{-1} \psi \, \underline{P}_5 \, .$$

Notice that in our formalism the strong conjugation $\underline{\bar{C}}$ is a unitary operator when acting on the states space.

See also Edmonds (108), Lake and Roeder (109), Pavšič and Recami (110), Recami (10), Recami and Rodrigues (74).

Here we want to show that, when considering the fundamental particles of matter as *extended objects*, the (geometrical) operation with reflects the *internal* space-time of a particle is equivalent to the ordinary operation $\underline{C}$ which reverses the sign of all its additive charges (18).

8.2 Charge conjugation and internal space-time reflection

Following Pavšič and Recami (18), let us consider in the ordinary space-time: (i) the extended object (particle) $\underline{a}$, such that the interior of its "world-tube" is a finite portion of M_4; (ii) the two operators space-reflection, $\mathcal{P}$, and time-reversal, $\mathcal{T}$, that act (w.r.t. the particle world-tube W) both on the *external* and on the *internal* space-time:

$$\begin{aligned} \mathcal{P} &= \mathcal{P}_E \, \mathcal{P}_I = \mathcal{P}_I \, \mathcal{P}_E \, ; \\ \mathcal{T} &= \mathcal{T}_E \, \mathcal{T}_I = \mathcal{T}_I \, \mathcal{T}_E \, , \end{aligned} \qquad (51)$$

where $\mathcal{P}_I(\tau_I)$ is the internal and $\mathcal{P}_E(\tau_E)$ the external space-reflection (time-reversal). The ordinary parity $\underline{P}$ and time-reversal $\underline{T}$ act on the contrary only on the external space-time:

$$P \equiv \mathcal{P}_E ; \quad T \equiv \tau_E . \tag{52}$$

The effects of $\mathcal{P}_E$, $\mathcal{P}_I$ and $\mathcal{P}$ on the world-tube W of $\underline{a}$ are shown in Figs. 19; and the analogous effects of τ_E, τ_I, τ in Figs. 20.

Let us now depict W as a sheaf of world-lines w representing —let us say— its constituentes (Fig. 21a). In Fig. 21 we show, besides the c.m. world-line, also $w_1 = A$ and $w_2 = B$. The operation $\mathcal{P}\tau$ will transform W into a second world-tube $\tilde{W}$ consisting of the transformed world-lines $\tilde{w}$ (see Fig. 21b). Notice that each $\tilde{w}$ points in the opposite time-direction and occupies (w.r.t. the c.m. world-line) the position symmetrical to the corresponding w.

If we apply the Stückelber-Feynman "switching" (Sect.2.1), each world-line $\tilde{w}$ transforms into a new world-line $\bar{w}$ (cf. Fig. 21c) which points in the positive time-direction, but represents now an <u>anti</u>-constituent. Let us now explicitely generalize the "switching principle" for extended particles as follows: We identify the sheaf $\bar{W}$ of the world-lines $\bar{w}$ with the antiparticles $\underline{\bar{a}}$, i.e. $\bar{W}$ with the world-tube of $\underline{\bar{a}}$. This corresponds to assume that the overall time-direction of a particle as a whole coincides with the time-direction of its "constituents". A preliminary conclusion is that the antiparticle $\underline{\bar{a}}$ of $\underline{a}$ can be regarded (from the chronotopical, geometrical point of view) as derived form the reflection of the internal space-time of $\underline{a}$.

Let us repeat what precedes in a more rigorous way, following our Sect.2, i.e. recalling that the transformation $\underline{L} = -\mathbb{1}$ is an actual (even if anti-chronous) Lorentz transformation, corresponding to the 180° space-time "rotation": $\bar{P}\bar{T} \equiv -\mathbb{1}$. Now, to apply $\bar{P}\bar{T}$ from the active point of view to the world-tube W of Fig.21a means to rotate it (by 180°, in four dimensions) into $\tilde{W}$ (Fig.21b); such a rotation effects also a reflection of the internal 3-space of particle $\underline{a}$, transforming it —among the others— into its mirror image. The same result would be got by applying $\underline{PT}$ from the passive point of view to the space-time in Fig.21a.

Then, we *generalize* the "Switching Principle" to the case of extended objets by applying it to the world-tube $\tilde{W}$ of Fig. 21b. The world-tube $\tilde{W}$ does represent an (internally "mirrored") particle not only going backwards in time, but also carrying negative energy; therefore, the "switching" does rigorously transform $\tilde{W}$ into $\bar{W}$ (Fig. 21c), the anti-world-tube $\bar{W}$ representing $\underline{\bar{a}}$.

In conclusion:

$$-\mathbb{1} \equiv \bar{P}\bar{T} = \mathcal{P}_E\tau_E\,\mathcal{P}_I\tau_I = \mathcal{P}\tau = PT\mathcal{P}_I\tau_I \tag{53}$$

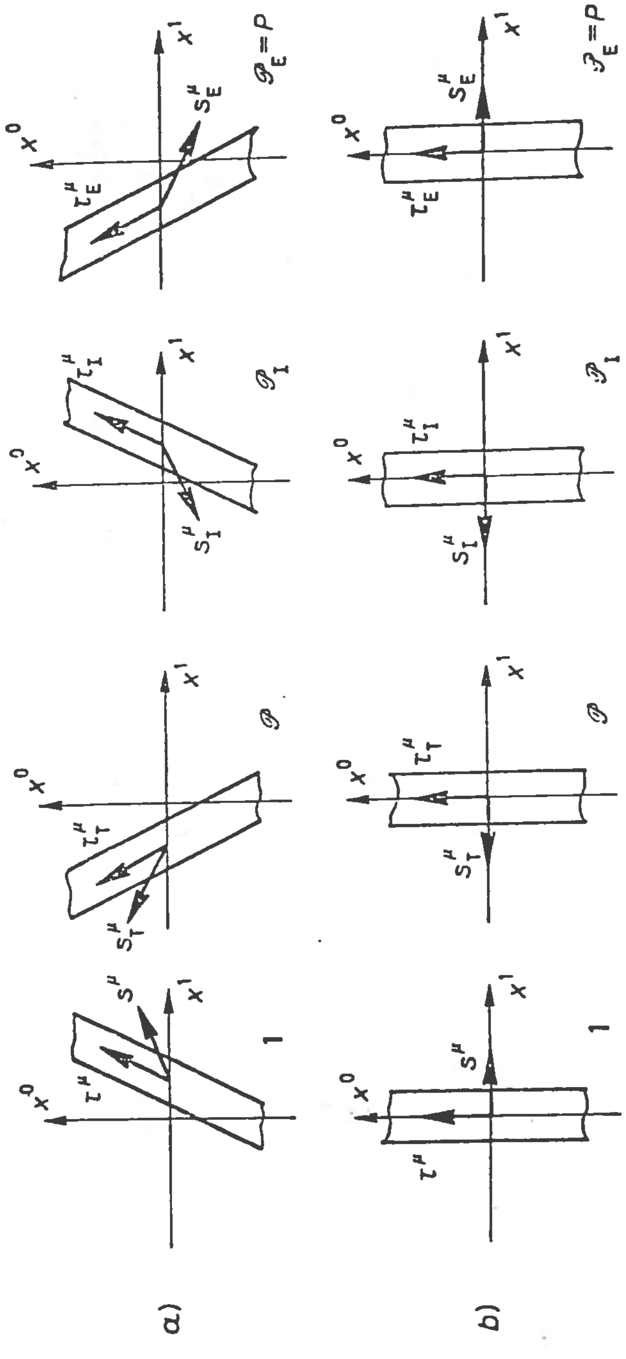
a)
b)
x^0
x^1
τ^μ
s^μ
1
τ_T^μ
s_T^μ
$\mathcal{P}$
τ_I^μ
s_I^μ
$\mathcal{P}_I$
τ_E^μ
s_E^μ
$\mathcal{P}_E = P$

FIG. 19

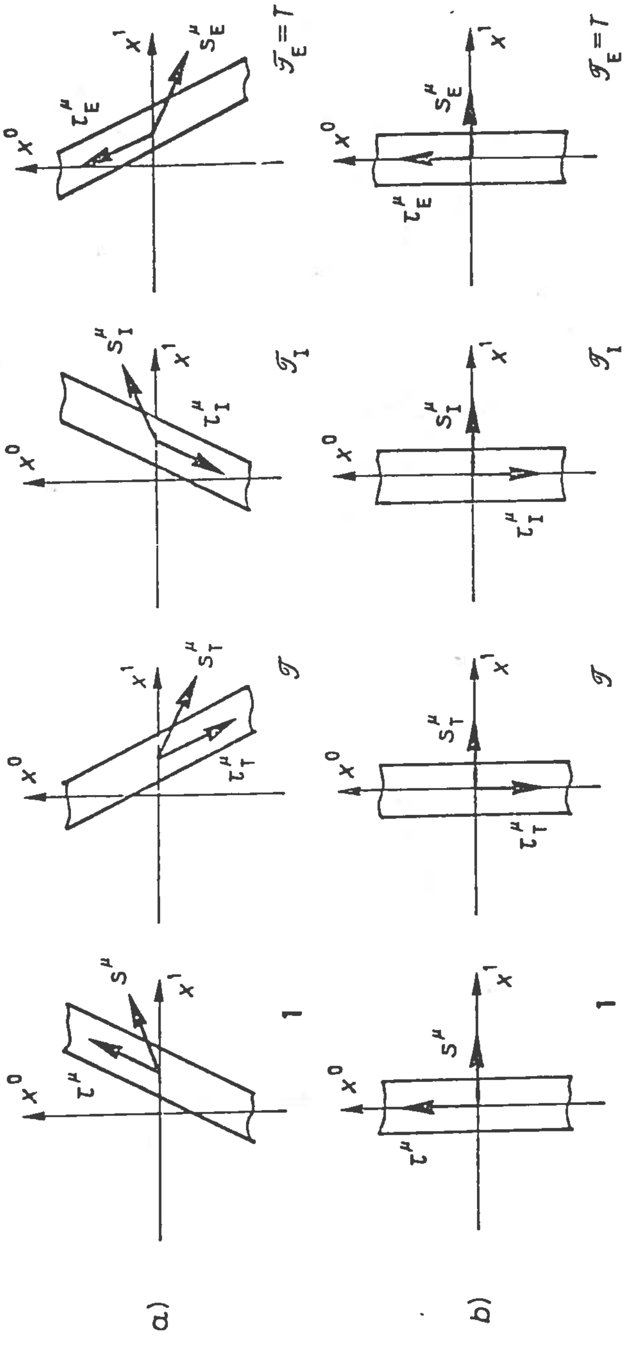
a)
x^0
x^1
τ^μ
s^μ
1
τ_T^μ
s_T^μ
$\mathcal{T}$
τ_I^μ
s_I^μ
$\mathcal{T}_I$
τ_E^μ
s_E^μ
$\mathcal{T}_E = T$
b)

FIG. 20

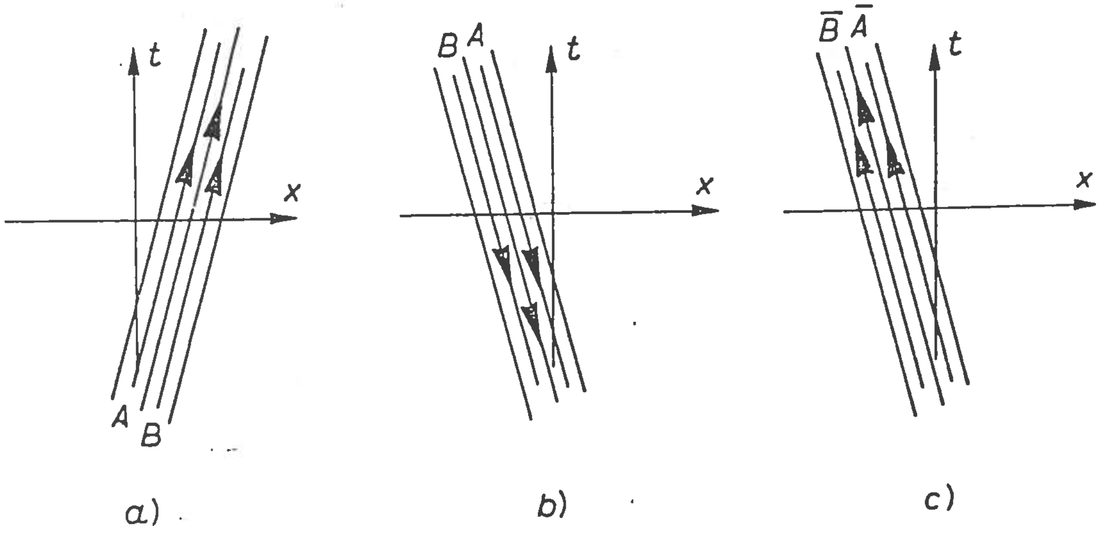

Fig.21

wherefrom, since $\underline{\bar{P}}\,\underline{\bar{T}} = \underline{CPT}$ (Sect.2.3), one derives:

$$C = P_I \mathcal{T}_I \, . \tag{54}$$

As already anticipated, we have therefore shown the operation $\underline{C}$, which inverts the sign of (all) the additive charges of a particle, to be equivalent to the (geometrical) operation of reflecting its internal space-time.

Also the results reported in this Section support the opinion that in theoretical physics we should advantageously substitute the new operations $\bar{P} \equiv \mathcal{P}$ and $\bar{T} \equiv \mathcal{T}$ for the ordinary operations $\underline{P}$ and $\underline{T}$, which are merely *external* reflections (for instance, only the former belong to the Full Lorents Group) : besides our Sect. 2, cf. also Review I, Recami (3) , and Costa de Beauregard (111).

Let us finally mention that (besides the $\underline{CPT}$ theorem, derived from the mere SR) from $\underline{ER}$ only it is possible to get also the so-called "crossing relations" (15,17,25)

9. ACKNOWLEDGMENTS

The author is grateful to R.Mignani,G.D.Maccarrone and L.R.Baldini for continuous collaboration over the years; and to J.Bell,S.Lo Nigro, M.Mavsic, and W.A.Rodrigues for many useful discussions. Thanks are due to the "Reparto Tipografico (INFN,Frascati) for the skilful,patient, kind collaboration.

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