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E. RECAMI :

CLASSICAL TACHYONS AND POSSIBLE APPLICATIONS : A REVIEW

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ABSTRACT

After having shown that ordinary Special Relativity can be adjusted to describe both particles and antiparticles, we present a review of tachyons, with particular attention to their classical theory.

We first present the extension of Special Relativity to tachyons in two dimensions, an elegant model-theory which allows a better understanding also of ordinary physics. We then pass to the four-dimensional results (particularly on tachyon mechanics) that can be derived without assuming the existence of Superluminal reference-frames. We discuss moreover the localizability and the unexpected apparent shape of tachyonic objects, and carefully show (on the basis of tachyon kinematics) how to solve the common causal paradoxes.

In connection with General Relativity, particularly the problem of the apparent superluminal expansions in astrophysics is reviewed. Later on we examine the important issue of the possible role of tachyons in elementary particle physics and in quantum mechanics.

At last we tackle the (still open) problem of the extension of relativistic theories to tachyons in four dimensions, and review the electromagnetic theory of tachyons: a topic that can be relevant also for the experimental side.

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1.- INTRODUCTION

1.1.- Foreword

The subject of Tachyons, even if still speculative, may deserve some attention for reasons that can be divided into a few categories, two of which we want preliminarly to mention right now: (i) the larger scheme that one tries to built up in order to incorporate space-like objects in the relativistic theories can allow a better understanding of many aspects of the ordinary relativistic physics, even if Tachyons would not exist in our cosmos as "asymptotically free" objects; (ii) Superluminal classical objects can have a role in elementary particle interactions (and perhaps even in astrophysics); and it might be tempting to verify how far one can go in reproducing the quantum-like behaviour at a classical level just by taking account of the possible existence of faster-than-light classical particles.

At the time of a previous review (Recami and Mignani 1974a, hereafter called Review I) the relevant literature was already conspicuous. During the last ten years such literature grew up so much that new reviews are certainly desirable; but for the same reason writing down a comprehensive article is already an overhelming task. We were therefore led to make a tight selection, strongly depending on our personal taste and interests. We confined our survey, moreover, to questions related to the classical theory of Tachyons, leaving aside for the moment the various approaches to a Tachyon quantum field theory. From the beginning we apologize to all the authors whose work, even if important, will not find room in the present review; we hope to be able to give more credit to it on another occasion. In addition, we shall adhere to the general rule of skipping here quotation of the papers already cited in Review I, unless useful to the self-containedness of the present paper.

1.2.- Plan of the review

This article is divided in five parts, the first one having nothing to do with tachyons. In fact, to prepare the ground, in Part I (Sect.2) we shall merely show that Special Relativity - even without tachyons - can be given a form such to describe both particles and anti-particles. Part II is the largest one: initially, after some historical remarks and having revisited the Postulates of Special Relativity, we present a review of the elegant "model-theory" of tachyons in two dimensions; passing then to four dimensions, we review the main results of the classical theory of tachyons that do not depend on the existence of Superluminal reference-frames (or that are al least independent of the explicit form of the Superluminal Lorentz "transformations"). In particular, we discuss how tachyons would look like, i.e. their apparent "shape". Last but not least, all the common causality problems are thoroughly solved, on the basis of the previously reviewed tachyon kinematics. Part III deals with tachyons in General Relativity; in particular the question of the apparent superluminal expansions in astrophysics is reviewed. Part IV shows the interesting, possible role of tachyons in elementary particle physics and in quantum theory. In Part V, the last one, we face the (still open) problem of the Superluminal Lorentz "transformations" in four dimensions, by introducing for instance an auxiliary six-dimensional space--time, and finally present the electromagnetic theory of tachyons: a theory that can be relevant also from the "experimental" point of view.

1.3.- Previous reviews

In the past years other works were devoted to review some aspects of our subject. As far as we know, besides Review I (Recami and Mignani 1974a), the following papers may be mentioned: Caldirola and Recami (1980); Recami (1979a, 1978a); Kirch (1977); Barashenkov (1975); Kirzhnits and Sakonov (1974); Recami (1973); Bolotovsky and Ginzburg (1972); Camenzind (1970); Feinberg (1970), as well as the short but interesting glimpse given at tachyons by Goldhaber and Smith (1975) in their review of all the hypothetical particles. At a simpler (or more concise) level, let us further list: Guasp (1983); Voulgaris (1976); Kreisler (1973, 1969); Velarde (1972); Gondrand (1971); Newton (1970); Bilaniuk and Sudarshan (1969a) and relative discussions (Bilaniuk et al. 1969, 1970); and a nice talk by Sudarshan (1968). On the experimental side, besides Goldhaber and Smith (1975), let us mention: Boratav (1980); Jones (1977); Berley et al. (1975); Carrol et al. (1975); Ramana Murthy (1972); Giacomelli (1970).

1.4.- Lists of references. Meetings. Books.

As to the existing bibliographies about tachyons, let us quote: (i) the references at pages 285-290 of Review I; at pages 592-597 in Recami (1979a); at pages 295--298 in Caldirola and Recami (1980); as well as in Recami and Mignani (1972) and in Mignani and Recami (1973); (ii) the large bibliographies by Perepelista (1980a,b); (iii) the list by Feldman (1974). However the last one, a librarian's compilation, lists some references (e.g. under the numbers 8,9,13,14,18,21-23) seemingly having not much to do with tachyons; while ref.38 therein (Peres 1969), e.g., should be associated with the comments it received from Baldo and Recami (1969). In connection with the experiments only, also the references in Bartlett et al. (1978) and Bhat et al. (1979) may be consulted.

As to meetings on the subject, to our knowledge: (i) a two-days meeting was held in India; (ii) a meeting (First Session of the Interdisciplinary Seminars) on "Tachyons and Related Topics" was held at Erice (Italy) in Sept. 1976; (iii) a "Seminar sur le Tachyons" exists at the Faculté des Sciences de Tours et de Poitiers (France), which organizes seminars on the subject.

With regards to books, we should mention: (i) the book by Terletsky (1968), devoted in part to tachyons; (ii) the book Tachyons, monopoles, and Related Topics (Amsterdam: North-Holland), with the proceedings of the Erice meeting cited above (see Recami ed 1978b).

PART I: PARTICLES AND ANTIPARTICLES IN SPECIAL RELATIVITY (SR)

2.- SPECIAL RELATIVITY WITH ORTHO- AND ANTI-CHRONOUS LORENTZ TRANSFORMATIONS

In this Part I we shall forget about Tachyons.

From the ordinary postulates of Special Relativity (SR) it follows that in such a theory - which refers to the class of Mechanical and Electromagnetic phenomena - the class of reference-frames equivalent to a given inertial frame is obtained by means of transformations L (Lorentz Transformations, LT) which satisfy the following sufficient requirements: (i) to be linear

$$x'^{\mu} = L^{\mu}_{\nu}\, x^{\nu} \quad ; \tag{1}$$

(ii) to preserve space-isotropy (with respect to electromagnetic and mechanical phenomena); (iii) to form a group; (iv) to leave the quadratic form invariant:

$$\eta_{\mu\nu}\, dx^{\mu}\, dx^{\nu} = \eta'_{\alpha\beta}\, dx'^{\alpha}\, dx'^{\beta} \quad . \tag{2}$$

From condition (i), if we confine ourselves to sub-luminal speeds, it follows that in eq.(2):

$$\eta'_{\alpha\beta} = \mathrm{diag}(+1,-1,-1,-1) = \eta_{\mu\nu} \quad . \tag{3}$$

Eqs.(1)-(3) imply that det L = 1; $(L^{o}_{o})^2 \geq 1$. The set of all subluminal (Lorentz) transformations satisfying all our conditions consists - as is well known - of four pieces, which form a noncompact, nonconnected group (the Full Lorentz Group). Wishing to confine ourselves to space-time "rotations" only, i.e. to the case det L = = +1, we are left with the two pieces

$$\{L_+^{\uparrow}\} \; : \; L_0^0 \geq +1 \; ; \qquad \det L = +1 \; ; \tag{4a}$$

$$\{L_+^{\downarrow}\} \; : \; L_0^0 \leq -1 \; ; \qquad \det L = -1 \; , \tag{4b}$$

which give origin to the group of the proper (orthochronous and antichronous) transformations

$$\underset{\sim}{\mathscr{L}}_+ \equiv \underset{\sim}{\mathscr{L}}_+^{\uparrow} \cup \underset{\sim}{\mathscr{L}}_+^{\downarrow} \equiv \{L_+^{\uparrow}\} \cup \{L_+^{\downarrow}\} \tag{5}$$

and to the subgroup of the (ordinary) proper orthochronous transformations

$$\underset{\sim}{\mathscr{L}}_+ \equiv \{L_+^{\uparrow}\} \; , \tag{6}$$

both of which being, incidentally, invariant subgroups of the Full Lorentz Group. For reasons to be seen later on, let us rewrite $\underset{\sim}{\mathscr{L}}_+$ as follows

$$\underset{\sim}{\mathscr{L}}_+ = \underset{\sim}{\mathscr{L}}_+^{\uparrow} \otimes \underset{\sim}{\mathscr{Z}}(2) \; ; \qquad \underset{\sim}{\mathscr{Z}}(2) \equiv \{\sqrt[2]{+1}\} \equiv \{+1, -1\} \; . \tag{5'}$$

We shall skip in the following, for simplicity's sake, the subscript + in the transformations $L_+^{\uparrow}$, $L_+^{\downarrow}$. Given a transformation $\bar{L}^{\downarrow}$, another transformation $\bar{L}^{\uparrow} \in \underset{\sim}{\mathscr{L}}_+^{\uparrow}$ always exists such that

$$\bar{L}^{\downarrow} = (-\mathbb{1}) \cdot \bar{L}^{\uparrow} \; , \qquad \forall \bar{L}^{\downarrow} \in \underset{\sim}{\mathscr{L}}_+^{\downarrow} , \tag{7}$$

and vice-versa. Such a one-to-one correspondence allows us to write formally

$$\underset{\sim}{\mathscr{L}}_+^{\downarrow} = - \underset{\sim}{\mathscr{L}}_+^{\uparrow} . \tag{7'}$$

It follows in particular that the central elements of $\underset{\sim}{\mathscr{L}}_+$ are: $\mathbb{C} = (+\mathbb{1}, -\mathbb{1})$.

Usually, even the piece (4b) is discarded. Our present aim is to show - on the contrary - that a physical meaning can be attributed also to the transformations (4b). Confining ourselves here to the active point of view (cf. Recami and Rodrigues 1982 and references therein), we wish precisely to show that the theory of SR, once based on the whole proper Lorentz group (5) and not only on its orthochronous part, will describe a Minkowski space-time populated by both matter and antimatter.

2.1.- The Stückelberg-Feynman "switching principle" in SR

Besides the usual postulates of SR (Principle of Relativity, and Light-Speed Invariance), let us assume - as commonly admitted, e.g. for the reasons in Garuccio et al. (1980), Mignani and Recami (1976a) - the following:

Assumption - "negative-energy objects travelling forward in time do not exist ". We shall give this Assumption, later on, the status of a fundamental postulate.

Let us therefore start from a positive-energy particle P travelling forward in time. As wellknown, any orthochronous LT (4a) transforms it into another particle still endowed with positive energy and motion forward in time. On the contrary, any antichronous (= non-orthochronous) LT (4b) will change sign - among the others - to the time-components of all the four-vectors associated with P. Any $L^{\downarrow}$ will transform P into a particle P' endowed in particular with negative energy and motion backwards in time (Fig.1).

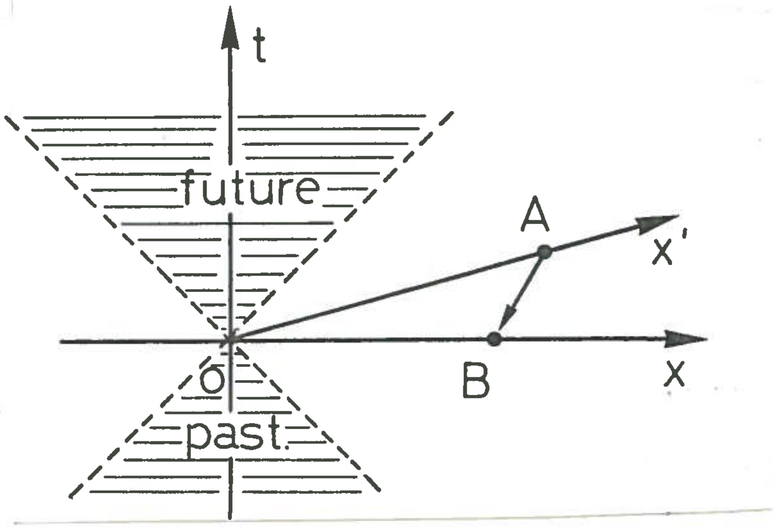


FIG. 1

In other words, SR together with the natural Assumption above implies that a particle going backwards in time (Gödel 1963) (Fig.1) corresponds in the four-momentum space, Fig. 2, to a particle carrying negative energy; and vice-versa, that changing the energy sign in one space corresponds to changing the sign of time in

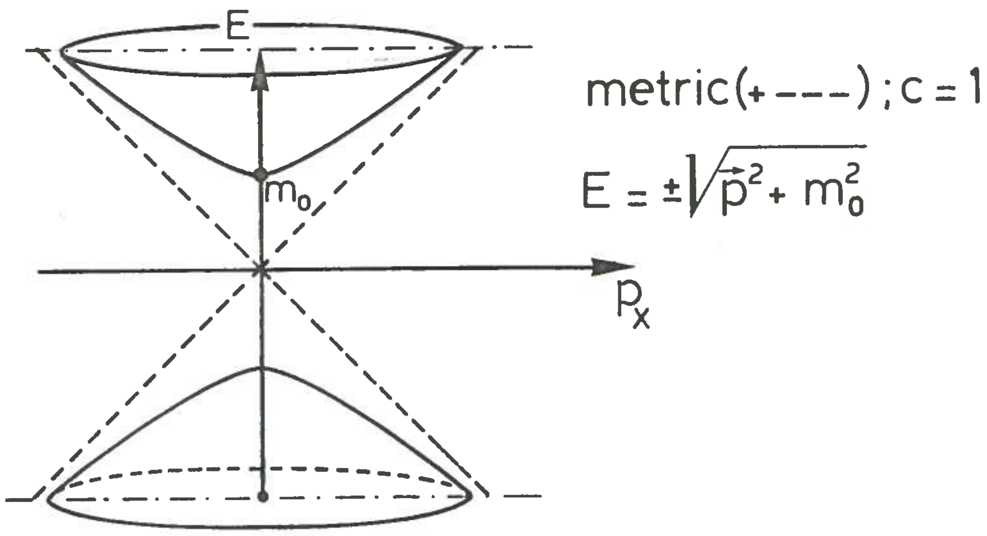


FIG. 2

the dual space. It is then easy to see that these two paradoxical occurrences ("negative energy" and "motion backwards in time") give rise to a phenomenon that any observer will describe in a quite orthodox way, when they are - as they actually are - simultaneous (Recami 1978c, 1979a and refs. therein).

Notice, namely, that: (i) every observer (a macro-object) explores space-time, Fig.1, in the positive t-direction, so that we shall meet B as the first and A as the last event; (ii) emission of positive quantity is equivalent to absorption of negative quantity, as $(-)\cdot(-) = (+)\cdot(+)$; and so on.

Let us now suppose (Fig. 3) that a particle P' with negative energy (and e.g. charge -e) moving backwards in time is emitted by A at time t_1 and absorbed by B at time $t_2 < t_1$. Then, it follows that at time t_1 the object A "looses" negative energy

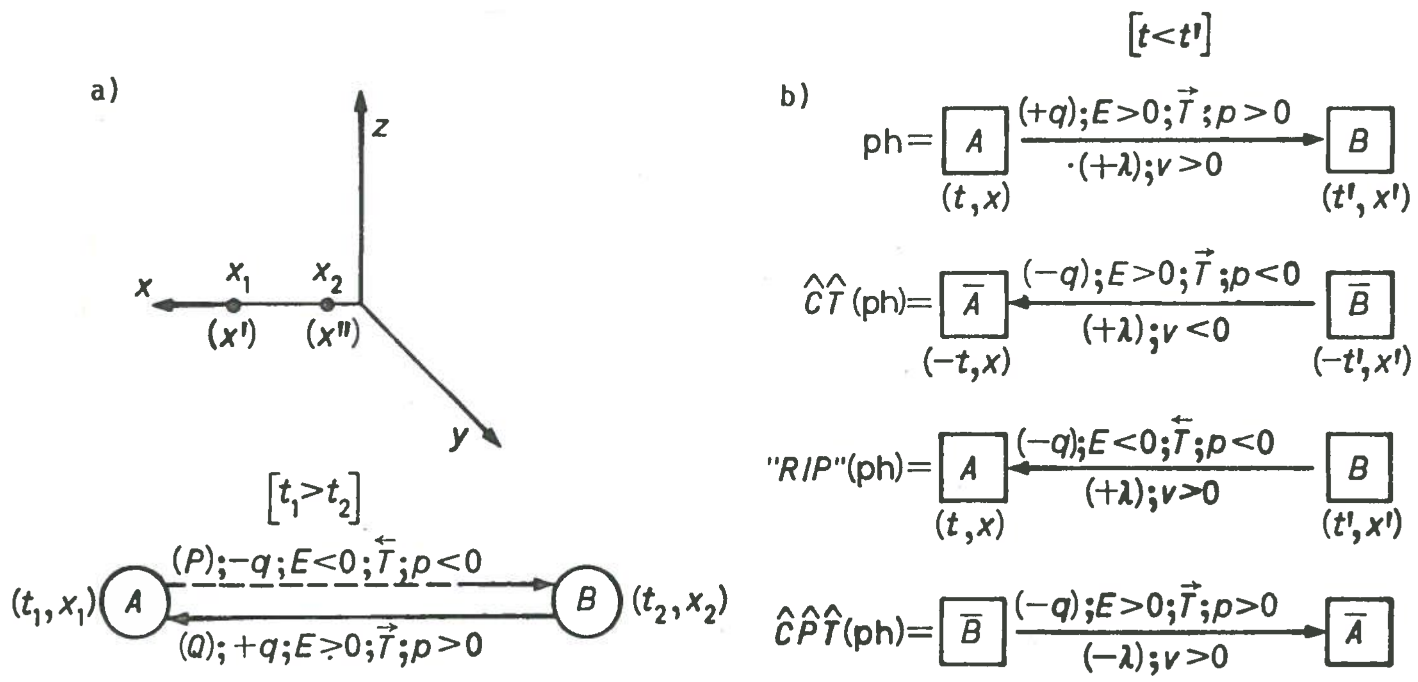


FIG. 3

and charge, i.e. *gains* positive energy and charge. And that at time $t_2 < t_1$ the object B "gains" negative energy and charge, i.e. *looses* positive energy and charge. The physical phenomenon here described is nothing but the exchange *from* B *to* A of a particle Q with *positive* energy, charge +e, and going *forward* in time. Notice that Q has, however, charges *opposite* to P'; this means that in a sense the present "switching procedure" (previously called "RIP") effects a "charge conjugation" C, among the others. Notice also that "charge", here and in the following, means *any* additive charge; so that our definitions of charge conjugation, etc., are more general than the ordinary ones (Review I, Recami 1978a). Incidentally, such a switching procedure has been shown to be equivalent to applying the chirality operation γ_5 (Recami and Ziino 1976). See also, e.g., Reichenbach (1971), Mensky (1976).

2.2.- Matter and Antimatter from SR

A close inspection shows the application of any antichronous transformation L, together with the switching procedure, to transform P into an object

$$Q \equiv \bar{P} \tag{8}$$

which is indeed the antiparticle of P. We are saying that the concept of antimatter is a purely relativistic one, and that, on the basis of the double sign in (c=1)

$$E = \pm \sqrt{p^2 + m^2} \tag{9}$$

the existence of antiparticles could have been predicted from 1905, exactly with the properties they actually exibited when later discovered, provided that recourse to the "switching procedure" had been made. We therefore maintain that the points of the lower hyperboloid sheet in Fig.2 - since they correspond not only to negative energy but also to motion backwards in time - represent the kinematical states of the antiparticle $\overline{P}$ (of the particle P represented by the upper hyperboloid sheet). Let us explicitly observe that the switching procedure exchanges the roles of source and detector, so that (Fig.1) any observer will describe B to be the source and A the detector of the antiparticle $\overline{P}$.

Let us stress that the switching procedure not only can, but must be performed, since any observer can do nothing but explore space-time along the positive time-direction. That procedure is merely the translation into a purely relativistic language of the Stückelberg (1941; see also Klein 1929) - Feynman (1949) "Switching principle". Together with our Assumption above, it can take the form of a "Third postulate": 'Negative-energy objects travelling forward in time do not exist; any negative-energy object P travelling backwards in time can and must be described as its anti-object $\overline{P}$ going the opposite way in space (but endowed with positive energy and motion forward in time)'. Cf. e.g. Caldirola and Recami (1980), Recami (1979a) and references therein.

2.3.- Further remarks

a) Let us go back to Fig.1. In SR, when based only on the two ordinary postulates, nothing prevents a priori the event A from influencing the event B. Just to forbid such a possibility we introduced our Assumption together with the Stückelberg-Feynman "Switching procedure". As a consequence, not only we eliminate any particle-motion backwards in time, but we also "predict" and naturally explain within SR the existence of antimatter.

b) The Third Postulate, moreover, helps solving the paradoxes connected with the fact that all relativistic equations admit, besides standard "retarded" solutions, also "advanced" solutions: The latter will simply represent antiparticles travelling the opposite way (Mignani and Recami 1977a). For instance, if Maxwell equations admit solutions in terms of outgoing (polarized) photons of helicity $\lambda = +1$, then they will admit also solutions in terms of incoming (polarized) photons of helicity $\lambda = -1$; the actual intervention of one or the other solution in a physical problem depending only on the initial conditions.

c) Eqs.(7), (8) tell us that, in the case considered, any $\bar{L}^{\downarrow}$ has the same kinematical effect than its "dual" transformation $\bar{L}^{\uparrow}$, just defined through eq.(7), except for the fact that it moreover transforms P into its antiparticle $\bar{P}$. Eqs.(7), (7') then lead (Mignani and Recami 1974a,b, 1975a) to write

$$-\mathbb{1} \equiv \bar{P}\bar{T} = CPT , \qquad (10)$$

where the symmetry operations $\bar{P}$, $\bar{T}$ are to be understood in the "strong sense": For instance, $\bar{T} \equiv$ reversal of the time-components of all fourvectors associated with the considered phenomenon (namely, inversion of the time <u>and</u> energy axes). We shall come back to this point. The discrete operations P, T have the ordinary meaning. When the particle P considered in the beginning can be regarded as an extended object, Pavšič and Recami (1982) have shown the "strong" operations $\bar{P}$, $\bar{T}$ to be equivalent to the space, time reflections acting on the space-time <u>both</u> external <u>and</u> internal to the particle world-tube.

Once accepted eq.(10), then eq.(7') can be written

$$\mathcal{L}_+^{\downarrow} = (\bar{P}\,\bar{T})\ \mathcal{L}_+^{\uparrow} \equiv (CPT)\,\mathcal{L}_+^{\uparrow} . \qquad (7'')$$

In particular, the total-inversion $\bar{L}^{\downarrow} = -\mathbb{1}$ transforms the process $a+b \rightarrow c+d$ into the process $\bar{d}+\bar{c} \rightarrow \bar{b}+\bar{a}$ without any change in the velocities.

d) All the ordinary relativistic laws (of Mechanics and Electromagnetism) are actually already covariant under the <u>whole</u> proper group $\mathcal{L}_+$, eq.(5), since they are CPT-symmetric besides being covariant under $\mathcal{L}_+^{\uparrow}$.

e) A few quantities, that happened (cf. Sect.5.17 in the following) to be Lorentz-invariant under the transformations $L^{\uparrow} \in \mathcal{L}_+^{\uparrow}$, are no more invariant under the transformations $L \in \mathcal{L}_+$. We have already seen this to be true for the <u>sign</u> of the additive charges, e.g. for the sign of the electric charge e of a particle P. The ordinary derivation of the electric-charge invariance is obtained by evaluating the integral flux of a current through a surface which, under $L^{\uparrow}$, <u>moves</u>, changing the angle formed with the current. Under $L^{\downarrow} \in \mathcal{L}_+^{\downarrow}$ the surface "rotates" so much with respect to the current (cf. also Figs.6,12 in the following) that the current enters it through the opposite face; as a consequence, the integrated flux (i.e. the charge) changes sign.

PART II: BRADYONS AND TACHYONS IN SR

3.- HISTORICAL REMARKS, AND PRELIMINARIES

3.1.- Historical remarks

Let us now take on the issue of Tachyons. To our knowledge (Corben 1975, Recami 1978a), the first scientist mentioning objects "faster than the Sun's light" was Lucretius (50 B.C., ca.), in his De Rerum Natura. Still remaining in pre-relativistic times, after having recalled e.g. Laplace (1845), let us only mention the recent progress represented by the noticeable papers by Thomson (1889), Heaviside (1892), Des Coudres (1900) and mainly Sommerfeld (1904, 1905).

In 1905, however, together with SR (Einstein 1905, Poincaré 1906) the conviction that the light-speed c in vacuum was the upper limit of any speed started to spread over the scientific community, the early-century physicists being led by the evidence that ordinary bodies cannot overtake that speed. They behaved in a sense like Sudarshan's (1972) imaginary demographer studying the population patterns of the Indian subcontinent: 'Suppose a demographer calmly asserts that there are no people North of the Himalayas, since none could climb over the mountain ranges! That would be an absurd conclusion. People of central Asia are born there and live there: They did not have to be born in India and cross the mountain range. So with faster-than-light particles' (Cf. Figs.4). Notice that photons are born, live and die just "on the top of the montain", i.e. always at the speed of light, without any need to violate SR, that is to say to accelerate from rest to the light-speed.

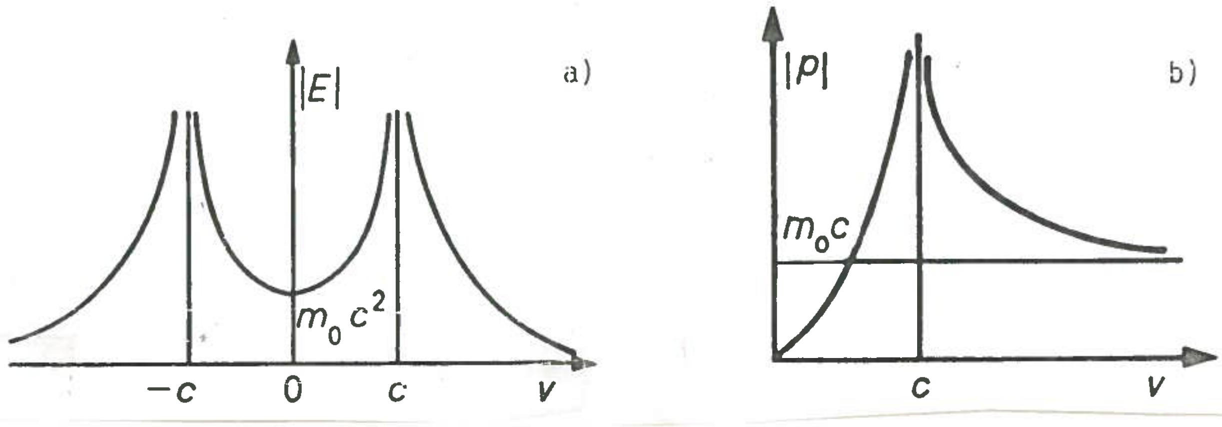


FIG. 4

Moreover, Tolman (1917) believed to have shown in his anti-telephone "paradox" (based on the already wellknown fact that the chronological order along a space-like path is not Lorentz-invariant) that the existence of Superluminal ($v^2 > c^2$) particles allowed information-transmission into the past. In recent times that "paradox" has been proposed again and again by authors apparently unaware of the existing literature (for instance, some of Rolnick's (1972; see also 1969) arguments had been already "answered" by Csonka (1970) before they appeared). Incidentally, we shall solve it in Sect. 9.1.

Therefore, except for the pioneering paper by Somigliana (1922; recently rediscovered by Caldirola et al. 1980), after the mathematical considerations by Majorana (1932) and Wigner (1939) on the space-like particles one had to wait untill the fifties to see our problem tackled again in the works by Arzeliès (1955,1957,1958), Schmidt (1958), Tangherlini (1959), and then by Tanaka (1960) and Terletsky (1960). It started to be fully reconsidered in the sixties: In 1962 the first article by Sudarshan and coworkers (Bilaniuk et al. 1962) appeared, and after that paper a number of physicists took up studying the subject - among whom, for instance, Jones (1963) and Feinberg (1967) in the USA and Recami (1968, 1969a) and colleagues (Olkhovsky and Recami 1968,1969,1970a,b,1971) in Europe.

The first experimental searches for Superluminal particles were carried out by Alväger et al. (1963,1965,1966).

As wellknown, Superluminal particles have been given the name "Tachyons" (T) by Feinberg (1967) from the Greek word ταχύς = fast. 'Une particule qui a un nom possède déjà un début d'existence' (A particle bearing a name has already taken on some existence) was later commented on by Arzeliès (1974). We shall call "Luxons" (ℓ), following Bilaniuk et al.(1962), the objects travelling exactly at the speed of light, like photons. At last, we shall call "Bradyons" (B) the ordinary subluminal ($v^2 < c^2$) objects, from the Greek word βραδύς = slow, as it was independently proposed by Cawley (1969), Barnard and Sallin (1969), and Recami (1970 ; see also Baldo et al. 1970).

Let us recall at this point that, according to Democritus of Abdera, everything that was thinkable without meeting contradictions did exist somewhere in the unlimited universe. This point of view - recently adopted also by M.Gell-Mann - was later on expressed in the known form 'Anything not forbidden is compulsory' (White 1939) and named the "totalitarian principle" (see e.g. Trigg 1970). We may adhere to this philosophy, repeating with Sudarshan that 'if Tachyons exist, they ought to be found. If they do not exist, we ought to be able to say why'.

3.2.- Preliminaries about Tachyons

Tachyons, or space-like particles, are already known to exist as internal, intermediate states or exchanged objects (see Sects. 6.13 and 13.2). Can thay also exist as "asymptotical free" objects?

We shall see that the particular - and unreplaceable - role in SR of the light -speed c in vacuum is due to its invariance (namely, to the experimental fact that c does not depend on the velocity of the source), and not to its being or not the maximal speed (Recami and Modica 1975, Kirzhnits and Polyachenko 1964, Arzeliês 1955).

However, one cannot forget that in his starting paper on Special Relativity Einstein - after having introduced the Lorentz transformations - considered a sphere moving with speed u along the x-axis and noticed that (due to the relative motion) it appears in the frame at rest as an ellipsoid with semiaxes

$$a_x = r\sqrt{1-\beta^2}\ ; \qquad a_y = a_z = r \qquad (\beta \equiv \frac{u}{c})\ . \tag{11}$$

Then Einstein (1905) added: 'Für u=c schrumpfen alle bewegten Objecte - vom "ruhenden" System aus betrachtet - in flächenhafte Gebilde zusammen. Für Überlichtgeschwindigkeiten werden unsere Überlegungen sinnlos; wir werden übrigens in der folgenden Betrachtungen finden, dass die Lichtgeschwindigkeiten spielt', which means (Scwartz 1977): 'For u=c all moving objects - viewed from the "stationary" system - shrink into plane-like structures. For superlight speeds our considerations become senseless; we shall find, moreover, in the following discussion that the velocity of light plays in our theory the rôle of an infinitely large velocity'. Einstein referred himself to the following facts: (i) for u>c, the quantity a_x becomes pure-imaginary: If $a_x \equiv a_x(u)$, then

$$a_x(U) = \pm\, ir\sqrt{\left|1-\frac{U^2}{c^2}\right|} \qquad (U^2>c^2)\ ; \tag{12}$$

(ii) in SR the speed of light v=c plays a rôle similar to the one played by the infinite speed $v=\infty$ in the Galilean Relativity (Galilei 1632, 1953).

Two of the aims of this review will just be to show how objection (i) - which touches a really difficult problem - has been answered, and to illustrate the meaning of point (ii). With regard to eq.(12), notice that a priori $\sqrt{\beta^2-1} = \pm i\sqrt{1-\beta^2}$, since $(\pm i)^2=-1$. Moreover, we shall always understand that $\sqrt{1-\beta^2}$ for $\beta^2>1$ represents the upper half-plane solution.

Since a priori we know nothing about Ts, the safest way to build up a theory for them is trying to generalize the ordinary theories (starting with the classical relativistic one, only later on passing to the quantum field theory) through "minimal extensions", i.e. by performing modifications as small as possible. Only after possesing a theoretical model we shall be able to start experiments: Let us remember that, not only good experiments are required before getting sensible ideas (Galilei

1632), but also a good theoretical background is required before sensible experiments can be performed.

The first step consists therefore in facing the problem of extending SR to Tachyons. In so doing, some authors limited themselves to consider objects both subluminal and Superluminal, always referred however to subluminal observers ("weak approach"). Other authors attempted on the contrary to generalize SR by introducing both subluminal observers (s) and Superluminal observers (S), and then by extending the Principle of Relativity ("strong approach"). This second approach is theoretically more worth of consideration (tachyons, e.g., get real proper-masses), but it meets of course the greatest obstacles. In fact, the extension of the Relativity Principle to Superluminal inertial frames seems to be straightforward only in the pseudo-Euclidean space-times M(n,n) having the same number n of space-axes and of time-axes. For instance, when facing the problem of generalizing the Lorentz transformations to Superluminal frames in four dimensions one meets no-go theorems as Gorini's et al. (Gorini 1971 and refs. therein), stating no such extensions exist which satisfy all the following properties: (i) to refer to the four-dimensional Minkowski space-time M_4=M(1,3); (ii) to be real; (iii) to be linear; (iv) to preserve the space isotropy; (v) to preserve the light-speed invariance; (vi) to possess the prescribed group--theoretical properties.

We shall therefore start by sketching the simple, instructive and very promising "model-theory" in two dimensions (n=1).

Let us first revisit, however, the postulates of the ordinary SR.

4.- THE POSTULATE OF SR REVISITED

Let us adhere to the ordinary postulates of SR. A suitable choice of Postulates is the following one (Review I; Maccarrone and Recami 1982a and refs. therein):

1) First Postulate - Principle of Relativity: 'The physical laws of Electromagnetism and Mechanics are covariant (= invariant in form) when going from an inertial frame f to another frame f' moving with constant velocity $\underset{\sim}{u}$ relative to f.

2) Second Postulate - "Space and time are homogeneous and space is isotropic". For future convenience, let us give this Postulate the form: 'The space-time accessible to any inertial observer is four-dimensional. To each inertial observer the 3-dimensional Space appears as homogeneous and isotropic, and the 1-dimensional Time appears as homogeneous'.

3) Third Postuale - Principle of Retarded Causality: 'Positive-energy objects travelling backwards in time do not exist; and any negative-energy particle P travel

ling backwards in time can and must be described as its antiparticle $\overline{P}$, endowed with positive energy and motion forward in time (but going the opposite way in space)'. See Sects. 2.1, 2.2.

The First Postulate is inspired to the consideration that all inertial frames should be equivalent (for a careful definition of "equivalence" see e.g. Recami (1979a)); notice that this Postulate does not impose any constraint on the relative speed $u \equiv |\underline{u}|$ of the two inertial observers, so that a priori $-\infty < u < +\infty$. The Second Postulate is justified by the fact that from it the conservation laws of energy, momentum and angular-momentum follow, which are well verified by experience (at least in our "local" space-time region); let us add the following comments: (i) The words homogeneous, isotropic refer to space-time properties assumed - as always - with respect to the electromagnetic and mechanical phenomena; (ii) Such properties of space-time are supposed by this Postulate to be covariant within the class of the inertial frames; this means that SR assumes the vacuum (i.e. space) to be "at rest" with respect to each inertial frame. The Third Postulate is inspired to the requirement that for each obsever the "causes" chronologically precede their own "effects" (for the definition of causes and effects see e.g. Caldirola and Recami 1980). Let us recall that in Sect.2 the initial statement of the Third Postulate has been shown to be equivalent - as it follows from Postulates 1) and 2) - to the more natural Assumption that 'negative-energy objects travelling forward in time do not exist'.

4.1.- Existence of an invariant speed

Let us initially skip the Third Postulate.

Since 1910 it has been shown (Ignatowski 1910, Frank and Rothe 1911, Hahn 1913 Lalan 1937, Severi 1955, Agodi 1973, Di Jorio 1974) that the postulate of the light-speed invariance is not strictly necessary, in the sense that our Postulates 1) and 2) imply the existence of an invariant speed (not of a maximal speed, however). In fact, from the first two Postulates it follows (Rindler 1969, Berzi and Gorini 1969, Gorini and Zecca 1970 and refs. therein, Lugiato and Gorini 1972) that one and only one quantity w^2 - having the physical dimensions of the square of a speed - must exist, which has the same value according to all inertial frames:

$$w^2 = \text{invariant} . \qquad (13)$$

If one assumes $w=\infty$, as done in Galilean Relativity, then one would get Galilei-Newton physics; in such a case the invariant speed is the infinite one: $\infty \oplus v = \infty$, where we symbolically indicated by $\oplus$ the operation of speed composition.

If one assumes the invariant speed to be finite and real, then one gets immediately Einstein's Relativity and physics. Experience has actually shown us the speed c of light in vacuum to be the (finite) invariant speed: $c \oplus v = c$. In this case, of course, the infinite speed is no more invariant: $\infty \oplus v = V \neq \infty$. It means that in SR the operation $\oplus$ is not the operation + of arithmetics.

Let us notice once more that the unique rôle in SR of the light-speed c in vacuum rests on its being invariant and not the maximal one (see e.g. Shankara 1974, Recami and Modica 1975); if tachyons - in particular infinite-speed tachyons - exist they could not take over the rôle of light in SR (i.e. they could not be used by different observers to compare the sizes of their space and time units, etc.), just in the same way as bradyons cannot replace photons. The speed c turns out to be a limiting speed; but any limit can possess *a priori* two sides (Fig. 4).

4.2.- The problem of Lorentz transformations

Of course one can substitute the light-speed invariance Postulate for the assumption of space-time homogeneity and space isotropy (see the Second Postulate).

In any case, from the first two Postulates it follows that the transformations connecting two generic inertial frames f, f', a priori with $-\infty < |\underset{\sim}{u}| < +\infty$

$$dx'_{\mu} = G^{\nu}_{\mu}\, dx_{\nu} \tag{14}$$

must (cf. Sect.2):

(i) transform inertial motion into inertial motion;

(ii) form a group $\underset{\sim}{\mathcal{G}}$;

(iii) preserve space isotropy;

(iv) leave the quadratic form invariant, except for its sign (Rindler 1966, p. 16; Landau and Lifshitz 1966a,b):

$$dx'_{\mu}\, dx'^{\mu} = \pm\, dx_{\mu}\, dx^{\mu} \; . \tag{15}$$

Notice that eq.(15) imposes - among the others - the light-speed to be invariant (Jammer 1979). Eq.(15) holds for any quantity dx_{μ} (position, momentum, velocity, acceleration, current, etc.) that be a G-fourvector, i.e. that behave as a fourvector under the transformations belonging to $\underset{\sim}{\mathcal{G}}$. If we explicitly confine ourselves to slower-than-light relative speeds, $u^2 < c^2$, then we have to skip in eq.(15) the sign minus, and we are left with eq.(2) of Sect.2. In this case, in fact, one can start from the identity transformation $G = \mathbb{1}$, which requires the sign plus, and then retain such a sign for continuity reasons.

On the contrary, the sign minus will play an important rôle when we are ready to go beyond the light-cone discontinuity. In such a perspective, let us preliminary clarify - on a formal ground - what follows (Maccarrone and Recami 1982a, 1984a).

4.3.- Orthogonal and Antiorthogonal Transformations: Digression

4.3.1.- Let us consider a space having, in a certain initial base, the metric $g^{\mu\nu}$, so that for vectors dx^{α} and tensors $M^{\alpha\beta}$ it is

$$dx^{\alpha} = g^{\alpha\beta} dx_{\beta} \; ; \qquad M^{\alpha\beta} = g^{\alpha\gamma} g^{\beta\delta} M_{\gamma\delta} \; .$$

When passing to another base, one writes

$$dx'^{\mu} = g'^{\mu\nu} dx'_{\nu} \; ; \qquad M'^{\alpha}_{\beta} = g'^{\alpha\gamma} M'_{\gamma\beta} \; .$$

In the two bases, the scalar products are defined

$$dx_{\alpha} dx^{\alpha} \equiv dx_{\alpha} g^{\alpha\beta} dx_{\beta} \; ; \qquad dx'_{\mu} dx'^{\mu} \equiv dx'_{\mu} g'^{\mu\nu} dx'_{\nu} \; ,$$

respectively.

Let us call A the transformation from the first to the second base, in the sense that

$$dx'^{\mu} = A^{\mu}_{\varrho} dx^{\varrho} \; ,$$

that is to say

$$dx^{\mu} = (A^{-1})^{\mu}_{\varrho} dx'^{\varrho} \; .$$

Now, if we impose that

$$dx_{\alpha} dx^{\alpha} = + dx'_{\mu} dx'^{\mu} \; , \text{ (assumption)} \tag{16}$$

we get

$$g_{\alpha\beta} = g'_{\mu\nu} A^{\mu}_{\alpha} A^{\nu}_{\beta} \; ; \tag{17}$$

however, if we impose that

$$dx_{\alpha} dx^{\alpha} = - dx'_{\mu} dx'^{\mu} \; , \text{ (assumption)} \tag{16'}$$

we get that

$$g_{\alpha\beta} = - g'_{\mu\nu} A^{\mu}_{\alpha} A^{\nu}_{\beta} \; . \tag{17'}$$

4.3.2.- Let us consider the case (16)-(17), i.e.

$$dx_\alpha\, dx^\alpha = + dx'_\mu\, dx'^\mu \;, \qquad \text{(assumption)} \qquad (16)$$

and let us look for the properties of transformations A which yield

$$g'_{\mu\nu} = + g_{\mu\nu} \,. \qquad \text{(assumption)} \qquad (18)$$

It must be

$$g_{\alpha\beta} = g_{\mu\nu} A^\mu_\alpha A^\nu_\beta \;; \quad \text{i.e.} \quad g_{\alpha\beta} = A^\mu_\alpha A_{\mu\beta} \;, \qquad (17)$$

wherefrom

$$g^{\gamma\alpha} g_{\alpha\beta} = g^{\gamma\alpha} A^\mu_\alpha A_{\mu\beta} = A^{\mu\gamma} A_{\mu\beta} = (A^T)^{\gamma\mu} A_{\mu\beta} \,. \qquad (19)$$

At this point, if we impose that in the initial base

$$g_{\mu\nu} \equiv \eta_{\mu\nu} \;, \qquad \text{(assumption)} \qquad (20)$$

then eq.(19) yields

$$\delta^\gamma_\beta = (A^T)^{\gamma\mu} A_{\mu\beta} = g^{\mu k} (A^T)^\gamma_k g_{\mu\sigma} A^\sigma_\beta = \delta^k_\sigma (A^T)^\gamma_k A^\sigma_\beta = (A^T)^\gamma_k A^k_\beta \;,$$

that is to say

$$(A^T)(A) = \mathbb{1} \,. \qquad (21)$$

4.3.3.- Now, in the case (16')-(17'), i.e.

$$dx_\alpha\, dx^\alpha = - dx'_\mu\, dx'^\mu \;, \qquad \text{(assumption)} \qquad (16')$$

when

$$g_{\alpha\beta} = - g'_{\mu\nu} A^\mu_\alpha A^\nu_\beta \;, \qquad (17')$$

let us investigate which are the properties of transformations A that yield

$$g'_{\mu\nu} = - g_{\mu\nu} \,. \qquad \text{(assumption)} \qquad (18')$$

In the particular case, again, when

$$g_{\mu\nu} \equiv \eta_{\mu\nu} \;, \qquad \text{(assumption)} \qquad (20)$$

it must be

$$g_{\alpha\beta} = - (-g_{\mu\nu}) A^\mu_\alpha A^\nu_\beta \;,$$

i.e. transformations A must still be orthogonal:

$$(A^T)(A) = \mathbb{1} \quad . \tag{21}$$

In conclusion, transformations A when orthogonal operate in such a way that

either: (i) $dx_\alpha\, dx^\alpha = +\, dx'_\mu\, dx'^\mu$ and $g'_{\mu\nu} = +\,\eta_{\mu\nu}$, (22a)

or: (ii) $dx_\alpha\, dx^\alpha = -\, dx'_\mu\, dx'^\mu$ and $g'_{\mu\nu} = -\,\eta_{\mu\nu}$. (22b)

4.3.4.- On the contrary, let us now require that

$$dx_\alpha\, dx^\alpha = -\, dx'_\mu\, dx'^\mu \qquad \text{(assumption)} \tag{16'}$$

when

$$g_{\alpha\beta} = -\, g'_{\mu\nu}\, A^\mu_\alpha\, A^\nu_\beta \quad , \tag{17'}$$

and simultaneously let us look for the transformations A such that

$$g'_{\mu\nu} = +\, g_{\mu\nu} \quad . \qquad \text{(assumption)} \tag{18}$$

In this case, when in particular assumption (20) holds, $g_{\mu\nu} \equiv \eta_{\mu\nu}$, we get that transformations A must be anti-orthogonal:

$$(A^T)(A) = -\,\mathbb{1} \quad . \tag{23}$$

4.3.5. - The same result (23) is easily obtained when assumptions (16) and (18') hold, together with condition (20).

In conclusion, transformations A when anti-orthogonal operate in such a way that

either: (i) $dx_\alpha\, dx^\alpha = -\, dx'_\mu\, dx'^\mu$ and $g'_{\mu\nu} = +\,\eta_{\mu\nu}$, (24a)

or (ii) $dx_\alpha\, dx^\alpha = +\, dx'_\mu\, dx'^\mu$ and $g'_{\mu\nu} = -\,\eta_{\mu\nu}$. (24b)

4.3.6.- For passing from sub- to Super-luminal frames we shall have (see the following) to adopt antiorthogonal transformations. Then, our conclusions (22) and (24) show that we will have to impose a sign-change either in the quadratic form (20'), or in the metric (22'), but not - of course - in both: Otherwise one would get, as known, an ordinary and not a Superluminal transformation (cf. e.g. Mignani and Recami 1974c). We expounded here such considerations, even if elementary since they arose some misunderstandings (e.g., in Kowalczyński 1984).

We choose to assume always (unless differently stated in an explicit way):

$$g'_{\mu\nu} = + g_{\mu\nu} . \tag{25}$$

Let us add the following comments. One could remember the theorems of Riemannian geometry (theorems so often used in General Relativity), which state the quadratic form to be positive-definite and the $g_{\mu\nu}$-signature to be invariant, and therefore wonder how it can be possible for our antiorthogonal transformations to act in a different way. The fact is that the pseudo-Euclidean (Minkowski) space-time is not a particular Riemannian manifold, but rather a particular Lorentzian (i.e. pseudo-Riemannian) manifold. The space-time itself of General Relativity (GR) is pseudo-Riemannian and not Riemannian (only space is Riemannian in GR): see e.g. Sachs and Wu (1980). In other words, the antiorthogonal transformations do not belong to the ordinary group of the so-called "arbitrary" coordinate-transformations usually adopted in GR, as outlined e.g. by Møller (1962). However, by introducing suitable scale-invariant coordinates (e.g. dilation-covariant "light-cone coordinates"), both sub- and Super-luminal "Lorentz transformations" can be formally written (Maccarrone et al. 1983) in such a way to preserve the quadratic form, its sign included (see Sect. 5.8).

Throughout this paper we shall adopt (when convenient) natural units ($c=1$); and (when in four dimensions) the metric-signature (+---), which will be always supposed to be used by both sub- and Super-luminal observers, unless differently stated in an explicit way.

5.- A MODEL-THEORY FOR TACHYONS: AN "EXTENDED RELATIVITY" (ER) IN TWO DIMENSIOS

Till now we have not taken account of tachyons. Let us finally take them into consideration, starting from a model-theory, i.e. from "Extended Relativity" (ER) (Maccarrone and Recami 1982a, Maccarrone et al. 1983, Barut et al. 1982, Review I) in two dimensions.

5.1.- A duality principle

We got from experience that the invariant speed is $w=c$. Once an inertial frame s_0 is chosen, the invariant character of the light-speed allows an exhaustive partition of the set $\{f\}$ of all inertial frames f (cf. Sect.4) into the two disjoint, complementary subsets $\{s\}$, $\{S\}$ of the frames having speeds $|u|<c$ and $|U|>c$ relative to s_0, respectively. In the following, for simplicity, we shall consider ourselves as "the observer s_0". At the present time we neglect the luminal frames ($u \equiv U = 0$)

as "unphysical". The First Postulate requires frames s and S to be equivalent (for such an extension of the criterion of "equivalence" see Caldirola and Recami 1980, Recami 1979a), and in particular observers S - if they exist - to have at their disposal the same physical objects (rods, clocks, nucleons, electrons, mesons,.....) than observers s. Using the language of multidimensional space-times for future convenience, we can say the first two Postulates to require that even observers S must be able to fill their space (as seen by themselves) with a "lattice-work" of meter--sticks and synchronized clocks (Taylor and Wheeler 1966). It follows that objects must exist which are at rest relatively to S and faster-than-light relatively to frames s; this, together with the fact that luxons ℓ show the same speed to any observers s or S, implies that the objects which are bradyons B(S) with respect to a frame S must appear as tachyons T(s) with respect to any frame s, and vice-versa:

$$B(S) = T(s) \; ; \quad T(S) = B(s) \; ; \quad \ell(S) = \ell(s) \; . \qquad (26)$$

The statement that the terms B,T,s,S do not have an absolute, but only a relative meaning, and eqs.(26), constitute the so-called duality principle (Olkhovsky and Recami 1971, Recami and Mignani 1972, 1973a, Migani et al. 1972, Antipa 1972, Mignani and Recami 1973).

This means that the relative speed of two frames s_1, s_2 (or S_1, S_2) will always be smaller than c; and the relative speed between two frames s, S will be always larger than c. Moreover, the above exhaustive partition is invariant when s_o is made to vary inside $\{s\}$ (or inside $\{S\}$), whilst the subsets $\{s\}$, $\{S\}$ get on the contrary interchanged when we pass from $s_o \in \{s\}$ to any frame $s_o \in \{S\}$.

The main problem is finding out how objects that are subluminal w.r.t. (= with respect to) observers S appear to observers s (i.e. to us). It is, therefore, finding out the (Superluminal) Lorentz transformations - if they exist - connecting the observations by S with the observations by s.

5.2.- Sub- and Super-luminal Lorentz transformations: Preliminaries

We neglect space-time translations, i.e. consider only restricted Lorentz transformations. All frames are supposed to have the same event as their origin. Let us also recall that in the chronotopical space Bs are characterized by time-like, ℓs by light-like, and Ts by space-like world-lines.

The ordinary, subluminal Lorentz transformations (LT) from s_1 to s_2, or from S_1 to S_2, are known to preserve the four-vector type. After Sect.5.1, on the contrary, it is clear that the "Superluminal Lorentz transformations" (SLT) from s to S, or from S to s, must transform time-like into space-like quantities, and vice-versa.

With the assumption (25) it follows that in eq.(15) the plus sign has to hold for LT's and the minus sign for SLTs:

$$ds'^2 = \pm\, ds^2 \; ; \qquad (u^2 \lessgtr 1) \tag{15}$$

therefore, in "Extended Relativity" (ER), with the assumption (25), the quadratic form

$$ds^2 \equiv dx_\mu \, dx^\mu$$

is a scalar under LTs, but is a pseudo-scalar under SLTs. In the present case, we shall write that LTs are such that

$$dt'^2 - dx'^2 = + (dt^2 - dx^2) \; ; \qquad (u^2 < 1) \tag{27a}$$

while for SLTs it must be

$$dt'^2 - dx'^2 = - (dt^2 - dx^2) \; . \qquad (u^2 > 1) \; . \tag{27b}$$

5.3.- Energy-momentum space

Since tachyons are just usual particles w.r.t. their own rest frames f, where the fs are Superluminal w.r.t. us, they will possess real rest-masses m_o (Recami and Mignani 1972, Leiter 1971a, Parker 1969). From eq.(27b) applied to the energy-momentum vector p^μ, one derives for free tachyons the relation

$$E^2 - p_x^2 = - m_o^2 < 0 \; , \qquad (m_o \text{ real}) \tag{28}$$

provided that p^μ is so defined to be a G-vector (see the following); so that one has (cf. Figs. 5)

$$p_\mu \, p^\mu = \begin{cases} + m_o^2 > 0 & \text{for bradyons (time-like case)} \quad (29a) \\ 0 & \text{for luxons (light-like case)} \quad (29b) \\ - m_o^2 < 0 & \text{for tachyons (space-like case).} \quad (29c) \end{cases}$$

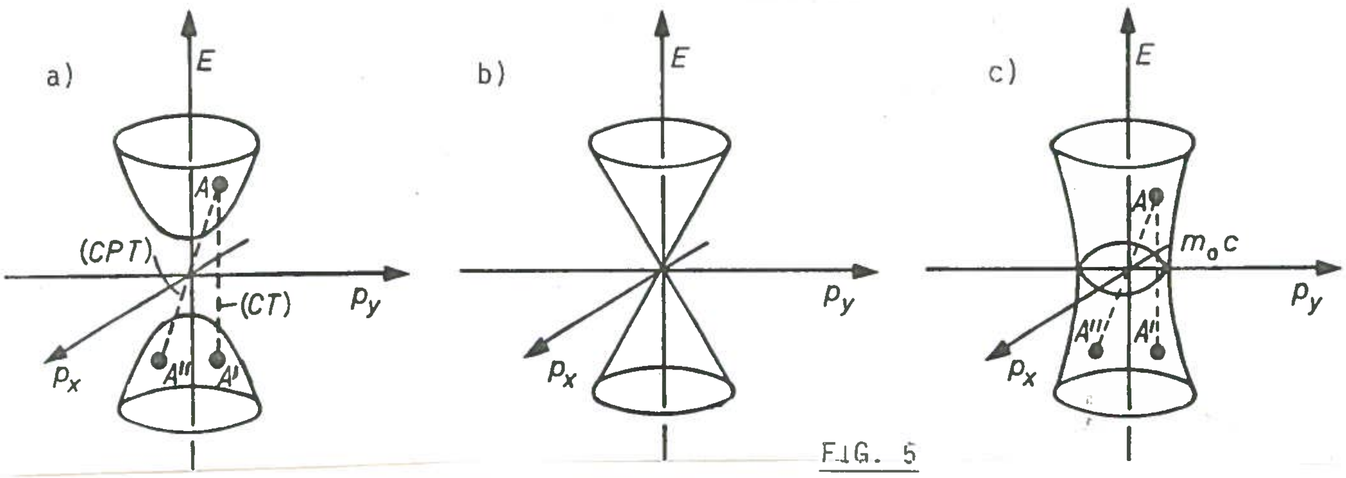


FIG. 5

Eqs.(27)-(29) tell us that the rôles of space and time and of energy and momentum get interchanged when passing from bradyons to tachyons (see Sect.5.6). Notice that in the present case (eqs.(29)) it is μ=0,1. Notice also that tachyons slow down when their energy increases and accelerate when their energy decreases. In particular, divergent energies are needed to slow down the tachyons' speed towards its (lower) limit c. On the contrary, when the tachyons' speed tends to infinity, their energy tends to zero; in ER, therefore, energy can be transmitted only at finite velocity. From Figs. 5a,c it is apparent that a bradyon may have zero momentum (and minimal energy m_0c^2), and a tachyon may have zero energy (and minimal momentum m_0c); however Bs cannot exist at zero energy, and tachyons cannot exist at zero momentum (w.r.t. the observers to whom they appear ar tachyons!). Incidentally, since transcendent (= infinite-speed) tachyons do not transport energy but do transport momentum (m_0c), they allow getting the rigid body behaviour even in SR (Bilaniuk and Sudarshan 1969, Review I, Castorina and Recami 1978). In particular, in elementary particle physics - see the following, e.g. Sects.6.7, 6.13 - they might a priori be useful for interpreting in the suitable reference frames the diffractive scatterings, elastic scatterings, etc. (Maccarrone and Recami 1980b and refs. therein).

5.4.- Generalized Lorentz transformations (GLT): Preliminaries

Eqs.(27a,b), together with requirements (i)-(iii) of Sect.4.2, finally imply the LTs to be orthogonal and the SLTs to be anti-orthogonal (Maccarone et al. 1983 and refs. therein):

$$G^TG = +\mathbb{1} \qquad \text{(subluminal case: } u^2<1) \; ; \tag{30a}$$

$$G^TG = -\mathbb{1} \qquad \text{(Superluminal case: } u^2>1) \; , \tag{30b}$$

as anticipated at the end of Sect.4.3. Both sub- and Super-luminal Lorentz transformations (let us call them "Generalized Lorentz transformations, GLT) result to be unimodular. In the two-dimensional case, however, the SLTs can a priori be special or not; to give them a form coherent with the four-dimensional case (see Sect.12; cf. also Sects.5.5, 5.6), one is led to adopt SLTs with negative trace: det SLT_2=-1. In four dimensions, however, all the GLTs will result to be unimodular and special:

$$\det G = +1 \, , \qquad \forall G \in \mathcal{G} \; . \tag{31}$$

5.5.- The fundamental theorem of (bidimensional) ER

We have now to write down the SLTs, satisfying the conditions (i)-(iv) of Sect 4.2. with the sign minus in eq.(15),still however with $g'_{\mu\nu} = g_{\mu\nu}$ (cf. Sect.4.3, and

Maccarrone and Recami 1982b), and show that the GLTs actually form a (new) group $\mathscr{G}$. Let us remind explicitly that an essential ingredient of the present procedure is the assumption that the space-time interval dx^{μ} is a (chronotipical) vector even with respect to $\mathscr{G}$: see eq.(14).

Any SLT from a sub- to a Super-luminal frame, $s \to S'$, will be identical with a suitable (ordinary) LT - let us call it the "dual" transformation - except for the fact that it must change time-like into space-like vectors, and vice-versa, according to eqs.(27b) and (25).

Alternatively, one could say that a SLT is identical with its dual subluminal LT, provided that we impose the primed observer S' to use the opposite metric-signature $g'_{\mu\nu} = - g_{\mu\nu}$, however without changing **the signs** into the definitions of time-like and space-like quantities! (Mignani and Recami 1974c, Shah 1977).

It follows that a generic SLT, corresponding to a Superluminal velocity U, will be formally expressed by the product of the dual LT corresponding to the subluminal velocity $u \equiv 1/U$, by the matrix $\mathscr{S} \equiv \mathscr{S}_2 \equiv i\mathbb{1}$, where here $\mathbb{1}$ is the two-dimensional identity:

$$\mathrm{SLT}(U) = \pm \mathscr{S} \cdot \mathrm{LT}(u) \qquad\qquad (32)$$

$$\left(u \equiv \frac{1}{U};\ u^2 < c^2;\ U^2 > c^2\right)$$

$$\mathscr{S} = i\mathbb{1}\,. \qquad\qquad (33)$$

Transformation $\mathscr{S} \equiv \mathscr{S}_2 \in \mathscr{G}$ plays the rôle of the "transcendent SLT" since for $u \to 0$ one gets $\mathrm{SLT}(U \to \infty) = \pm \mathscr{S}$. The double sign in eq.(32) is required by condition (ii) of Sect.4.2; in fact, given a particular subluminal Lorentz transformation L(u) and the SLT = + iL(u), one gets

$$\left[iL(u)\right]\left[iL^{-1}(u)\right] \equiv \left[iL(u)\right]\left[iL(-u)\right] \equiv - \mathbb{1}\,. \qquad\qquad (34a)$$

However

$$\left[iL(u)\right]\left[- iL^{-1}(u)\right] \equiv \left[iL(u)\right]\left[- iL(-u)\right] \equiv + \mathbb{1}\,. \qquad\qquad (34b)$$

Eqs.(34) shows that

$$\left[iL(u)\right]^{-1} = - iL^{-1}(u) \equiv - iL(-u)\,.$$

5.6.- Explicit form of the Superluminal Lorentz transformations (SLT) in two dimensions

In conclusion, the Superluminal Lorentz transformations $\pm$ iL(u) form a group $\mathscr{G}$ together with both the orthochronous and the antichronous subluminal LTs of Sect.2: see Fig. 6. Namely, if $\mathscr{Z}(n)$ is the discrete group of the n-th roots of unity, then the new group $\mathscr{G}$ of GLTs can be formally written down as

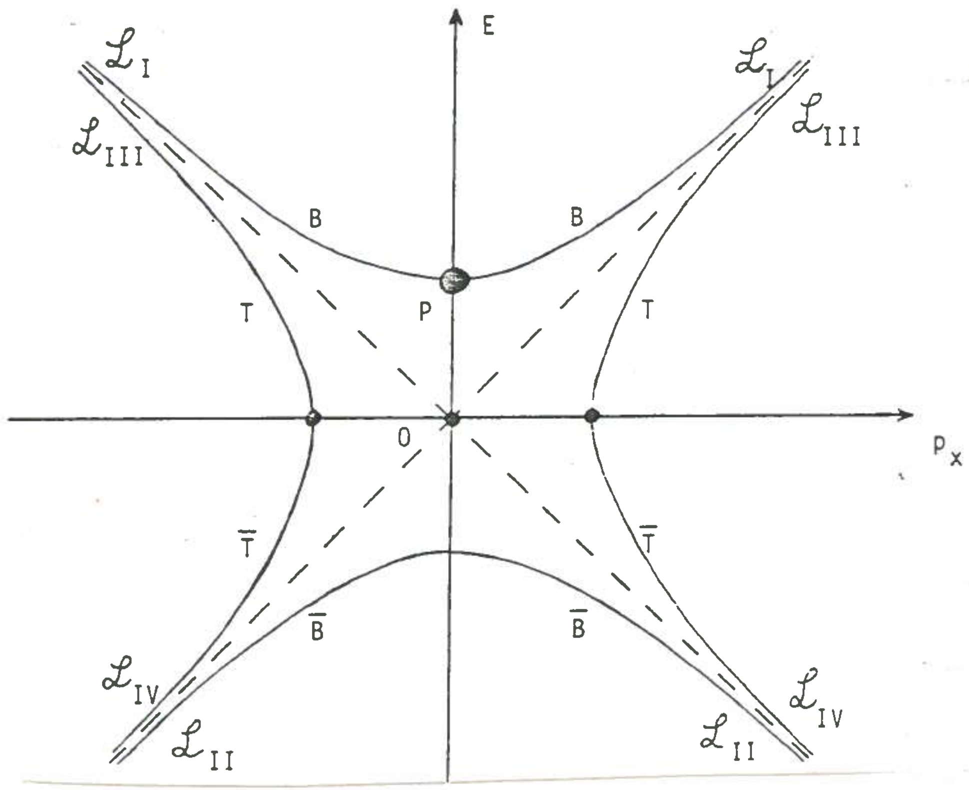


FIG. 6

$$\underset{\sim}{\mathcal{G}} = \underset{\sim}{\mathcal{Z}}(4) \otimes \underset{\sim}{\mathcal{L}}{}_+^{\uparrow} \quad ; \tag{35}$$

$$\underset{\sim}{\mathcal{Z}}(4) = \{\sqrt[4]{1}\} = \{+1,-1,+i,-i\} \quad , \tag{36}$$

where $\underset{\sim}{\mathcal{L}}{}_+^{\uparrow}$ represents <u>here</u> the <u>bidimensional</u> proper orthochronous Lorentz group. Eq. (35) should be compared with eq.(5'). It is

$$\begin{aligned} G \in \underset{\sim}{\mathcal{G}} &\Rightarrow -G \in \underset{\sim}{\mathcal{G}} \; , \qquad \forall G \in \underset{\sim}{\mathcal{G}} \; ; \\ G \in \underset{\sim}{\mathcal{G}} &\Rightarrow \mathcal{S} G \in \underset{\sim}{\mathcal{G}} \; , \qquad \forall G \in \underset{\sim}{\mathcal{G}} \; . \end{aligned} \tag{37}$$

The appearance of imaginary units into eqs.(33)-(36) is <u>only</u> formal, as it can be guessed from the fact that the transcendent operation $\mathcal{S}_2 \equiv \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$ goes into $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ through a "congruence" trasnformation (Maccarrone et al. 1983):

$$\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} = M \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M^T \quad . \tag{38}$$

Actually, the GLTs given by eqs.(32)-(33), or (35)-(36), simply represent (Review I, p.232-233) <u>all</u> the space-time pseudo-rotations for $0 \leqslant \alpha \leqslant 360°$: see Figs.7. To show this, let us write down explicitly the SLTs in the following formal way

$$\begin{aligned} dt' &= \pm i \, \frac{dt - udx}{\sqrt{1-u^2}} \; ; \\ dx' &= \pm i \, \frac{dx - udt}{\sqrt{1-u^2}} \; . \end{aligned} \qquad \text{(Superluminal case ; } u^2 < 1) \tag{39}$$

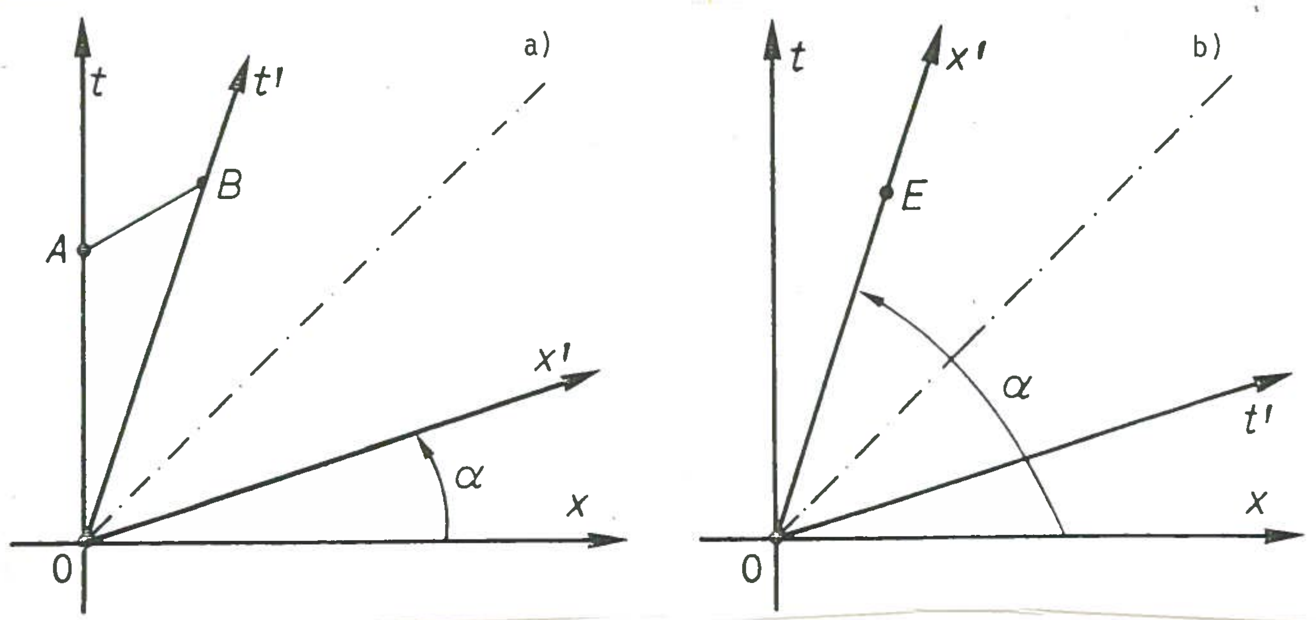


FIG. 7

The two-dimensional space-time $M(1,1) \equiv (t,x)$ can be regarded as a complex-plane; so that the imaginary unit

$$i \equiv \exp\left[\tfrac{1}{2} i\pi\right] \tag{40}$$

operates there as a 90° pseudo-rotation. The same can be said, of course, for the operation $\mathscr{S}_2$; in accord with eq.(38). Moreover, with regard the axes x', t', x, t, both observers s_0, S' will agree in the case of a SLT that: $t' \equiv x$; $x' \equiv t$. It follows that eq.(39) can be immediately rewritten

$$\begin{aligned} dt' &= \pm \frac{dx - u\,dt}{\sqrt{1-u^2}} \; ; \\ dx' &= \pm \frac{dt - u\,dx}{\sqrt{1-u^2}} \; , \end{aligned} \qquad \text{(Superluminal case; } u^2 < 1) \tag{39'}$$

where the rôles of the space and the time coordinates apper interchanged, but the imaginary units disappeared.

Let us now take advantage of a very important symmetry property of the ordinary Lorentz boosts, expressed by the identities

$$\begin{aligned} \frac{dx - u\,dt}{\sqrt{1-u^2}} &\equiv - \frac{dt - U\,dx}{\sqrt{U^2-1}} \; ; \\ \frac{dt - u\,dx}{\sqrt{1-u^2}} &\equiv - \frac{dx - U\,dt}{\sqrt{U^2-1}} \; . \end{aligned} \qquad (U \equiv 1/u) \tag{41}$$

Eqs.(39') eventually write

$$dt' = \mp \frac{dt - Udx}{\sqrt{U^2-1}} \; ; \qquad dx' = \mp \frac{dx - Udt}{\sqrt{U^2-1}} \; , \qquad \text{(Superluminal case; } U^2 > 1) \tag{39"}$$

which can be assumed as the canonical form of the SLTs in two dimensions. Let us observe that eqs.(39') or (39") yield for the speed of s_o w.r.t. S':

$$x \equiv 0 \quad \Rightarrow \quad \frac{dx'}{dt'} = \pm \left(-\frac{1}{u}\right) = \mp U \; , \qquad \begin{bmatrix} u^2 < 1 \; ; \; U^2 > 1 \\ U \equiv 1/u \end{bmatrix} \tag{42}$$

where u, U are the speeds of the two dual frames s, S'. This confirms that eqs.(39') (39") do actually refer to Superluminal relative motion. Even for eqs.(39) one could have derived that the G-vectorial velocity $u^\mu \equiv dx^\mu/d\tau_o$ (see the following) changes under transformation (39) in such a way that $u'_\mu u'^\mu = -u_\mu u^\mu$; so that from $u_\mu u^\mu = +1$ it follows $u'_\mu u'^\mu = -1$ (that is to say, bradyonic speeds are transformed into tachyonic speeds). We could have derived the "reinterpreted form" (39')-(39") from the original expression (39) just demanding that the second frame S' move w.r.t. s_o with the Superluminal speed U=1/u, as required by eq.(32).

The group $\mathcal{G}$ of the GLTs in two dimensions can be finally written (Fig.6):

$$\mathcal{G} = \{_+L^\uparrow\} \cup \{_-L^\uparrow\} \cup \{_-\mathcal{S}L^\uparrow\} \cup \{_+\mathcal{S}L^\uparrow\} \; ; \tag{35'}$$

$$\mathcal{S} \equiv \mathcal{S}_2 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{36'}$$

Notice that the transcendent SLT, $\mathcal{S}$, when applied to the motion of a particle, just interchanges the values of energy and impulse, as well as of time and space: cf. also Sects 5.2, 5.3 (Review I; see also Vyšín 1977a,b).

5.7.- Explicit form of GLTs

The LTs and SLTs together, i.e. the GLTs, can be written of course in a form covariant under the whole group $\mathcal{G}$; namely, in "G-covariant" form, they can be written (Figs.7):

$$dt' = \pm \frac{dt - udx}{\sqrt{|1-u^2|}} \; ; \qquad dx' = \pm \frac{dx - udt}{\sqrt{|1-u^2|}} \; , \qquad \text{(Generalized case; } -\infty < u < +\infty) \tag{43}$$

or rather (Recami and Mignani 1973a), in terms of the continuous parameter $\vartheta \in [0, 2\pi]$,

$$dt' = \Omega\gamma_o(dt - dx\,\mathrm{tg}\vartheta)\;; \qquad \left[\begin{array}{c} -\infty < u < +\infty \\ 0 \leq \vartheta \leq 2\pi \end{array}\right] \tag{43a}$$
$$dx' = \Omega\gamma_o(dx - dt\,\mathrm{tg}\vartheta)\;,$$

with

$$u \equiv \mathrm{tg}\vartheta\;, \qquad \Omega \equiv \Omega(\vartheta) = \frac{\cos\vartheta}{|\cos\vartheta|}\,\delta^2\;;$$
$$\delta = +\sqrt{\frac{1-\mathrm{tg}^2}{|1-\mathrm{tg}^2\vartheta|}}\;; \qquad \gamma_o \equiv +(|1-\mathrm{tg}^2\vartheta|)^{-1/2}\;, \tag{43b}$$

where the form (43a) of the GLTs explicitly shows how the signs in front of t', x' succeed one another as functions of u, or rather of ϑ (see also the Figs.2-4 and 6 in Review I).

Apart from Somigliana's early paper, only recently rediscovered (Caldirola et al. 1980), the eqs.(39"), (43) first appeared in Olkhovsky and Recami (1970b, 1971), Recami and Mignani (1972), Mignani et al. (1972), and then - independently - in a number of subsequent papers: see e.g. Antippa (1972) and Ramanujam and Namasivayam (1973). Eqs.(39'), (39") have been shown by Recami and Mignani (1972) to be eqivalent to the pioneering - even if more complicated - equations by Parker (1969). Only in Mignani et al. (1972), however, it was first realized that eqs.(39)-(43) need their double sign, necessary in order that any GLT admits an inverse transformation (see also Mignani and Recami 1973a).

5.8.- The GLTs by discrete scale transformations

If you want, you can regard eqs.(39')-(39") as entailing a "reinterpretation" of eqs.(39), - such a reinterpretation having nothing to do, of course, with the Stückelberg-Feynman "switching procedure", also known as "reinterpretation principle" ("RIP"). - Our interpretation procedure, however, not only is straightforward (cf. eqs.(38),(40)), but has been also rendered automatic in terms of new, scale-invariant "light-cone coordinates" (Maccarrone et al. 1983).

Let us first rewrite the GLTs in a more compact form, by the language of the discrete (real or imaginary) scale transformations (Pavšič and Recami 1977, Pavšič 1978):

$$ds'^2 = \varrho^2 ds^2\;; \qquad \varrho^2 = \pm 1\;; \tag{15'}$$

notice that, in eq.(36), $\mathscr{Z}(4)$ is nothing but the discrete group of the dilations D: $x'_\mu = \varrho x_\mu$ with $\varrho^2 = \pm 1$. Namely, let us introduce the new (discrete) dilation-invariant coordinates (Kastrup 1962):

$$\eta^{\mu} \equiv k\, x^{\mu} \,, \qquad (k = \pm 1, \pm i) \tag{44}$$

k being the <u>intrinsic</u> scale-factor of the considered object; and let us observe that, under a dilation D, it is $\eta'_{\mu} = \eta_{\mu}$ with $\eta'_{\mu} = k' x'_{\mu}$, while $k' = \varrho^{-1} k$. Bradyons (antibradyons) correspond to k=+1 (k=-1), whilst tachyons and antitachyons correspond to $k = \pm i$. It is interesting that in the present formalism the quadratic form $d\sigma^2 \equiv d\eta_{\mu} d\eta^{\mu}$ is invariant, its sign included, under <u>all</u> the GLTs:

$$d\sigma'^2 = +\, d\sigma^2 \,, \qquad \forall G \in \mathscr{G} \,. \tag{15"}$$

Moreover, under an orthochronous Lorentz transformation $L \in \mathscr{L}_+^{\uparrow}$, it holds that $\eta'_{\mu} = L_{\mu\nu} \eta^{\nu}$; $k' = k$.

It follows - when going back to eq.(14), i.e. to the coordinates x^{μ}, k - that the generic GLT ≡ G can be written in two dimensions

$$G_r = k'^{-1} L k \; ; \quad L \in \mathscr{L}_+^{\uparrow} \,. \qquad \left[\begin{array}{l} \varrho^2, k^2 = \pm 1 \\ k' = \varrho^{-1} k \end{array} \right] \tag{45}$$

5.9.- The GLTs in the "light-cone coordinates". <u>Automatic</u> interpretation

It is known (Bjorken et al. 1971) that the ordinary subluminal (proper, orthochronous) boosts along x can be written in the generic form:

$$d\xi' = \alpha\, d\xi \; ; \quad d\zeta' = \alpha^{-1} d\zeta \; ; \quad dy' = \frac{\alpha}{|\alpha|} dy \; ; \quad dz' = \frac{\alpha}{|\alpha|} dz \; ;$$

$$\frac{\alpha - \alpha^{-1}}{\alpha + \alpha^{-1}} = u \; ; \quad u^2 < 1 \,, \qquad (0 < \alpha < +\infty)$$

in terms of the light-cone coordinates (Fig. 8):

$$\xi \equiv t - x \; ; \quad \zeta \equiv t + x \; ; \quad y \; ; \quad z \,. \tag{46}$$

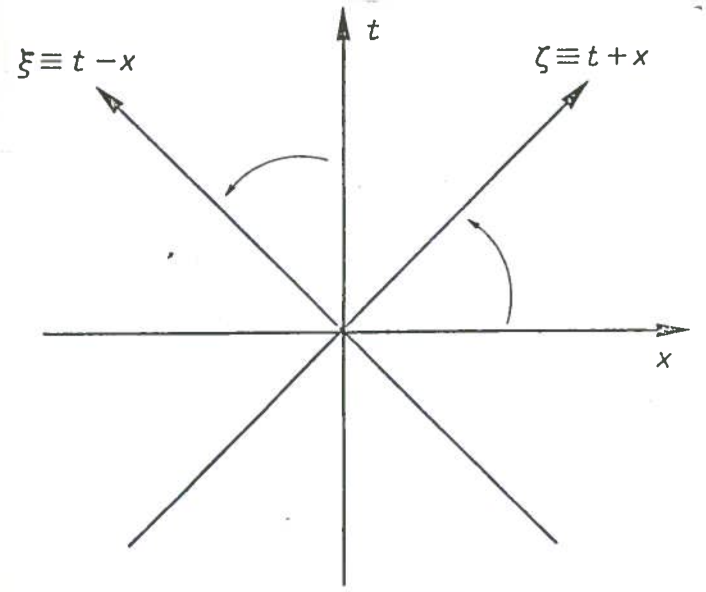


FIG. 8

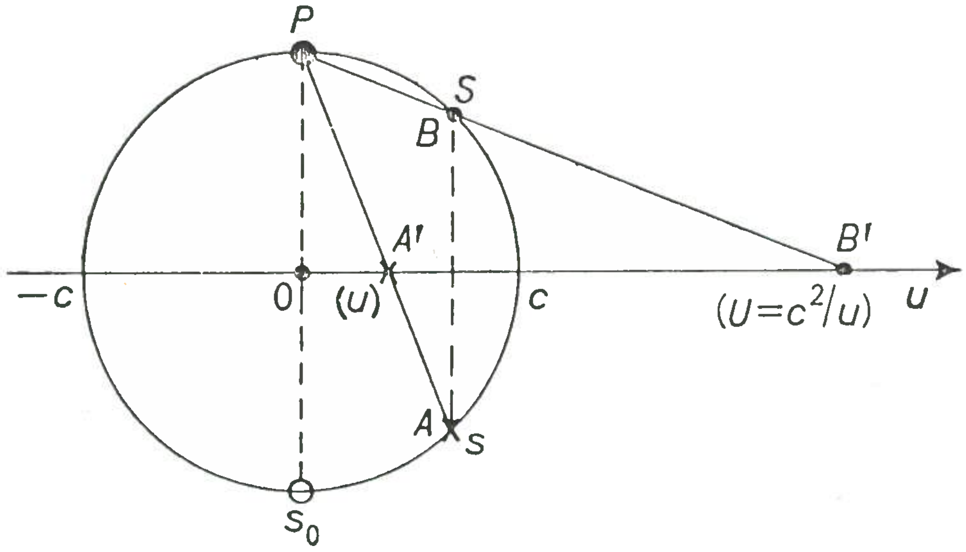


FIG. 9

It is interesting that the orthochronous Lorentz boosts along x just correspond to a dilation of the coordinates ξ, ζ (by the factors α and α^{-1}, respectively, with α any positive real number). In particular for $\alpha \to +\infty$ we have $u \to c^{-}$ and for $\alpha \to 0^{+}$ we have $u \to -(c^{-})$. It is apparent that $\alpha = e^{R}$, where R is the "rapidity".

The proper antichronous Lorentz boosts correspond to the negative real α values (which still yield $u^2 < 1$).

Recalling definitions (44), let us eventually introduce the following scale-invariant "light-cone coordinates":

$$\varphi \equiv \eta^{(o)} - \eta^{(1)} \; ; \quad \psi \equiv \eta^{(0)} + \eta^{(1)} \; ; \quad \eta^{(2)} \; ; \quad \eta^{(3)} \; . \tag{47}$$

In terms of coordinates (47), all the two-dimensional GLTs (both sub- and Superluminal) can be expressed in the synthetic form (Maccarrone et al. 1983):

$$\begin{aligned} &d\varphi' = \alpha d\varphi \; ; \quad d\psi' = \alpha^{-1} d\psi \; ; \\ &k' = \varrho^{-1} k \; ; \quad \alpha \equiv \varrho a \; ; \quad a \in (0, +\infty) \; ; \quad \varrho^2 = \pm 1 \; , \end{aligned} \quad (|u| \lesseqgtr 1) \tag{48}$$

and all of them preserve the quadratic form, its sign included: $\varphi' \psi' = \varphi \psi$.

The point to be emphasized is that eqs.(48) in the Superluminal case yield directly eq.(39"), i.e. they automatically include the "reinterpretation" of eqs.(39). Moreover, eqs.(48) yield

$$\begin{aligned} &u = \frac{\alpha - \alpha^{-1}}{\alpha + \alpha^{-1}} \; ; \\ &\alpha = \varrho a \; ; \quad \varrho^2 = \pm 1 \; ; \end{aligned} \quad (u^2 \lesseqgtr 1 \; ; \quad 0 < a < +\infty) \tag{49}$$

i.e. also in the Superluminal case they forward the correct (faster-than-light) relative speed without any need of "reinterpretation".

5.10.- An application

As an application of eqs.(39"), (43), let us consider a tachyon having (real) proper-mass m_o and moving with speed V relatively to us; then we shall observe the relativistic mass

$$m = \frac{m_o}{(|1-V^2|)^{1/2}} \equiv \frac{-im_o}{(1-V^2)^{1/2}} \equiv \frac{m_o}{(V^2-1)^{1/2}}\ , \qquad (V^2>1;\ m_o\ \text{real})$$

and, more in general (in G-covariant form):

$$m = \pm \frac{m_o}{(|1-v^2|)^{1/2}}\ , \qquad (-\infty < v < +\infty) \tag{50}$$

so as anticipated in Fig. 4a.

For other applications, see e.g. Review I; for instance: (i) for the generalized "velocity composition law" in two dimensions see eq.(33) and Table I in Review I; (ii) for the generalization of the phenomenon of Lorentz contraction/dilation see Fig. 8 of Review I.

5.11.- Dual Frames (or Objects)

Eqs.(32) and follows, show that a one-to-one correspondence

$$v \longleftrightarrow \frac{c^2}{v} \tag{51}$$

can be set between subluminal frames (or objects) with speed $v<c$ and Superluminal frames (or objects) with speed $V \equiv c^2/v > c$. In such a particular conformal mapping (inversion) the speed c is the "united" one, and the speeds zero, infinite correspond to each other. This clarifies the meaning of observation (ii), Sect.3.1, by Einstein. Cf. also Fig.9, which illustrates the important eq.(32). In fact (Review I) the relative speed of two "dual" frames s, S (frames dual one to the other are characterized in Fig.9 by AB being orthogonal to the u-axis) is infinite; the figure geometrically depicts, therefore, the circumstance that $(s_o \rightarrow S) = (s_o \rightarrow s)(s \rightarrow S)$, i.e. the fundamental theorem of the (bidimensional) "Extended Relativity": 'The SLT: $s_o \rightarrow S(U)$ is the product of the LT: $s_o \rightarrow s(u)$, where $u \equiv 1/U$, by the transcendent SLT': Cf. Sect. 5.5, eq.(32). (Mignani and Recami 1973a).

Even in more dimensions, we shall call "dual" two objects (or frames) moving along the same line with speeds satisfying eq.(51):

$$vV = c^2 , \qquad (51')$$

i.e. with infinite relative speed. Let us notice that, if p^μ and P^μ are the energy-momentum vectors of the two objects, then the condition of infinite relative speed writes in G-invariant way as

$$p_\mu P^\mu = 0 . \qquad (51'')$$

5.12.- The "Switching Principle" for tachyons

The problem of the double sign in eq.(50) has been already taken care of in Sect.2 for the case of bradyons (eq.(9)).

Inspection of Fig. 5c shows that, in the case of tachyons, it is enough a (suitable) ordinary subluminal orthochronous Lorentz transformation $L^\uparrow$ to transform a positive-energy tachyon T into a negative-energy tachyon T'. For simplicity let us here confine ourselves, therefore, to transformations $L \equiv L^\uparrow \in \mathscr{L}_+^\uparrow$, acting on free teachyons. (See also, e.g., Marx 1970).

On the other hand, it is wellknown in SR that the chronological order along a space-like path is not $\mathscr{L}_+^\uparrow$-invariant.

However, in the case of Ts it is even clearer than in the bradyon case that the same transformation L which inverts the energy-sign will also reverse the motion-direction in time (Review I, Recami 1973, 1975, 1979a, Caldirola and Recami 1978; see also Garuccio et al. 1980). In fact, from Fig. 10 we can see that for going from a positive-energy state T_i to a negative-energy state T'_f it is necessary to

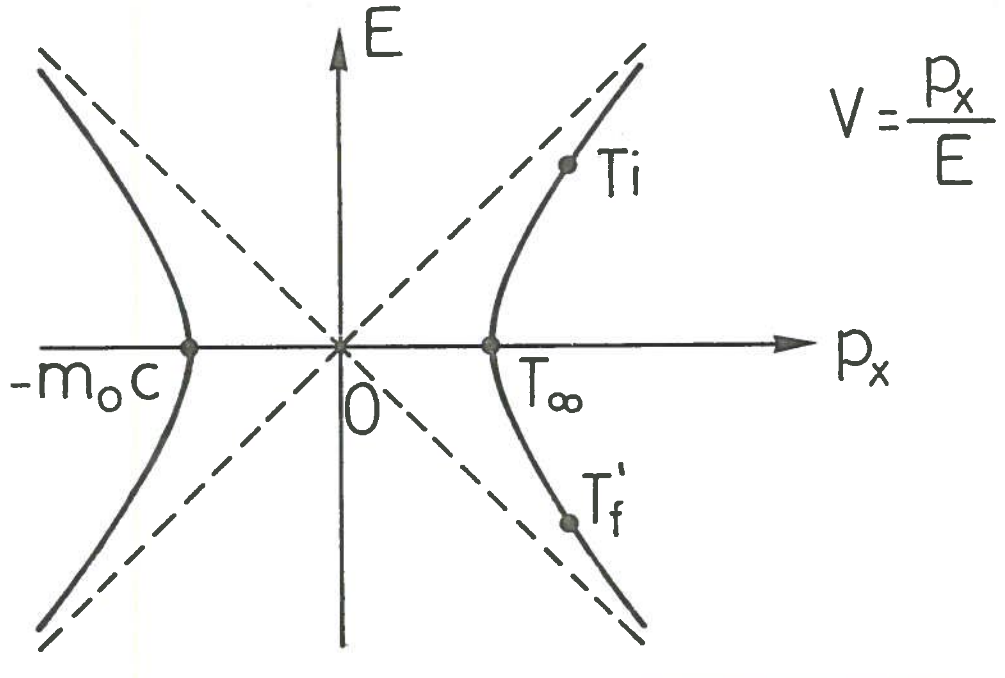


FIG. 10

bypass the "transcendent" state T_∞ (with $V=\infty$). From Fig. 11a we see moreover that, given in the initial frame s_o a tachyon T travelling e.g. along the positive x-axis

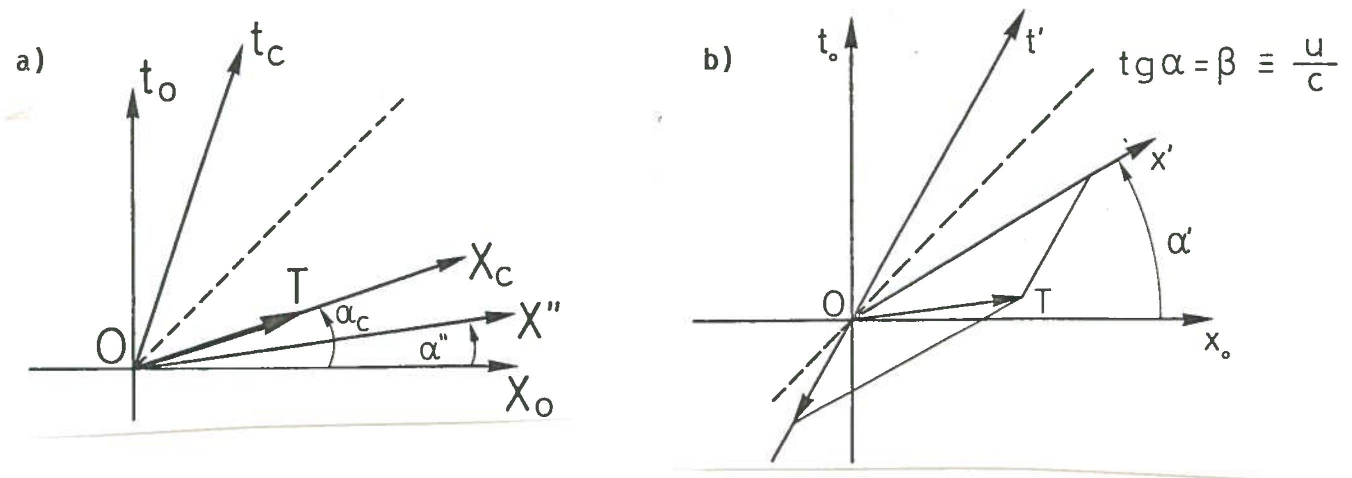


FIG. 11

with speed V_o, the "critical observer" (i.e. the ordinary subluminal observer $s_c \equiv$ $\equiv (t_c,x_c)$ seeing T with <u>infinite</u> speed) is simply the one whose space-axis x is superimposed to the world-line OT; its speed u_c w.r.t. s_o, along the positive x-axis, is evidently

$$u_c = c^2/V_o \; ; \qquad u_c V_o = c^2 \; , \qquad \text{("critical frame")} \tag{52}$$

dual to the tachyon speed V_o. Finally, from Fig.10 and Fig. 11b we conclude that any "trans-critical" observer $s' \equiv (t',x')$ such that $u'V_o > c^2$ will see the tachyon T not only endowed with negative energy, but also travelling backwards in time. Notice, incidentally, that nothing of this kind happens when $uV_o < 0$, i.e. when the final frame moves in the direction opposite to the tachyon's.

Therefore Ts display negative energies in the same frames in which they would appear as "going backwards in time", and vice-versa. As a consequence, we can - and must - apply also to tachyons the Stückelberg-Feynman "switching procedure" exploited in Sects.2.1-2.3. As a result, point A' (Fig.5c) or point T_f' (Fig.10) do <u>not</u> refer to a "negative-energy tachyon moving backwards in time", but rather to an <u>anti-tachyon</u> $\overline{T}$ moving the opposite way (in space), forward in time, and with positive energy. Let us repeat that the "switching" never comes into the play when the sign of u is opposite to the sign of V_o (Review I, Recami 1978c, 1979a, Caldirola and Recami 1980).

The "Switching Principle" has been first applied to tachyons by Sudarshan and coworkers (Bilaniuk et al. 1962; see also Gregory 1961, 1962).

Recently Schwartz (1982) gave the switching procedure an interesting formalization, in which - in a sense - it becomes "automatic".

5.13.- Sources and Detectors. Causality

After the considerations in the previous Sect. 5.12, i.e. when we apply our Third Postulate (Sect.4) also to tachyons, we are left with no negative energies (Recami and Mignani 1973b) and with no motions backwards in time (Maccarrone and Recami 1980a,b and refs. therein).

Let us remind, however, that a tachyon T can be transformed into an antitachyon $\bar{T}$ "going the opposite way in space" even by (suitable) ordinary subluminal Lorentz transformations $L \in \mathscr{L}_+^\uparrow$. It is always essential, therefore, when dealing with a tachyon T, to take into proper consideration also its source and detector, or at least to refer T to an "interaction-region". Precisely, when a tachyon overcomes the divergent speed, it passes from appearing e.g. as a tachyon T entering (leaving) a certain interaction-region to appearing as the antitachyon $\bar{T}$ leaving (entering) that interaction-region (Arons and Sudarshan 1968, Dhar and Sudarshan 1968, Glück 1969, Baldo et al. 1970, Camenzind 1970). More in general, the "trans-critical" transformations $\bar{L} \in \mathscr{L}_+^\uparrow$ (cf. Figs.11 and Sect.5.12) lead from a T emitted by A and absorbed by B to its $\bar{T}$ emitted by B and absorbed by A (see Figs.1 and 3b, and Review I).

The already mentioned fact (Sect.2.2.) that the Stückelberg-Feynman-Sudarshan "switching" exchanges the rôles of source and detector (or, if you want, of "cause" and "effect") led to a series of apparent "causal paradoxes" (see e.g. Thoules 1969, Rolnick 1969, 1972, Benford 1970, Strnad 1970, Strnad and Kodre 1975) which - even if easily solvable, at least in microphysics (Caldirola and Recami 1980 and refs. therein, Maccarrone and Recami 1980a,b; see also Recami 1978a,c, 1973 and refs. therein, Trefil 1978, Recami and Modica 1975, Csonka 1970, Baldo et al. 1970, Sudarshan 1970, Bilaniuk and Sudershan 1969b, Feinberg 1967, Bilaniuk et al. 1962) - gave rise to much perplexity in the literature.

We shall deal with the causal problem in due time (see Sect.9), since various points should rather be discussed about tachyon mechanics, shape and behaviour, before being ready to propose and face the causal "paradoxes". Let us here anticipate that, - even if in ER the judgement about which is the "cause" and which is the "effect", and even more about the very existence of a "causal connection", is relative to the observer -, nevertheless in microphysics the law of "retarded causality" (see our Third Postulate) remains covariant, since any observers will always see the cause to precede its effect.

Actually, a sensible procedure to introduce Ts in Relativity is postulating both (a) tachyon existence and (b) retarded causality, and then trying to build up an ER in which the validity of both postulates is enforced. Till now we have seen that such an attitude - which extends the procedure in Sect.2 to the case of tachyons - has already produced, among the others, the description within Relativity of both matter and antimatter (Ts and $\bar{T}$s, and Bs and $\bar{B}$s).

5.14.- Bradyons and Tachyons. Particles and Antiparticles

Fig. 6 shows, in the energy-momentum space, the existence of two different "symmetries", which have nothing to do one with the other.

The symmetry particle/antiparticle is the mirror symmetry w.r.t. the axis E=0 (or, in more dimensions, to the hyperplane E=0).

The symmetry bradyon/tachyon is the mirror symmetry w.r.t. the bisectors, i. e. to the two-dimensional "light-cone".

In particular, when we confine ourselves to the proper orthochronous subluminal transformations $L^{\uparrow} \in \mathscr{L}_{+}^{\uparrow}$, the "matter" or "antimatter" character is invariant for bradyons (but not for tachyons).

We want at this point to put forth explicitly the following simple but important argumentation. Let us consider the two "most typical" generalized frames: the frame at rest, $s_0 \equiv (t,x)$, and its dual Superluminal frame (cf. eq.(51) and Fig. 9), i.e. the frame $S'_\infty \equiv (t',x')$ endowed with infinite speed w.r.t. s_0. The world-line of S'_∞ will be of course superimposed to the x-axis. With reference to Fig. 7b, observer S'_∞ will consider as time-axis t' our x-axis and as space-axis x' our t-axis and vice-versa for s_0 w.r.t. S'_∞. Due to the "extended principle of relativity" (Sect.4), observers s_0, S'_∞ have moreover to be equivalent.

In space-time (Fig.1) we shall have bradyons and tachyons going both forward and backwards in time (even if for each observer - e.g. for s_0 - the particles travelling into the past have to bear negative energy, as required by our Third Postulate). The observer s_0 will of course interpret all - sub- and Super-luminal - particles moving backwards in his time t as antiparticles; and he will be left only with objects going forward in time.

Just the same will be done, in his own frame, by observer S'_∞, since to him all - sub- or Super-luminal - particles travelling backwards in his time t' (i.e. moving along the negative x-direction, according to us) will appear endowed with negative energy. To see this, it is enough to remember that the transcendent transformation $\mathscr{S}$ does exchange the values of energy and momentum (cf. eq.(38), the final part of Sect.5.6, and Review I). The same set of bradyons and tachyons will be therefore described by S'_∞ in terms of particles and antiparticles all moving along its positive time-axis t'.

But, even if axes t' and x coincide, the observer s_0 will see bradyons and tachyons moving (of course) along both the positive and the negative x-axis! In other words, we have seen the following: The fact that S'_∞ sees only particles and antiparticles moving along its positive t'-axis does not mean at all that s_0 sees only bradyons andtachyons travelling along his positive x-axis! This erroneous belief entered, in connection with tachyons, in the (otherwise interesting) two-dimensional approach by Antippa (1972), and later on contributed to lead Antippa and Everett (1973) to violate space-isotropy by conceiving that even in four dimensions tachyons had to move just along a unique, privileged direction - or "tachyon corridor" -: see Sect.14.1 in the following.

5.15.- Totally Inverted Frames

We have seen that, when a tachyon T appears to overcome the infinite speed (Figs.11a,b), we must apply our Third Postulate, i.e. the "switching procedure". The same holds of course when the considered "object" is a reference frame.

More in general, we can regard the GLTs expressed by eqs.(35')-(36') from the passive, and no more from the active, point of view (Recami and Rodrigues 1982). Instead of Fig.6, we get then what depicted in Fig. 12. For future convenience, let us use the language of multi-dimensionaL space-times. It is apparent that the four subsets of GLTs in eq.(36') describe the transitions from the initial frame s_0 (e.

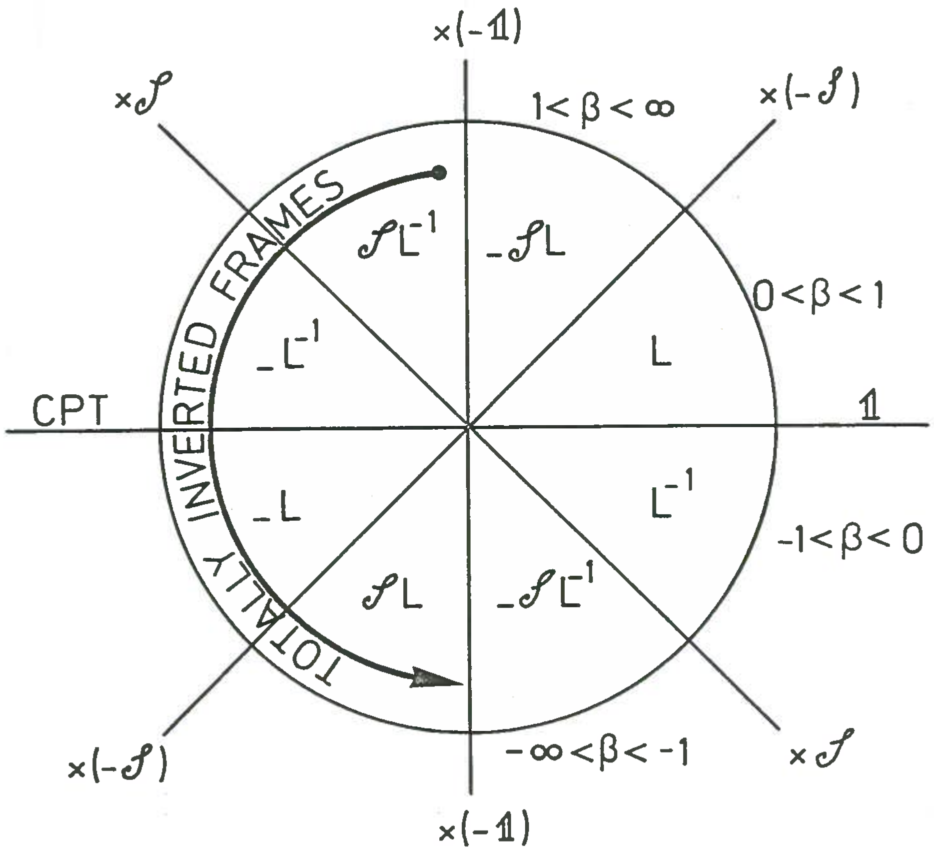


FIG. 12

g. with right-handed space-axes) not only to all frames f^R moving along x with all possible speeds $u \in (-\infty,+\infty)$, but also to the "totally inverted" frames $\tilde{f}^L=(-\mathbb{1})f^R = (\overline{P}\overline{T})f^R$, moving as well along x with all possible speeds u (cf. Figs.2-6 and 11 in Review I). With reference to Fig.9, we can say loosely speaking that, if an ideal frame f could undergo a whole trip along the axis (circle) of the speeds, than - after having overtaken $f(\infty) \equiv f(U=\infty)$ - it would come back to rest with a left-handed set of space-axes and with particles transformed into antiparticles. For further details, see Recami and Rodrigues (1982) and refs. therein.

5.16.- About CPT

Let us first remind (Sect.5.5) that the product of two SLTs (which is always a subluminal LT) can yield a transformation both orthochronous, $L^{\uparrow} \in \mathscr{L}_+^{\uparrow}$, and antichronous, $(-\mathbb{1})L^{\uparrow} \equiv (\overline{P}\overline{T})L^{\uparrow} = L^{\downarrow} \in \mathscr{L}_+^{\downarrow}$ (cf. Sect.2.3). We can then give eq.(10) the following meaning within ER.

Let us consider in particular (cf. Figs.7a,b) the antichronous GLT(ϑ=180°) = $= -\mathbb{1} = \overline{P}\overline{T}$. In order to reach the value ϑ=180° starting from ϑ=0 we must bypass the case ϑ=90° (see Fig. 12), where the switching procedure has to be applied (Third Postulate). Therefore:

$$\mathrm{GLT}(\vartheta=180°) = -\mathbb{1} \equiv \overline{P}\overline{T} = CPT \ . \tag{53}$$

The "total inversion" $-\mathbb{1} \equiv \overline{P}\overline{T}$ = CPT is nothing but a particular "rotation" in space-time, and we saw the GLTs to consist in all the space-time "rotations" (Sect. 5.6). In other words, we can write: CPT $\in \mathscr{G}$, and the CPT-theorem may be regarded as a particular, explicit requirement of SR (as formulated in Sect.2), and *a fortiori* of ER (Mignani and Recami 1974b, 1975a, and refs therein, Recami and Ziino 1976, Pavšič and Recami 1982). Notice that, in our formalization, the operator CPT is linear and unitary.

Further considerations will be added in connection with the multidimensional cases (see Sects.11.1-11.3).

5.17.- Laws and Descriptions. Interactions and Objects

Given a certain phenomenon ph, the principle of relativity (First Postulate) requires two different inertial observers O_1, O_2 to find that ph is ruled by the same physical laws, but it does not require at all O_1, O_2 to give the same description of ph (cf. e.g. Review I; p.555 in Recami 1979a; p.715, Appendix, in Recami and Rodrigues 1982).

We have already seen in ER that, whilst the "Retarded Causality" is a law (corollary of our Third Postulate), the assignment of the "cause" and "effect" labels is relative to the observer (Camenzind 1970); and is to be considered a description-detail (so as, for instance, the observed color of an object).

In ER one has to become acquainted with the fact that many description-details, which by chance were Lorentz-invariant in ordinary SR, are no more invariant under the GLTs. For example, what already said (see Sect.2.3, point e)) with regard to the possible non-invariance of the sign of the additive charges under the transformations $L \in \mathscr{L}_+$ holds a fortiori under the GLTs, i.e. in ER. Nevertheless, the total charge of an isolated system will have of course to be constant during the time -evolution of the system - i.e. to be conserved - as seen by any observer (cf. also Sect.15).

Let us refer to the explicit example in Fig. 13 (Feinberg 1967, Baldo et al. 1970), where the pictures (a), (b) are the different descriptions of the same inter

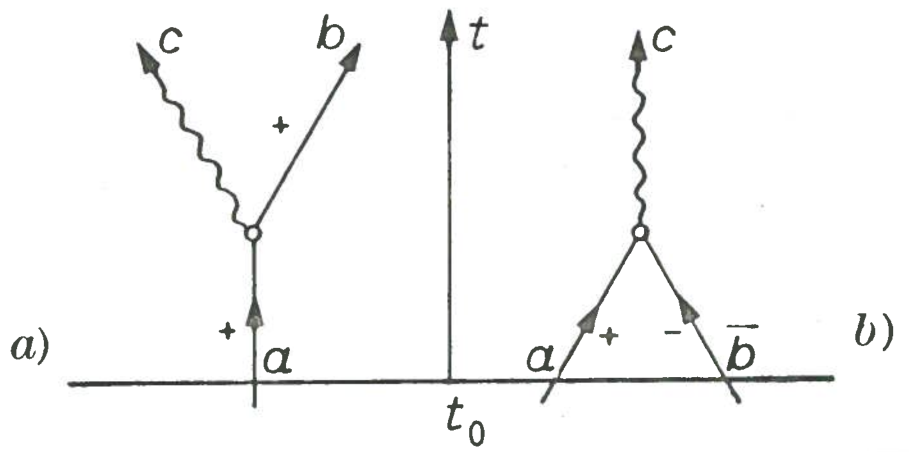


FIG. 13

action given by two different (generalized) observers. For instance, (a) and (b) can be regarded as the descriptions, from two ordinary subluminal frames O_1, O_2, of one and the same process involving the tachyons a, b (c can be a photon, e.g.). It is apparent that, before the interaction, O_1 sees one tachyon while O_2 sees two tachyons. Therefore, the very number of particles - e.g. of tachyons, if we consider only subluminal frames and LTs - observed at a certain time-instant is not Lorentz-invariant. However, the total number of particles partecipating in the reaction either in the initial or in the final state is Lorentz-invariant (due to our initial three Postulates). In a sense, ER prompts us to deal in physics with interactions rather than with objects (in quantum-mechanical language, with "amplitudes" rather than with "states"); (cf. e.g. Glück 1969, Baldo and Recami 1969).

Long ago Baldo et al. (1970) introduced however a vector-space $H = \mathscr{H} \otimes \bar{\mathscr{H}}$, direct product of two vector-spaces $\mathcal{H}$ and $\overline{\mathcal{H}}$, in such a way that any Lorentz transformation was unitary in the H-space even in presence of tachyons. The spaces $\mathcal{H}$ ($\overline{\mathcal{H}}$) were defined as the vector-spaces spanned by the states representing particles and antiparticles only in the initial (final) state. Another way out, at the classical level, has been recently put forth by Schwartz (1982).

5.18.- SR with tachyons in two dimensions

Further developments of the classical theory for tachyons in two dimensions, after what precedes, can be easily extracted for example from: Review I and refs. therein; Recami (1978b, 1979a), Corben (1975,1976,1978), Caldirola and Recami (1980), Maccarrone and Recami (1980b, 1982a), Maccarrone et al. (1983).

We merely refer here to those papers, and references therein. But the many po

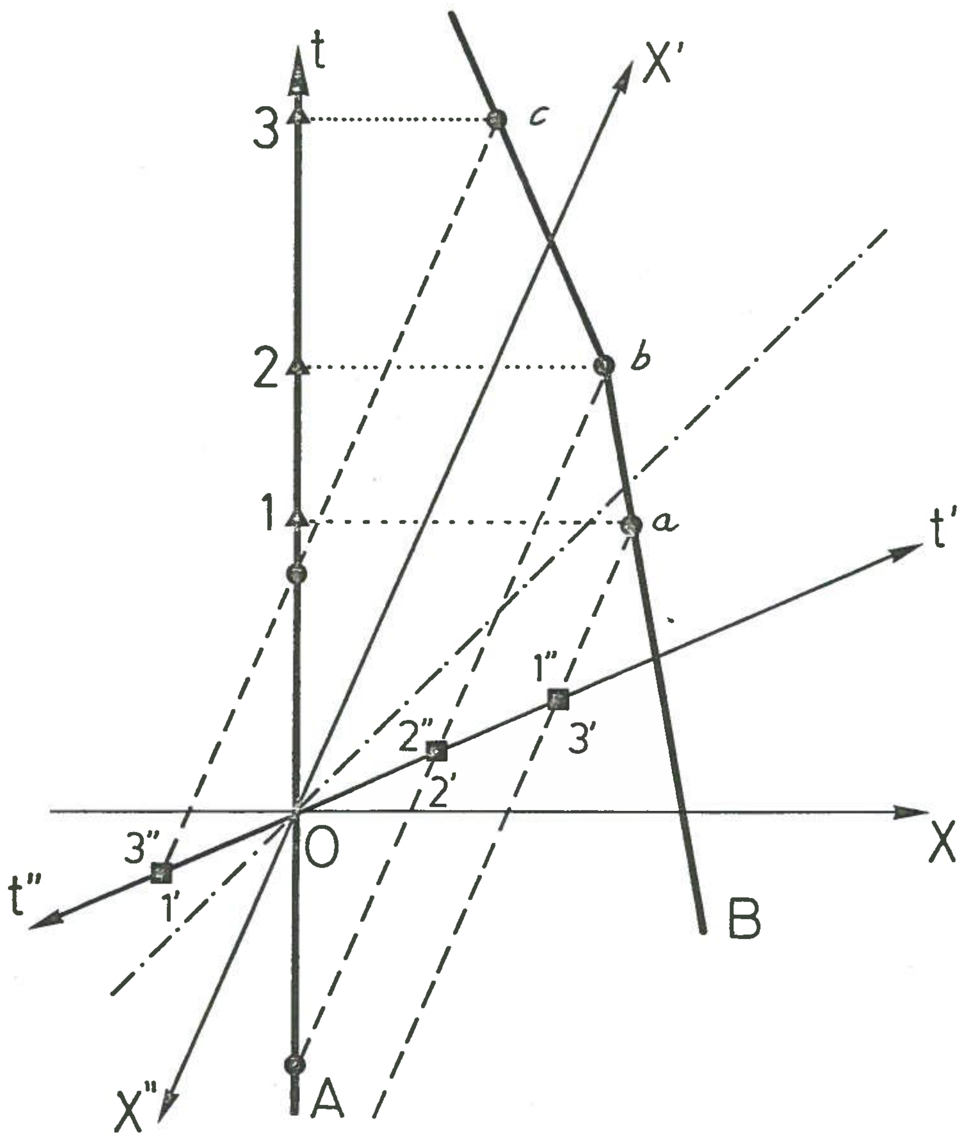


FIG. 14

sitive aspects and meningful results of the two-dimesnional ER - e.g. connected with the deeper comprehension of the ordinary relativistic physics that it affords - will be apparent (besides from Sect.5) also from the future Sections dealing with the multi-dimensional cases.

In particular, further subtelities of the so called "causality problem" (a problem already faced in Sects.5.12-5.14) will be tackled in Sect.9.

Here we shall only make the following (simple, but important) remark. Let us consider two (bradyonic) bodies A, B that for instance - owing to mutual attraction - accelerate while approaching each other. The situation is sketched in Fig. 14 where A is chosen as the reference-frame s ≡ (t,x) and, for simplicity, only one discrete velocity-change is depicted. From a Superluminal frame they will be described either as two tachyons that decelerate while approaching each other (in the frame S" ≡ (t",x")), or as two (anti)tachyons that accelerate while receding one from the other (frame S' ≡ (t',x')). Therefore, we expect that two tachyons from the kinematical point of view will seem to suffer a repulsion, if they attract each other in their own rest-frames (and in the other frames in which they are subluminal); we shall however see that such a behaviour of tachyons may be still considered - from the energetical and dynamical points of view - as due to an attraction.

Before going on, let us explicitly remark that the results of the model-theory in two dimensions strongly prompt us to attempt building up a similar theory (based as far as possible on the same Postulates) also in more dimensions.

6.- TACHYONS IN FOUR DIMENSIONS: RESULTS INDEPENDENT OF THE EXISTENCE OF SLTs

6.1.- Caveats

We have seen that a model-theory of ER in two dimensions can be straightforwardly built up (Sect. 5).

We have also anticipated (Sect.3.2) that the construction of an ER is straightforward as well in the pseudo-Euclidean space-times M(n,n), and in Sect.14.3 we shall approach the case n=3 (Mignani and Recami 1976b, Maccarrone and Recami 1982a, 1984a).

In the 4-dimensional Minkowski space-time M(1,3), however, if we want a priori to enforce the Principle of Relativity for both sub- and Super-luminal (inertial) frames, it comes the following (cf. Figs. 7a,b). Our own world-line coincides with our time-axis t; the world-line t' of a transcendent (infinite speed) free tachyon moving along the x-axis will coincide, on the contrary, with our x-axis (in our language). The transcendent observer would then call time-axis (t') what we call x-axis, and analogously would consider our axes t,y,z as his three space-axes x',y',z' Conversely, due to our first two Postulates (i.e. to the requirements in Sect.4.2), he would seem to possess one space-axis and three time-axes! (Maccarrone and Recami 1982a,b and refs. therein, Recami 1979a). This point constitutes the problem of the 4-dimensional ER, i.e. of the SLTs in four dimensions. We shall deal with it in Sect.14.

In four dimensions, however, we can start as a first step by studying here the behaviour of tachyons within the "weak approach" (Sect.3.2), i.e. confining preliminarily the observers to be all subluminal. In this Section, therefore, we shall only assume the existence of sub- and Super-luminal (observed) objects. Tachyons are the space-like ones, for which in four dimensions it is $ds^2 \equiv dt^2 - d\mathbf{x}^2 < 0$ and (m_o real) :

$$p_\mu p^\mu \equiv E^2 - \mathbf{p}^2 = - m_o^2 < 0 \, . \qquad (29c)$$

To go on, therefore, we need only the results in Sects.5.12, 5.13, which do not imply any SLT. Those results remain moreover valid in four dimensions (see Sects. 5.12 and 2.1), provided that one takes into account the fact that the relevant speed is now the component V_x of the tachyon velocity $\mathbf{V}$ along the (subluminal) boost-direction (Review I; Maccarrone et al. 1983 p.108; Maccarrone and Recami 1984a, Sect.8). Namely, if $\mathbf{u}$ is the (subluminal) boost-velocity, then the new observer s' will see instead of the initial tachyon T an antitachyon T travelling the opposite way ("switching principle") if and only if (Maccarrone and Recami 1980b)

$$\mathbf{u} \cdot \mathbf{V} > c^2 \, . \qquad (52')$$

Remember once more that if $\mathbf{u} \cdot \mathbf{V}$ is negative the "switching" does never come into the play.

As an example of results that do not depend on the very existence of SLTs, let us consider some tachyon kinematics.

6.2.- On Tachyon Kinematics

Let us first explore the unusual and unexpected kinematical consequences of the mere fact that in the case of tachyons (see eq.(29c)) it holds

$$|E| = + \sqrt{\mathbf{p}^2 - m_o^2} \, , \qquad (m_o \text{ real} \; ; \quad V^2 > 1) \, , \qquad (54)$$

as partially depicted in Fig. 4.

To begin with, let us recall (Feinberg 1967, Dhar and Sudarshan 1968, Review I) that a bradyon at rest - for instance a proton p -, when absorbing a tachyon or antitachyon t, may transform into itself: $p + t \rightarrow p$. This can be easily verified (see the following) in the rest-frame of the initial proton. It can be similarly verified that, in the same frame, the proton cannot decay into itself plus a tachyon. However, if we pass from that initial frame to another subluminal frame moving e.g. along the x-axis with positive speed $u \equiv u_x > 1/V_x$ (where V_x, assumed to be positive too, is the velocity x-component of t), we know from Sects.5.12-5.14 that in the new frame the tachyon t entering the above reaction will appear as an outgoing antitachyon: $p \rightarrow p + \bar{t}$. In other words, a proton in flight (but not at rest!) may a priori be seen to decay into itself plus a tachyon (or antitachyon).

Let us examine the tachyon kinematics with any care, due to its essential rôle in the proper discussion of the causality problems.

6.3.- "Intrinsic emission" of a tachyon

Firtly, let us describe (Maccarrone and Recami 1980a,b and refs. therein) the phenomenon of "intrinsic emission" of a tachyon, as seen in the rest-frame of the emitting body, and in generic frames as well. Namely, let us first consider in its rest-frame a bradyonic body C, with initial rest-mass M, which emits towards a second bradyonic body D a tachyon (or antitachyon) T, endowed with (real) rest-mass m and 4-momentum $p \equiv (E_T, \underset{\sim}{p})$ and travelling with speed V in the x-direction. Let M' be the final rest-mass of the body C. The 4-momentum conservation requires

$$M = \sqrt{\underset{\sim}{p}^2 - m^2} + \sqrt{\underset{\sim}{p}^2 + M'^2} \qquad \text{(rest-frame)} \tag{55}$$

that is to say

$$2M|\underset{\sim}{p}| = \sqrt{[m^2 + (M'^2 - M^2)]^2 + 4m^2M^2}\ , \tag{55'}$$

wherefrom it follows that a body (or particle) C cannot emit in its rest-frame any tachyon T (whatever its rest-mass m be), unless the rest-mass M of C "jumps" classically to a lower value M' such that $(E_T = +\sqrt{\underset{\sim}{p}^2 - m^2})$:

$$\Delta \equiv M'^2 - M^2 = -m^2 - 2ME_T\ , \qquad \text{(emission)} \tag{56}$$

so that

$$-M^2 < \Delta \leqslant -\underset{\sim}{p}^2 \leqslant -m^2\ . \qquad \text{(emission)} \tag{57}$$

Eq.(55') can read

$$V = \sqrt{1 + 4m^2M^2/(m^2 + \Delta)^2}\ . \tag{55''}$$

In particular, since infinite-speed Ts carry zero energy but non-zero impulse, $|\underset{\sim}{p}| = m_0 c$, then C cannot emit any transcendent tachyon without lowering its rest--mass; in fact, in the case of infinite-speed T emission, i.e. when $E_T=0$ (in the rest-frame of C), eq.(56) yields

$$\Delta = - m^2 . \qquad (V = \infty \;;\; E_T = 0) \qquad (58)$$

Since emission of transcendent tachyons (antitachyons) is equivalent to absorption of transcendent antitachyons (tachyons), we shall get again eq.(58) also as a limiting case of tachyon absorption (cf. eq.(64)).

It is essential to notice that Δ is, os course, an <u>invariant</u> quantity; in fact, in a generic frame f eq.(56) can be read

$$\Delta = - m^2 - 2p_\mu p^\mu , \qquad (59)$$

where p^μ is now the initial 4-momentum of body C w.r.t. the generic frame f. It is still apparent that $- M^2 < \Delta \leqslant - m^2$. If we recall (cf. eq.(51")) that two objects having infinite relative speed possess orthogonal 4-momenta

$$p_\mu p^\mu = 0 \qquad (51")$$

we get again eq.(58) for the case in which T is transcendent w.r.t. body C.

6.4.- Warnings

The word "emission" in eq.(57) aims at indicating - let us repeat - an <u>intrinsic, "proper"</u> behaviour, in the sense that it refers to emission (as seen) in the rest-frame of the emitting body or particle. In suitably moving frames f, such an 'emission' can even appear as an absorption.

Conversely, other (suitably moving) frames f' can observe a T-emission from C (in flight), which does <u>not</u> satisfy inequalities (57) since it corresponds in the rest-frame of C to an (intrinsic) <u>absorption</u>.

However, if - in the moving frame f - inequalities (57) appear to be satisfied, this implies that in the C-rest-frame the process under exam is a tachyon emission, both when f observers an actual emission and when f observes on the contrary an absorption. We can state the following theorem:

<u>Theorem 1</u>: 'Necessary and sufficient condition for a process, observed either as the emission or as the absorption of a tachyon T by a bradyon C, to be a tachyon -emission in the C rest-frame - i.e. to be an <u>"intrinsic emission"</u> - is that during the process C <u>lowers</u> its rest-mass (invariant statement!) in such a way that : $- M^2 < \Delta \leqslant - m^2$, where M, m, Δ are defined above'.

Let us anticipate that, in the case of "intrinsic absorption", relation (62') will hold instead of relation (57); and let us observe the following. Since the (invariant) quantity Δ in the relation (62') can assume also positive values (contrary to the case of eqs.(56)-(57)), if an observer f sees body C to increase its rest-mass in the process, then the "proper description" of the process can be nothing but an intrinsic absorption.

Let us stress once again that the body C, when in flight, can appear to emit suitable tachyons without lowering (or even changing) its rest-mass: In particular, a particle in flight can a priori emit a suitable tachyon t transforming into itself. But in such cases, if we pass in the rest-frame of the initial particle, the "emitted" tachyon appears then as an absorbed antitachyon $\bar{t}$.

At last, when Δ in eqs.(56)-(59) can assume only known discrete values (so as in elementary particle physics), then - once M is fixed - eq.(56) imposes a link between m and E_T, i.e. between m and $|\mathbf{p}|$.

6.5.- "Intrinsic absorption" of a tachyon

Secondly, let us consider (Maccarrone and Recami 1980a,b) our bradyon C, with rest-mass M, to absorb now in its rest-frame a tachyon (or antitachyon) T' endowed with (real) rest-mass m, 4-momentum $p \equiv (E_T, \mathbf{p})$, emitted by a second bradyon D, and travelling with speed V (e.g. along the x-direction).

The 4-momentum conservation requires that

$$M + \sqrt{\mathbf{p}^2 - m^2} = \sqrt{\mathbf{p}^2 + M'^2}\,, \qquad \text{(rest frame)} \qquad (60)$$

wherefrom it follows that a body (or particle) C at rest can a priori absorb (suitable) tachyons both when increasing or lowering its rest-mass, and when conserving it. Precisely, eq.(60) gives

$$|\mathbf{p}| = \frac{1}{2M}\sqrt{(m^2 + \Delta)^2 + 4m^2M^2} \qquad \text{(rest frame)} \qquad (61)$$

which corresponds to

$$\Delta = -m^2 + 2ME_T \qquad (62)$$

so that

$$-m^2 \leq \Delta \leq \infty\,. \qquad \text{(absorption)} \qquad (62')$$

Eq.(61) tells us that body C in its rest-frame can absorb T' only when the tachyon speed is

$$V = \sqrt{1 + 4m^2M^2/(m^2 + \Delta)^2}\,. \qquad (63)$$

Notice that eq.(62) differs from eq.(56), such a difference being in agreement with the fact that, if bradyon C moves w.r.t. tachyon T', then - in the C-rest-frame - eq.(60) can transform into eq.(55): Cf. Sects.5.12-5.14. Eqs.(61),(63) formally coincide, on the contrary, with eqs.(55'),(56"), respectively; but they refer to different domains of Δ: In eq.(55") we have $\Delta < - m^2$, while in eq.(63) we have $\Delta > - m^2$.

In particular eq.(63) yields that C can absorb (in its rest-frame) infinite-speed tachyons only when $m^2 + \Delta = 0$, i.e.

$$V = \infty \Leftrightarrow \Delta = - m^2 \qquad \text{(rest-frame)} \tag{64}$$

in agreement with eq.(58), as expected.

Quantity Δ, of course, is again invariant. In a generic frame f eq.(62) can be written

$$\Delta = - m^2 + 2p_\mu P^\mu \tag{65}$$

P^μ being now the initial C-fourmomentum in f. Still $\Delta \geq - m^2$. Notice also here that the word absorption in eq.(62') means "intrinsic absorption", since it refers to 'absorption (as seen) in the rest-frame of the absorbing body or particle'. This means that, if a moving observer f sees relation (62) being satisfied, the "intrinsic" description of the process, in the C-rest-frame, is a tachyon absorption, both when f observes an actual absorption and when f observes on the contrary an emission. Let us state the following theorem:

Theorem 2: 'Necessary and sufficient condition for a process, observed either as the emission or as the absorption of a tachyon T' by a bradyon C, to be a tachyon-absorption in the C-rest-frame - i.e. to be an "intrinsic absorption" - is that $- m^2 \leq \Delta < + \infty$'. In the particular case $\Delta = 0$, one simply gets

$$2ME_T = m^2 . \qquad (M' = M)$$

When Δ in eqs.(61)-(65) can assume only known discrete values (so as in elementary particle physics) then - once M is fixed - eqs.(61)-(65) provide a link between m and E_T (or $|\underline{p}|$, or V).

6.6.- Remarks

We shall now describe the tachyon-exchange between two bradyonic bodies (or particles) A and B, because of its importance not only for causality but possibly also fo particle physics. We have to write down the implications of the 4-momentum conservation at A and at B; in order to do so we need choosing a unique frame wherefrom to describe the processes both at A and at B.Let us choose the rest-frame of A.

However, before going on, let us explicitly remark the important fact that, when bodies A and B exchange one tachyon T, the unusual tachyon kinematics is such that the "intrinsic descriptions" of the processes at A and at B (in which the process at A is described from the rest-frame of A and the process at B is now described from the rest-frame of B) can a priori be of the following four types (Maccarrone and Recami 1980a,b):

(i) emission - absorption ;
(ii) absorption - emission ;
(iii) emission - emission ; (66)
(iv) absorption - absorption .

Notice that the possible cases are not only (i) and (ii). Case (iii) can take place only when the tachyon-exchange happens in the receding phase (i.e. while A,B are receding from each other); case (iv) can take place only when the tachyon-exchange happens in the approaching phase (i.e. when A,B are approaching to each other).

Let us repeat that the descriptions (i)-(iv) above do not refer to one and the same observer, but on the contrary add together the "local" descriptions of observers A and B.

6.7.- A preliminary application

For instance, let us consider an elastic scattering between two (different) particles a, b. In the c.m.s., as wellknown, a and b exchange momentum but no energy. While no bradyons can be the realistic carries of such an interaction, an infinite-speed tachyon T can be on the contrary a suitable interaction-carrier (notice that T will appear as a finite-speed tachyon in the a, b rest-frames). However, if a, b have to retain their rest-mass during the process, then the tachyon-exchange can describe that elastic process only when "intrinsic ansorptions" take place both at a and at b (and this can happen only when a, b are approaching to each other).

6.8.- Tachyon exchange when $\underline{u}\cdot\underline{V}\leq c^2$. Case of "intrinsic emission" at A

Let $\underline{V}$, $\underline{u}$ be the velocities of the tachyon T and the bradyonic body C, respectively, in the rest-frame of A. And let us consider A, B to exchange a tachyon (or antitachyon) T when $\underline{u}\cdot\underline{V}<c^2$. In the rest-frame of A we can have either intrinsic emission or intrinsic absorption from the bradyonic body A. Incidentally, the case $\underline{u}\cdot\underline{V}<c^2$ includes both tachyon exchanges in the "approaching phase" (for intrinsic T emission at A), and in the "recession phase" (for intrinsic T absorption at A).

Let us first confine ourselves to the case when one observes in the A-rest--frame an (intrinsic) tachyon emission from A. In such a case both A and B will see the exchanged tachyon to be emitted by A and absorbed by B. In fact, the observer B would see an antitachyon $\bar{T}$ (travelling the opposite way in space w.r.t. tachyon T, according to the "switching principle") only when $\underset{\sim}{u}\cdot\underset{\sim}{V} > c^2$, whilst in the present case $\underset{\sim}{u}\cdot\underset{\sim}{V} < c^2$.

Imposing the 4-momentum conservation at A, we get in the A-rest-frame all the eqs.(55)-(59), where for future clarity a subscript A should be introduced to identify the quantities (M_A, M'_A, Δ_A, p^{μ}_A) pertaining to A.

Let us remain in the rest-frame of A, and study now the kinematical conditions under which the tachyon T emitted by A can be absorbed by the second body B. Let M_B and $P_B \equiv (M_B, \underset{\sim}{P}_B)$ be rest-mass and 4-momentum of body B, respectively. Then:

$$\sqrt{\underset{\sim}{P}_B^2 + M_B^2} + \sqrt{\underset{\sim}{p}^2 - m^2} = \sqrt{(\underset{\sim}{P}_B + \underset{\sim}{p})^2 + M_B'^2} \quad , \tag{67}$$

where M'_B is the B final mass. Let us define $\Delta_B = M_B'^2 - M_B^2$, which reads: $\Delta_B = - m^2 + 2\tilde{m}\tilde{M}_B(1 - uV\cos\alpha)$, where $\tilde{m} \equiv E_T$, $\tilde{M}_B \equiv E_B \equiv \sqrt{\underset{\sim}{P}_B^2 + M_B^2}$ are the relativistic masses of T and B, respectively, and $\alpha = \widehat{\underset{\sim}{u}\underset{\sim}{V}}$. The invariant quantity Δ_B in a generic frame f would be written

$$\Delta_B = - m^2 + 2p_{\mu}P^{\mu}_B \tag{68}$$

with p_{μ} , P^{μ}_B the T and B fourmomenta in f. At variance with the process at A (intrinsic emission: eq.(56)), now Δ_B can a priori be both negative and positive or null:

$$- m^2 \leqslant \Delta_B < + \infty \quad . \qquad \text{(intrinsic absorption)} \tag{69}$$

Notice that, if relation (69) is verified, then the process at B will appear in the B-rest-frame as an (intrinsic) absorption, whatever the description of the process given by f may be. Of course the kinematics associated with the eq.(67) is such that Δ_B can even be smaller than $- m^2$; but such a case ($uV\cos\alpha > 1$) would correspond to intrinsic emission at B (and no more to intrinsic absorption).

In conslusion, the tachyon exchange here considered is allowed when in the A-rest-frame the following equations are simultaneously satisfied:

$$\begin{aligned} \Delta_A &= - m^2 - 2M_A E_T \quad , \\ \Delta_B &= - m^2 + 2E_T E_B(1 - \underset{\sim}{u}\cdot\underset{\sim}{V}) \quad , \end{aligned} \tag{70}$$

with

$$- M_A^2 < \Delta_A < - m^2 \; ; \qquad \Delta_B > - m^2 \; . \tag{70'}$$

When B is at rest w.r.t. A we recover Sect.6.5.

Differently from Δ_A, quantity Δ_B can even vanish; in this case the second of eqs.(70) simplifies into $2E_T E_B(1 - \underset{\sim}{u}\cdot\underset{\sim}{V}) = m^2$. In the very particular case when both $\underset{\sim}{P}_B$ and Δ_B are null, we get $V = \sqrt{1 + 4M_B^2/m^2}$. Further details can be found in Maccarrone and Recami (1980b), which constitutes the basis also of Sects.6.9-6.13.

6.9.- The case of "intrinsic absorption" at A (when $\underset{\sim}{u}\cdot\underset{\sim}{V} \leq c^2$)

Let us consider tachyon exchanges such that the process at A appears, in the A rest-frame, as an (intrinsic) absorption. The condition $u\cdot V < c^2$ then implies body B to appear as emitting the tachyon T both in the A-rest-frame and in its own rest--frame.

The present case, therefore, is just the symmetrical of the previous one (Sect.6.8); the only difference being that we are now in the rest-frame of the absorbing body A. In conclusion, this tachyon-exchange is allowed when eqs.(70) are simultaneously satisfied, but with

$$\Delta_A \geq -m^2 \; ; \qquad -M_B^2 < \Delta_B \leq -m^2 \; . \tag{71}$$

In the particular case in which B moves along the same motion-line than T (along the x-axis, let us say) so that $\underset{\sim}{P}_B//(\pm\underset{\sim}{p})$, then

$$2M_B^2|\underset{\sim}{p}| = E_B\sqrt{(m^2+\Delta_B)^2 + 4m^2M_B^2} \mp (m^2+\Delta_B)|\underset{\sim}{P}_B| \; ; \qquad (\underset{\sim}{P}_B//(\pm\underset{\sim}{p})) \tag{72}$$

whilst for the analogous situation of the case in Sect.6.8 we would have obtained (owing to evident symmetry reasons) eq.(72) with opposite signs in its r.h.s. Moreover, when B is at rest w.r.t. body A, so that $\underset{\sim}{P}_B=0$, we recover mutatis mutandis eq.(55'), still with $-M_B^2 < \Delta_B < -m^2$.

6.10.- Tachyon exchange with $\underset{\sim}{u}\cdot\underset{\sim}{V} \geq c^2$. Case of "intrinsic emission" at A

Still in the A-rest-frame, let us now consider A, B to exchange a tachyon T when $\underset{\sim}{u}\cdot\underset{\sim}{V} \geq c^2$. Again we can have either "intrinsic emission" or "intrinsic absorption" at A. The present cases differ from the previous ones (Sects.6.8, 6.9) in the fact that now - due to the "switching procedure" (cf. the Third Postulate) - any process described by A as a T emission at A and a T absorption at B is described in the B-rest-frame as a $\overline{T}$ absorption at A and a $\overline{T}$ emission at B, respectively.

Let us analyse the case of "intrinsic emission" by body A. Due to the condition $\underset{\sim}{u}\cdot\underset{\sim}{V} > c^2$ (cf. eq.(52')) and to the consequent "switching", in the rest-frame of

B one then observes an antitachyon $\bar{T}$ absorbed by A. Necessary condition for this case to take place is that A, B be receding from each other (i.e., be in the "recession phase").

In any case, for the process at A (in the A-rest-frame) we get the same kinematics already expounded in Sects.6.8 and 6.3.

As to the process at B, in the A rest-frame the body B is observed to absorb a tachyon T, so that eq.(67) holds. In the B rest-frame, however, one observes an (intrinsic) $\bar{T}$ emission, so that Theorem 1 is here in order: Namely, $-M_B^2 < \Delta_B \leq -m^2$. Notice that, when passing from the A to the B rest-frame (and applying the switching procedure), in eq.(67) one has: i) that quantity E_T changes sign, so that quantity $\sqrt{p^2-m^2}$ appears added to the r.h.s., and no longer to the l.h.s.; ii) that the tachyon 3-momentum $\underset{\sim}{p}$ changes sign as well (we go in fact from a tachyon T with impulse $\underset{\sim}{p}$ to its antitachyon $\bar{T}$ with impulse $-\underset{\sim}{p}$).

In conclusion, the tachyon exchange is kinematically allowed when the two eqs.(70) are simultaneously verified, but now with

$$- M_A^2 < \Delta_A \leq - m^2 \; ; \qquad - M_B^2 < \Delta_B \leq - m^2 \; . \tag{73}$$

In the particular case when $\underset{\sim}{P}_B$ and $\underset{\sim}{p}$ are collinear (we can have only $\underset{\sim}{P}_B // \underset{\sim}{p}$: recession phase), we get

$$2M_B^2 |\underset{\sim}{p}| = E_B \sqrt{(m^2+\Delta_B)^2 + 4m^2 M_B^2} + (m^2+\Delta_B)|\underset{\sim}{P}_B| \; . \qquad (\underset{\sim}{P}_B // \underset{\sim}{p}) \tag{74}$$

with Δ_B in the range given by eq.(73).

6.11.- The case of "intrinsic absorption" at A (when $\underset{\sim}{u} \cdot \underset{\sim}{V} \geq c^2$)

Due to the present condition $\underset{\sim}{u} \cdot \underset{\sim}{V} \geq c^2$, and to the consequent "switching", if we observe the body A in its own rest-frame to absorb (intrinsically) a tachyon T, then in the B-rest-frame we shall observe an antitachyon $\bar{T}$ emitted by A. Necessary conditon for this case to take place is taht A, B be approaching to each other (i.e., be in the "approaching phase").

In any case, for the process at A in the A-rest-frame we obtain the same kinematics as expounded in Sects.6.9 and 6.5. As to the process at B, in the A-rest-frame the body B is observed to emit a tachyon T:

$$\sqrt{\underset{\sim}{P}_B^2 + M_B^2} = \sqrt{\underset{\sim}{p}^2 - m^2} + \sqrt{(\underset{\sim}{P}_B - \underset{\sim}{p})^2 + M_B'^2} \; ; \tag{75}$$

in the B-rest-frame, however, one would observe an (intrinsic) T absorption, so that it must be $\Delta_B \geq - m^2$.

In conclusion, the present tachyon exchange is kinematically allowed when eqs.(70) are satisfied, but now with

$$\Delta_A \geqslant - m^2 \; ; \qquad \Delta_B \geqslant - m^2 \; . \tag{76}$$

In the particular case in which $\underset{\sim}{P}_B$ and $\underset{\sim}{p}$ are collinear, we can have only $(-\underset{\sim}{P}_B)//\underset{\sim}{p}$ (approaching phase), and we get

$$2M_B^2|\underset{\sim}{p}| = E_B \sqrt{(m^2+\Delta_B)^2 + 4m^2M_B^2} - (m^2+\Delta_B)|\underset{\sim}{P}_B| \; , \qquad (\underset{\sim}{P}_B//(-\underset{\sim}{p})) \tag{77}$$

with $\Delta_B \geqslant - m^2$.

Finally, let us recall that in the present case ("intrinsic absorptions" at B and at A) both quantities Δ_A, Δ_B <u>can</u> vanish. When $\Delta_A=0$, we simply get $2M_AE_T = m^2$. In the particular case when $\Delta_B=0$ one gets: $2E_TE_B(\underset{\sim}{u}\cdot\underset{\sim}{V}-1) = m^2$, and then : $|\underset{\sim}{p}| = (m/2M_B^2)\left[E_B(m^2+4M_B^2)^{\frac{1}{2}} - m|\underset{\sim}{P}_B|\right]$.

6.12.- Conclusions about the tachyon exchange

With regard to the process at B, the kinematical results of Sects.6.8-6.11 yield what follows (Maccarrone and Recami 1980b):

$$\underset{\sim}{u}\cdot\underset{\sim}{V} \leqslant c^2 : \qquad \Delta_B = \begin{cases} -m^2+2p_\mu P_B^\mu \geqslant -m^2 \; ; \\ -m^2-2p_\mu P_B^\mu \leqslant -m^2 \; ; \end{cases} \tag{78a}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{V} \geqslant c^2 : \qquad \Delta_B = \begin{cases} -m^2+2p_\mu P_B^\mu \leqslant -m^2 \; ; \\ -m^2-2p_\mu P_B^\mu \geqslant -m^2 \; . \end{cases} \tag{78b}$$

More specifically, the kinematical <u>conditions</u> for a tachyon to be <u>exchangeable</u> between A and B can be summarized as follows (notice that the case $\underset{\sim}{u}\cdot\underset{\sim}{V} < c^2$ <u>includes</u> of course the case $\underset{\sim}{u}\cdot\underset{\sim}{V} < 0$):

a) in the case of "intrinsic emission" at A:

$$\begin{aligned} \underset{\sim}{u}\cdot\underset{\sim}{V} < c^2 \;&\Rightarrow\; \Delta_B > - m^2 \;\Rightarrow\; \text{intrinsic absorption at B;} \\ \underset{\sim}{u}\cdot\underset{\sim}{V} > c^2 \;&\Rightarrow\; \Delta_B < - m^2 \;\Rightarrow\; \text{intrinsic emission at B;} \end{aligned} \tag{79}$$

b) in the case of "intrinsic absorption" at A:

$$\begin{aligned} \underset{\sim}{u}\cdot\underset{\sim}{V} < c^2 \;&\Rightarrow\; \Delta_B < - m^2 \;\Rightarrow\; \text{intrinsic emission at B;} \\ \underset{\sim}{u}\cdot\underset{\sim}{V} > c^2 \;&\Rightarrow\; \Delta_B > - m^2 \;\Rightarrow\; \text{intrinsic absorption at B.} \end{aligned} \tag{80}$$

6.13.- Applications to elementary particle physics: examples. Tachyons as "internal lines"

Let us recall that, when elementary interactions are considered to be mediated by exchanged objects, no ordinary (bradyonic) particles can be the classical, "realistic" carriers of the transferred energy-momentum. On the contrary, classical tachyons - in place of the so-called virtual particles - can a priori act as the actual carriers of the fundamental subnuclear interactions.

For instance, any elastic scattering can be regarded as classically ("realistically") mediated by a suitable tachyon exchange during the approaching phase of the two bodies (cf. Sect.6.7). In such a case, eqs.(70),(76) read, always in the A-rest-frame ($\Delta_A = \Delta_B = 0$):

$$E_T = m^2/2M_A \; ; \qquad E_B = M_A/(\underset{\sim}{u}\cdot\underset{\sim}{V} - 1) \tag{81}$$

where the angular-momentum conservation is not considered. In the c.m.s. we would have $|\underset{\sim}{P}_A| = |\underset{\sim}{P}_B| = |\underset{\sim}{P}|$ and

$$\cos\vartheta_{c.m.} = 1 - \frac{m^2}{2|\underset{\sim}{P}|^2} \; , \qquad \text{(elastic scattering)} \tag{82}$$

so that (once $|\underset{\sim}{P}|$ is fixed) for each tachyon-mass m we get a particular $\vartheta_{c.m.}$; if m assumes only discrete values - as expected from the duality principle, Sect. 5.1 - then ϑ results to be classically "quantized", apart from the cylindrical symmetry.

More in general, for each discrete value of the tachyon-mass m, the quantity $\vartheta_{c.m.}$ assumes a discrete value too, which is merely a function of $|\underset{\sim}{p}|$. These elementary considerations neglect the possible mass-width of the tachyonic "resonances" (e.g. of the tachyon-mesons). Let us recall from Sects.5.3, 6.7 that in the c.m.s. any elastic scattering appears classically as mediated by an infinite-speed tachyon having $p^\mu = (0,\underset{\sim}{p})$, with $|\underset{\sim}{p}| = m$. Moreover, eqs.(81) impose a link between m and the direction of $\underset{\sim}{p}$, or rather between m and $\alpha \equiv \widehat{\underset{\sim}{p}\underset{\sim}{P}}$ (where we can choose $\underset{\sim}{P}=\underset{\sim}{P}_B$; remember that $\underset{\sim}{P}_B = -\underset{\sim}{P}_A$) :

$$\cos\alpha_{c.m.} = \frac{m}{2|\underset{\sim}{P}|} \; ; \tag{83}$$

again we find (once $|\underset{\sim}{P}|$ is given, and if the intermediate-tachyon masses are discrete) that also the exchanged 3-momentum results to be (classically) "quantized" in both its magnitude and direction. In particular, for each discrete value of m, also the exchanged 3-momentum assumes one discrete direction (except, again, for the cylindrical symmetry), which is a function only of $|\underset{\sim}{P}|$.

It is essential to notice that such results cannot be obtained at the classical level when confining ourselves only to ordinary particles, for the mere fact that bradyons are not allowed by kinematics to be the interaction-carriers.

Of course, also the non-elastic scattering can be regarded as mediated by suitable tachyon exchanges. We shall come back to this in the following (Sect.13.2).

6.14.- On the Variational Principle: a tentative digression

After having expounded some tachyon mechanics in Sects.6.2-6.12, let us turn a bit our attention to the action S for a free object. In the ordinary case it is $S = \alpha\int_a^b ds$; for a free tachyon let us, rather, write

$$S = \alpha\int_a^b |ds| \quad . \tag{84}$$

By analogy with the bradyonic case, we might assume for a free tachyon the Lagrangian (c=1)

$$L = + m_o \sqrt{\underset{\sim}{V}^2 - 1} \;, \qquad (\underset{\sim}{V}^2 > 1) \tag{85}$$

and therefore evaluate, in the usual way,

$$\underset{\sim}{p} \equiv \frac{\partial L}{\partial \underset{\sim}{V}} = + \frac{m_o \underset{\sim}{V}}{\sqrt{\underset{\sim}{V}^2-1}} \equiv m\underset{\sim}{V} \tag{86}$$

which suggest eq.(50) to hold in the four-dimensional case to:

$$m = \frac{m_o}{\sqrt{\underset{\sim}{V}^2-1}} \quad . \tag{50'}$$

If the tachyon is no more free, we can write as usual

$$\underset{\sim}{F} = \frac{d\underset{\sim}{p}}{dt} = \frac{d}{dt}\left(\frac{m_o \underset{\sim}{V}}{\sqrt{\underset{\sim}{V}^2-1}}\right) \quad . \tag{87}$$

By chosing the reference-frame, at the considered time-instant t, in such a way that $\underset{\sim}{V}$ is parallel to the x-axis, i.e. $|\underset{\sim}{V}| = V_x$, we then get

$$F_x = +m_o\left[\frac{1}{\sqrt{V^2-1}} - \frac{\underset{\sim}{V}^2}{\sqrt{(\underset{\sim}{V}^2-1)^3}}\right]a_x = - \frac{m_o}{(\underset{\sim}{V}^2-1)^{3/2}}\,a_x \tag{88a}$$

and

$$F_y = + \frac{m_o}{\sqrt{\underset{\sim}{V}^2-1}}\,a_y \;; \qquad F_z = + \frac{m_o}{\sqrt{\underset{\sim}{V}^2-1}}\,a_z \;. \tag{88b}$$

The sign in eq.(88a) is consistent with the ordinary definition of work $\mathscr{L}$:

$$d\mathscr{L} \equiv + \underset{\sim}{F} \cdot d\underset{\sim}{\ell} \tag{89}$$

and the fact that the total energy of a tachyon increases when its speed decreases (cf. Figs.4 and 10).

Notice, however, that the proportionality constant between force and acceleration does change sign when passing from the longitudinal to the transverse components.

The tachyon total energy E, moreover, can still be defined as

$$E \equiv \underset{\sim}{p} \cdot \underset{\sim}{V} - L = \frac{m_o c^2}{\sqrt{\underset{\sim}{V}^2 - 1}} \equiv mc^2 \tag{90}$$

which, together with eq.(50'), extends to tachyons the relation $E=mc^2$.

However, the following comments are in order at this point. An ordinary time-like (straight) line can be bent only in a space-like direction; and it gets shorter. On the contrary, if you take a space-like line and, keeping two points on it fixed, bend it slightly in between in a space-like (time-like) direction, the bent line is longer (shorter) than the original straight line (see e.g. Dorling 1970). For simplicity, let us here skip the generic case when the bending is partly in the time-like and partly in a space-like direction (even if such a case looks to be the most interesting). Then, the action integral $\int_a^b |ds|$ of eq.(84) along the straight (space-like) line is minimal w.r.t. the "space-like" bendings and maximal w.r.t. the "time-like" bendings. A priori, one might then choose for a free tachyon, instead of eq.(85), the Lagrangian

$$L = - m_o \sqrt{\underset{\sim}{V}^2 - 1} \; , \tag{85'}$$

which yields

$$\underset{\sim}{p} \equiv \frac{\partial L}{\partial \underset{\sim}{V}} = - \frac{m_o \underset{\sim}{V}}{\sqrt{\underset{\sim}{V}^2 - 1}} \equiv - m\underset{\sim}{V} \; . \tag{86'}$$

Eq.(86') would be rather interesting, at the light of the previous Sect.6.13 (cf. also Sect.13.2), i.e. when tachyons are substituted for the "virtual particles" as the carriers of the elementary particle interactions. In fact, the (classical) exchange of a tachyon endowed with a momentum antiparallel to its velocity would generate an attractive interaction.

For non-free tachyons, from eq.(86') one gets

$$\underset{\sim}{F} = \frac{d\underset{\sim}{p}}{dt} = - \frac{d}{dt} \left(\frac{m_o \underset{\sim}{V}}{\sqrt{\underset{\sim}{V}^2 - 1}} \right) \tag{87'}$$

and therefore, when $|\underset{\sim}{V}| = V_x$,

$$F_x = + \frac{m_o}{(\underset{\sim}{V}^2-1)^{3/2}} a_x \quad ; \tag{88'a}$$

$$F_y = - \frac{m_o}{\sqrt{\underset{\sim}{V}^2-1}} a_y \quad ; \qquad F_z = - \frac{m_o}{\sqrt{\underset{\sim}{V}^2-1}} a_z \ . \tag{88'b}$$

Due to the sign in eq.(88'a), it is now necessary to define the work $\mathscr{L}$ as

$$d\mathscr{L} \equiv - \underset{\sim}{F}\cdot d\underset{\sim}{\ell} \ , \tag{89'}$$

and analogously the total energy E as

$$E \equiv - (\underset{\sim}{p}\cdot\underset{\sim}{V} - L) = \frac{m_o c^2}{\sqrt{\underset{\sim}{V}^2-1}} = mc^2 \ . \tag{90'}$$

6.15.- On radiation tachyons

Many other results, actually independent of the very existence of SLTs, will appear in the following Sections 9-13.

Here, as a further example, let us report tha fact that a tachyon - when seen by means of its electromagnetic emissions (see the following, and Review I, Baldo et al. 1970) - will appear in general as occupying two positions at the same time (Recami 1974,1977b,1978a,1979a, Barut et al. 1982; see also Grøn 1978). Let us start by considering a macro-object C emitting spherical electromagnetic waves (Fig. 15c). When we see it travelling at constant Superluminal velocity $\underset{\sim}{V}$, because of the distortion due to the large relative speed $|\underset{\sim}{V}| > c$, we shall observe the electromagnetic waves to be internally tangent to an enveloping cone Γ having as its axis the motion-line of C (Recami and Mignani 1972, Review I); even if this cone has nothing to do with Cherenkov's (Mignani and Recami 1973b). This is analogous to what happens with an airplane moving at a constant supersonic speed in the air. A first observation is the following: as we hear a sonic boom when the sonic contact with the supersonic airplane does start (Bondi 1964), so we shall analogously see an "optic boom" when we first enter in radio-contact with the body C, i.e. when we meet the Γ-cone surface. In fact, when C is seen by us under the angle (Fig.15a)

$$V \cos\alpha = c \qquad\qquad ((V = |\underset{\sim}{V}|) \tag{91}$$

all the radiations emitted by C in a certain time-interval around its position C_o reach us simultaneously. Soon after, we shall receive at the same time the light emitted from suitable couples of points, one on the left and one of the right of C_o. We shall thus see the initial body C, at C_o, split in two luminous objects C_1, C_2 which will then be observed to recede from each other with the Superluminal "trans-

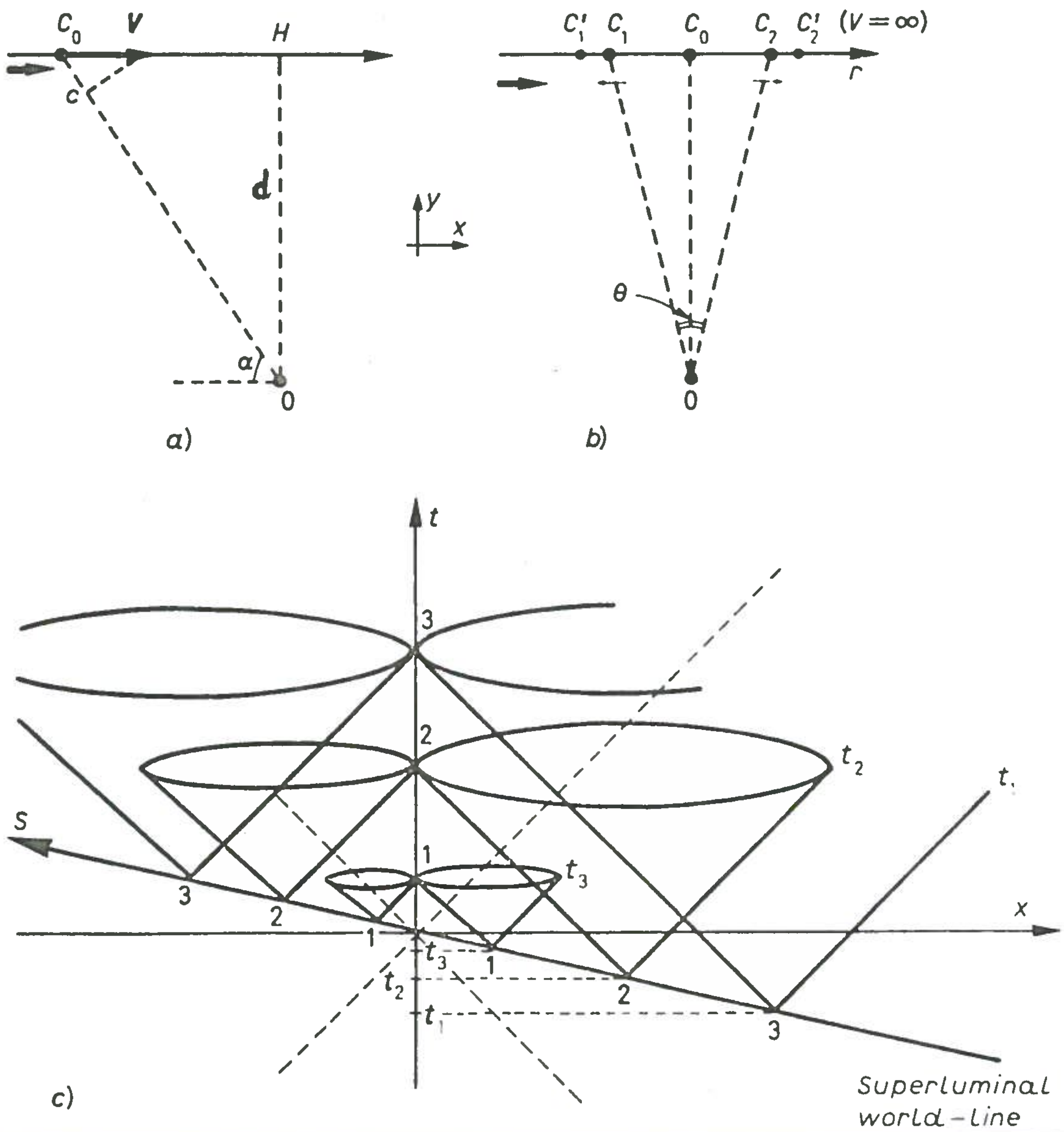


FIG. 15

verse" relative speed W (Recami et al.1976, Barut et al.1982):

$$W = 2b\,\frac{1+d/bt}{(1+2d/bt)^{\frac{1}{2}}}\;; \qquad b \equiv \frac{V}{\sqrt{V^2-1}}\;, \qquad (V^2>1) \tag{92}$$

where $d \equiv \overline{OH}$, and t=0 is just the time-instant when the observer enters in radiocontact with C, or rather sees C at C_0. In the simple case in which C moves with almost infinite speed along r (Fig. 15b), the apparent relative speed of C_1 and C_2 varies in the initial stage as $W \simeq (2cd/t)^{\frac{1}{2}}$, where now $\overline{OH} = \overline{OC_0}$ while t=0 is still the instant at which the observer sees $C_1 = C_2 = C_0$.

We shall come back to this subject when dealing with astrophysics (Sect.12.4); see also the interesting paper by Lake and Roeder (1975).

Here let us add the observation that the radiation associated with one of the images of C (namely, the radiation emitted by C while approaching us, in the simple case depicted in Fig. 15c) will be received by us in the reversed chronological order; cf. Mignani and Recami (1973a), Recami (1977b).

It may be interesting to quote that the circumstance, that the image of a tachyon suddenly appears at a certain position C_0 and then splits into two images, was already met by Bacry (1972) and Bacry et al. (1972) while exploiting a group-theoretical definition of the motion of a charged particle in a homogeneous field; definition which was valid for all kind of particles (bradyons, luxons, tachyons). Analogous solutions, simulating a pair-production, have been later on found even in the subluminal case by Barut (1978b), when exploting non-linear evolution equations, and by Sala (1979), by merely taking account of the finite speed of the light which carries the image of a moving subluminal object. Sala (1979) did even rediscover - also in subluminal cases - that one of the two images can display a time-reversed evolution.

At this point, we might deal with the problem of causality for tachyons (since the most relevant aspects of that problem do arise w.r.t. the class of the subluminal observers). We shift such a question, however, to Sect.9, because we want preliminarily to touch the problem of tachyon localization.

7.- FOUR-DIMENSIONAL RESULTS INDEPENDENT OF THE EXPLICIT FORM OF THE SLTs: INTRODUCTION

7.1.- A preliminary assumption

Let us start from our three Postulates (Sect.4). Also in four dimensions, when attempting to generalize SR to Superluminal frames, the fundamental requirement of such an "extended relativity" (cf. Sects.4.2, 4.3, as well as 5.1, 5.2) is that the SLTs change time-like into space-like tangent vectors, and vice-versa, i.e. invert the quadratic-form sign.

Let us assume in these Sects.7, 8 that such "transformations" exist in four dimensions (even if at the price of giving up possibly one of the properties (i)-(vi) listed at about the end of Sect.3.2). Their actual existence has been claimed for instance by Shah (1977, 1978) within the "quasi-catastrophes" theory.

7.2.- G-vectors and G-tensors

If we require also that the SLTs form a new group $\mathcal{G}$ together with the subluminal (ortho- and anti-chronous) Lorentz transformations, the following remarks are then in order. Eqs.(14)-(15) introduce the four-position x^{μ} as a G-vector; in other words, by definition of GLTs, quantity x^{μ} is a four-vector not only w.r.t. the group $\mathscr{L}_{+}^{\uparrow}$, but also w.r.t. the whole group $\mathscr{G}$. As a consequence, the "scalar product" $dx_{\mu}dx^{\mu}$ behaves as a pseudo-scalar under the SLTs.

Under SLTs it is $ds'^2 = -ds^2$; it follows that quantity $u^{\mu} \equiv dx^{\mu}/ds$, a Lorentz-vector, is not a G-vector. In order to define the four-velocity as a G-vector we must set

$$u^{\mu} \equiv dx^{\mu}/d\tau_0 \tag{93a}$$

where τ_0 is the proper time. Analogously for the four-acceleration: $a^{\mu} = du^{\mu}/d\tau_0$; and so on. We can expect that also the electromagnetic quantities A^{μ} (Lorentz-vector) and $F^{\mu\nu}$ (Lorentz-tensor) do not have a priori to be any more a G-vector and a G-tensor, respectively: Cf. Sect.15.

However, once $T^{\mu\nu}$ is supposed to be a G-tensor, then under a SLT it is

$$T'^{\mu\nu} = G^{\mu}_{\alpha}\, G^{\nu}_{\beta}\, T^{\alpha\beta} \quad , \tag{93b}$$

wherefrom it follows that the ordinary invariants

$$T_{\mu\nu}T^{\mu\nu} \quad ; \qquad \varepsilon_{\mu\nu\alpha\beta}T^{\mu\nu}T^{\alpha\beta} \tag{93c}$$

are still invariant (even under SLTs). This holds, of course, only for even-rank tensors.

As already mentioned, if we define u^{μ} by eq.(93a), so to be a G-fourvector, then under a SLT the quantity $u^2 \equiv u_{\mu}u^{\mu}$ becomes $u'^2 = -u^2$. That is to say, after a SLT a bradyonic velocity has to be seen as a tachyonic velocity, and vice-versa, in agreement with eqs.(26).

Let us add here, at this point, that sometimes in the literature it has been avoided the explicit use of a metric tensor by making recourse to Einstein's notations, and by writing the generic chronotopical vector as $x \equiv (x_0,x_1,x_2,x_3) \equiv (ct,ix,iy,iz)$, so that $g_{\mu\nu} = \delta_{\mu\nu}$ ("Euclidean metric"). Thus, one does not have to distinguish between covariant and contravariant components. In such a case, since one has practically to deal with a complex manifold, the quadratic form which is Lorentz-invariant is to be defined as the scalar product of the first vector by the complex conjugate of the second vector:

$$\text{quadratic-form} \equiv (dx,\overline{dy}) = dx_{\mu}\, dy^{\mu} \quad ; \tag{93d}$$

in particular, the invariant square-interval would be $ds^2 \equiv (dx,\overline{dx}) = dx_{\mu}\, dx^{\mu}$.

8.- ON THE SHAPE OF TACHYONS

8.1.- Introduction

We have already noticed that a tachyon - observed by means of its light-signals - will generally appear as occupying two positions at the same time (Sect.6.14 and Figs. 15).

Still at a preliminary level, let us moreover recall that free bradyons always admit a particular class of subluminal reference-frames (their rest-frames) wherefrom they appear - in Minkowski space-time - as "points" in space extended in time along a line. On the contrary, free tachyons always admit a particular class of subluminal (w.r.t. us) reference-frames - the critical frames - wherefrom they appear with divergent speed $V=\infty$, i.e. as "points" in time extended in space along a line (cf. Figs.7,11). Considerations of this kind correspond to the fact that the "localization" groups (little groups) of the timelike and spacelike representations of the Poincaré group are SU(3) and SU(2,1), respectively (see e.g. Barut 1978a), so that tachyons are not expected to be localizable in our ordinary space (cf. also Peres 1970, Cawley 1970, Duffey 1975, 1980, Vyšín 1977a, Souček 1981).

It is therefore worthwhile to study the shape of tachyons in detail, following Barut et al. (1982).

8.2.- How would tachyons look like?

Let us consider an ordinary bradyon $P=P_B$ which for simplicity be intrinsically spherical (in particular point-like), so that when at rest its "world-tube" in Minkowski space-time is represented by $0 \leq x^2+y^2+z^2 \leq r^2$. When P_B moves with subluminal speed v along the x-axis (Fig. 16), its four-dimensional shape (i.e. its world-tube equation) becomes:

$$0 \leq \frac{(x-vt)^2}{1-v^2} + y^2+z^2 \leq r^2 \qquad (v^2 < 1) \qquad (94a)$$

and in Lorentz-invariant form,

$$0 \leq \frac{(x_\mu p^\mu)^2}{p_\mu p^\mu} - x_\mu x^\mu \leq r^2 , \quad (v^2 < 1) \qquad (94b)$$

where $x^\mu \equiv (ct,x,y,z)$ and p^μ is the 4-momentum.

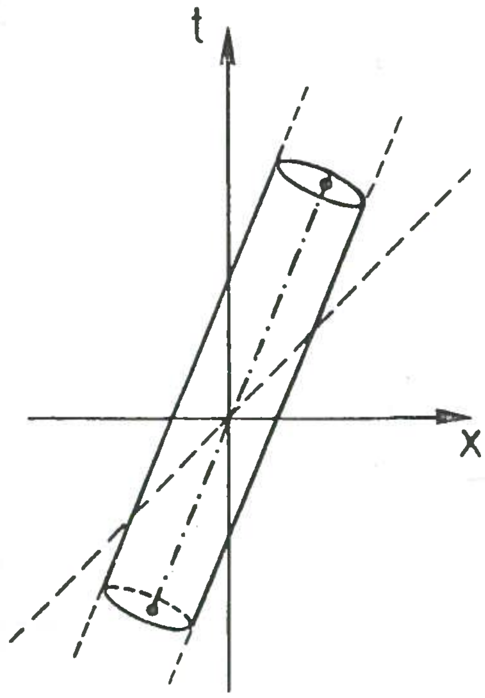


FIG. 16

Let us now take into examination also the space-like values of the 4-momentum p^{μ}, still considering however only subluminal observers s: We shall regard in these Sections the SLTs, as well as the ordinary LTs, from the active point of view only. By an active SLT let us transform the initial P_B into a final tachyon $P \equiv P_T$ endowed with Superluminal speed V along x. Due to Sect.7.1, one can expect that eq.(94b) will transform for P_T into

$$0 \leq x_{\mu}x^{\mu} - \frac{(x_{\mu}p^{\mu})^2}{p_{\mu}p^{\mu}} \leq r^2 , \qquad (V^2 > 1) \qquad (95)$$

where p^{μ} has been regarded as a G-fourvector (for both Bs and Ts it will be defined $p^{\mu} \equiv m_0 u^{\mu} = m_0 dx^{\mu}/d\tau_0$: see Sect.14.13). Notice however the following: If a SLT is requested to change the sign of the quadratic form $ds^2 = dx_{\mu}dx^{\mu}$, this means that it will change the type of all the "tangent vectors" (i.e., for example, the sign of $p_{\mu}p^{\mu}$) but does not mean at all that it will change sign also to $x_{\mu}x^{\mu}$; this happens only if the SLTs: $dx_{\mu} \rightarrow dx'_{\mu}$ are linear. [Actually, if such a linear SLT has constant coefficients (as required by homogeneity and isotropy), then it is linear also the transformation $\mathscr{T}$: $x_{\mu} \rightarrow x'_{\mu}$; cf. e.g. Rindler(1966)]. Therefore, to go from eq.(94b) to eq.(95) it is necessary to assume explicitly that SLTs exist which change sign both to $dx_{\mu}dx^{\mu}$ and to $x_{\mu}x^{\mu}$. Eq.(95) then yields the four-dimensional shape of tachyon P_T.

In the initial frame, eq.(95) writes:

$$0 \geq - \frac{(x-Vt)^2}{V^2-1} + y^2 + z^2 \geq - r^2 . \qquad (V^2 > 1) \qquad (96)$$

In conclusion, if the world-tube of P_B was supposed to be unlimited, - i.e. if P_B was supposed to be infinitely extended in time, - then tachyon P_T appears as occupying the whole space bound by the double, unlimited cone $\mathscr{C}_0$: $y^2 + z^2 = (x-Vt)^2/(V^2-1)$ and the two-sheeted rotation hyperboloid $\mathscr{H}_0$: $y^2 + z^2 = (x-Vt)^2/(V^2-1) - r^2$, where the latter is asymptotic to the former; see Figs.17. As time elapses, eq.(96) yields the relativistic shape of our tachyon; the whole structure in Figs.17 (and 18) rigidly moving along x - of course - with the speed V. Notice that the cone semi-angle α is given by

$$\mathrm{tg}\, \alpha = (V^2 - 1)^{\frac{1}{2}} . \qquad (97)$$

Let us fix our attention on the external surface of P. When it is at rest, the surface is spherical; when subluminal, it becomes an ellipsoid (Fig. 19b); when Superluminal, such a surface becomes a two-sheeted hyperboloid (Fig. 19d). Fig. 19c refers to the limiting case when the speed tends to c, i.e. when either $v \rightarrow 1^-$ or $V \rightarrow 1^+$. Incidentally, let us remind that even in ER the light-speed in vacuum goes

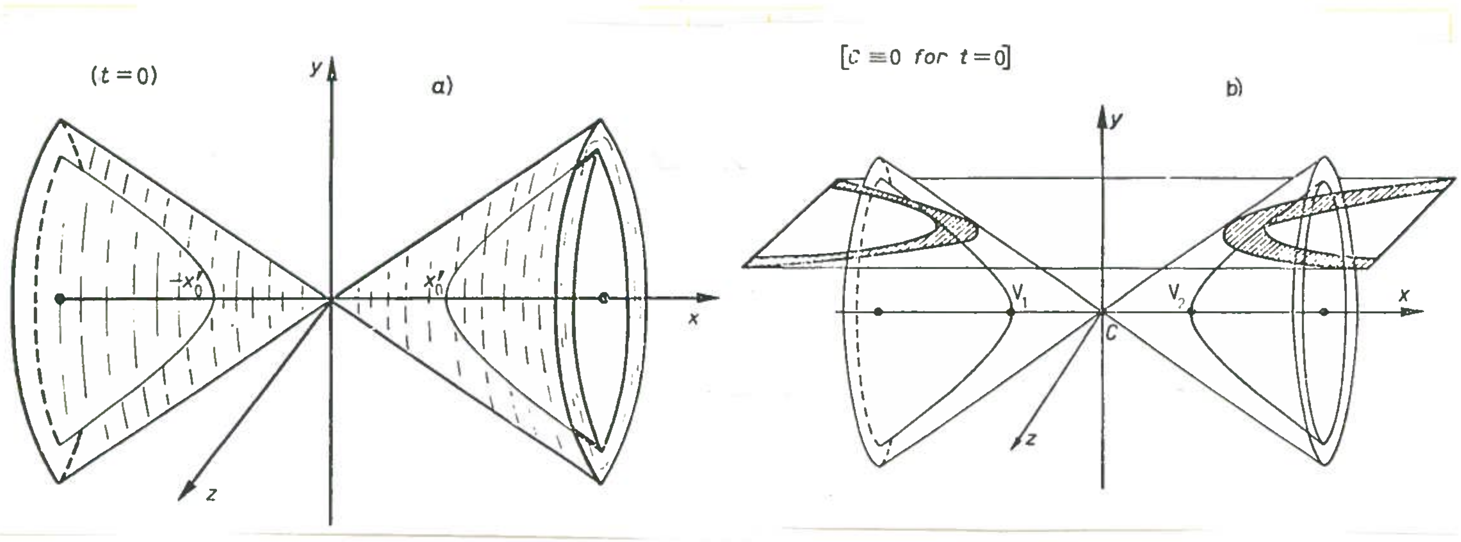


FIG. 17

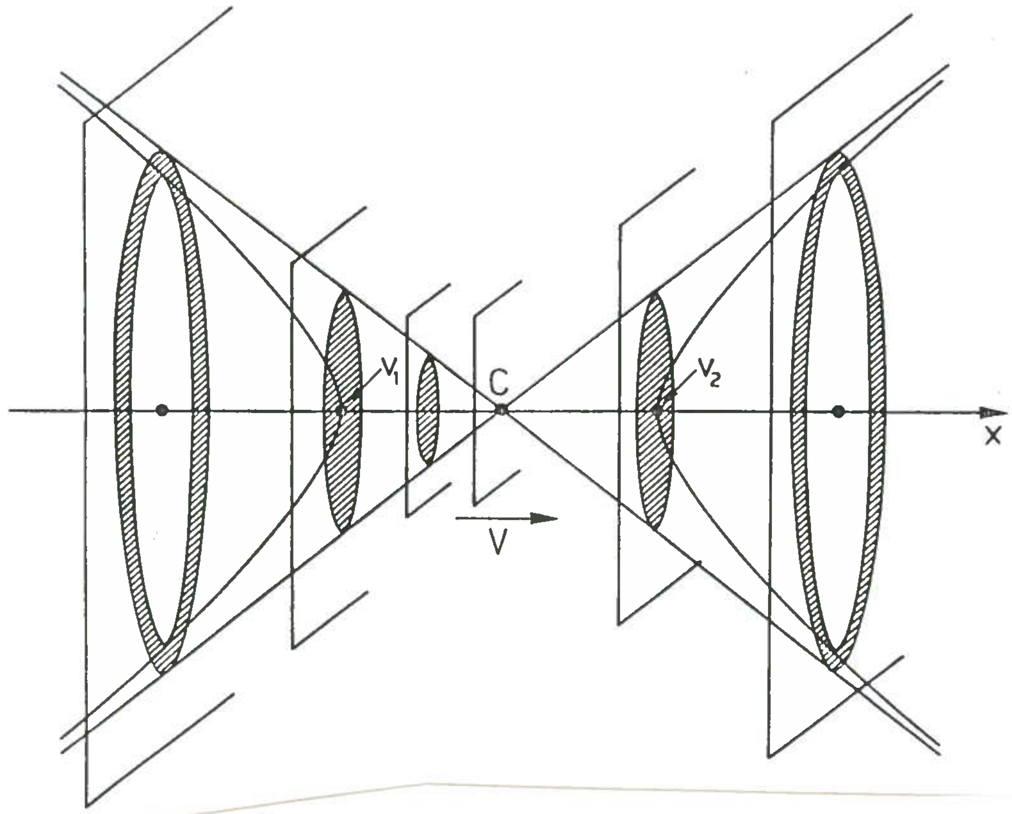


FIG. 18

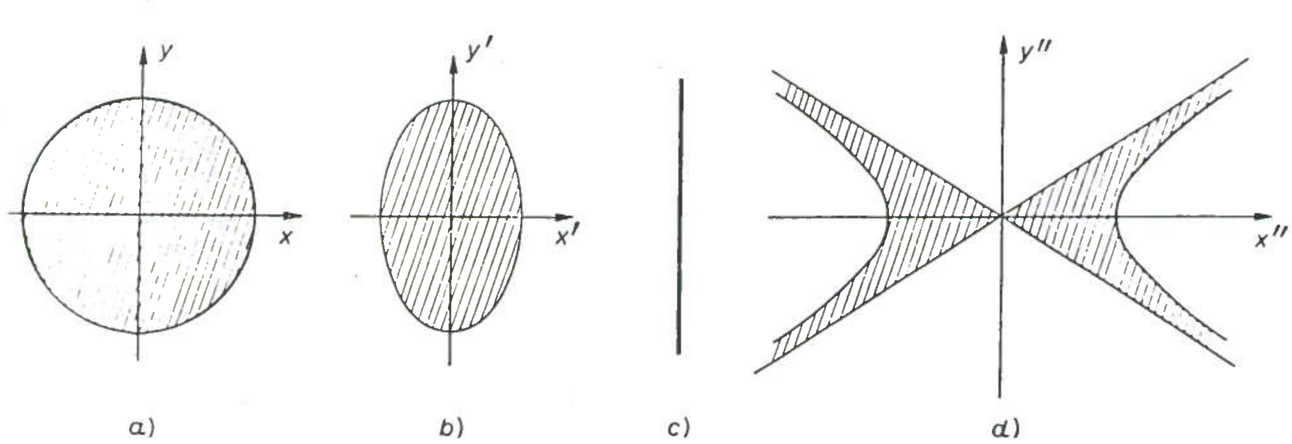


FIG. 19

on being the invariant speed, and can be crossed neither from the left, nor from the right.

Let us make a comment. Tachyons appeared to be more similar to fields than to particles. It would be desirable to find out the space-time function yielding the density distribution of a tachyon. For instance, when the tachyon-shape just reduces to the cone $\mathscr{C}_0$, it would be interesting to work out the L^2-function of x and t yielding the tachyon density-distribution over $\mathscr{C}_0$.

8.3.- Critical comments on the Preliminary Assumption

In connection with Sects.7.1 and 8.2 a critical warning is in order, since we saw at the end of Sect.3.2 (and shall better see in the following) that real linear SLTs: $dx_\mu \to dx'_\mu$ which meet the requirements (ii)-(iv) of Sect.4.2 do not exist in four dimensions. We therefore expect that real transformations $\mathscr{T}$: $x_\mu \to x'_\mu$ mapping points of M_4 into points of M_4 (in such a way that $ds^2 \to -ds^2$) do not exist as well; otherwise real linear SLTs: $dx_\mu \to dx'_\mu$ should exist.

Let us state it differently. Eq.(95) was derived under the hypothesis that SLTs do exist in four dimensions which change the sign both of the quadratic form $dx_\mu dx^\mu$ and of the quantity $x_\mu x^\mu$. This means that the SLTs: $dx_\mu \to dx'_\mu$ transforming $dx_\mu dx^\mu \to -dx_\mu dx^\mu$ have to be linear. In the case of SLTs linear and real, it would exist as a consequence in M_4 a point-to-point transformation $\mathscr{T}$: $x_\mu \to x'_\mu$, and furthermore linear (Rindler 1966).

The results in this Sect.8 seem to show, however, that in M_4 we meet mappings that transform manifolds into manifolds (e.g., points into surfaces). This seems to predict to us that our SLTs: $dx_\mu \to dx'_\mu$ in M_4 will be linear, but not real.

For such non-real SLTs we shall suggest in Sect.14.16 an interpretation-procedure that will lead us from linear non-real SLTs to real non-linear SLTs; cf. e.g. Fig.5 in Maccarrone and Recami (1982a, 1984a). The latter SLTs, actually, cannot be integrated, so that no $\mathscr{T}$: $x_\mu \to x'_\mu$ can be found in this case (Šmrz 1984).

Let us explicitly mention that non-linear SLTs: $dx_\mu \to dx'_\mu$ can exist which, nevertheless: (i) do transform inertial motion into inertial motion (e.g., the inertial motion of a point into the inertial motion of a cone); (ii) preserve space isotropy and space-time homogeneity; (iii) retain the light-speed invariance (cf. also Sects.8.2, 8.4).

8.4.- On the space extension of tachyons

In the limiting case when it is intrinsically point-like, tachyon P_T reduces to the cone $\mathscr{C}_0$ only, and we shall see P_T to be a double cone, infinitely extended in space (Recami and Maccarrone 1980, Barut et al. 1982). But this happens only if the corresponding bradyon P_B exists for $-\infty < t < +\infty$. On the contrary, if the life-time (and extension) of P_B are <u>finite</u>, the space-extension (and life) of P_T are <u>finite</u> too. Namely if P_B in its rest-frame is spherical, is born at time $\bar{t}_1$ and is absorbed at time $\bar{t}_2$, then the corresponding tachyon P_T possesses a finite space-extension (Recami and Maccarrone 1980,1983). Under the present hypotheses, in fact, one has to associate with eqs.(93)-(94) suitable <u>limiting</u> space-like hypersurfaces, which simply become the hyperplanes $t=\bar{t}_1$ and $t=\bar{t}_2$ when P_B is at rest (Fig. 20). The generic Lorentz-invariant equation for a hyperplane is

$$x_\mu u^\mu = K , \qquad (K = \text{constant}) \qquad (98)$$

Due to Sect.7.1 we get that eq.(98) keeps its form even under an active SLT : $x_\mu u'^\mu = K'$. The relevant fact is that we passed from a time-like u_μ to a space-like u'_μ, so that the hyperplanes $x_\mu u'^\mu = K'$ are now to be referred to two spatial and one temporal basis-vectors (Fig. 21). Such hyperplanes represent ordinary planes (orthogonal to the x-axis, in our case) which move parallely to themselves <u>with the subluminal speed</u> $v' = 1/V$, as it follows from their orthogonality to u'_μ.

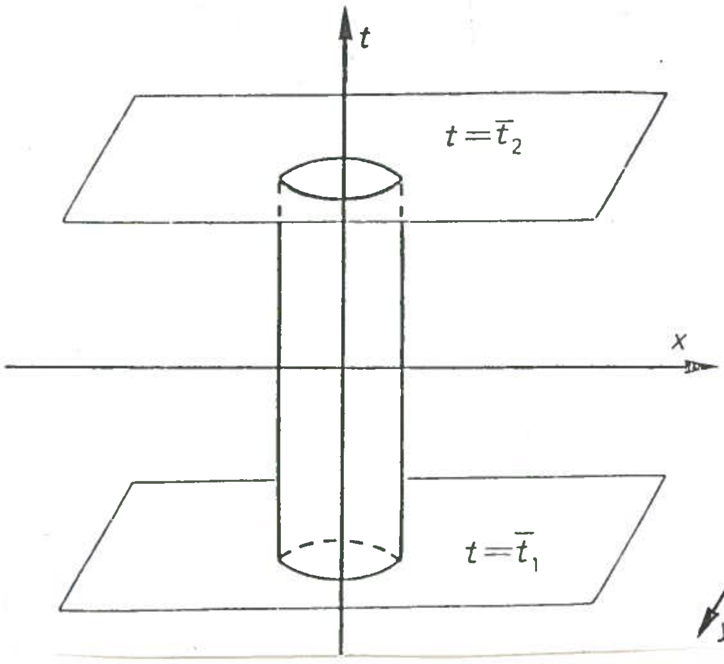


FIG. 20

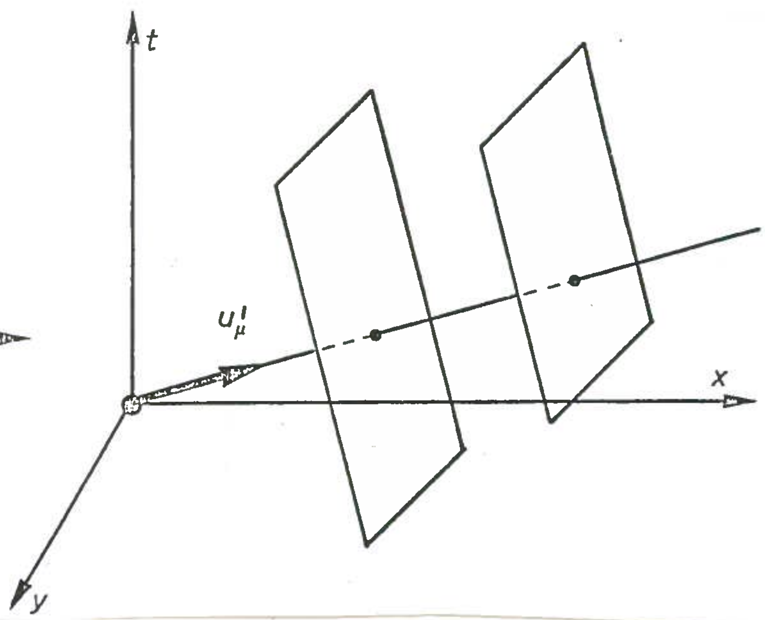


FIG. 21

In conclusion, in the tachyon case ($V^2 > 1$), one has to associate with eqs.(95)-(96) the additional contraints

$$-\bar{t}_2\sqrt{V^2-1} + xV \leqslant t \leqslant -\bar{t}_1\sqrt{V^2-1} + xV ;$$

the shape of a realistic tachyon P_T, obtained from a finite life-time bradyon P_B, is got therefore by imposing on the structure $\mathscr{C}_0 + \mathscr{H}_0$ in Figs.17-18 the following constraints

$$x_1 \equiv \bar{t}_1 \frac{\sqrt{V^2-1}}{V} + \frac{t}{V} \leq x \leq \bar{t}_2 \frac{\sqrt{V^2-1}}{V} + \frac{t}{V} \equiv x_2 \ . \tag{99}$$

It seems to follow that our realistic tachyon is constituted not by the whole structure in Figs.17-18, but only by its portion confined inside a mobile "window", i.e. bound by the two planes $x=x_1$ and $x=x_2$. As we saw, this "window" travels with the speed v' dual to the tachyon speed V

$$v' = \frac{1}{V} \qquad (V^2 > 1 \ ; \ \ v'^2 < 1) \tag{100}$$

and, if V is constant, its width is constant too ($\bar{\Delta t} = \bar{t}_2 - \bar{t}_1$):

$$x = \bar{\Delta t} \sqrt{1 - v'^2} \ . \qquad (v' \equiv 1/V) \tag{101}$$

Chosen a fixed position $x=\bar{x}$, such a "window" to cross the plane $x=\bar{x}$ will take a time independent of $\bar{x}$ (if V is still constant):

$$\Delta t = \bar{\Delta t} \sqrt{V^2-1} \equiv \bar{\Delta t} \frac{\sqrt{1-v'^2}}{v'} \ . \tag{102}$$

The problem of the time-extension of such "realistic" tachyons does not seem to have been yet clarified.

If P_B is not intrinsically spherical, but ellipsoidal, then P_T will be bound by a double cone $\mathscr{C}$ and a two-sheeted hyperboloid $\mathscr{H}$ devoid, this time, of cylindrical symmetry: Cf. Barut et al. (1982). Those authors investigated also various limiting cases. Let us mention that when $V \to \infty$ (while $\bar{t}_1$, $\bar{t}_2$ and r remain finite) the "window" becomes fixed: $\bar{x}_1 \equiv c\bar{t}_1 < x < c\bar{t}_2 \equiv x_2$.

We may conclude that, if the life-time of P_B is very large (as it is usually for macroscopic and even more for cosmic objects), then the corresponding tachyon description is essentially the old one given in Sect.8.1, and P_T can be associated with actual Superluminal motion. If, on the contrary, the life-time of P_B is small w.r.t. the observation-time of the corresponding tachyon (as it commonly happens in the microscopic domain), then P_T would actually appear to travel with the subluminal (dual) speed $v'=1/V$; even if P_T is associated with a structure $\mathscr{C} + \mathscr{H}$ travelling with the Superluminal speed V. In fact the magnitude of its "group velocity" (i.e. the speed of its "front") is given by eq.(100). However, within the "window" confining the real portion of the tachyon (which possibly carries the tachyon energy and momentum, just as

P_B carried energy-momentum only between $t=\bar{t}_1$ and $t=\bar{t}_2$), there will be visible a "structure" evolving at Superluminal speed, associable therefore with a tachyonic "phase-velocity". What precedes is based on Maccarrone and Recami (1982b), but similar results - even if got from quite different starting points - were put forth by Fox et al. (1969,1970). See also Alagar Ramanujam et al.(1983); Souček (1981), Kowalczyński (1979), Schulman (1971), Coleman (1970).

8.5.- Comments

The tachyons' characteristics exploited in the previous Sect.8.4 remind us once more (cf. e.g. Sect.6.13) of the ordinary quantum particles with their "de Broglie waves": In that case too phase-velocity and group-velocity obey eq.(100).

To investigate this connection (Recami and Maccarrone 1983) let us recall the ordinary definitions of Compton wave-length λ_C and de Broglie wave-length λ_{dB} ($\beta^2 < 1$):

$$\lambda_C \equiv \frac{\hbar}{m_o c} \; ; \qquad \lambda_{dB} \equiv \frac{\hbar}{|\mathbf{p}|} = \lambda_C \frac{\sqrt{1-\beta^2}}{\beta} = \frac{\lambda}{\beta} \; , \qquad (103a)$$

where we introduced the new "wave-length" λ

$$\lambda \equiv \frac{\hbar}{E/c} = \lambda_C \sqrt{1-\beta^2} \qquad (\beta^2 \quad 1) \qquad (103b)$$

satisfying the relation

$$\frac{1}{\lambda^2} = \frac{1}{\lambda^2_{dB}} + \frac{1}{\lambda^2_C} \; . \qquad (103c)$$

Eqs.(103) suggest of course the following kinematical interpretation: Let λ_C represent the intrinsic size of the considered (subluminal, quantum) particle: then $\lambda = \lambda_C \sqrt{1-\beta^2}$ is the particle size along its motion-line in the frame where it travels with speed $v \equiv \beta c$; and $\lambda_{dB}/c = \lambda/v$ is then the time spent by the particle to cross in the same frame a plane orthogonal to its motion-line.

Let us now examine our eqs.(101)-(102). In eq.(101) it is natural to identify

$$\Delta x = \lambda' = \lambda'_C \sqrt{1-\beta'^2} \qquad (\beta' \equiv v'/c \; ; \quad v' \equiv \frac{1}{V} \; ; \quad v'^2 < 1) \qquad (104a)$$

wherefrom

$$\lambda'_C = c\overline{\Delta t} \; . \qquad (104b)$$

Then, from eq.(102):

$$\lambda'_{dB} = \lambda'_C \frac{\sqrt{1-\beta'^2}}{\beta'} = \frac{\lambda'}{\beta'} \quad . \qquad (\beta' \equiv v'/c \ ; \quad v' \equiv \frac{1}{V} \ ; \quad v'^2 < 1) \qquad (104c)$$

By comparing eqs.(104) with eqs.(103), one recognizes that the characteristics of a classical tachyon actually fit the "de Broglie relations" $v'=1/V$ and $\lambda'_{dB}=\lambda'/\beta'$, with $\lambda' \equiv \Delta x$. However, a classical (realistic) tachyon T obeys all the eqs.(97) only provided that one attributes to the tachyon (or, rather, to its "real" portion confined within the mobile subluminal "window") a proper-mass m_o depending on its intrinsic (proper) life-time, namely such that

$$\frac{\lambda'_C}{c} = \frac{\hbar}{m_o c^2} = \overline{\Delta t} \quad \Rightarrow \quad m_o = \frac{\hbar}{c^2 \overline{\Delta t}} \quad . \qquad (105)$$

Notice that eq.(105) corresponds to the case $E_o \cdot \overline{\Delta t} = E \cdot \Delta x/c = \hbar$, with $E_o = m_o c^2$; $E = m_o c^2 / \sqrt{1-(v'/c)^2}$. Notice moreover that the wavelength of the de Broglie wave associated with a tachyon has an upper limit (Grøn 1979), which is essentially equal to its Compton wavelength: $(\lambda'_{dB})_{max} = h/(m_o c) = 2\pi\lambda'_C$.

9. THE CAUSALITY PROBLEM

As mentioned at the end of Sect.6.15, the discussion that will follow in this Sect.9 is independent of the very existence of the SLTs, since the most relevan causal problems arise when describing tachyons (and bradyons) from the ordinary subluminal frames. We wanted, however, to face the causality problem for tachyons in Relativity only after having at least clarified that tachyons are not trivially localizable in the ordinary space (cf. Sects.8.2-8.5; see also Shay and Miller 1977). Actually, a tachyon T is more similar to a field than to a particle, as we already noticed at the end of Sect.8.2. There are reasons however to believe that, in general, most of the tachyon mass be concentrated near the center C of T (Figs.17b, 18): At least, we shall refer in the following to the cases in which tachyons can be regarded as "almost localized" in space. In what follows, therefore, we shall essentially make recourse only to the results in Sects.5.12-5.14 (which, incidentally, have been seen to hold also in four dimensions) and to our results about tachyon kinematics (Sect.6). As mentioned above, we shall confine ourselves only to the subluminal observers (in presence, of course, of both bradyons and tachyons) and, for simplicity, to the orthochronous Lorentz Transformations only.

The results in Sects. 5.12-5.14, in particular, showed us that each observer will always see only tachyons (and antitachyons) moving with positive energy forward in time. As expounded in Sects.5.13 and 5.17, however, this success is obtained at the price of releasing the old conviction that judgement about what is "cause" and what is "effect" is independnent of the observer; in Sect.5.17 we assignment of the "source" and "detector" labels is to be regarded as a description--detail. As anticipated in Sect.5.13, this fact led to the proposal of a series of seeming "causal paradoxes", that we are going to discuss and (at least "in microphysics") to solve.

9.1.- Solution of the Tolman-Regge Paradox

The oldest paradox is the "anti-telephone" one, originally proposed by Tolman (1917 ; see also Bohm 1965) and then reproposed by many authors (cf. Sect.3.1). Let us refer to its most recent formulation (Regge 1981), and spend some care in solving it since it is the kernel of many other paradoxes.

9.1.1.- The paradox - In Figs. 22 the axes t and t' are the world-lines of two devices A and B respectively, able to exchange tachyons and moving with constant relative speed u, ($u^2 < 1$). According to the terms of the paradox (Fig.22a), A sends tachyon 1 to B (in other words, tachyon 1 is supposed to move forward in time w.r.t. A).

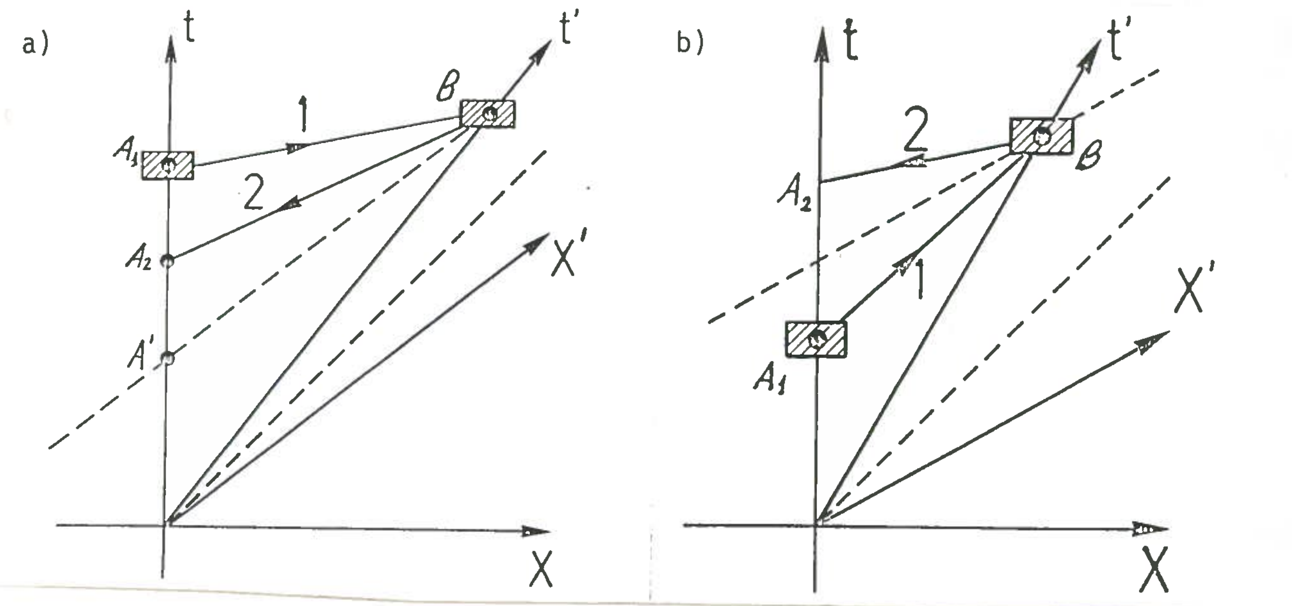


FIG. 22

The apparatus B is constructed so to send back a tachyon 2 to A as soon as it receives a tachyon 1 from A. If B has to emit (in its rest-frame) tachyon 2, then 2 must move forward in time w.r.t. B, that is to say its world-line BA_2 must have a slope smaller than the x'-axis slope BA' (where BA'//x'); this means that A_2 must stay abo ve A'. If the speed of tachyon 2 is such that A_2 falls between A' and A_1, it seems that 2 reaches back to A (event A_2) before the emission of 1 (event A_1). This appears to realize an anti-telephone.

9.1.2.- The solution - First of all, since tachyon 2 moves backwards in time w.r.t. A, the event A_2 will appear to A as the emission of an antitachyon $\bar{2}$. The observer "t" will see his apparatus A (able to exchange tachyons) emit successively towards B the antitachyon $\bar{2}$ and the tachyon 1.

At this point, some supporters of the paradox (overlooking tachyon kinematics, as well as relations (66)) would say that, well, the description forwarded by observer "t" can be orthodox, but then the device B is no longer working according to the premises, because B is no longer emitting a tachyon 2 on receipt of tachyon 1. Such a statement would be wrong, however, since the fact that "t" sees an "intrinsic emission" at A_2 does not mean that "t'" will see an "intrinsic absorption" at B. On the contrary, we are just in the case of Sect.6.10: intrinsic emission by A, at A_2, with $\underset{\sim}{u} \cdot \underset{\sim}{V}_{\bar{2}} > c^2$, where $\underset{\sim}{u}$ and $\underset{\sim}{V}_{\bar{2}}$ are the velocities of B and $\bar{2}$ w.r.t. A, respectively; so that both A and B suffer an intrinsic emission (of tachyon 2 or of antitachyon $\bar{2}$) in their own rest-frames.

But the terms of the paradox were cheating us even more, and ab initio. In fact Fig.22a makes clear that, if $\underset{\sim}{u} \cdot \underset{\sim}{V}_{\bar{2}} > c^2$, then for tachyon 1 a fortiori $\underset{\sim}{u} \cdot \underset{\sim}{V}_1 > c^2$, where $\underset{\sim}{u}$ and $\underset{\sim}{V}_1$ are the velocities of B and 1 w.r.t. A. Due to Sect.6.10, therefore, observer "t'" will see B emit also tachyon 1 (or, rather, antitachyon $\bar{1}$). In conclusion the proposed chain of events does not include any tachyon absorption by B.

For body B to absorb tachyon 1 (in its own rest-frame), the world-line of 1 ought to have a slope larger than the x'-axis slope (see Fig.22b). Moreover, for body B to emit ("intrinsically") tachyon 2, the slope of 2 should be smaller than x'-axis'. In other words, when the body B, programmed to emit 2 as soon as it receives 1, does actually do so, the event A_2 does regurarly happen after A_1 (cf. Fig.22b).

9.1.3. The moral - The moral of the story is twofold: (i) one should never mix together the descriptions of one phenomenon yielded by different observers, otherwise - even in ordinary physics - one would immediately meet contradictions: in Fig.22a, e.g., the motion-direction of 1 is assigned by A and the motion-direction of 2 is assignad by B; this is illegal; (ii) when proposing a problem about tachyons, one must comply (Caldirola and Recami 1980) with the rules of tachyon kinematics (Maccarrone and Recami 1980b), just as when formulating the text of an ordinary problem one must comply with the laws of ordinary physics (otherwise the problem in itself is "wrong").

Most of the paradoxes proposed in the literature suffered the above shortcomings; for a late, remarkable example, see Girard and Marchildon (1984).

Notice that, in the case of Fig.22a, neither A nor B regard event A_1 as the cause of event A_2 (or vice-versa). In the case of Fig.22b, on the contrary, both A and B consider event A_1 to be the cause of event A_2: but in this case A_1 does chronologically precede A_2 for both observers, in agreement with the relativistic covariance of the Law of Retarded Causality. We shall come back to such considerations.

9.2.- Solution of the Pirani Paradox

A more sophisticated paradox was proposed, as wellknown, by Pirani (1970). It was substantially solved by Parmentola and Yee (1971), on the basis of the ideas initially expressed by Sudarshan (1970), Bilaniuk and Sudarshan (1969b), Csonka (1970), etc.

9.2.1.- The paradox - Let us consider four observers A,B,C,D having given velocities in the plane (x,y) w.r.t. a fifth observer s_0. Let us imagine that the four observers are given in advance the instruction to emit a tachyon as soon as they receive a tachyon from another observer, so that the following chain of events (Fig. 23) takes place. Observer A initiates the experiment by sending tachyon 1 to B; observer B immediately emits tachyon 2 towards C; observer C sends tachyon 3 to D; and observer D sends tachyon 4 back to A, with the result - according to the paradox - that A receives tachyon 4 (event A_1) before having initiated the experiment by emitting tachyon 1 (event A_2). The sketch of this "Gedankenexperiment" is in Fig. 23, where oblique vectors represent the observer velocities w.r.t. s_0 and lines parallel to the Cartesian axes represent the tachyon paths.

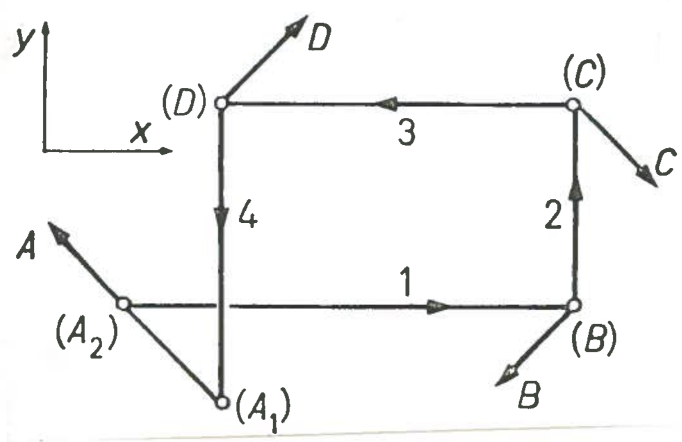


FIG. 23

9.2.2.- The solution - The above paradoxical situation arises once more by mixing together observations by four different observers. In fact, the arrow of each tachyon line simply denotes its motion direction w.r.t. the observer which emitted it. Following the previous Sect.9.1, it is easy to check that Fig. 23 does not represent the actual description of the process by any observer. It is necessary to investigate, on the contrary, how each observer describe the event chain.

Let us pass, to this end, to the Minkowski space-time and study the description given e.g. by observer A. The other observers can be replaced by objects (nuclei, let us say) able to absorb and emit tachyons. Fig. 24 shows that the absorption of 4 hap

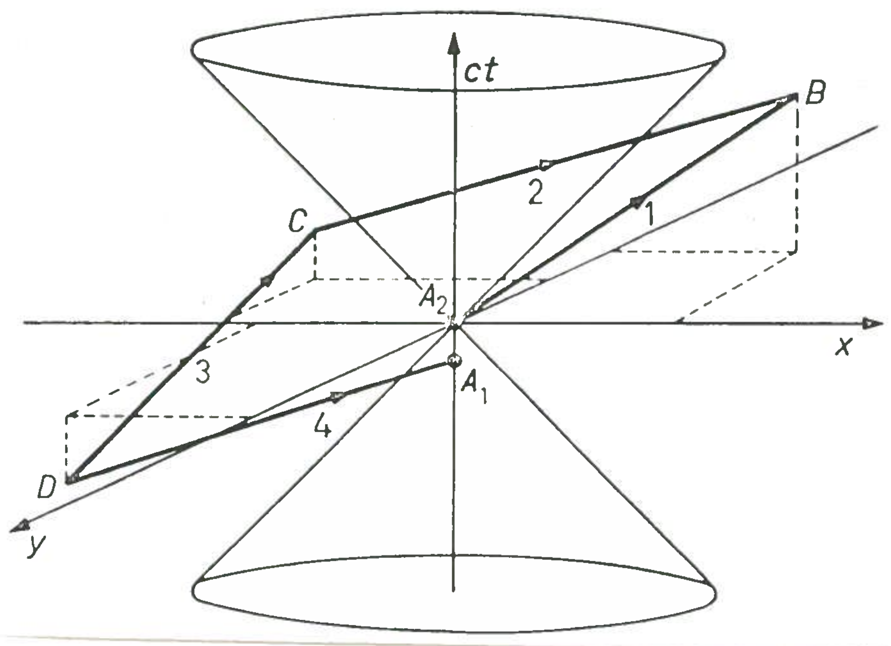


FIG. 24

pens before the emission of 1; it might seem that one can send signals into the past of A. However (cf. Sects.5.12-5.14 and Sect.6, as well as Recami 1973,1978c), observer A will actually see the sequence of events in the following way: The event at D consists in the creation of the pair $\bar{3}$ and 4 by the object D; tachyon 4 is then absorbed at A_1, while $\bar{3}$ is scattered at C (transforming into tachyon $\bar{2}$); the event A_2 is the emission, by A itself, of tachyon 1 which annihilates at B with tachyon $\bar{2}$. Therefore, according to A, one has an initial pair-creation at D, and a final pair--annihilation at B, and tachyons 1, 4 (as well as events A_1, A_2) do not appear causally correlated at all. In other words, according to A, the emission of 1 does not initiate any chain of events that brings to the absorption of 4, and we are not in the presence of any "effect" preceding its own "cause".

Analogous, orthodox descriptions would be forwarded by other observers. For instance, the tachyons' and observers' velocities chosen by Pirani (1970) are such that all tachyons will actually appear to observer s_0 as moving in directions opposite to the ones shown in Fig. 23.

9.2.3.- Comments - The comments are the same as in the previous Sect.9.1. Notice that the "ingredients" that allow us to give the paradox a solution are always the "switching principle" (Sect.5.12; see also Schwartz 1982) and the tachyon relativistic kinematics (Sect.6).

9.2.4..- "Strong version" and its solution - Let us formulate Pirani's paradox in its strong version. Let us suppose that tachyon 4, when absorbed by A at A_1, blows up the whole lab of A, eliminating even the physical possibility that tachyon 1 (believed to be the sequence starter) is subsequently emitted (at A_2). Following Root and Trefil (1970; see also Trefil 1978) we can see on the contrary how, e.g., observers s_o and A will really dscribe the phenomenon.

Observer s_o will see the lab of A blow up after emission (at A_1) of the antitachyon $\overline{4}$ towards D. According to s_o, therefore, the antitachyon $\overline{1}$ emitted by B will proceed beyond A (since it is not absorbed at A_2) and will eventually be absorbed at some remote sink-point U of the universe. By means of a LT, starting from the description by s_o, we can obtain (Caldirola and Recami 1980) the description given by A.

Observer A, after having absorbed at A_1 the tachyon 4 (emitted at D together with $\overline{3}$), will record the explosion of his own laboratory. At A_2, however, A will cross the flight of a tachyonic "cosmic ray" 1 (coming from the remote source U), which will annihilate at B with the antitachyon $\overline{3}$ scattered at C, i.e. with the antitachyon $\overline{2}$.

9.3.- Solution of the Edmonds Paradox

The seeming paradoxes arising from the relativity of the judgment about "cause" and "effect" have been evidenced by Edmonds (1977b) in a clear (and amusing) way, with reference to the simplest tachyon process: the exchange of tachyons between two ordinary objects, at rest one w.r.t. the other.

9.3.1.- The paradox - We build a long rocket sled with a "tachyon-laser" at the left end and a "target-flower" at the right end. A short lever sticks out of the side of the "laser". If we trip the lever, the tachyon laser emits a very sharp, intense burst of tachyons for which we measure the speed of, let's say, V. These tachyons then hit the "flover" and blast it into pieces. The flower absorbs all the tachyons in the pulse as it explodes, so that the tachyons disappear.

Now we accelerate the sled (with "charged" tachyon-laser and flower attached to it) up to an incoming speed of $-v=-v_x$ relative to our frame, and then turn off its rocket engines. Moreover, we form a long line of "astronauts" floating in space along the x-axis (i.e. along the rocket-sled motion-line). Each astronaut has a "roulette wheel" in his one hand, and keeps spinning his gambling wheel untill he gets, say, the number 13. When he happens to do so, he quikly put out a stick in front of him which could beat the trigger on the moving laser. No one in our frame knows when a given astronaut will get 13 to come up. Some astronauts may get 13, but too far down the line, or find the trigger has already passed them when they get it. But, finally, someone gets the right number, puts out his stick, finds that the lever is almost at his position and he triggers the laser.

Once the laser fires, the observer travelling with the sled sees - so as before - a burst of tachyons actually travelling from the laser to the flower. If the sled is moving slowly enough ($vV < c^2$), then we also - together with the "astronauts" - see the flower blow up at a time later then the time at which the laser fires. However, if the sled is fast enough ($vV > c^2$), we see a pulse of antitachyons going from the flower to the laser: Namely, we would see the flower to blow up before the laser fires. Therefore, the astronaut who triggers the laser sees the laser immediately "swallowing" a pulse of antitachyons coming from the flower. In other words, the lucky astronaut will conclude that the flower had to know in advance who was going to get 13 (so that it can blow up and create the antitachyon-pulse just at the right time, in order for the beam to arrive at the lucky astronaut as he gets the number 13 to come up for him).

9.3.2.- The solution - Since "source" and "detector" are supposed by Edmonds to be at rest one w.r.t. the other, according to both laser and flower - i.e. in the lab - there are no problems about the flight-direction of the tachyons. However, if we choose other observers (as the astronauts) they will in reality see the laser absorb antitachyons $\overline{T}$ coming from the flower (and not to fire tachyons T towards the flower). We have simply to accept it, since we learned (cf. e.g. Sect.5.17) that only the "principle of retarded causality" (Third Postulate) is a law, and therefore has to be valid for each observer; whilst the assignement of the labels "source" and "detector" is a description-detail, which does not have to be relativistically invariant.

Then, to answer Edmonds (Recami 1977a), let us show by an example that seeming paradoxes as the one above arise also in ordinary Special Relativity (due to the Lorentz non-invariance of the descriptions). Let us therefore forget about tachyons in the following example.

Let us suppose we are informed about a cosmic fight taking place between two different kinds of extraterrestial beings, each one driving his own rocket, where the rocket colors are violet for the first and green for the second species. Let us suppose moreover that we know the "green men" to possess an inviolable natural instinct that makes them peaceful; on the contrary the "violet men" possess an agressive, warrior instinct. When we observe the interplanetary battle by our telescope, it can well happen - due to the Doppler effect, i.e. due to the "observation distorsions" caused by the relative motions - that, when a "violet man" fires his gun and strikes a green rocket, the violet color appears to us as green, and vice-versa, because of the rocket motions. Then, according to the spirit of Edmonds' paradox, we should deduce that an inviolable law of nature has been badly violated (the instictive law of those extraterrestrial beings). Within SR, however, we already know how to clarify the whole story. We "observe" at first a seeming violation of natural laws; but, if we know the relevant physics (i.e. SR and the rocket velocities), we can determine the "intrinsic (proper) colors" of the rockets in their own rest-frames, and solve any doubts.

In other words, any observer is capable of understanding the physical world in terms of his own observations only, provided that he is equipped with a suitable theory (he uses his knowledge of SR, in this case).

Going back to the tachyon "paradox", we conclude that the lucky astronaut, when knowing tachyon mechanics (i.e. the ER), can calculate the tachyons' direction in the flower rest-frame and find out the "intrinsic behaviour" of the flower. The astronaut will find that in the flower-frame the tachyons are not emitted, but absorbed by the flower; even if the relative speed produces a high "distorsion" of the observed phenomenon. In analogy with our example, it is not important that the flower seem to the astronauts to precognize the future, but that the flower "intrinsically" does not.

The discussion of this paradox reminded us that: (i) one can scientifically observe (or observe, tout court) the natural world only if he is endowed with theoretical instruments, besides experimental and sensorial instruments; (ii) the "intrinsic properties" (so as the color) of a body appear to a moving observer distorted by the relative motion; if high relative speeds are involved, that distortion can be large as well).

Let us add a further comment.

9.3.3.- Comment - In the case of a bradyon exchange, in which the roles of "source" and "detector" are independent of the observer, the emitter and receiver are well represented by a male and a female object, respectively. Such a habit is however misleading in the case of a tachyon exchange, in which the same object can now apppear as the emitter, now as the receiver, depending on the observer. Devices such as "guns" ought to be avoided in the "Gedankenexperimente" regarding the exchange of tachyons. A round-shaped device, such as a sphere, should be the right one for representing objects able to emit/absorb tachyons.

9.4.- Causality "in Micro-" and "in Macro-physics"

Let us go on investigating the paradoxes arising when two bradyonic objects A, B exchange tachyons T, since there we meet in nuce all the problems than one encoun ters in the more complicated processes.

Let us consider, namely, the situation in which "laser" (A) and "flower" (B) are no more at rest one w.r.t. the other.

Such a situation is much more problematic. Nevertheless, no real problems are actually present (cf. Sect.6) as far as the tachyon production is supposed to be a "spontaneous", uncontrollable phenomenon, just as particle production in elementary particle physics. By convention, let us refer to this as the case "of microphysics".

Problems arise, however, when the tachyon production is a priori regarded as controllable (we shall refer to this latter as the case "of macrophysics"). We are going to analyse such problem by means of two paradoxes.

The first one was proposed by Bell (1979).

9.5.- The Bell Paradox and its solution

9.5.1.- The paradox - By firing tachyons you can commit a "perfect murder". Suppose that A purposes killing B, without risking prosecution. When he happens to see B together with a "witness" C, he aims his tachyon-pistol at the head of B, untill B and C (realizing the danger) start running away with speed, say, u. Then, A chooses to fire tachyonic projectiles T having a speed V such that $uV > c^2$. In the A rest-frame, tachyons T reach B soon and are absorbed by B's head, making him die. Due to the fact that $uV > c^2$ (and to Sects.5.12 and 6), however, the witness C - when questioned by the police - will have to declare that actually he only saw antitachyons $\overline{T}$ come out of B's head and be finally absorbed by A's pistol. The same would be confirmed by B himself, were he still able to give testimony.

9.5.2.- The solution; and comments - Let us preliminarily notice that B and C (when knowing tachyon mechanics) could at least revenge themselves on A by making A surely liable to prosecution: they should simply run towards A! (cf. Sects.5.12 and 6).

But let us analyse our paradox, as above expounded. Its main object is emphasizing that, when A and B are moving one w.r.t. the other, both A and B can observe "intrinsic emissions" in their respective rest-frames (Sect.6.10). It follows that it seems impossible in such cases to decide who is actually the beginner of the process; i.e., who is the cause of the tachyon exchange. There are no grounds, in fact, for privileging A or B.

In a pictoresque way - as Bell put it - it seems that, when A aims his pistol at B (which is running away) and decides to fire suitable tachyons T, then B is "obliged" to emit antitachyons $\overline{T}$ from his head and die.

To approach the solution, let us first rephrase the paradox (following the last lines of Sect.9.3) by substituting two spherical objects for A's pistol and B's head. About the properties of the emitters/absorbers of tachyons we know a priori only the results got in Sect.6; but, since this paradox simply exploits a particular aspect of the two-body interactions via tachyon exchange, we have just to refer to those results. Their teaching may be interpreted as follows, if we recall that we are assuming tachyon-production to be controllable (otherwise the paradox vanishes). The tachyon exchange takes place only when A, B possess suitable "tachyonic aptitudes", so as an electric discharge takes place between A and B only if A, B possess electrical charges (or, rather, are at different potential levels). In a sense, the couple of "spherical objects" A, B can be regarded as resembling a Van-de-Graaf generator. The tachyon-spark is exchanged between A and B, therefore, only when observer A gives his sphere (the "pistol") a suitable "tachyonic charge", or raises it to a suitable "tachyonic potential". The person responsible for the tachyon discharge between A and B (which may cause B to die) is therefore the one who intentionally prepares or modifies the "tachyonic properties" of his sphere: i.e., in the case above, it is A. In the same way, if one raises a conducting sphere A to a positive (electrostatic) potential high enough w.r.t. the earth to provoke a thunderbolt between A and a pedestrian B, he shall be the guilty murderer, even if the thunderbolt-electrons actually start from B and end at A.

Notice that we have been always considering tachyons emissions and absorptions, but never tachyon scatterings, since - while we know the tachyon mechanics for the former, simple processes - we do not know yet how tachyons interact with the (ordinary) matter.

9.6.- Signals by modulated tachyon beams: Discussion of a Paradox

9.6.1.- The paradox - Still "in macrophysics", let us tackle at last a more sophisticated paradox, proposed by ourselves (Caldirola and Recami 1980), which can be used to illustrate the most subtle hints contained in the "causality" literature (cf. e. g. Fox et al. 1969, 1970).

Let us consider two ordinary inertial frames s=(t,x) and s'=(t',x') moving one w.r.t. the other along the x-direction with speed $u < c$, and let us suppose that s sends - in its own frame - a signal along the positive x-direction to s' by means of a modulated tachyon beam having speed $V > c^2/u$ (Fig. 25). According to s' the tachyon-beam will actually appear as an antitachyon-beam emitted by s' itself towards s. We can imagine that observer s, when meeting s' at O, hands him a sealed letter and

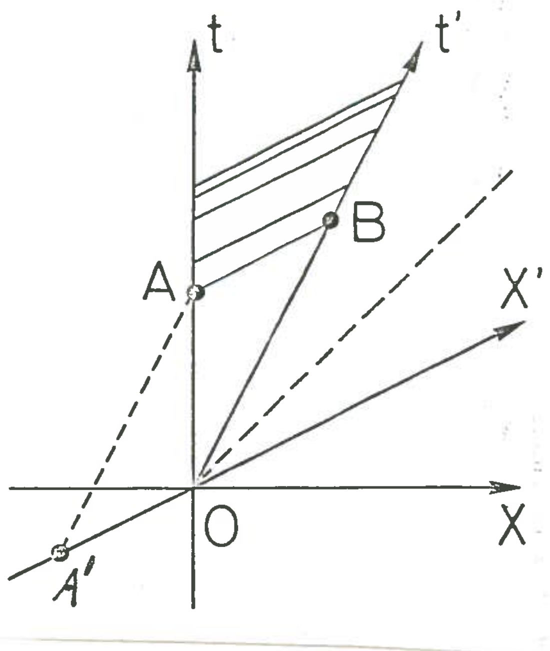


FIG. 25

tells him the following: 'By means of my "tachyon-radio" A and starting at time t, I will transmit to your "tachyon-radio" B a multi-figured number. The number is written inside the envelope, to be opened only after the transmission'.

Notice that the "free-will" of s' is not jeopardized nor under question, since s' can well decide to not switch on his tachyon-radio B. In such a case we would be back to the situation in Sect.9.3: In fact, s would see his tachyons T bypass s' without being absorbed and proceed further into the space; s', on the contrary, would see antitachyons $\overline{T}$ coming from the space and reaching A. If s' knows extended relativity, he can transform his description of the phenomenon into the "intrinsic description" given by s, and find out that s is "intrinsically" emitting a signal by tachyons T. He can check that the signal carried by those tachyons T corresponds just to the number written in advance by s.

The paradox is actually met when s' does decide to switch on his tachyon-radio B. In fact (if t' is the Lorentz-transformed value of t, and $\Delta t' = \overline{A'O}/V'$) the observer s' at time $t' - \Delta t'$ would see his radio not only broadcast the foretold multi-figured number (exatly the one written in the sealed letter, as s' can check straight after), but also emit simultaneously antitachyons $\overline{T}$ towards s: That is to say, transmit the same number to s by means of antitachyons. To make the paradox more evident, we can imagine s to transmit by the modulated tachyon-beam one of Bethoven's symphonies (whose number is shut up in advance into the envelope) instead of a plain number.

Further related paradoxes were discussed by Pavšič and Recami (1976).

9.6.2.- Discussion - Let us stress that s' would see the antitachyons $\overline{T}$ emitted by his radio B travel forward in time, endowed with positive energy. The problematic situation above arises only when (the tachyon-emission being supposed to be controllable) a well-defined pattern of correlated tachyons is used by s as a signal. In such a case, s' would observe his tachyon-radio B behave very strangely and unexpectedly, i.e. to transmit (by antitachyons $\overline{T}$) just the signal specified in advance by s in the sealed letter. He should conclude the intentional design of the tachyon exchange to stay on the side of s; we should not be in the presence of a real causality violation, however, since s' would not conclude that s is sending signals backward in time to him. We would be, on the contrary, in a condition similar to the one studied in Sect.

9.5.2. The paradox has actually to do with the unconventional behaviour of the sources/detectors of tachyons, rather than with causality; namely s", observing his apparatus B finds himself in a situation analogous to the one (Fig. 26) in which we possessed a series of objects b and saw them slip out sucked and "aspired" by A (or in which we possessed a series of metallic pellets and saw them slip out attracted by a variable, controllable electromanet A).

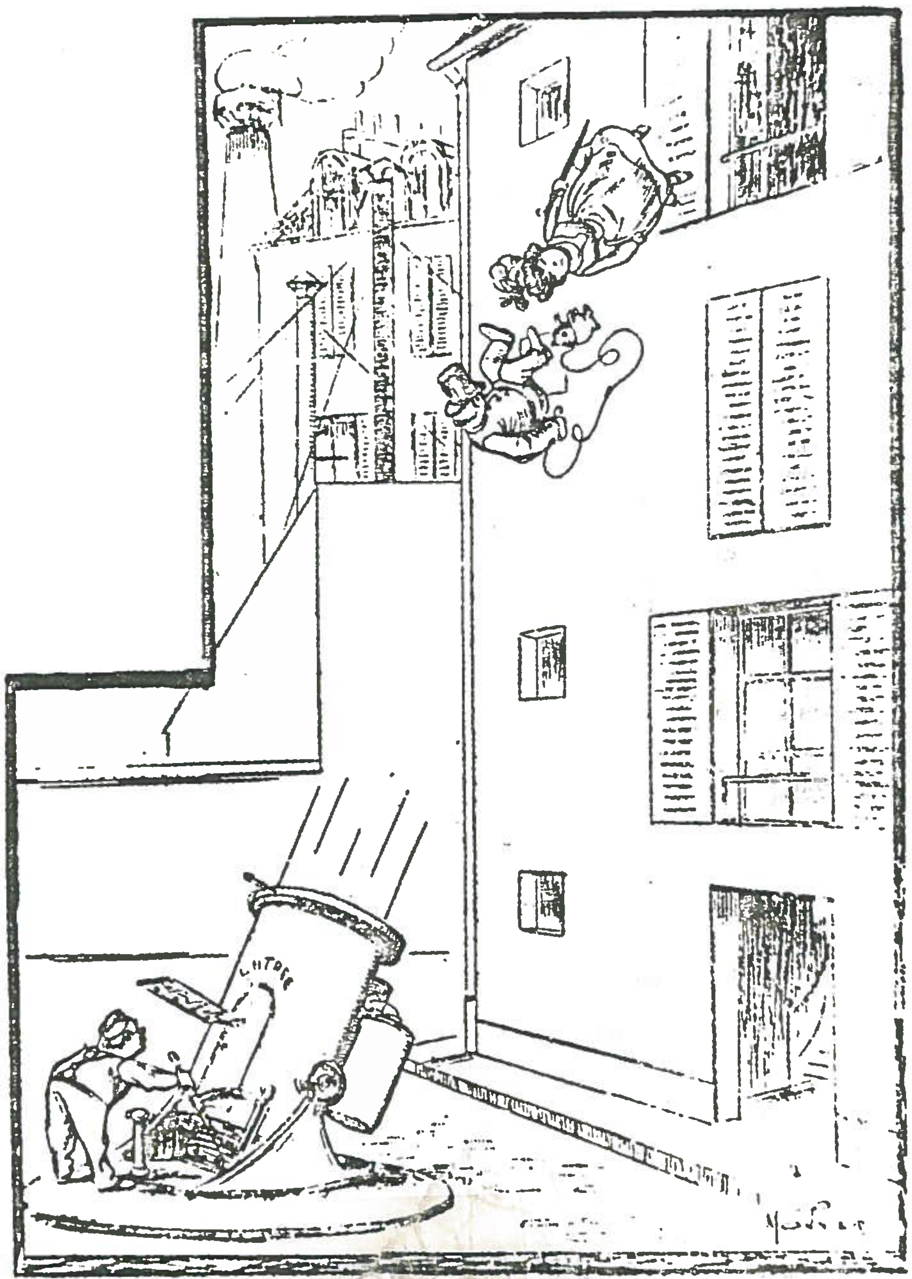


FIG. 26

From the behaviour of tachyon-radios in the above Gedankenexperiment it seems to follow that we are in need of a theory-formalization similar to Wheeler and Feynman's (1945, 1949; see also Flato and Guenin 1977, and Gott III 1974a). In particular, no tachyons can be emitted if detectors do not yet exist in the universe that will be able sooner or later to absorbs them. This philosophy, as we already saw many times, is a must in ER, since tachyon physics cannot be developed without taking always into

attitude might be actually adopted, even if unnecessary. See also, e.g. Havas (1974), and Rolnick (1974).

paradoxical situations arising when one deals with macro-tachyons are furthermore discussed).

We mentioned in the previous discussion (Sect.9.6.2) that the behaviour of tachyon sources/detectors might appear paradoxical to us for the mere fact that we are not accustomed to it. To shed some light on the possible nature of such difficulties, let us report at last the following anecdote (Csonka 1970), which does not involve contemporary prejudices: 'For ancient Egyptians, who know only the Nile and its tributaries, which all flow South to North, the meaning of the word "south" coincided with the one of "up-stream", and the meaning of the word "north" coincided with the one of "down-stream". When Egyptians discovered the Euphrates, which unfortunately happens to flow North to South, they passed through such a crisis that it is mentioned in the stele of Tuthmosis I, which tells us about that inverted water that goes down-stream (i.e. towards the North) in going up-stream'. See also, e.g.,Hilgevoord (1960).

9.7.- On the Advanced Solutions

Relativistic equations (both classical and quantal) are known to admit in general advanced, besides retarded, solutions. For instance, Maxwell equations predict both retarded and advanced electromagnetic radiations. Naïvely, advanced solutions have been sometimes regarded as actually representing motions backwards in time. On the contrary, we know from the "switching principle" (Sect.2.1) and the very structure of SR (see Part I, Sect.2) that the "advanced" waves or objects are nothing but anti-waves travelling in the opposite space-direction.

Within ER, actually, when an equation admits a solution corresponding to (outgoing) particles or photons, then a class of suitable GLTs transform such a solution into another one corresponding to (incoming) antiparticles or (anti)-photons. In other words, if an equation is G-covariant, it must admit also of solutions relative to incoming antiparticles or photons, whenever it admits of solutions relative to outgoing particles or photons.

This means that all G-covariant relativistic equations must admit both retarded and advanced solutions. When confining ourselves to subluminal velocities $u^2, v^2 < 1$, the ordinary relativistic equations already satisfy such a requirement, for the reasons discussed in Part I (see in particular Sect.2.3, point d)).

We could however ask ourselves why do we usually observe only, e.g., the outgoing, rather than the incoming radiation. The clue to the question is in taking into account the initial conditions: In ordinary macrophysics some initial conditions are by far more probable than others. For instance, the equations of fluid-dynamics allow to have on the sea surface both outgoing circular concentric waves and incoming circular waves tending to a center. It is known, however, that the initial conditions yielding the former are more likely to be met than those yielding the latter case.

10.- TACHYON CLASSICAL PHYSICS (RESULTS INDEPENDENT OF THE SLTs' EXPLICIT FORM)

According to Sect.5.1, the laws of classical physics for tachyons are to be derived just by applying a SLT to the ordinary, classical laws of bradyons (this statement has been sometime referred to as the "Rule of extended relativity": cf. Parker 1969 and Recami and Mignani 1974a). To proceed with, we need nothing but the Assumption in Sect.7.1; i.e. we need only assuming that SLTs exist which carry time-like into space-like tangent vectors, and vice-versa.

It is noticeable that tachyon classical physics can be obtained in terms of purely real quantities.

Sects.10.1 and 10.2 below do contain important improvements w.r.t. Review I.

10.1.- Tachyon Mechanics

For example, the fundamental law of bradyon dynamics reads

$$F^{\mu} = c\,\frac{d}{ds}\left(m_o c\,\frac{dx^{\mu}}{ds}\right) \equiv \frac{d}{d\tau_o}\left(m_o\,\frac{dx^{\mu}}{d\tau_o}\right) . \qquad (\beta^2 < 1) \qquad (106)$$

Notice that eq.(106) in its first form is only Lorentz-covariant, while in its second form is G-covariant (cf. Sect.7.2).

Even for tachyons, then, we shall have (Recami and Mignani 1974a)

$$F^{\mu} = +\frac{d}{d\tau_o}(m_o u^{\mu}) \equiv +\frac{dp^{\mu}}{d\tau_o} , \qquad (\beta^2 > 1) \qquad (107)$$

where m_o is the tachyon (real) rest-mass and, anticipating Sect.14.13, we defined $p^{\mu} \equiv m_o u^{\mu}$ also for tachyons. Equation (107) is the relativistic Newton Law written in G-covariant form: i.e. it is expected to hold for $\beta^2 \gtrless 1$. It is essential to recall, however, that u^{μ} is to be defined $u^{\mu} \equiv dx^{\mu}/d\tau_o$ just as in eq.(93a). Quantity $d\tau_o$, where τ_o is the proper-time, is of course G-invariant; on the contrary, $ds = \pm c d\tau_o$ for bradyons, but $ds = \pm icd\tau_o$ for tachyons (cf. Sects.2.2 and 4.3).

Equation (107) agrees with eqs.(87) and (87') of Sect.6.14, where we set $\underset{\sim}{F} = d\underset{\sim}{p}/dt$, and suggests that for tachyons $dt = \pm d\tau_o/\sqrt{\beta^2-1}$ (see Review I), so that in G-covariant form $dt = \pm d\tau_o(|1-\beta^2|)^{-1/2}$.

For the tachyon case, let us notice the following. If at the considered time-instant t we choose the x-axis so that $V = |\underset{\sim}{V}| = V_x$, then only the force-component F_x will make work. We already mentioned that the total energy of a tachyon decreases when its speed increases, and vice-versa (see Figs.4a and 10); it follows that F_x when applied to a tachyon will actually make a positive, elementary work $d\mathscr{L}$ only if a_x is anti-parallel to the elementary displacement dx, i.e. if sign(a_x) = - sign(dx). In other words, $d\mathscr{L}$ in the case of a force $\underset{\sim}{F}$ applied to a tachyon must be defined (cf. Sect.6.14) so that

$$d\mathscr{L} = - \frac{m_o}{(V^2-1)^{3/2}} a_x dx \tag{108}$$

where a_x and dx posses of course their own signs. Equation(108) does agree both with the couple of eqs.(88a), (89) and with the couple of eqs.(88'a), (89').

It is evident that, with the choice (Review I) represented by eqs.(89) and (85) of Sect.6.14, we shall have ($v=v_x$; $V=V_x$):

$$F_x = + \frac{m_o}{(1-v^2)^{3/2}} a_x \qquad \text{for bradyons ;} \tag{109a}$$

$$F_x = - \frac{m_o}{(V^2-1)^{3/2}} a_x \qquad \text{for tachyons .} \tag{109b}$$

On the contrary, still with the choice (89)-(85), we shall have

$$F_{y,z} = + \frac{m_o}{(|1-\beta^2|)^{1/2}} a_{y,z} \tag{109c}$$

for both bradyons and tachyons. Actually, under our hypotheses ($v=v_x$, $V=V_x$), the transverse force-components $F_{y,z}$ do not make any work; therefore, one had no reasons a priori for expecting any change in eq.(109c) when passing from bradyons to tachyons.

10.2.- Gravitational interactions of Tachyons

In any gravitational field a bradyon feels the (attractive) gravitational 4-force

$$F^{\mu} = - m_o \Gamma^{\mu}_{\varrho\sigma} \frac{dx^{\varrho}}{ds} \frac{dx^{\sigma}}{ds} \quad . \qquad (\beta^2 < 1) \tag{110}$$

In G-covariant form, then, eq.(110) will write (Review I, Mignani and Recami 1974d, Recami and Mignani 1974a, Recami 1977b):

$$F^{\mu} = -\frac{m_o}{c^2}\,\Gamma^{\mu}_{\varrho\sigma}\,\frac{dx^{\varrho}}{d\tau_o}\,\frac{dx^{\sigma}}{d\tau_o}\,, \qquad (\beta^2 \gtrless 1) \qquad (111)$$

since the Christoffel symbols behave like (third-rank) tensors under any linear transformations of the coordinates. Eq.(111) hold in particular for a tachyon in any gravitational field (both when originated by tachyonic and by bradyonic sources).

Analogously, the equation of motion for both bradyons and tachyons in a gravitational field will still read (Review I), in G-covariant form,

$$a^{\mu} + \Gamma^{\mu}_{\varrho\sigma}\, u^{\varrho}u^{\sigma} = 0\,, \qquad (\beta^2 \gtrless 1) \qquad (112)$$

with $a^{\mu} \equiv d^2x^{\mu}/d\tau_o^2$.

Passing to General Relativity, this does agree with the Equivalence Principle: Bradyons, photons and tachyons follow different trajectories in a gravitational field, which depend only on the initial (different) four-velocities and are independent of the masses.

Going back to eqs.(111), we may say that also tachyons are attracted by a gravitational field. However, such an "attraction" has to be understood from the energetical and dynamical point of view only.

In fact, if we consider for simplicity a tachyon moving radially w.r.t. a gravitational source, due to eq.(109b) (i.e., due to the couples of equations either (88a)-(89), or (88'a)-(89')) it will accelerate when receding from the source, and decelerate when approaching the source. From the kinematical point of view, therefore, we can say that tachyons seem to be gravitationally repelled. Analogous results were put forth by Vaidya (1971), Raychaudhuri (1974), Honig et al. (1974) and so on.

In the case of a bradyonic source, what precedes agrees with the results obtained within General Relativity: see e.g. Saltzman and Saltzman (1969), Gregory (1972), Hettel and Helliwell (1973), Sum (1974), Narlikar and Sudarshan (1976) , Narlikar and Dhurandhar (1976), Comer and Lathrop (1978), Maltsev (1981), Ciborowski (1982), Finkelstein et al. (1983), Cao Shenglin et al. (1984), etc.

10.3.- About Cherenkov Radiation

Let us consider a tiny spherically symmetric electric charge P, in particular point-like. From Sect.8.2 (cf. Figs.17 and 18) we know that, when endowed with constant Superluminal speed V (e.g. along x): (i) its shape transforms into a double cone $\mathscr{C}_o$; (ii) its equipotential surfaces (spherical surafaces in the rest-frame) transform into two-sheeted hyperboloyds asymptotic to $\mathscr{C}_o$. Such is the result of the "distortion" due to the high relative speed V; cf. Fig. 27 (see also Gladkikh 1978a,b, Terletsky 1978, Corben 1975, 1974, Gott III 1974, Fleury et al. 1973, as well as Shankara 1979). Notice explicitly that we are here dealing with the equipotential surfaces of the initial electrostatic field and with their transforms under a SLT; completely different would be the case (cf. Sect.14.1) of the electromagnetic waves actually emitted by a source (initial spherical waves will have to transform again into spherical waves!).

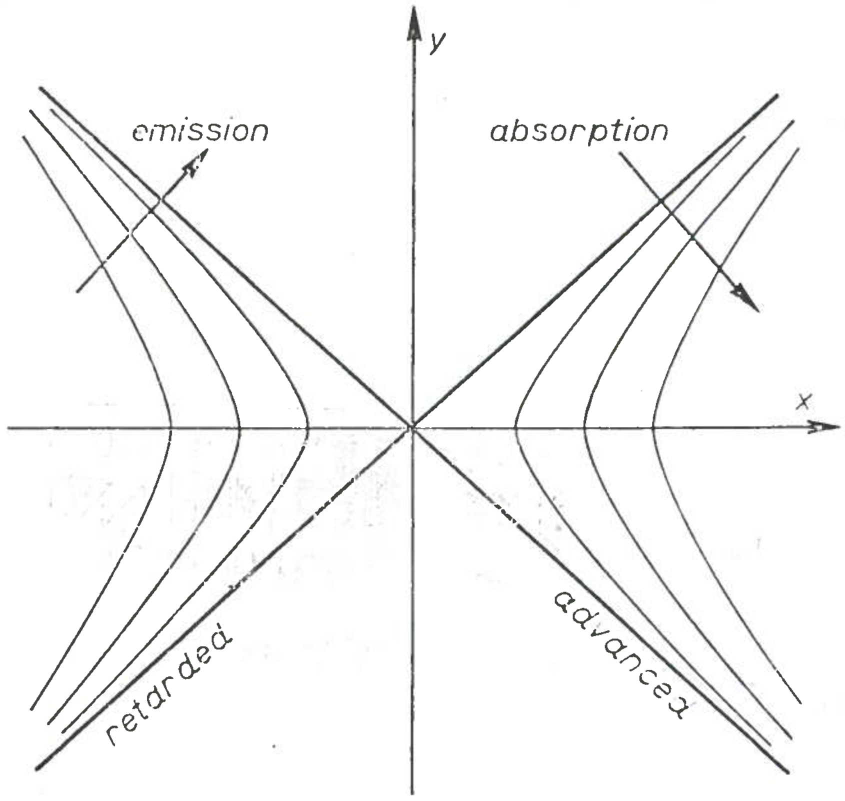


FIG. 27

The asymptotic double cone $\mathscr{C}_0$ in Fig.27 has nothing to do with Cherenkov's, since no actual radiation energy is globally emitted by $P=P_T$ during its inertial Superluminal motion; in fact, one may say that the seeming emission associated with the "retarded" cone is exactly counterbalanced by the seeming absorption associated with the "advanced" cone (Barut et al. 1982). Moreover, Cherenkov radiation is known to come not from the "radiating" particle itself, but from the charges of the (material) medium; so that the expression 'Cherenkov radiation in vacuum' is itself meaningless: unless one provides a suitable theory about the vacuum structure (which is not expected to be done within the present classical approach. See Mignani and Recami 1973b).

Incidentally, it would be nice to know (cf. also the end of Sect.8.2) the L^2-function of the space-time coordinates yielding the distribution over $\mathscr{C}_0$ of the tachyon charge density. Afterwards, on the basis of the Maxwell equations for tachyons (see Sect.15.1 in the following) and for a constant speed V, it would be interesting to find out solutions for $\underset{\sim}{E}(t,\underset{\sim}{x})$ and $\underset{\sim}{H}(t,\underset{\sim}{x})$ corresponding to a null global flux of radiation at infinity.

Since we do not know yet the explicit form of the SLTs in four dimensions, we can resort to the two-dimensional formulae (Sect.5.6) to check at least in that case the aboveseen prediction that constant speed tachyons do not emit Cherenkov radiation in the vacuum. Let us consider a free tachyon P_T in the vacuum. It will appear as a free bradyon P_B to any Superluminal observer S; according to S the energy lost by P_B through Cherenkov emission is therefore zero: $dE/d\ell = 0$. If we transform such a law by means of a SLT, e.g. by the transcendent 2-dimensional SLT, we get again: $dE'/d\ell' = 0$. Provided that the "electromagnetic vacuum" is invariant under SLTs (apart from tachyons), we have verified that free tachyons are not expected to emit Cherenkov in vacuum (Mignani and Recami 1973b; see also, e.g., Ey and Hurst 1977, Kirch 1977, Bulbeck and Hurst 1984).

10.4.- About Doppler Effect

In the two-dimensional case (Sect.5.7), the Doppler-effect formula for a sub- or a Super-luminal source, moving along the x-axis, will be (Mignani and Recami 1973a):

$$\nu = \nu_0 \frac{\sqrt{|1-u^2|}}{1 \pm u}, \qquad (-\infty < u < +\infty) \qquad (113a)$$

where the sign - (+) corresponds to approach (recesion). The consequences are depicted in figures like Fig. 23 of Review I. For Superluminal approach, ν happens to be negative, so as explained by our Fig.15c. Let us moreover observe that, in the case of recesion, the same Doppler shift is associated both with $\bar{u} < c$ and with $\bar{U} = 1/\bar{u} > c$ (Mignani and Recami 1974a, Recami 1977b).

In the four-dimensional case, if the observer is still located at the origin, eq.(113a) is expected to generalize (Recami and Mignani 1974a,e) into

$$\nu = \nu_0 \frac{\sqrt{|1-u^2|}}{1+u\cos\alpha}, \qquad (-\infty < u < +\infty) \qquad (113b)$$

where $\alpha = \widehat{\underset{\sim}{u}\underset{\sim}{\ell}}$, vector $\underset{\sim}{\ell}$ being directed from the observer to the source. Let us notice from Sect.6.15 (eq.(91)), incidentally, that when an observer starts receiving radiation from a Superluminal pointlike source C (at C_0, i.e. in the "optic-boom" situation), the received radiation is infinitely blue-shifted.

10.5.- Electromagnetism for Tachyons: Preliminaries

The problem of extending electromagnetism to tachyons is not straightforward, since one does not know a priori whether the "electromagnetic tensor" $F^{\mu\nu}$ has to be still a tensor under the SLTs; cf. Sect.7.2 (quantity $F^{\mu\nu}$ is a tensor under the transformation group $\underset{\sim}{\mathscr{L}}_+$, but may not behave any more as a tensor under a larger transformation group).

If one assumes $F^{\mu\nu}$ to be a G-tensor, then ordinary Maxwell equations keep their form also for tachyons (Recami and Mignani 1974a, p.277):

$$\partial_\nu F^{\mu\nu} = j^\mu \;; \qquad \partial_\nu \tilde{F}^{\mu\nu} = 0 \,, \tag{114}$$

where $j^\mu = \varrho_0 u^\mu$ is the 4-current of both sub- and Super-luminal electric charges (and where the "tilde" indicates the dual tensor). Such a choice is the one adopted by Corben (1974,1975,1976,1978a). It corresponds to assume that the electric and magnetic fields $\underset{\sim}{E}$, $\underset{\sim}{H}$ transform under SLTs just as in eq.(101) of Review I, or sim.

If one, on the contrary, does not assume a priori that $F^{\mu\nu}$ is still a tensor even under $\underset{\sim}{\mathscr{G}}$, then one has first of all to determine or choose the behaviour either of the components of $\underset{\sim}{E}$, $\underset{\sim}{H}$, or of the electromagnetic 4-potential A^μ under SLTs. At this stage, let us observe what follows. In Sect.7.2 we noticed that two different kinds of "4-vectors" are easily met when trying to extend SR: the ones like $u^\mu \equiv dx^\mu/d\tau_0$ that are also G-vectors, and the ones like $w^\mu \equiv dx^\mu/ds$ that are Lorentz-vectors, but under a SLT, $S^{\mu\nu}$, (when $ds^2 \to -ds^2$) transform as follows:

$$w_\mu \to \pm\, i S_\mu^{\nu} w_\nu \equiv \pm\, i w'_\mu \quad . \tag{115}$$

When we write down the ordinary Maxwell equations for a purely subluminal 4-current $j_\mu(s)$ in terms of the 4-potential A_μ

$$\Box A_\mu = j_\mu(s) \;; \qquad (v^2 < c^2) \tag{116a}$$

$$\partial_\mu A^\mu = 0 \,, \tag{116b}$$

where we imposed the Lorentz gauge and, as usual, $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$, the left and write hand sides of eq.(116a) can be both vectors of the same "kind", or not. In the former case, Maxwell equations are G-covariant, and we are back to eqs.(114). In the latter case, however, the eqs.(116) under a SLT, $(S^{-1})_{\mu\nu}$, become (except, possibly, for a sign: see Mignani and Recami 1975b):

$$\Box A'_\mu = -\, i j_\mu(S) \;; \qquad \partial_\mu A'^\mu = 0 \,, \qquad (V'^2 > c^2) \tag{117}$$

where $j'_\mu(s) \equiv j_\mu(S)$ represents the Superluminal 4-current.

In such a second, alternative case - which, incidentally, is supported by our discussion of tachyon electrodynamics to follow in Sect.15 - when in presence of both sub- and Super-luminal 4-currents we may therefore write:

$$\Box (A_\mu + A'_\mu) = j_\mu(s) - i j_\mu(S) \,. \qquad (v^2 \gtrless c^2)$$

If we introduce the complex 4-potential $\bar{A}_\mu \equiv A_\mu + A'_\mu \equiv A_\mu + iB_\mu$, and the complex 4-current $\bar{J}_\mu = j_\mu(s) - i\, j_\mu(S)$, the previous equations write (Mignani and Recami 1975b)

$$\Box \bar{A}_\mu = \bar{J}_\mu \; ; \qquad \partial_\mu \bar{A}^\mu = 0 \,, \qquad (v^2 \gtrless c^2) \tag{118}$$

which extend Maxwell equations, eqs.(116), to the case when both slower and faster-than-light currents are present. By defining

$$A_\mu \equiv \mathrm{Re}\bar{A}_\mu \; ; \qquad B_\mu \equiv -\mathrm{Im}\bar{A}_\mu = + iA'_\mu \equiv + i(S^{-1})_{\mu\nu} A^\nu \,, \tag{119}$$

we can write the generalized equations for the extended 4-potential as follows:

$$\begin{aligned} &\Box (A_\mu - iB_\mu) = j_\mu(s) - i j_\mu(S) \; ; \\ &\partial^\mu (A_\mu - iB_\mu) = 0 \,, \end{aligned} \qquad (v^2 \gtrless c^2) \tag{118'}$$

which can of course be split into two real equations.

We shall come back to the problem of the generalized Maxwell equations for bradyons and tachyons in Sect.15.2, where we shall show that B_μ can be formally identified with the "second 4-potential" introduced by Cabibbo and Ferrari (1962) for magnetic monopoles (Amaldi 1968, Ferrari 1978).

11.- SOME ORDINARY PHYSICS IN THE LIGHT OF ER

11.1.- Introduction. Again about CPT

Looking for the SLTs in the ordinary space-time will pose us a new problem: finding out the transcendent transformation $\mathscr{S} \equiv \mathscr{S}_4$ which generalizes eq.(32) of Sect.5.5 to the 4-dimensional case. However, after what we saw in Part I (Sect.2), we are already prepared to accept (cf. Sects.5.16 and 5.6) that

$$G \in \mathscr{G} \;\Rightarrow\; -G \in \mathscr{G} \,, \qquad \forall G \in \mathscr{G} \tag{37'}$$

even in four-dimensions.

Actually, from Figs.5c and 6 (now understood to hold in four-dimensions), we see that: (i) an ordinary LT=L_S can carry from Ts to $\overline{T}$s; (ii) if a SLT = L_S exists that carries from Bs to Ts, then the subluminal transformation $L_S^{-1} L_s L_S$ will carry from Bs to $\overline{B}$s. Our general results in Sect.2 (e.g., eq.(10)) imply therefore that eq.(53) will be valid also in four-dimensions (Mignani and Recami 1974b):

$$-\mathbb{1} \equiv \overline{P}\overline{T} \equiv CPT \in \mathcal{G} \quad ; \tag{53'}$$

in connection with eq.(53') see all the remarks already expounded in Sect.2.3.

As a consequence, the generalized group $\mathcal{G}$ in Minkowski space-time is expected to be the <u>extension</u> (Pavšič and Recami 1977) of the proper, orthochronous (4-dimensional) Lorentz group $\mathcal{L}_+^\uparrow$ by means of the two operations CPT = $-\mathbb{1}$ and $\mathcal{S}$:

$$\mathcal{G} = \mathcal{E}(\mathcal{L}_+^\uparrow, \text{CPT}, \mathcal{S}) \; . \tag{120}$$

In our formalism, the operation CPT is a linear (classical) operator in the pseudo-Euclideal space, and will be a unitary (quantum-mechanical) operator when acting on the states space: cf. eq.(53'), and see Recami (1979a), Costa de Beauregard (1983).

From what precedes, and from Figs.5 and 6, we may say that even in the 4-dimensional energy-momentum space we have two symmetries: (i) the one w.r.t. the hyperplane E=0, corresponding to the transition particle ⇄ antiparticle; and (ii) the one w.r.t. the light-cone, expected to correspond to the transition bradyon ⇄ tachyon.

In any case, the "switching procedure" (Sects.2 and 5.12) will surely have to be applied for both bradyons and tachyons also in four-dimensions. Let us therefore reconsider it in a more formal way.

11.2.- Again about the "Switching procedure"

This and the following Section do <u>not</u> depend on the existence of tachyons. They depend essentially on our Part I.

We shall indicate by "SWP" the <u>switching</u> procedure (previously often called "RIP"). Let us also call strong conjugation $\overline{C}$ the discrete operation

$$\overline{C} \equiv CM_o \tag{121}$$

where C is the conjugation of all additive charges and M_o the rest-mass conjugation (i.e. the reversal of the rest-mass sign). Recami and Ziino (1976) showed that formally (cf. Fig.3b)

$$\text{"SWP"} = \overline{C} \; .$$

Then by considering m_o as a <u>fifth</u> coordinate, besides the ordinary four (Einstein and Bergmann 1938), and shifting to the language of quantum mechanics, they recognized that $P_5 \equiv \overline{C}$, quantity P_5 being the chirality operation, so that

$$\text{"SWP"} \equiv P_5 \; ; \tag{122}$$

in fact, when dealing as usual with states with definite parity, one may write $\overline{C}^{-1} \psi \overline{C} \equiv \gamma^5 \psi \equiv P_5^{-1} \psi P_5$. Notice that in our formalism the strong conjugation $\overline{C}$ is a unitary operator when acting on the states space (cf. also Vilela-Mendes 1976).

For details and further developments see e.g., besides the abovequoted papers, Edmonds (1974a,b), Lake and Roeder (1975), Pavšič and Recami (1977), Recami (1978a), Recami and Rodrigues (1982).

Here we want only to show that, when considering the fundamental particles of matter as extended objects, the (geometrical) operation which reflects the internal space-time of a particle is equivalent to the ordinary operation C which reverses the sign of all its additive charges (Pavšič and Recami 1982).

11.3.- Charge conjugation and internal space-time reflection

Following Pavšič and Recami (1982), let us consider in the ordinary space-time: (i) the extended object (particle) a, such that the interior of its "world-tube" is a finite portion of M_4; (ii) the two operators space-reflection, $\mathscr{P}$, and time-reversal, $\mathscr{T}$, that act (w.r.t. the particle world-tube W) both on the external and on the internal space-time:

$$\mathscr{P} = \mathscr{P}_E \mathscr{P}_I = \mathscr{P}_I \mathscr{P}_E \; ; \qquad \mathscr{T} = \mathscr{T}_E \mathscr{T}_I = \mathscr{T}_I \mathscr{T}_E \; , \tag{123}$$

where $\mathscr{P}_I (\mathscr{T}_I)$ is the internal and $\mathscr{P}_E (\mathscr{T}_E)$ the external space-reflection (time-reversal). The ordinary parity P and time-reversal T act on the contrary only on the external space-time:

$$P \equiv \mathscr{P}_E \; ; \qquad T \equiv \mathscr{T}_E \; .$$

The effects of $\mathscr{P}_E$, $\mathscr{P}_I$ and $\mathscr{P}$ on the world-tube W of a are shown in Figs.28; and the analogous effects of $\mathscr{T}_E$, $\mathscr{T}_I$, $\mathscr{T}$ in Figs. 29.

Let us now depict W as a sheaf of world-lines w representing - let us say - its constituents (Fig. 30a). In Figs. 30 we show, besides the c.m. world-line, also $w_1 = A$ and $w_2 = B$. The operation $\mathscr{P}\mathscr{T}$ will transform W into a second world-tube $\widetilde{W}$ consisting of the transformed world-lines $\widetilde{w}$ (see Fig.30b). Notice that each $\widetilde{w}$ points in the opposite time-direction and occupies (w.r.t. the c.m. world-line) the position symmetrical to the corresponding w.

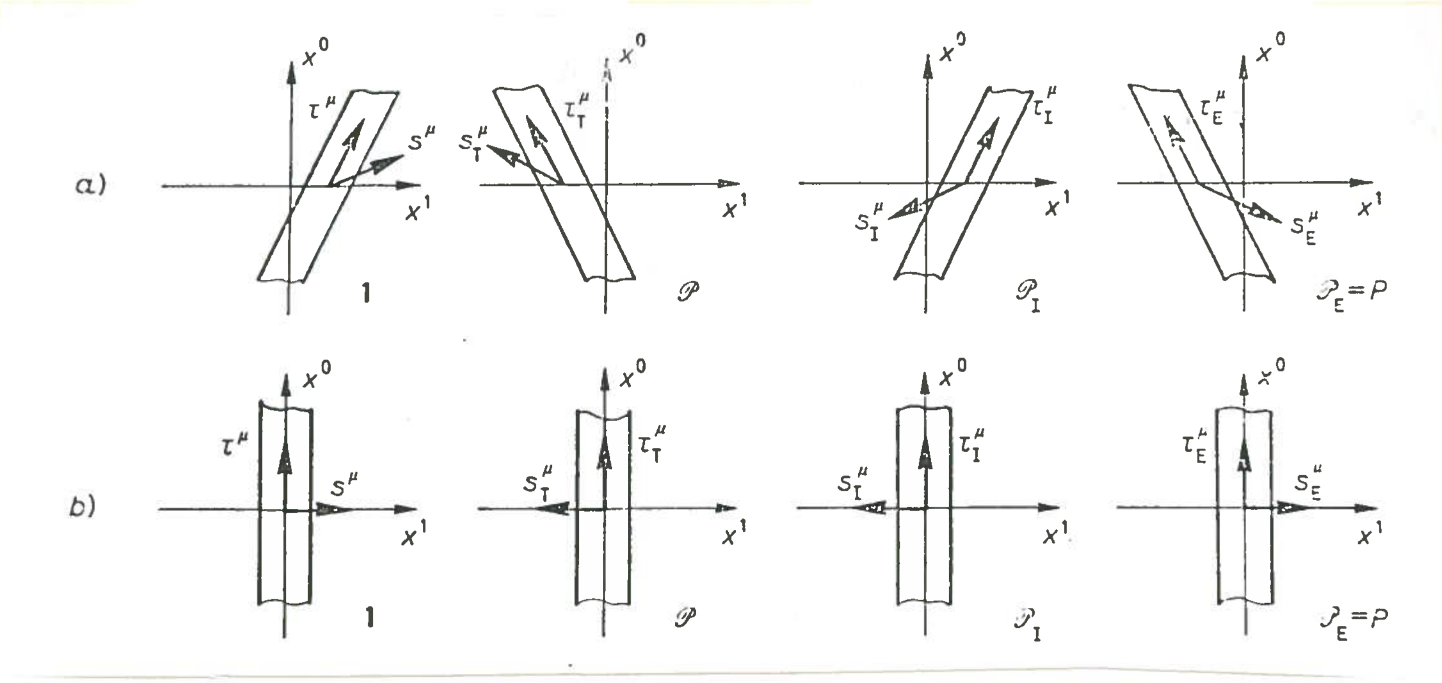


FIG. 28

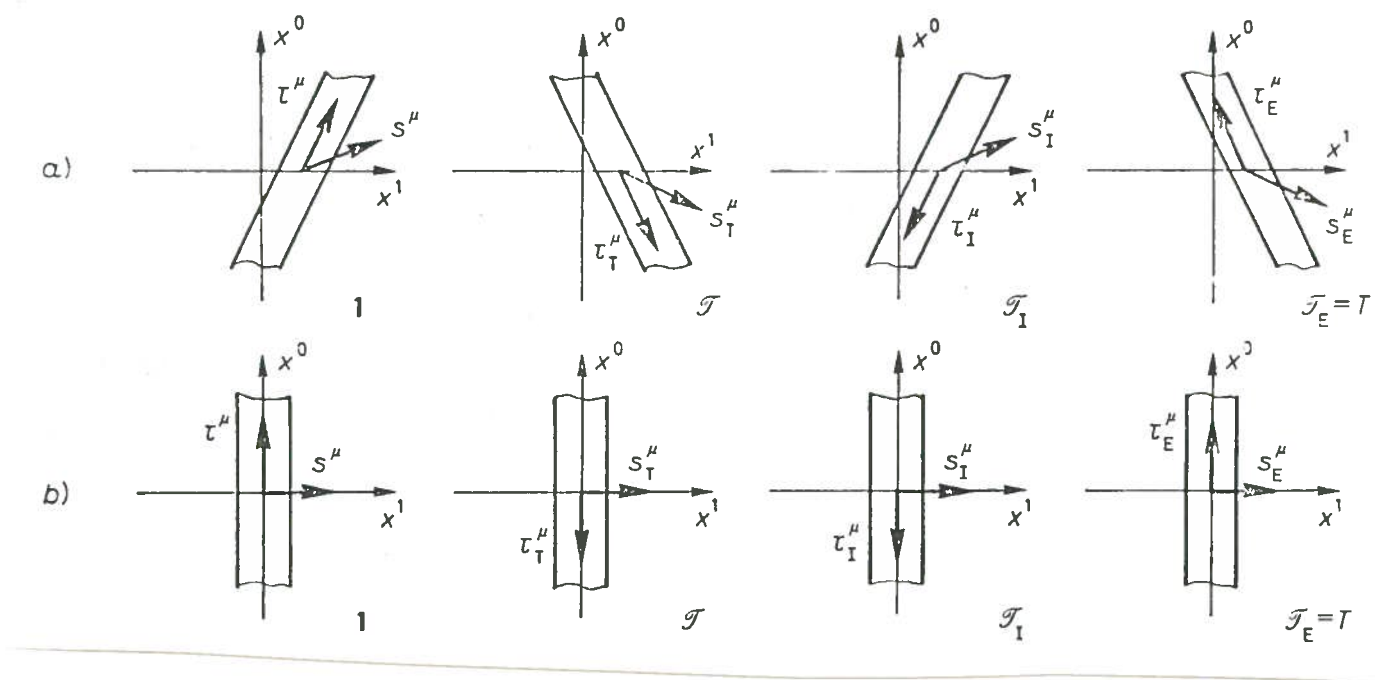


FIG. 29

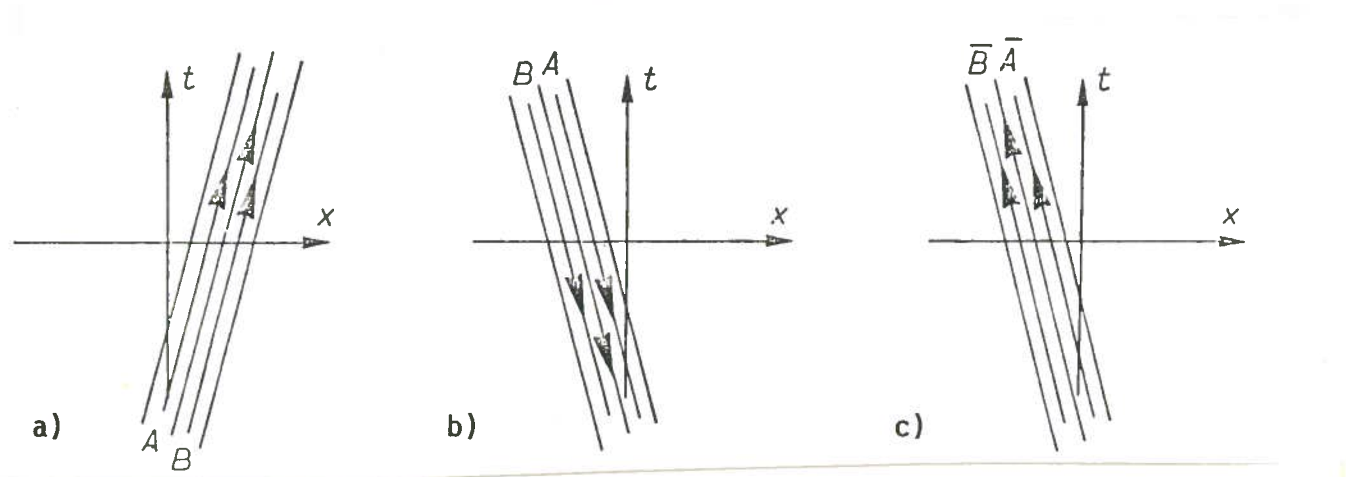


FIG. 30

If we apply the Stückelberg-Feynman "switching" (Sect.2.1), each world-line $\tilde{w}$ transforms into a new world-line $\bar{w}$ (cf. Fig.30c) which points in the positive time--direction, but represents now an anti-constituent. Let us now explicitely generalize the "switching principle" for extended particles as follows: We identify the sheaf $\bar{W}$ of the world-lines $\bar{w}$ with the antiparticle $\bar{a}$, i.e. $\bar{W}$ with the world-tube of $\bar{a}$. This corresponds to assume that the overall time-direction of a particle as a whole coincides with the time-direction of its "constituents". A preliminary conclusion is that the antiparticle $\bar{a}$ of a can be regarded (from the chronotopical, geometrical point of view) as derived from the reflection of the internal space-time of a.

Let us repeat what precedes in a more rigorous way, following our Sect.2, i.e. recalling that the transformation $L=-\mathbb{1}$ is an actual (even if antichronous) Lorentz transformation, corresponding to the 180° space-time "rotation": $\bar{P}\bar{T}\equiv-\mathbb{1}$. Now, to apply $\bar{P}\bar{T}$ from the active point of view to the world-tube W of Fig.30a means to rotate it (by 180°, in four-dimensions) into $\tilde{W}$ (Fig.30b); such a rotation effects also a reflection of the internal 3-space of particle a, transforming it - among the others - into its mirror image. The same result would be got by applying $\bar{P}\bar{T}$ from the passive point of view to the space-time in Fig.30a.

Then, we generalize the "Switching Principle" to the case of extended objects by applying it to the world-tube $\tilde{W}$ of Fig.30b. The world-tube $\tilde{W}$ does represent an (internally "mirrored") particle not only going backwards in time, but also carrying negative energy; therefore, the "switching" does rigorously transform $\tilde{W}$ into $\bar{W}$ (Fig. 30c), the anti-world-tube $\bar{W}$ representing $\bar{a}$.

In conclusion:

$$-\mathbb{1} \equiv \bar{P}\bar{T} = \mathcal{P}_E \mathcal{T}_E \mathcal{P}_I \mathcal{T}_I = \mathcal{P}\mathcal{T} = PT\mathcal{P}_I \mathcal{T}_I \qquad (124)$$

wherefrom, since $\bar{P}\bar{T}$ = CPT (Sect.2.3), one derives:

$$C = \mathcal{P}_I \mathcal{T}_I \, . \qquad (125)$$

As already anticipated, we have therefore shown the operation C, which inverts the sign of (all) the additive charges of a particle, to be equivalent to the (geometrical) operation of reflecting its internal space-time.

Also the results reported in this Section support the opinion that in theoretical physics we should advantageously substitute the new operations $\bar{P}\equiv\mathcal{P}$ and $\bar{T}\equiv\mathcal{T}$ for the ordinary operations P and T, which are merely external reflections (for instance, only the former belong to the Full Lorentz Group). Besides our Sect.2, cf. e.g. Review I, Recami (1978c), and also Costa de Beauregard (1984).

11.4.- Crossing Relations

Besides the CPT theorem, derived from the mere SR, from ER only it is possible to get also the socalled "crossing relations". Let us first recall that cross-sections and invariant scattering amplitudes can be defined (Recami and Mignani 1974a) even at a classical, purely relativistic level.

We are going to show (Mignani and Recami 1974a, 1975a) that - within ER - the same function is expected to yield the scattering amplitudes of different processes like

$$a + b \longrightarrow c + d \,, \tag{126a}$$

$$a + \bar{c} \longrightarrow \bar{b} + d \,, \tag{126b}$$

in correspondence, of course, to the respective, different domains of the kinematical variables.

Let a,b,c,d, be bradyonic objects w.r.t. a frame s_o. The two reactions (126a), (126b) among Bs are two different processes p_1, p_2 as seen by us; but they can be described as the same interaction $d_S \equiv d_1 \equiv d_2$ among Ts by two suitable, different Superluminal observers S_1, S_2 (Review I, Recami 1979a, Caldirola and Recami 1980). We can get the scattering amplitude $A(p_1)$ of p_1 by applying the $SLT(S_1 \rightarrow s_o) = L_1$ to the amplitude $A_1(d_1)$ found by S_1 when observing the scattering p_1; i.e. $A(p_1) = L_1[A_1(d_1)]$. Conversely, we may get the scattering amplitude $A(p_2)$ of p_2 by applying the $SLT(S_2 \rightarrow s_o) = L_2$ to the amplitude $A_2(d_2)$ found by S_2 when observing the scattering p_2; i.e.: $A(p_2) = L_2[A_2(d_2)]$. But, by hypothesis, $A_1(d_1) = A_2(d_2) = A(d_S)$. Then, it follows - roughly speaking - that

$$A(p_1) = A(p_2) \tag{127}$$

for all reactions among bradyons of the kind (126a) and (126b).

Actually, in ordinary QFT the requirement (127) is satisfied by assuming the amplitude A to be an analitic function that can be <u>continued</u> from the domain of the invariant variables relative to (126a) to the domain relative to (126b). However, our requirement (127), imposed by ER on the processes (126), has a more general nature, besides being purely relativistic in character. For further details see Review I.

At last, new "crossing-type relations" were derived from ER; they might serve to check the relativistic covariance of weak and strong interaction (which *a priori* don't have to be relativistically covariant): cf. Mignani and Recami (1974a, 1975a).

11.5.- Further results and remarks

Some results already appeared above; see e.g. Sect.9.7 on the interpretation of the "Advanced Solutions".

Many further results will appear in Part IV (Sect.13), in connection with QM and elementary particle physics: let us mention the ones related with the vacuum decays, virtual particles, a Lorentz-invariant boostrap for hadrons, the wave-particle dualism, etc.

Here, let us only add the following preliminary observations.

Let us consider (Fig. 31) two bodies A and B which exchange (w.r.t. a frame s_0) a trascendent tachyon T_∞ moving along the x-axis. From Fig.3 and Sect.6 we have

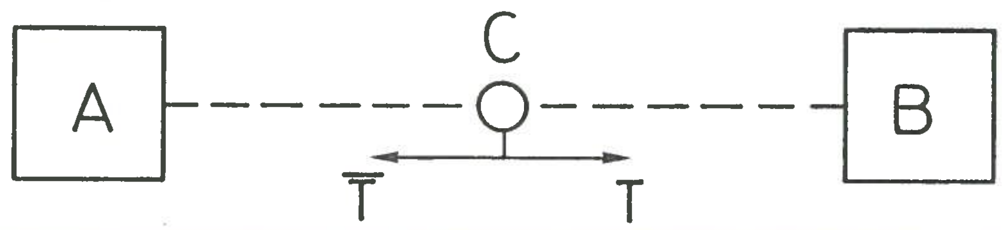


FIG. 31

seen that for transcendent particles the motion direction along AB is not defined. In such a limiting case, we can consider T_∞ either as a tachyon $T(v=+\infty)$ going from A to B, or equivalently as an antitachyon $\overline{T}(v=-\infty)$ going from B to A (cf. also Figs. 3). In QM language, we could write (Pavšič and Recami 1976):

$$|T_\infty\rangle = a|T(v=+\infty)\rangle + b|\overline{T}(v=-\infty)\rangle; \qquad a^2+b^2=1 . \tag{128}$$

Alternatively, it will be immediately realized that s_0 can interpret his observations also as due to a <u>pair-creation</u> of infinite-speed tachyons T and $\overline{T}$ (travelling along x) at any point C of the x-axis between A and B (Mignani and Recami 1976a, Edmonds 1976, Caldirola and Recami 1980): for instance, as the creation of a transcendent tachyon T travelling towards (and absorbed by) B and of a transcendent antitachyon $\overline{T}$ travelling towards (and absorbed by) A. Actually, for each observer the vacuum can become classically unstable <u>only</u> by emitting two (or more) <u>infinite-speed</u> tachyons, in such a way that the total 3-momentum of the emitted set is zero (the total energy emitted would be automatically zero: see Figs.4, 5 and 6).

It is interesting to check - cf. Sect.5.6 and eq.(52) of Sect.5.12 - that any (subluminal) observer s_1, moving along x w.r.t. s_0 in the direction A to B, will just see a unique (finite-speed) antitachyon $\overline{T}_1$ emitted by B, passing through point C without any interaction, and finally absorbed by A. On the contrary, any observer s_2 moving along x w.r.t. s_0 in the direction B to A, will just see a unique (finite-speed) tachyon T_2 emitted by A, freely travelling from A to B (without any interaction at C), and finally absorbed by B.

In what precedes we may consider the masses of A and B so large that the kinematical constraints, met in Sect.6, get simplified. In such a case, s_0, s_1 and s_2 will all see an elastic scattering of A and B.

As we have seen above, any observer s_0 can describe the particular process ph under examination in terms either of a vacuum decay or of a suitable tachyon emission by one of the two nearby bodies A, B. One can alternatively adopt one of those two languages. More generally, the probability of such vacuum decays must be related to the transcendent-tachyon emission-power (or absorption-power) of matter.

Furthermore, if A and B can exchange tachyons even when they are very far from each other, any observer s' (like s_1 and s_2) moving w.r.t. s_0 will describe ph in terms either of an incoming, suitable tachyonic cosmic ray or of the emission of a suitable, finite-speed tachyon by a material object. One of the consequence, in brief, is that the tachyon cosmic flux is expected to have for consistency a Lorentz-invariant 4-momentum distribution, just as depicted in Figs.10 and 5c. The large majority of "cosmic" tachyons ought then appear to any observer as endowed with speed very near to the light-speed c (see also Vigier 1979, Kamoi and Kamefuchi 1977). On this respect, it may be interesting to recall that an evaluation of the possible cosmic flux of tachyons yielded - even if very rough - a flux close to the neutrinos' one (Mignani and Recami 1976a).

As an elementary illustration of other possible considerations, let us at last add the following. If s_0 observes the process

$$a \rightarrow b + \bar{t} \,, \tag{129a}$$

where $\bar{t}$ is an antitachyon, then - after a suitable LT - the new observer s' can describe the same process as

$$a + t \rightarrow b \,. \tag{129b}$$

If, in eq.(129a), the emitted $\bar{t}$ had travelled till absorbed by a (near or <u>far</u>) detector U, then in eq.(129b) t must of course be regarded as emitted by a (near or <u>far</u>) source U.

If $\Delta\tau$ is the mean-life of particle a for the decay (129a), measured by s_0, it will be the Lorentz transform of the average time $\Delta\tau'$ that particle a must spend according to s' before absorbing a "cosmic" tachyon t and transforming into b.

PART III: GENERAL RELATIVITY AND TACHYONS

12.- ABOUT TACHYONS IN GENERAL RELATIVITY (GR)

12.1.- Foreword, and some bibliography

Space-like geodesics are "at home" in General Relativity (GR), so that tachyons have often been implicit ingredients of this theory.

Some papers dealing with tachyons in GR have been already quoted in Sect.10.2; other papers are Fuller and Wheeler (1962), Foster and Ray (1972), Ray and Foster (1973), Leibowitz and Rosen (1973), Banerjee (1973), Gott III (1974a,b), Arcidiacono (1974), Goldoni (1975a,b,c, 1978), Davies (1975), Lake and Roeder (1975), Ray and Zimmerman (1976,1877), Pavšič and Recami (1977), De Sabbata et al.(1977), Banerjee and Choudhuri (1977), Srivastava and Pathak (1977), Srivastava (1977), Gurevich and Tarasevich (1978), Kowalczyński (1978), Recami (1978a), Camenzind (1978), Milewski (1978), Johri and Srivastava (1978), Dhurandar (1978), Dhurandar and Narlikar (1978), Castorina and Recami (1978), Narlikar and Dhurandar (1978), Recami and Shah (1979), Dadhich (1979), Miller (1979), Ljubicic et al.(1979), Prasad and Sinha (1979), Ray(1980), Shanks (1980), Talukdar et al.(1981), Banerji and Mandal (1982), Mann and Moffat (1982), Srivastava (1982,1984), Ishikawa and Miyashita (1983), Nishioka (1983), Gurin (1983,1984).

For instance, Sum (1974) calculated - see Sect.10.2 - the deflection of a neutral tachyon (coming e.g. from infinity) in the field of a gravitating body like the Sun. He found the deflection towards the Sun to decrease monotonically for increasing tachyon speeds, and at infinite speed to be half as much as that for photons. Later on Comer and Lathrop (1978) noticed that the ordinary principle-of-equivalence calculation for the deflection of light by the Sun yields, by construction, only the deflection relative to the trajectories of infinitely fast particles (purely spatial geodesics); the total deflection will thus be the sum of the deflection given by the principle of equivalence and the deflection of the infinite-speed tachyons. This does solve and eliminate the puzzling discrepancy between the deflection of light evaluated by Einstein in 1911, using the principle of equivalence only, and the one calculated four years later using the full theory of GR.

In the first calculation Einstein (1911) found a deflection of one-half the correct value, since the remaining one-half is exactly forwarded by the deflection of the transcendent tachyons.

We shall here confine ourselves only to two topics: (i) tachyons and black-holes; (ii) the apparent Superluminal expansions in astrophysics.

Let us recall that the space-times of SR and of GR are pseudo-Riemannian (Sect. 4.3.5); a priori, one may thus complete the ordinary GR transformation group (Møller 1962, Sachs and Wu 1980) by adding to it coordinate transformations which invert the geodesic type.

12.2.- Black-holes and Tachyons

12.2.1.- Foreword - Black-holes (see e.g. Hawking and Ellis 1973) are naturally linked to tachyons, since they are a priori allowed in classical physics to emit only tachyons. Black-holes (BH) offer themselves, therefore, as suitable sources and detectors (see Sects.5.12-5.14) of tachyons; and tachyonic matter could be either emitted and reabsorbed by a BH, or exchanged between BHs (see Pavšič and Recami 1977, De Sabbata et al.1977, Narlikar and Dhurandar 1978, Castorina and Recami 1978, Recami 1979a, Recami and Shah 1979, Barut et al.1982). This should hold also for hadrons (Sect.6.13) if thay can actually be regarded as "strong BHs" (Ammiraju et al.1983, Recami 1982a, Castorina and Recami 1978, Salam 1978, Salam and Strathdee 1978, Caldirola et al.1978).

12.2.2.- Connections between BHs and Ts - But the connection between BHs and tachyons is deeper, since the problem of the transition outside/inside the Laplace-Schwarzschild horizon in GR is mathematically analogous to the problem of the transition bradyon/tachyon in SR (Recami 1978a,1979a). Let us start by recalling some results in the Appendix B of Hawking and Ellis (1973). The vacuum metric in the spherically symmetric case reads

$$ds^2 = -F^{-2}(t,r)dt^2 + X^2(t,r)dr^2 + Y^2(t,r)\, d\Omega \tag{130}$$

with $d\Omega \equiv d\vartheta^2 + \sin^2\vartheta\, d\varphi^2$. When $Y^{;a}Y_{;a} < 0$, eq.(130) becomes ($G = c = 1$):

$$ds^2 = -(1 - \frac{2m}{r})dt^2 + (1 - \frac{2m}{r})^{-1}dr^2 + r^2 d\Omega \tag{131a}$$

which is the known, unique (static) Schwarzschild metric for $r > 2m$. When $Y^{;a}Y_{;a} > 0$, eq.(130) yields on the contrary the (spatially homogeneous) solution

$$ds^2 = -(1 - \frac{2m}{t})dr^2 + (1 - \frac{2m}{t})^{-1}dt^2 + t^2 d\Omega \tag{131b}$$

which is (part of) the Schwarzschild solution for $r < 2m$, since the transformation $t \rightleftarrows r$ carries eq.(131b) into the form (131a) with $r < 2m$ (see also Goldoni 1975c).

In other words, the solution (131a) holds a priori for $r \lessgtr 2m$; inside the horizon, however, it is "reinterpreted" into the form (131b), by inverting the roles of t and r. In such a way one obtains that the metric does not change signature. In the two-dimensional case, however, we have seen (Sect.5.6) that the trasnformation $t \rightleftarrows x$ is just the effect of eqs.(39") when $U \to \infty$, i.e. is just the transcendent (Superluminal) Lorentz transformation (cf. also eq.(39')). And in four dimensions the operation $t \rightleftarrows r$ would have the same effect expected from a (4-dimensional) transcendent "transformation" (see Sect.3.2): it seems to lead to a manifold described by three time-like coordinates and one space-like coordinate... Such is the problem that one meets to avoid that change of signature; a problem that shows up more clearly when eqs.(131) are written down in Cartesian coordinates (De Sabbata et al.1977). That this is not a trivial problem is shown also by the difficulties met as soon as one eliminates the privileged rôle of the radial coordinate r by destroying the spherical symmetry. Actually, when analysing non-spherically symmetric perturbations, coordinate-independent "singular surfaces" do arise (Mysak and Szekeres 1966, Israel 1967, Janis et al.1968). Clarifying such questions would mean solving also the mathematical problem of the "SLTs" in four dimensions.

12.2.3.- On Pseudo-Riemannian geometry - In the spherically symmetric case (when it is easy to single out the "privileged" space-coordinate r, to be coupled with t),one can resort to the Szekeres-Kruskal coordinates. If we set

$$
\left.\begin{aligned}
u &\equiv \sqrt{|r/2m - 1|}\cdot \exp\left(\frac{r}{4m}\right)\cdot \cosh\left(\frac{t}{4m}\right) \;; \\
v &\equiv \sqrt{|r/2m - 1|}\cdot \exp\left(\frac{r}{4m}\right)\cdot \sinh\left(\frac{t}{4m}\right) \;,
\end{aligned}\right\} \quad (r \gtrless 2m) \qquad (132)
$$

defined for $r \gtrless 2m$, then the Szekeres-Kruskal coordinates are chosen as follows:

$$u_> \equiv u \;; \qquad v_> \equiv v \qquad \text{for} \quad r > 2m \qquad (133a)$$

outside the horizon, and

$$u_< \equiv v \;; \qquad v_< \equiv u \qquad \text{for} \quad r < 2m \qquad (133b)$$

inside the horizon. But again, when crossing the horizon, we avoid having to deal with a change of signature only at the price of passing from coordinates (133a) to (133b), that is to say, of applying to the (everywhere defined) coordinates (132) a transformation of the kind (39') with u=0; i.e. a Superluminal-type (transcendent) transformation of the kind (39") with $U \to \infty$.

We reached the point where it becomes again essential the fact that the space-time of GR is pseudo-Riemannian (Sachs and Wu 1980), and not Riemannian. Namely, if one wishes to make use of the theorems of Riemannian geometry, one has to limit the group of the admissible coordinate-transformations: see Møller (1962) p.234, Camenzind (1970), Halbern and Malin (1969). This was overlooked, e.g., by Kowalczyński (1984).

In a pseudo-Riemannian - or Lorentzian - space-time we may have coordinate transformations even changing the ds^2-sign. Therefore, in order to be able to realize whether we are dealing with a "bradyon" or a "tachyon" we must - given an initial set of coordinates (α, β, γ, δ) and a space-point P - confine ourselves to the general coordinate-transformations which comply with the following requirement. If coordinates (α, β, γ, δ) define at P a local observer O, then a new set of coordinates (α', β', γ', δ') is acceptable only if it defines at the same P a second local observer O', which (locally) moves slower than light w.r.t. O. To use Møller's (1962) words, any "reference frame" in GR can be regarded as a moving fluid; and we must limit ourselves only to the general coordinate-transformations leading to a "frame" (α', β', γ', δ') that can be pictured as a real fluid: This means that the velocities of the "points of reference" - the fluid particles - must always be smaller than c relative to the local inertial observer. This has to hold of course also for the initial "frame" (α, β, γ, δ). For instance, once we introduce everywhere the coordinates (132), we cannot pass (inside the horizon) to coordinates (133b).

In terms of the coordinates (131a), or rather of the coordinates (132), defined everywhere (for $r \gtrless 2m$), a falling body which is a bradyon B in the external region would seemingly be a tachyon T in the internal region (see also Goldoni 1975c). This agrees with the fact that, when adopting suitable coordinates bearing a particularly direct physical meaning, many authors verified that any falling body does reach the light-speed c - in those coordinates - on any Schwarzschild surfaces (see e.g. Zeldovich and Novikov 1971, Markley 1973, Jaffe and Shapiro 1974, Cavalleri and Spinelli 1973,1977,1978, Milewski 1978).

In particular, the coordinates r,t of the distant observer have no direct significance when looking at the speed of a falling body. For instance, De Sabbata et al. (1977), following Saltzman and Saltzman (1969), choose at each space-point P (r, ϑ, φ constant) outside the horizon the local frame $\Sigma(\underset{\sim}{X},T)$ at rest with respect to the horizon and to the Schwarzschild metric ($\partial g^{\mu\nu}/\partial T=0$). Of course frames Σ are not inertial. Then one immediately gets (see e.g. the book by Lightman et al 1975) that the stationary observer Σ measures the velocity $dR/dT = (1-2m/r)^{-1} dr/dt$ so that, independently of the initial velocity, this locally measured speed approaches that of light as r approaches 2m. It should not look strange that a falling body would reach the light-speed for r=2m w.r.t. the local stationary frame Σ_∞, since the local inertial frame would also move with the speed of light w.r.t. Σ_∞. Let us recall within SR that, given a frame s_o, if we are in presence of a body B with speed $v=c-\varepsilon_1 \to c$

and of a second frame s' with speed $u=c-\varepsilon_2 \to c$ where $\varepsilon_2=\varrho\varepsilon_1$ (for simplicity we refer to the case of collinear motions), the speed v' of B w.r.t. s' will be

$$v' = c\,\frac{1-\varrho}{1+\varrho}\;; \qquad \varrho \equiv \frac{\varepsilon_2}{\varepsilon_1}\,, \tag{134}$$

which can yield <u>any</u> real values. If $\varrho=0$, then $v'\to c$; but if $\varrho=1$, then $v'\to 0$. And, when $v \to c$, the energy of the falling body B does not diverge in Σ_∞; actually, the total energy E of a test-particle B is invariant in the local frames Σ. For instance, in the frames Σ where dT is orthogonal to the space-hyperplane, it is $E = m_o \sqrt{g_{oo}}/\sqrt{1-v^2}$.

<u>12.2.4.- A reformulation</u> - Obviously, part of what precedes does not agree with the conventional formulation of GR based on Riemannian geometry, where space-time is supposed to be a smooth, para-compact, simply-connected manifold with metric. Recami and Shah (1979) proposed a new formulation, where ("metric-induced") <u>changes of topology</u> are allowed when passing from a space-time path to another (see also Schmutzer 1968, Ivanenko 1979, Rosen 1970, Wheeler 1968, Göbel 1976). Within such a formulation, they concluded that an "external observer" will deem a falling body to be a bradyon for $r>2m$ and a tachyon for $r<2m$. Conversely, a body which is a tachyon for $r>2m$ will be deemed a bradyon for $r<2m$; but it will of course be able to come out from the BH, transforming again into a tachyon (cf. also Cunningham 1975).

Notice that, a priori, the "external" observer should be able to get information about the BH interior by means of tachyons. It should be repeated once more that tachyonic trajectories are perfectly "at home" in GR.

The motion of a tachyon penetrating the horizon has been studied e.g. in Fuller and Wheeler (1962: see Appendix and Fig.6), Raychaudhuri (1974), Narlikar and Dhurandhar (1976).

<u>12.3.- The apparent superluminal expansions in Astrophysics</u>

The theoretical possibility of Superluminal motions in astrophysics has been considered since long (Gregory 1965,1972, Mignani and Recami 1974d,e, Recami 1974, 1977b,1978a,d,1979a).

Experimental investigations, started long ago as well (Smith and Hoffeit 1963, Knight et al.1971), led at the beginning of the Seventies to the claim that radio-interferometric observations had revealed - at least in the two quasars 3C279,3C273 and in the Seyfert Type I galaxy 3C120 - expansion of small radio components at velocities apparently a few times greater than that of light (Whitney et al.1971, Cohen et al.1971, Shaffer et al.1972, Shapiro et al.1973). The first claim were followed by extensive collections of data, all obtained by very-long-baseline-interferometry (VLBI) system with many radio-telescopes; reviews of the experimental data can be found in Cohen et al.(1977), Kellerman (1980), and Cohen and Unwin (1982); see also Schillizzi and de Bruyn (1983). The result is, grosso modo, that the nucleus of seven strong radiosources (six quasars, 3C273, 3C279, 3C345, 3C179, NRAO-140, BLLac, and one galaxy, 3C120) consists of two components which appear to recede from each other with Superluminal relative speeds ranging from a few c to a few tens c (Pauliny-Toth et al.1981). A result so puzzling that the journal Nature even devoted one of its covers (April 2, 1981) to the Superluminal expansion exhibited by quasar 3C273. Simplifying it, the experimental situation can be summarized as follows:

(i) the Superluminal relative motion of the two components is always a collinear recession;

(ii) such Superluminal "expansion" seems endowed with a roughly constant velocity, which does not depend on the observed wave-length;

(iii) the flux density ratio for the two components, F_1/F_2, does depend on the (observed) wavelength and time.

Apparently, those strong radiosources exhibit a compact inverted-spectrum core component (usually variable), and one extended component which separate from the core with Superluminal velocity. But it is not yet clear whether the compact core is indeed stationary or it too moves. The extended component seem to become weaker with time and more rapidly at high frequencies.

The most recent results, however, seem to show that - at least in quasar 3C345 - the situation may be more complex (Unwin et al.1983, Readhead et al.1983, Biretta et al.1983, Porcas 1983). In the same quasar an "extended component" does even appear to accelerate away with time (Moore et al.1983; see also Pearson et al.1981).

Many theoretical models were soon devised to explain the apparent Superluminal expansions in an orthodox way (Rees 1966, Whitney et al.1971, Cavaliere et al.1971, Dent 1972, Sanders 1974, Epstein and Geller 1977, and so on). Reviews of the orthodox models can be found in Blandfort et al.(1977), Scheuer and Readhead (1979), Marscher and Scott (1980), Orr and Browne (1982), Porcas (1983).

The most successful and therefore most popular models resulted to be:

a) The relativistic jet model: A relativistically moving stream of plasma is supposed to emanate from the core. The compact core of the "superluminal" sources is identifid with the base of the jet and the "moving" component is a shock or plasmon moving down the jet. If the jet points at a small angle α towards the observer, the apparent separation speed becomes Superluminal since the radiation coming from the knot has to travel a shorter distance. Namely, if v is the knot speed w.r.t. the core, the apparent recesion speed will be (c=1): $w = v\sin\alpha/(1-v\cos\alpha)$, with $v \geqslant w/(1+w^2)^{\frac{1}{2}}$. The maximal probability for a relativistic jet to have the orientation required for producing the apparent Superluminal speed $\bar{w}$ - independently of the jet speed v - is $P(\bar{w}) = (1+\bar{w}^2)^{-1} < 1/\bar{w}^2$ (Blandford et al.1977, Finkelstein et al.1983, Castellino 1984). The relativistic jet models, therefore, for the observed "superluminal" speeds suffer from statistical objections, even if selecting effects can play in favour of them (see e.g. Porcas 1981, Science News 1981, Pooley 1981, Pearson et al.1981).

b) The "Screen" models: The "superluminal" emissions are triggered by a relativistic signal coming from a central source and "illuminating" a pre-existing screen. For instance, for a spherical screen of radius R illuminated by a concentric spherical relativistic signal, the distant observer would see a circle expanding with speed $w \simeq 2c(R-ct)/(2Rct-c^2t^2)^{\frac{1}{2}}$; such a speed will be superluminal in the time-interval $0 < t < \frac{1}{2}(2-\sqrt{2})R/c$ only. When the screen is a ring the observer would see an expanding double source. The defect of such models is that the apparent expansion speed will be $w \geqslant \bar{w}$ (with $\bar{w} \gg 2c$) only for a fraction $c^2/\bar{w}^2$ of the time during which the radio-source exibits its variations. Of course one can introduce "oriented" screens - or ad hoc screens -, but they are statistically unfavoured (Bladford et al.1977, Castellino 1984).

c) Other models: many previous (unsucessful) models have been abandoned. The gravitational lens models did never find any observational support, even if a new type of model (where the magnifying lens is just surrounding the source) has been recently suggested by Liaofu and Chongming (1984).

In conclusion, the orthodox models are not too much successful, especially if the more complicated Superluminal expansions (e.g. with acceleration) recently observed will be confirmed.

It may be of some interest, therefore, to explore the possible alternative models in which actual Superluminal motions take place (cf. e.g. Mignani and Recami 1974e).

12.4.- The model with a unique (Superluminal) source

The simplest Superluminal model is the one of a unique Superluminal source. In fact we have seen in Sect.6.15 (see Fig.15) that a unique Superluminal source C will appear as the creation of a pair of sources collinearly receding from each other with relative speed $W > 2c$. This model immediately explains some gross features of the "su perluminal expansions"; e.g., why converging Superluminal motions are never seen, and the high luminosity of the "superluminal" component (possibly due to the optic-boom effect mentioned in Sect.6.15; see also Recami 1977b,1979a), as well as the oscillations in the received overall intensity (perhaps "beats"; cf. Recami 1977b). Since, moreover, the Doppler effect will be different for the two images C_1, C_2 of the same source C (Sect.10.4), a priori the model may even explain why F_1/F_2 does depend on the observed wavelength and on time (see Sect.12.3, point (iii)).

Such a model for the "superluminal expansions" was therefore proposed long ago (Recami 1974,1977b,1978a,d,1979a; Mignani and Recami 1974a, Recami et al. 1976, Grøn 1978, Barut et al.1982). What follows is mainly due to Recami, Maccarrone, Castellino. Many details can be found in the M.S. thesis work by Castellino (1984), where e.g. the case of an extended source C is thoroughly exploited.

12.4.1.- The model - With reference to Fig.15a and Sect.6.15, let us first consider the case of an expanding universe (homogeneous isotropic cosmology). If we call $\overline{C_0 0} \equiv s = db$, with $b \equiv V/\sqrt{V^2-1}$, the observed angular rate of recession of the two images C_1 and C_2 as a function of time will be

$$\dot{\vartheta}(t) \equiv \omega = \frac{2bc}{s} \frac{1+A}{(1+2A)^{\frac{1}{2}}} \quad ; \qquad A \equiv \frac{s}{b^2 ct} , \tag{135}$$

provided that s is the "proper distance" between C_0 and O at the epoch of the radiation reception by O, and t is the time at which O receives those images. Let us repeat that ω is the separation angular velocity of C_1 and C_2, observed by O, in the case of a space-time metric

$$ds^2 = c^2dt^2 - R^2(t)\cdot(dr^2 + r^2d\Omega)$$

where R=R(t) is the (dimensionless) scale-factor. Notice that $\dot{\vartheta}(t) \to \infty$ for $t \to 0$.

If we call t^* and t the emission time and the reception time, respectively, then the observed frequency ν (see Sect.10.4 and eq.(113b)) and the received radiation intensity I will be given of course by (Recami 1974, Recami et al.1976, Castellino 1984):

$$\nu = \nu_0 \frac{\sqrt{V^2-1}}{|1-V\cos|} \cdot \frac{R(t^*)}{R(t)} \quad ; \qquad I = \frac{(V^2-1)W_0}{4\pi s^2(1-V\cos\alpha)^2} \left(\frac{R(t^*)}{R(t)}\right)^2 , \tag{136}$$

where ν_0 is the intrinsic frequency of emission and W_0 is the emission power of the source in its rest-frame. Quantity s is again the source-observer "proper distance" (Weinberg 1972, p.415) at the reception epoch.

Let us pass to the case of a non-pointlike source C. Let for simplicity C be one-dimensional with size ℓ w.r.t. the observer O (Fig.15a), and move with speed V in the direction r of its own length. Let us call x the coordinate of a generic point of r, the value x=0 belonging to H. As in Sect.6.15, be t=0 the instant when the observer O enters in radiocontact with C.

Once the two (extended) images C_1 and C_2 get fully separated (i.e., for $t>\ell/V$), if the intrinsic spectral distribution $\Sigma(\nu_0)$ of the source C is known, one can evaluate the differential intensities $dI_1/d\nu$ and $dI_2/d\nu$ observed for the two images (Recami et al.1976, Castellino 1984). For the moment let us report only that, due to the extension of the moving images, for each emitted frequency ν_0 the <u>average</u> observed frequencies will be

$$<\nu_1> = \frac{2\nu_0}{\sqrt{V^2-1}(1-\alpha_2/\alpha_1)} \frac{R(t^*)}{R(t)} \; ; \qquad <\nu_2> = \frac{\alpha_2}{\alpha_1}<\nu_1> \; , \tag{137}$$

quantities α_1, α_2 being the observed angular sizes of the two images, with $\alpha_1 > \alpha_2$. Moreover $\ell/d = \frac{1}{2}V^2(\alpha_1-\alpha_2)$.

<u>12.4.2.- Corrections due to the curvature</u> - Let us consider the corrections due to the curvature of the universe, which can be important if the observed expansions are located very far. Let us consider, therefore, a curved expanding cosmos (closed Friedmann model), where the length element $d\ell$ is given by $d\ell^2=dr^2(1-r^2/a^2)^{-1}+r^2d\Omega$, quantity a=a(t) being the curvature radius of the cosmos. Again, some details can be found in Recami et al.(1976) and Castellino (1984). For instance, the apparent angular velocity of separation $\dot{\vartheta}(t)$ between the two observed images C_1 and C_2 (cf. eq.(135)) becomes ($h\equiv r/a$):

$$\dot{\vartheta}(t) \equiv \omega \simeq b \, \frac{1+bh}{b+h} \left(\frac{2c}{rt}\right)^{1/2} (1-h^2)^{1/4} \tag{138}$$

quantities r and a being the "radial coordinate" of C_0 and the universe radius, respectively, at the present epoch [r=a sin(s/a), where s is the "proper distance" of C_0; moreover $a=c/(H\sqrt{2q-1})$; H = Hubble constant; q ≡ deceleration parameter; and $ct \ll r$]. Further evaluations in the abovequoted literature.

<u>12.4.3.- Comments</u> - The eq.(135) yields apparent angular velocities of separation two or three orders of magnitude larger than the experimental ones. It is then necessary to make recourse to eq.(138), which includes the corrections due to the universe curvature; actually, eq.(138) can yield arbitrarily small values of $\dot{\vartheta}(t)$ provided that $h \to 1$, i.e. $r \to a$. To fit the observation data, however, one has to attribute to the "superluminal expansions" values of the radial coordinate r very close to a.

Such huge distances would explain why the possible blue shifts - often expected from the local motion of the Superluminal source C (cf. end of Sect.10.4) - appear masked by the cosmological red-shift (Notice, incidentally, that a phenomenon as the one here depicted can catch the observer's attention only when the angular separation ϑ between C_1 and C_2 is small, i.e. when C_1 and C_2 are still close to C_o). But those same large distances make also this model improbable as an explanation of the observed "superluminal" expansions, at least in the closed models. One could well resort,then, to open Friedmann models. In fact, the present model with a unique (Superluminal) source is appealing since it easily explains: (a) the appearance of two images with Superluminal relative speed ($W>2c$); (b) the fact that only Superluminal expansions (and not approaches) are observed; (c) the fact that W is always Superluminal and practically does not depend on ν; (d) the relative-motion collinearity; (e) the fact that the flux-densities ratio does depend on ν and t, since the observed flux differential intensities for the two images as a function of time are given by the formulae (Castellino 1984):

$$\frac{dI_i}{d\nu} = \frac{V^2-1}{4\pi d^2 VL} \int_{\nu/M_i(t)}^{\nu/m_i(t)} \frac{\Sigma(\nu_o)d\nu_o}{\nu_o F} , \qquad (i=1,2)$$

$$F \equiv \left[V^{-2} (\sqrt{V^2-1}\, \frac{R(t)}{R(t^*)} \frac{\nu}{\nu_o} + 1)^2 - 1\right]^{1/2} ; \qquad (139)$$

the integration extrema being

$$m^1_2(t) \equiv K \left[VG\, [VTG']^{-1/2} \pm 1\right] \qquad (140a)$$

$$M^1_2(t) \equiv K \left[\frac{V(G-L)}{[VT(G'-2L) + L(L-2\sqrt{V^2-1})]^{1/2}} \pm 1\right] \qquad (140b)$$

where d is the "proper distance" $\overline{OH}$ at the reception epoch (Fig.15a); $L \equiv \ell/d$; $T \equiv ct/d$; $K \equiv \sqrt{V^2-1}\, R(t^*)/(R(t)$; $G \equiv \sqrt{V^2-1} + VT$; and $G' \equiv 2G-VT$. All eqs. (139)-(140) become dimensionally correct provided that V/c is substituted for V.

But the present model remains disfavoured since: (i) the Superluminal expansion seems to regard not the whole quasar or galaxy, but only a "nucleus" of it; (ii) at least in one case (3C273) an object was visible there, even before the expansion started; (iii) it is incompatible with the acceleration seemengly observed at least in another case (3C345).

Nevertheless, we exploited somewhat this question since: (A) in general, the above discussion tells us how it would appear a unique Superluminal cosmic source;

(B) it might still regard part of the present-type phenomenology; (C) and, chiefly, it must be taken into account even for each one of the Superluminal, far objects considered in the following models.

12.5.- The models with more than one radio sources

We recalled in Sect.12.2 that black-holes can classically emit (only) tachyonic matter, so that they are expected to be suitable classical sources - and detectors - of tachyons (Pavšič and Recami 1977, De Sabbata et al.1977, Narlikar and Dhurandhar 1978, Recami 1979a, Recami and Shah 1979, Barut et al.1982). Notice that, vice-versa, a tachyon entering the horizon of a black-hole can of course come out again from the horizon. As we already said, the motion of a space-like object penetrating the horizon has been already investigated, within GR, in the existing literature (see the end of Sect.12.2.4).

We also saw in Sect.5.18 (Fig.14) and in Sect.10.2 that, in a "subluminal" frame, two tachyons may seem - as all the precedent authors claimed - to repel each other from the kinematical point of view, due to the novel features of tachyon mechanics (Sect.10.1: eqs.(109b,c)). In reality, they will gravitationally attract each other, from the energetical and dynamical points of view (Sect.10.2; see also Fig.4a).

From Sect.10.2 a tachyon is expected to behave the same way also in the gravitational field of a bradyonic source. If a central source B (e.g., a black-hole) emits e.g. a Superluminal body T, the object T under the effect of gravity will loose energy and therefore accelerate away (Sect.5.3). If the total energy $E = m_o c^2/\sqrt{V^2-1}$ of T is larger than the gravitational binding energy $\overline{E}$, it will escape to infinity with infinite (asymptotically constant) speed.(Since at infinite speed a tachyon possesses zero total energy - see Fig.5c, and Sect.6.14 -, we may regard its total energy as all kinetic). If on the contrary $E<\overline{E}$, then T will reach infinite speed (i.e. the zero total-energy state) at a finite distance; afterwards the gravitational field will not be able to subtract any more energy to T, and T will start going back towards the source B, appearing now - actually - as an antitachyon $\overline{T}$ (Sects.5.12 and 11.2). It should be remembered (Sect.11.5 and eq.(128)) that at infinite speed the motion direction is undefined, in the sense that the transcendent tachyon can be described either as a tachyon T going back or as an antitachyon $\overline{T}$ going forth, or vice-versa.

We shall see, on another occasion (Sect.13.2), that a tachyon subjected e.g. to a central attractive elastic force F=-kx can move periodically back and forth with a motion analogous to the harmonic one, reversing its direction at the points where it has transcendent speed, and alternatively appearing - every half an oscillation -

now as a tachyon and now as an antitachyon. Let us consider, in general, a tachyon T moving in space-time (Fig. 32) along the space-like curved path AP, so to reach at P the zero-energy state. According to the nature of the force fields acting on T, after P it can proceed along PB (just as expected in the above two cases, with attractive central forces), or along PC, or along PD. In the last case, T would appear to annihilate at P with an antitachyon emitted by D and travelling along the curved world-line DP (Sects.5.12 and 11.2; see also Davies 1975, p.577).

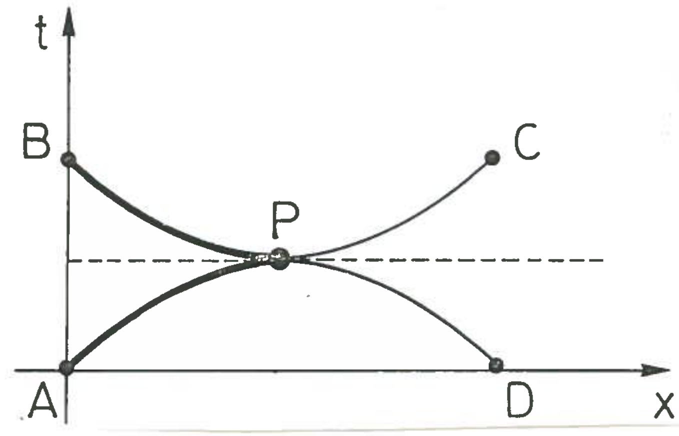


FIG. 32

It is clear that the observed "superluminal" expansions can be explained: (i) either by the splitting of a central body into two (oppositely moving) collinear tachyons T_1 and T_2; or by the emission from a central source B of: (ii) a tachyon T, or (iii) of a couple of tachyons T_1 and T_2 (in the latter case, T_1 and T_2 can for simplicity's sake be considered as emitted in opposite directions with the same speed). On this respect, it is interesting that Ne'eman (1974) regarded quasars - or at least their dense cores - as possible white holes, i.e. as possible "lagging cores" of the original expansion.

For simplicity, let us confine ourselves to a flat stationary universe.

12.5.1.- The case (ii) - In the case (ii), be O the observer and α the angle between BO and the motion-direction of T. Neglecting for the moment the gravitational interactions, the observed apparent relative speed between T and B will of course be (see Fig. 33):

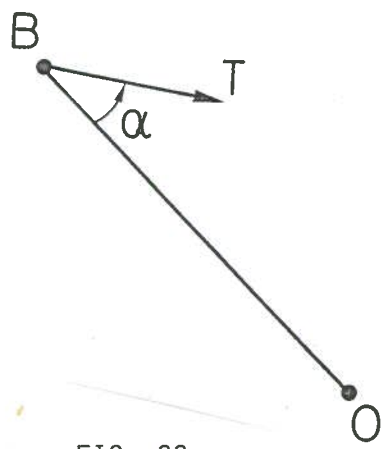


FIG. 33

$$W = \frac{V \sin\alpha}{1 - V\cos\alpha} \quad . \qquad (V > 1) \qquad (141)$$

Let us assume $V > 0$; then, $W > 0$ will mean recession of T from B, but $W < 0$ will mean approach. Owing to the cylindrical symmetry of our problem w.r.t. BO, let us confine ourselves to values $0 < \alpha < 180°$. Let us mention once more that $W \rightarrow \infty$ when $\cos\alpha \rightarrow 1/V$ ("optic-boom" situation). If the emission angle α of T from B w.r.t. BO has the value $\alpha = \alpha_b$, with $\cos\alpha_b = 1/V$ $(0 < \alpha_b < 90°$; b ≡ "boom"), tachyon T appears in the optic-boom phase; but the recession speed of T from B would be too high in this case, as we saw in the previous Section.

Incidentally, to apply the results got in Sect.12.4 to the Superluminal object T (or T_1 and T_2 in the other cases (i),(iii)), one has to take account of the fact that the present tachyons are born at a finite time, i.e. do not exist before their emission from B. It is then immediate to deduce that we shall observe: (a) for $\alpha > \alpha_b$, i.e. for $\alpha_b < \alpha < 180°$, the object T to recede from B; but (b) for $0 < \alpha < \alpha_b$, the object T to approach B. More precisely, we shall see T receding from B with speed $W > 2$ when

$$\frac{V-\sqrt{5V^2-4}}{2(V+1)} < \mathrm{tg}\,\frac{\alpha}{2} < \frac{V+\sqrt{5V^2-4}}{2(V+1)} \quad ; \qquad \arccos\frac{1}{V} < \alpha < 180° \ . \tag{142}$$

It should be noticed that eq.(141) can yield values $W > 2$ whenever $V > 2/\sqrt{5}$: in particular, therefore, for all possible values $V > 1$ of V. Due to eqs.(142), the "emission--direction" α of T must be however contained inside a certain suitable solid angle: $\alpha_1 < \alpha < \alpha_2$; such a solid angle always including, of course, the optic-boom direction α_b. For instance, for $V \to 1$ we get $0 \leqslant \mathrm{tg}\frac{\alpha}{2} < 1/2$; $\alpha \geqslant \alpha_b = 0$, wherefrom:

$$0 \leqslant \alpha < 53.13° \ ; \qquad\qquad (V \to 1) \tag{143}$$

in such a case, we shall never observe T approaching B. On the contrary, for $V \to \infty$ we get $\frac{1}{2}(1-\sqrt{5}) < \mathrm{tg}\,\frac{\alpha}{2} < \frac{1}{2}(1+\sqrt{5})$; $\alpha_b = 90° \leqslant \alpha < 180°$; wherefrom $-63.44° < \alpha < 116.57°$; $\alpha \geqslant 90°$, that is to say: $90° \leqslant \alpha < 116.57°$. If we add the requirement, e.g., $W < 50$, in order that $2 < W < 50$, we have to exclude in eq.(143) - for $V \to 1$ - only the tiny angle $0 < \alpha < 2.29°$, so that in conclusion

$$2.29° < \alpha < 53.13° \ . \qquad\qquad (V \to 1)$$

The same requirement $2 < W < 50$ will not affect - on the contrary - the above result $90° \leqslant \alpha < 116.57°$ for the case $V \to \infty$.

Similar calculations were performed by Finkelstein et al.(1983).

The present case (ii) suffers some difficulties. First, for $\alpha > \alpha_2$ (for instance, for $53° \leqslant \alpha < 180°$ in the case $V \to 1$) we should observe recession-speeds with $1 < W < 2$, which is not supported by the data; but this can be understood in terms of the Doppler-shift selective effects (see Sect.10.4; and Blandford et al. 1977). Second, for $\alpha < \alpha_b$ one should observe also Superluminal approaches; only for $V \simeq 1$ ($V \gtrsim 1$) it is $\alpha_b \simeq 0$ and therefore such Superluminal approaches are not predicted.

In conclusion, this model (ii) appears acceptable only if the emission mechanism of T from B is such that T has very large kinetic energy, i.e. speed $V \gtrsim 1$.

12.5.2.- The cases (i) and (iii) - Let us pass now to analyse the cases (i) and (iii), still assuming for simplicity T_1 and T_2 to be emitted with the same speed V in opposite directions. Be α again in the range (0,180°). In these cases, one would observe faster-than-light recessions for $\alpha > \alpha_b$. When $\alpha < \alpha_b$, on the contrary, we would observe a unique tachyon $T \equiv T_1$ reaching the position B, bypassing it, and continuing its motion (as $T \equiv T_2$) beyond B with the same velocity but with a new, different Doppler-shift.

One can perform calculations analogous to the ones in Sect.12.5.1; see also Finkelstein et al.(1983).

In case (i), in conclusion, we would never observe Superluminal approaches. For $\alpha < \alpha_b$ we would always see only one body at a time (even if $T \equiv T_2$ might result as a feeble radiosource, owing to the red-shift effect): the motion of T would produce a variability in the quasar. For $\alpha > \alpha_b$, as already mentioned, we would see a Superluminal expansion; again, let us recall that the cases with $1 < W < 2$ (expected for large angles α only) could be hidden by the Doppler effect.

Case (iii) is not very different from the case (ii). It becomes "statistically" acceptable only if, for some astrophysical reasons, the emitted tachyonic bodies T_1 and T_2 carry very high kinetic energy ($V \gtrsim 1$).

12.6.- Are "superluminal" expansions Superluminal ?

If the emitted tachyonic bodies T (or T_1 and T_2) carry away a lot of kinetic energy ($V \gtrsim 1$), all the models (i), (ii), (iii) may be acceptable from the probabilistic point of view.

Contrariwise, only the model (i) - and the model (iii), if B becomes a weak radiosource after the emisssion of T_1, T_2 - remain statistically probable, provided that one considers that the Doppler effect can hide the objects emitted at large angles (say, e.g., between 60° and 180°). On this point, therefore, we do not agree with the conclusions in Finkelstein et al.(1983).

In conclusion, the models implying real Superluminal motions investigated in Sect.12.5 seem to be the most probable for explaining the apparent "superluminal expansions"; especially when taking account of the gravitational interactions between B and T, or T_1 and T_2 (or among T_1, T_2, B).

Actually, if we take the gravitational attraction between B and T (Sect.102) into account - for simplicity, let us confine ourselves to the case (ii) - we can easily explain the accelerations, probably observed at least for 3C345 and maybe for 3C273 (Shenglin and Yongzhen 1983).

Some calculations in this direction have been recently performed by Shenglin et al.(1984) and Cao (1984). But those authors did not compare correctly their evaluations with the data, since they overlooked that - because of the finite value of the light-speed - the images' apparent velocities do not coincide with the sources' real velocities. The values W_0 calculated by those authors, therefore, have to be corrected by passing to the values $W = W_0 \sin\alpha/(1-\cos\alpha)$; only the values of W are to be compared with the observation data.

All the calculations, moreover, ought to be corrected for the universe expansion. However, let us recall (Sect.12.4) that in the homogeneous isotropic cosmologies - "conformal" expansions -, the angular expansion rates are not expected to be modified by the expansion, at least in the ordinary observational conditions. While the corrections due to the universe curvature would be appreciable only for very distant objects.

PART IV: TACHYONS IN QUANTUM MECHANICS AND ELEMENTARY PARTICLE PHYSICS

13.- THE POSSIBLE ROLE OF TACHYONS IN ELEMENTARY PARTICLE PHYSICS AND QUANTUM MECHANICS

In this review we purposed (Sect.1.1) to confine ourselves to the classical theory of Tachyons, leaving aside their possible quantum field theories (cf. e.g. Broido and Taylor 1968). We have already met, however, many instances of the possible rôle of tachyons in elementary particle physics. And we want to develop some more such an aspect of tachyons in the present Section.

In Sect.1.1. we mentioned moreover the dream of reproducing the quantum behaviour at a classical level, i.e. within a classical physics including tachyons (and suitable extended-type models of elementary particles). In the present Section we shall put forth also some hints pointing in such a direction.

Let us finally mention that we noticed (in Sect.8.2) tachyons themselves to be more similar to fields than to particles.

13.1.- Recalls

We have already seen that ER allows a clearer understanding of high energy physics: in Sect.11.4 we derived from it, e.g., the so-called Crossing Relations.

Actually, the predicting power of the pure SR (even without tachyons) with regard to elementary particle physics is larger than usually recognized. Once one deve lops SR as we did in Part I, one succeeds in expalaining - within SR alone - not only the existence of antiparticles (Sect.2 and Sect.5.14), but also of the CPT symmetry (Sects.5.16 and 11.1), as well as of a relation between charge conjugation and internal space-time reflection (Sect.11.3). For the interpretation of advanved solutions, see Sect.9.7.

As to tachyons and elementary particle physics, we recall the results in Sects 6.2, 6.3 and particularly 6.13, where we mentioned the possible rôle of tachyons as "internal lines" in subnuclear interactions. For the connections between tachyons and Wheeler-Feynman type theories, see Sect.9.6.2. In Sect.11.5, at last, we discussed the relevance of tachyons for a classical description of the vacuum decay and fluctuation properties.

13.2.- "Virtual Particles" and Tachyons. The Yukawa potential

We already saw in Sect.6.13 that tachyons can be substituted for the so-called "virtual partilces" in subnuclear interactions; i.e. that tachyons can be the "realistic" classical carriers of elastic and inelastic interactions between elementary particles (Sudarshan 1968, Recami 1968, Clavelli et al.1973; see also all the Refs.(8) and (9) in Maccarrone and Recami 1980b).

Actually, it is known that the "virtual particles" exchanged between two elementary particles (and therefore realizing the interaction) must carry a negative fourmomentum square, for simple kinematical reasons (Review I):

$$t \equiv p_\mu \, p^\mu \equiv E^2 - \underset{\sim}{p}^2 < 0 \,, \tag{144}$$

just as it happens for tachyons (cf. e.g. Sect.6.1, eq.(29c)).

Long ago it was checked (Recami 1969 , Olkhovsky and Recami 1969) whether virtual objects could really be regarded as faster than light, at least within the so-called peripheral models "with absorption" (see e.g. Dar 1964). To evaluate the effect of the absorptive channels in the one-particle-exchange-models, one has to cut out the low partial waves from the Born amplitude. Namely, an impact-parameter (Fourier-Bessel) expansion of the Born amplitudes is used, and a cut-off is imposed at a minimal radius R which is varied to fit the experimental data. While considering - for example - different cases of $p\bar{p}$ interactions via K-meson exchange, values of R were found ranging from 0.9 to 1.1 fm, i.e. much larger than the K-meson Compton wavelength. The same kind of model (at a few GeV/c; with form factors) was also applied to pion-nucleon reactions via ϱ-meson exchange; and also for the ϱ a value (R=0.8 fm) much greater than the ϱ-meson Compton wavelength was found. Even if such rough tests are meaningful only within those models, one deduced the virtual K and ϱ mesons of the nucleon cloud to travel faster than light: for instance, in the first case, for $t=-m_K^2$, one finds $\langle v \rangle > 1.75\ c$.

According to Wigner (1976), 'there is no reason to believe that interaction cannot be transmitted faster than light travels'. This possibility was considered in detail by Van Dam and Wigner (1965, 1966) already in the Sixties. See also Agudin (1971), Costa de Beauregard (1972), Mathews and Seetharaman (1973), Flato and Guenin (1977) and Shirokov (1981).

And any action-at-a-distance theory (see e.g. Sudarshan 1970d, Volkov 1971, Leiter 1971b, Hoyle and Narlikar 1974) implies the existence of space-like objects, since the infinite speed is not invariant (Sect.4.1).

Moreover, *if* hadrons can really be considered as "strong black-holes" (Sect. 12.2.1.), than strong interactions can classically be mediated anly by a tachyon-exchange; i.e., the strong field "quanta" should be Superluminal.

In any case, we *can* describe at a classical level the virtual cloud of the hadrons as made of tachyons (see also Sudarshan 1970b), provided that such tachyons, once emitted, are - "strongly" - attracted by the source hadron, in analogy with what we discussed for the ordinary gravitational case (Sect.12.5). For the description in terms of a "strong gravity" field, see e.g. Salam (1978), Sivaram and Sinha (1979), Recami (1982a,b) and refs. therein, and Ammiraju et al.(1983). In fact, if the attraction is strong enough, the emitted tachyons will soon reach the zero-energy (infinite-speed) state; and afterwards (cf. Fig. 32) they will go back as antitachyons, till reabsorbed by the source hadron. Notice that transcendent tachyons can only *take* energy from the field. Notice, moreover, that classical tachyons subjected to an attractive central field can move back and forth in a kind of tachyonic harmonic motion (see Fig.34), where the inversion points just correspond to the infinite speed (cf. Sect. 12.5; see also Aharonov et al.1969).

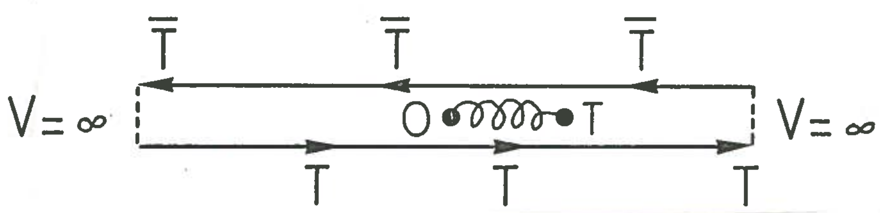


FIG. 34

Finally, let us consider a hadron emitting and reassorbing (classical) tachyons. It will be surrounded by a cloud of outgoing and incoming tachyons. In the "continuos" approximation (and spherically symmetric case), that cloud can be described by the spherical waves:

$$I \propto \left| \frac{\exp(\pm i m r)}{r} \right|^2 . \qquad (145)$$

We are of course confining ourselves to subluminal frames only. We can find out, however, the results forwarded by ER formally by putting for tachyons $m = \pm i\mu$ (μ real). It is noticeable that from eq.(145) we get, then, the Yukawa potential by setting $m = +i\mu$ for the outgoing and $m = -i\mu$ for the incoming waves:

$$I \propto \left| \frac{\exp(-\mu r)}{r} \right|^2 ; \qquad (145')$$

in other words, at the static limit, the Yukawa potential can be regarded as the "continuous" (classical) description of a flux of outgoing tachyons and incoming antitachyons: see Castorina and Recami (1978). See also Hadjioannou (1966), Ferretti and Verde (1966), Yamamoto (1976), Eriksen and Vøyenli (1976), Flato and Guenin (1977), and Federighi (1983).

When two hadrons come close to each other, one of the cloud tachyons - instead of being reabsorbed by the mother hadron - can be absorbed by the second hadron; or vice versa (this statement is frame dependent). That would be the simplest hadron-hadron interaction. The actual presence of a tachyon exchange would produce a resonance peak in the scattering amplitude as a function of the momentum transfer $t \equiv (p_1 - p_2)^2$ (Sudarshan 1969a,b,1970c). Precisely, it would produce a "negative t enhancement", fixed when $s \equiv (p_1 + p_2)^2$ varies, and possibly to be found also in other similar processes (Dhar and Sudarshan 1968, Glück 1969, Baldo et al.1970); unless the tachyons appear to possess a very large width (Bugrij et al.1972; see also Królikowski 1969). A positive theoretical evidence was put forth by Gleeson et al.(1972a). See also Van der Spuy (1973), Jue (1973), Akiba (1976), Enatsu et al.(1978), Review I p. 266, and Baldo et al.(1970).

Before closing this Section, let us recall that long ago (Recami 1968, 1969a) it was suggested that the unstable particles ("Resonances"), bearing masses $M^* = M + i\Gamma$ formally complex, might be compounds of bradyons and tachyons. We shall come back to this point in Sect.13.5 (See also, e.g., Sudarshan 1970d, Edmonds 1974, Keszthelhyi and Nagy 1974).

More in general, for the possible connections between Superluminal motions and the "quantum potential" (Bohm and Vigier 1954,1958), see for instance Vigier (1979, 1980). see also Stapp (1977) and d'Espagnat (1981).

13.3.- Preliminary applications

If subnuclear interactions are considered as mediated by quanta, no ordinary (bradyonic) particles can be the carriers of the transferred energy-momentum. We have seen, on the contrary, that classical tachyons can a priori act as the carriers of those interactions.

As preliminary examples or applications, let us consider the vertex $\Delta_{33} \rightarrow p + \pi_T$ of a suitable one-particle-exchange diagram, and suppose the exchanged particle (internal line) π_T to be a tachyonic pion, instead of a "virtual" pion. Then, from Sects.6.3 and 6.8 we should get: $(1232)^2-(938)^2=(140)^2+2\times 1232\times\sqrt{c^2|\underset{\sim}{p}|^2-(140)^2}$, and therefore (Maccarrone and Recami 1980b):

$$|\underset{\sim}{p}|_{\pi_T} = 287 \text{ MeV/c} \; ; \qquad E_{\pi_T} = 251 \text{ MeV} \; , \tag{146}$$

so that, in the c.m. of the $\Delta_{33}(1232)$, the total energy of the tachyon pion is predicted to be centered around 251 MeV.

Again, let us consider the decay $\pi \rightarrow \mu + \nu_T$ under the hypothesis, now, that ν_T be a tachyon neutrino, with $m_\nu \gtrsim 0$; $v \gtrsim c$. It has been shown by Cawley (1972) that such an hypothesis is not inconsistent with the experimental data, and implied for the muon-neutrino a mass $m_{\nu_T} \lesssim 1.7$ MeV. In the two limiting cases, from Sects.6.3 and 6.8 in the c.m. of the pion we should get (Maccarrone and Recami 1980b):

$$m_\nu = 0 \quad \Rightarrow \quad |\underset{\sim}{p}|_\nu = 29.79 \text{ MeV/c} \; ; \qquad v_\nu = c \; ; \tag{147a}$$

$$m_\nu = 1.7 \quad \Rightarrow \quad |\underset{\sim}{p}|_\nu = 29.83 \text{ MeV/c} \; ; \qquad v_\nu = 1.0016 \, c \; . \tag{147b}$$

Let us recall once more from Sect.6.13 that <u>for instance</u> any elastic scattering can be "realistically" mediated by a suitable tachyon-exchange during the approaching phase of the two bodies. In the c.m.f. ($|\underset{\sim}{P}_A| = |\underset{\sim}{P}_B| \equiv |\underset{\sim}{P}|$) we would obtain eq.(82):

$$\cos \vartheta_{\text{c.m.}} = 1 - \frac{m_o^2}{2|\underset{\sim}{p}|^2} \; ; \tag{82}$$

so that, for each discrete value of the tachyon rest-mass m_o (Sect.5.1) the quantity ϑ too assumes a <u>discrete</u> value, which is merely a function of $|\underset{\sim}{P}|$. We have always neglected, however, the mass-width of the tachyons.

For further considerations about tachyons and virtual fields see e.g. Van der Spuy (1978), and Souček et al.(1981).

Tachyons can also be the exchanged particles capable of solving the classical-physics paradoxes connected with pair creation in a constant electric field (Zeldovich 1974a p.342, and 1972).

For joint probability distributions in phase-space for tachyons see e.g. ger (1978 and refs. therein), where the ordinary formalism was generalized to the relativistic case and shown to yield a unified description of bradyons and tachyons.

13.4.- Classical vacuum-unstabilities

We saw in Sect.11.5 that the vacuum can become unstable, at the classical level, by emitting couples of zero-energy (infinite speed) tachyons T and $\overline{T}$. For a discussion of this point (and of the possible connection between the cosmic tachyon-flux and the tachyon emittance of ordinary matter) we refer the reader to Sect.11.5 (and Fig.31). See also Mignani and Recami (1976a), as well as Fig.32 in our Sect.12.5.

Here let us observe that the probability of such a decay of a vacuum "bubble" into two collinear transcendent tachyons (T and $\overline{T}$) is expressible, according to Fermi's golden rule (Fermi 1951), as $\tau^{-1} \propto m_o c/(8\pi h)$, where m_o is the tachyon rest-mass (both tachyons T and $\overline{T}$ must have the same rest-mass, due to the impulse conservation; remember that for transcendent tachyons $|\underset{\sim}{p}| = m_o c$); but we are unable to evaluate the proportionality constant.

More interesting appears considering, in two dimensions (Sect.5), an ordinary particle $P \equiv P_B$ harmonically oscillating in a frame f' around the space-origin O'. If the frame f' moves Superluminally w.r.t. another frame $f_o(t,x)$, in the second frame the world-line of point O' is a space-like straight-line $\mathscr{S}$, and the world-line of the harmonic oscillator $P = P_T$ (now a tachyon with variable velocity) is depicted in Fig. 35. Due to what we saw in Sects.5.12-5.14, - as well as in Sect.11, Sect.12.5 and

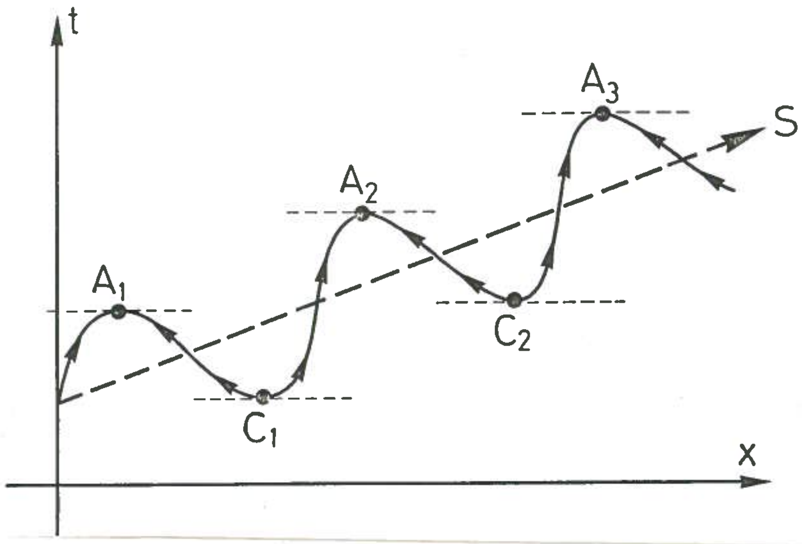


FIG. 35

Sect.13.2 -, the "subluminal" observer f_o will see a vacuum fluctuation propagating in space, with vacuum decays (pair-creations of transcendent tachyons) in correspondence with the events $C_1, C_2, C_3, \ldots$, and with analogous pair-annihilations (of transcendent tachyons) in correspondence with the events $A_1, A_2, A_3, \ldots$ (Fig.35). Cf.also

Wimmel (1971b), and Catara et al.(1982). Notice that each vacuum unstability C is just a vacuum decay into a tachyon T and an antitachyon $\overline{T}$, having the same rest-mass and oppositely moving with infinite speed; such a process is perfectly allowed by classical mechanics (see e.g. Sect.11.5). Analogously, each event A is nothing but the annihilation (into a "vacuum bubble") of a transcendent T-$\overline{T}$ pair.

This is another example of classical description of a typically "quantal" phenomenon, i.e. of a phenomenon usually regarded as belonging to the realm of quantum field theory (QFT). See also, e.g. Nambu (1950), Mannheim (1977), Fukuda (1977), Shay and Miller (1978), and Souček (1981).

Let us remark, at this point, that in ordinary theories the possible presence of tachyons is not taken into explicit account. It follows that the ordinary vacuum is not relativistically invariant, if tachyons on the contrary exist (and, let us repeat, if account of them is not explicitly taken): cf. e.g. Sect.5.17 and Fig.13. The fact that in the usual theories the ordinary concept of empty space may not be Lorentz invariant was particularly stressed by Nielsen (1979); who noticed that, if some large region in space is empty of tachyons as observed from one frame, there is no guarantie that it will be so seen from another frame of reference. Nielsen et al.(see e.g. Nielsen and Ninomiya 1978, Nielsen 1977) also developed non-invariant theories, even if independently of the above observations.

13.5.- A Lorentz-invariant Bootstrap

The idea that tachyons may have a rôle in elementary particle structure has been taken over by many authors (see e.g. Recami 1968,1969a, Hamamoto 1974, Akiba 1976, Rafanelli 1976,1978, Van der Spuy 1978, Castorina and Recami 1978, Szamosi and Trevisan 1978; see also Rosen and Szamosi 1980, and the Refs.(8) and (9) in Maccarrone and Recami 1980b).

One of the most interesting results is probably the one by Corben, who succeeded in building up a Lorentz-invariant "bootstrap" of hadrons or of hadronic "Resonances" (Corben 1977a,b, 1978a,b). Let us describe his approach by following initially Castorina and Recami (1978).

Corben started from the known fact that a free bradyon with rest-mass M and a free tachyon T with rest-mass m can trap each other in a relativistically invariant way; if $M > m$ the compound particle is always a bradyon B^*. If the two particles have infinite relative speed, and P,p are their fourmomenta, then (Sect.11.5):

$$P_\mu p^\mu = 0 \Leftrightarrow P \perp p . \qquad (148a)$$

In such a case the mass M^* of the compound bradyon B^* is (Sect.6.3 and 6.5):

$$M^* = \sqrt{M^2 - m^2} \quad , \tag{148b}$$

as easily follows from eqs.(58)-(59), or from eqs.(64)-(65).

Let us now assume that, inside the "composite hadron", the tachyon T feels a strong field similar to the gravitationsl one (see e.g. Recami 1982a and refs. therein); let us assume moreover that the trapped tachyon has already reached an equilibrium state and is revolving along a circumference around the bradyon B (see also Stephas 1983). From Sects.6.14 and 10.1, we then derive that any bradyon-tachyon compound - in its lowest energy state ("ground state") - is expected to be constituted by a tachyon T having divergent speed w.r.t. the bradyon B, so that condition (148a) is satisfied. T reaches in fact its minimal potential energy when its speed diverges; i.e., the fundamental state of the system corresponds to a transcendent periodic motion of T. One also derives that the trapping force, which holds T on a circular orbit, tends to zero when T tends to have infinite speed. In such a case the interaction is negligible, even if the "self-trapping" keeps itself. Under condition (148a), therefore, one may consider the B-T compound as a couple of two free particles!

Actually Corben (1978a), by using the quantum language, considered two particles satisfying the equations $(M > m)$:

$$\Box \psi_B = K^2 \psi_B \quad ; \qquad (K \equiv Mc/\hbar) \tag{149a}$$

$$\Box \psi_T = - k^2 \psi_T \quad ; \qquad (k \equiv mc/\hbar) \tag{149b}$$

and such that, if $\psi \equiv \psi_B \psi_T$,

$$\Box \psi = (K^2 - k^2)\psi \quad ; \qquad (\sqrt{K^2 - k^2} \equiv M^* c/\hbar) \; . \tag{149c}$$

Eq.(149c) comes from postulating the invariant interaction $\partial_\mu \psi_B \partial_\mu \psi_T = 0$, which is nothing but the quantum-field version of condition (148a); in fact, applied to the eigenstates of energy and momentum, it just implies eq.(148a). (Cf. also eqs. (149c) and (148b)). Plane time-like and space-like waves can therefore "lock" to form a plane wave, that is time-like when $M > m$. Notice that everything still holds when we substitute $D_\mu^{(\alpha)} \equiv \partial_\mu - \frac{ie_\alpha}{\hbar c} A_\mu$ for ∂_μ.

It would not be possible to combine two time-like states in this way, because applying the condition $\partial_\mu \psi_B \partial_\mu \psi_T = 0$ (or $D_\mu^{(B)} \psi_B D_\mu^{(T)} \psi_T = 0$) to such states leads to imaginary momenta and exponentially increasing (not normalizable) wave-functions. This corresponds, of course, to the classical fact that condition (148a) cannot be satisfied by two bradyons.

On the contrary, a bradyon B can combine in a relativistically invariant way with more than one tachyon to yield another bradyon B^*. Actually, due to conditions of the type of eq.(148a), it can trap no more than three tachyons, getting eventually the mass

$$M^* = \sqrt{M^2 - m_1^2 - m_2^2 - m_3^2} \tag{148c}$$

provided that it is real. In such a situation, the three transcendent tachyons T_1, T_2, T_3 can be imagined as moving circularly around the axes x,y,z, respectively (the circle centers always coinciding with B). Going back to the quantum-field language (Corben 1977a,1978b), the extra conditions $\partial_\mu \psi_i \, \partial_\mu \psi_j = 0$ $(i,j=1,2,3;\ i \neq j)$ require the functions $\partial_\mu \psi_i$ to be orthogonal to each other in space. More generally, setting $M=m_0$, the conditions $\partial_\mu \psi_\alpha \, \partial_\mu \psi_\beta = 0$ $(\alpha, \beta = 0,1,2,3;\ \alpha \neq \beta)$ imply that no more than three space-like states can be superimposed on one time-like state to yield another particle (Cf. also Preparata 1976, Hoh 1976, Pagels 1976).

In QFT a bradyon at rest is described, as usual, by a wave-function periodic in time and independent of position. A transcendent tachyon, on the contrary, corresponds to a wave-function static in time and periodic in space: a lattice (cf. also Sect.8). Incidentally, the interaction between a bradyon and a transcendent tachyon is therefore analogous to the scattering of a wave by a diffracting grating (Corben 1978a). The three values of the lattice spacings in the three directions of space may be regarded as corresponding to the masses of the three space-like states that can combine in the above way with one time-like state (Corben 1978b).

By resorting to eqs.(148b,c) and to suitable quantum number considerations, Corben (1977a,b, 1978a,b) found masses and quantum numbers of a host of hadrons as composed of other (sub- and Super-luminal) hadrons, thus realizing a relativistically invariant bootstrap (Chew 1968). There are a number of examples which appear to verify this, especially in the spectrum of the K particles and the s=0 mesons that preferentially decay into $K\bar{K}$: we refer the reader to the interesting tables published by Corben in his abovementioned papers, which also contain further details and comments. Corben found also the mass-differences among the members of various isospin multiplets by binding Superluminal leptons to suitable subluminal hadrons.

It would be interesting trying to generalize such an approach even to the quark level.

Actually, many authors suggested that quarks - more generally, the elementary--particle constituents - might be tachyons (see e.g. Hamamoto 1972, Mignani and Recami 1975b p.539, Guenin 1976, Souček 1979a,b; see also Brown and Rho 1983). Rafanelli (1974,1976,1978) showed that in classical relativistic physics there exists the possibility for a description of an elementary particle which has constituents, if those constituents are tachyons. Free spinning tachyons are then the candidates for elementary particle constituents. And in the range of Superluminal velocities the theory of a free spinning point particle admits uniquely a lineraly rising trajectory, naturally yielding the constituent confinement (see also Sect.12.2 and Recami 1982a).

Moreover, we shall see - Sect.15.2 - that the duality between electric and magnetic charges is possibly a particular aspect of the bradyons/tachyons duality; and authors as Tze (1974) and Barut (1978c) underlined the connections between electromagnetic and dual strings; possibly, a link can thus be found between tachyons and hadron structure (Mignani and Recami 1975b). Let us add that, more generally, quarks have been identified (Parisi 1978) with non-conventional "monopoles", i.e. with the monopoles of the field which mediates strong interactions inside hadrons.

Aside, it stands the "electromagnetic" approach by Jehle (1971,1972), who noticed that - while the introduction of a gauge variable single-valued in space implies charge conservation but does not change the physical situation - a "pseudogauge" transformation (with a variable ϑ which is single-valued only modulo 2π) is equivalent on the contrary to the introduction of a quantized flux hc/e (see also Bernardini 1982). It is, namely, a transformation from $A'_k = 0$, $\partial_k \vartheta' = 0$ to A_k, ϑ_k, where $A_k-(\hbar c/e)\partial_k\vartheta = 0$ is invariant; and, because of $(e/\hbar c)\oint \sum_{1,3} A_k dr^k = \pm 2\pi$, one may assume the flux line (singularity of $\partial_k\vartheta$) to be a closed loop. A more detailed analysis shows that such singularity loops spinning with velocities larger than c permit a consistent formulation of leptons, as well as of quarks and hadrons, in terms of electromagnetic fields and their probability amplitude distributions. The topological structure of those fields (toroidal field lines) represents the internal quantum numbers in particle physics.

13.6.- Are classical tachyons slower-than-light quantum particles?

We have seen in Sect.8.4 that, if a tachyon P_T has a very large intrinsic (i.e., measured in its own rest-frame) life-time $\overline{\Delta t}(P_B)$, just as it is for macroscopic and even more cosmic objects, then P_T can actually be associated with Superluminal motion (Sect.8.1). On the contrary, we saw that, if the intrinsic life-time $\overline{\Delta t}(P_B)$ of P_T is small w.r.t. the observation time-duration of tachyon P_T (as it commonly happens in microphysics), then P_T will actually appear endowed with a slower-than-

-light "front-velocity", or "group velocity", v_g. Only its "phase velocity" V_φ will be Superluminal: $v_g V_\varphi = 1$; cf. eq.(100).

In Sect.8.5 we noticed some formal analogies between these classical tachyons and the "de Broglie particles" met in QFT. The analogies become more strict when we analyze the appearance of a tachyonic particle endowed with an additional oscillatory movement, for example (and for simplicity) along the motion-line (Garuccio 1984). Let us recall that the "shape" of a tachyon (Sect.8.2) depends also on its speed V; namely, the semi-angle α of the cone $\mathscr{C}_0$ is given (Sect.8.2) by: $\mathrm{tg}\,\alpha = (V^2-1)^{-\frac{1}{2}}$. In such cases, the microphysical tachyon P_T will really appear as a bradyonic object associated with a kind of wave (having Superluminal phase-velocity). Cf. also Tanaka (1960), Schroer (1971), Streit and Klauder (1971), Murphy (1971), Naranan (1972), Gott III (1974), Strnad and Kodre (1975), Thankappan (1977), and particularly Robinett (1978).

At each time-instant, the real portion - which does carry energy-momentum - of such a wave is the one contained inside a certain moving "window" (see eq.(99), Sect.8.4): the whole wave may be possibly regarded, in a sense, as a "pilot wave". On this respect, it may become enlightening describing the scattering of two tachyonic particles P_T, P'_T; i.e. of two microphysical bradyons P_B, P'_B observed from a Superluminal frame.

13.7.- About tachyon spin

It is known that the little group of a space-like vector (cf. e.g. Jordan 1978) is isomorphic to SO(1,2), the Lorentz group in a pseudo-Euclidean space-time with one time-like and two space-like dimensions (Sect.8.1). Since SO(1,2) is non-compact, its unitary (irreducible) representations are infinite-dimensional, except for the one-dimensional representation. It was often concluded that, thus, either a tachyon has no spin (i.e., it is a scalar particle), or it has an infinite number of polarization states (Camenzind 1970).

However, after the results in Sects.5.9 and 11 (see e.g. Sect.5.17), we are justified in resorting for tachyons to non unitary representations; which are finite-dimensional (see also Carey et al.1979). For instance, solving the "relativistic wave equations for any spin" in the case of space-like momentum, the finite-dimensional wave-functions form non-unitary representations of the little group SO(1,2). Also tachyons can therefore be associated with integer andi semi-integer spins. This complies better with the philosophy of ER (see e.g. Corben 1978a).

Here we refer, e.g., to Shay (1978); see also Wolf (1969), Marx (1970), Fleury et al.(1973), Yaccarini (1975), Pavšič and Recami (1976 p.184), Camenzind (1978), and Tanaka (1979). Wolf (1968) showed moreover that, if a Bargmann-Wigner equation holds for time-, light- and space-like particles, then W-spin conservation holds for all of them, and not only for time-like particles.

Let us mention, at last, that the ordinary relation between spin and statistics seem to be valid also for tachyons (Sudarshan and Mukunda 1970), but contrary opinions do exist (Feinberg 1967, Hamamoto 1972).

13.8.- Further remarks

In the present Sect.6 we have met some indications not only of the possible rôle of tachyons in elementary particle interaction (and perhaps even structure), but also of the eventual reproduction of quantum results within classical physics with tachyons. Let us list some more hints:

(i) Many relativistic wave equations - based on perfectly valid representations of the Lorentz group (Wigner 1939) - lead to space-like solutions: see e.g. Barut and Nagel (1977); see also Korff and Fried (1967). For example, in a quantum electrodynamics based on the Joos-Weiberg higher spin wave equations, some solutions for integer spin particles correspond to tachyons (Eeg 1973).

(ii) In particular, the infinite-component relativistic equations (Majorana 1932) lead also to space-like solutions (see e.g. Fronsdal 1968, Grodsky and Streater 1968). It is noteworthy that the time-like and space-like solutions of the infinite-component Majorana wave-equations, taken together, constitute a complete set of solutions (Abers et al. 1967, Mukunda 1969). Barut and Duru (1973) recalled that a wave equation with many mass and spin states can be interpreted as describing a composite system in a relativistically invariant way, and then investigated the composite system corresponding to the Majorana equation (by introducing the internal coordinates in the c.m.f.). They showed that the internal motion of the two constituents of that composite system can be either oscillatory-type or Kepler-type. While the time-like solutions of the Majorana equation correspond to bound-states of the internal motion, the space-like solutions correspond on the contrary to the scattering-states of the constituent "particles". This material was put on a more formal basis by Barut et al.(1979), thus providing a completely relativistic quantum-theory suitable to describe a composite object; such a result being obtained - let us repeat - only by accepting the space-like solutions too. In a further series of papers, Barut and Wilson underlined many other circumstances in which the presence of those solutions in the infinite-component equations is good and not evil.

(iii) In general, the existence of space-like components seem a natural and unavoidable feature of interacting fields (Stoyanov and Todorov 1968). For instance, it has been proved by Dell'Antonio (1961) and Greenberg (1962) that, if the Fourier transform of a lo cal field vanishes in a whatever domain od space-like vectors in momentum space, then the field is a generalized free field. But space-like components seem necessary even to give locality to the fields.

(iv) In connection with what we were saying in Sect.13.5 about the field-theoretical models of elementary particles (see e.g. Parisi 1978), let us recall that the dual resonance models led to conceive hadrons as non-local objects: strings. String models have been widely investigated at both the classical and quantum levels. And they predicted the presence of tachyons in the spectrum of states. To eliminate tachyons, one had to introduce an additional interaction of a particle with the vacuum and spontaneous vacuum transitions (see e.g. Volkov and Pervushin 1977).

More in general, field theories with tachyons are quite popular (Taylor 1976; see also Nielsen and Olesen 1978): but, by assuming the vacuum to be the ground state, an automatic procedure is usually followed to get rid of tachyons, or rather to turn them into bradyons (see e.g. Nielsen 1978).

Also in the case of the Salam-Weinberg type of models, the gauge symmetry is spontaneously broken by filling the vacuum with tachyons: in this case such tachyons are the Higgs-field particles. However, the vacuum is supposed once more to adjust itself so as to turn the tachyons into bradyons (Nielsen 1978).

In conclusion, in the conventional treatment of field theories, tachyons seem to exist only at a formal level. But the procedure itself to get rid of tachyons might be only "formal". In any case, the Higgs particles - yet to be observed experimentally - can be considered at least as tachyons which have been converted into bradyons.

(v) The standard theories with positive metric and purely local interaction have not been developed in a convincing way; Heisenberg considered the efforts in that direction to be largerly based on "wishful thinking". He was more favourable to Dirac's hypothesis of an indefinite metric in state space (Heisenberg 1972).

In quantum theory with an indefinite metric complex-mass states are permitted, and cannot be ignored (see e.g. Yamamoto 1969, 1970a,b, Gleeson and Sudarshan 1970, Jadczyk 1970, Yokoyama 1972, Toyoda 1973, Yamamoti and Kudo 1975). As we saw towards the end of Sect.13.2 (Recami 1968, 1969a), complex-mass objects may be related to tachyons: see e.g. Sudarshan (1970d,f), Van der Spuy (1971), Gleeson et al.(1972b), Marques and Swieca (1972); see also Das (1966), and Corben (1975).

(vi) Again, Wimmel (1971a,b) noticed that classical tachyons can appear to undergo a (classical) "tunnel effect", an effect ordinarily allowed only to quantum objects.

Let us recall that, more in general, the tunnel effect can be described within "classical physics" by extrapolation to imaginary time (cf. Sect.5.6): see e.g. McLaughin (1972), Freed (1972), Jackiw and Rebbi (1976), t'Hooft (1976); see also Bjorkeen and Drell (1964),

p. 86.

(vii) At last, let us mention that two number fields exist that are associative and contain "imaginary units" (both properties being apparently necessary in Quantum Mechanics (QM)): the complex and the quaternion number field. Starting from the beginning of QM (we mean from the de Broglie wave-particle dualism) and recalling the above Hurwitz theorem, Souček attempted the constructtion of a quaternion QM, besides the ordinary complex QM. He seemingly found that, as the latter describes bradyons, so the former describes tachyons. Namely, in the duality between complex and quaternion QM, there correspond bradyons and tachyons; the electrodynamic U(1) gauge field and the Yang-Mills SU(2) gauge field; and so on. See Souček (1981); see also Weingarten (1973), Edmonds (1977, 1972); Barret (1978).

PART V: THE PROBLEM OF SLTs IN MORE DIMENSIONS. TACHYON ELECTROMAGNETISM

14.- THE PROBLEM OF SLTs IN FOUR DIMENSIONS

We have already seen various times (Sects.3.2, 6.1 and 8.3) that the Lorentz transformations (LT) can be straightforwardly extended to Superluminal frames S only in pseudo-Euclidean space-times M(n,n) having the same number of space and time dimensions. In Sect.5 we developed a model-theory in two dimensions, i.e. in a M(1,1) space-time; and those nice results strongly prompted us to attempt building up a similar theory also in more dimensions, based as far as possible on the same Postulates (Sect.5.18). In four dimensions, M(1,3), the asymmetry in the numbers of the time and space dimensions carries in very delicate problems (Sect.5.18). And no fourdimensional extensions of LTs for the Superluminal case exist, that satisfy all the properties (i)-(vi) listed at the end of Sect.3.2 (cf. also Pahor and Strnad 1976).

By trials, it is easy to write down "Superluminal" Lorentz transformations (SLT) in four dimensions which are real: but they violate one of the remaining conditions (Sect. 3.2); see e.g. the interesting paper by Sen Gupta (1966; see also Saavedra 1970). The first proposal of real SLTs in four dimensions is due to Olkhovsky and Recami (1970; see also 1971); such a proposal, soon abandoned by those authors, was independently taken over again by Antippa and Everett (1971,1973), who were inspired in part by a belief shown by us to be probably erroneous (see end of Sect.5.14).

A way out has been already outlined in Sect.8.3; we shall come back to it later on.

Moreover, let us preliminarily observe that (see Figs.5 and 6) in the four-momentum space, e.g., the mirror symmetry w.r.t. the light cone is a mapping one-to-one almost everywhere, in the sense that the whole plane E=0 should be mapped onto the E-axis, and vice versa; but one might restore a one-to-one correspondence by associating a direction also with every object at rest (namely, the limiting direction of its motion when coming to rest); or, alternatively, by identifying all the points of the hyperplane E=0, i.e. by adding to the 3-velocity space only one ideal point at infinity.

14.1.- On the "necessity" of imaginary quantities (or more dimensions)

Let us start from some elementary considerations, assuming we want to introduce Superluminal reference-frames also "in four dimensions". If a light-burst springs out from the event O (Fig. 36), the subluminal observer $s_0=(t,\underset{\sim}{x})$ will observe a spherical light-wave expanding with time. The Superluminal observer S'_∞ moving w.r.t. s_0 along the x-axis with divergent speed (having, i.e., as time-axis t' the x-axis of frame s_0) would not observe a spherical wave any more, but a light-wave with the shape of a two-sheeted hyperboloid; unless the SLT which connects s_0 with S leads to imaginary quantities for the transverse coordinates, so to transform the hyperboloid back to a spherical surface (Recami and Maccarrone 1980). This shows that, if we want to preserve in ER the main characters of SR (e.g., the equivalence of all inertial frames), we have to release in part the reality condition by introducing also imaginary quantities

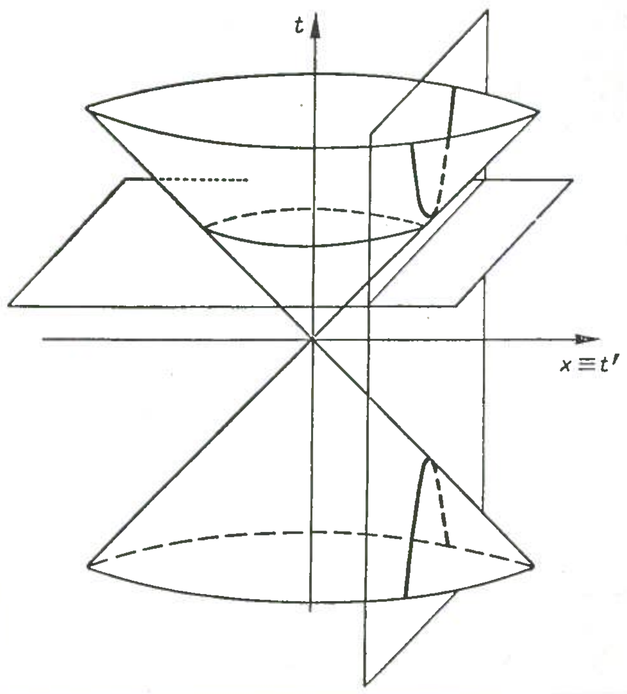


FIG. 36

(Recami and Mignani 1972,1973a, Corben 1974,1975,1976; see also Ramachandran et al.1972, and Alagar Ramanujam et al.1973); or - which is in a sense equivalent - to increase the number of space-time dimensions. Actually, Maccarrone and Recami (1982a,1984) had to introduce an auxiliary six-dimensional space-time $M_6\equiv M(3,3)$ as the abstract background in which the events are a priori allowed to happen. Later on, they went back - for each observer - to a four-dimensional space-time M(1,3) by assuming that each observer has access only to a suitable fourdimensional slice of M_6; even if a price has to be paid (in a sense, tachyons should then be described by three time-coordinates and one space-coordinate), as we shall see in the following.

14.2.- The formal expression of SLTs in four dimensions

What follows is mainly based on Maccarrone et al.(1983) and Maccarrone and Recami (1982a,1984), and references therein. Let us start from the Postulates of SR as put forth in Sect.4; let us recall in particular that we gave the Second Postulate the form: 'The space-time accessible to any inertial observer is four-dimensional. To each inertial observer the 3-dimensional Space appears as homogeneous and isotropic, and the 1-dimensional Time as homogeneous'. Let us recall also that the transformations $G^{\mu\nu}$ connecting (see eq. (14)) two generic inertial frames, f, f', a priori with $-\infty < |u| < +\infty$, must (cf. Sect. 4.2): (i) transform inertial motion into inertial motion; (ii) form a group $\mathbb{G}$; (iii) preserve space isotropy; (iv) leave the quadratic form invariant, except for its sign (see eq.(15)):

$$dx'_{\mu}\, dx'^{\mu} = \pm\, dx_{\mu}\, dx^{\mu} \, . \qquad (u^2 \lessgtr c^2) \qquad (15)$$

Let us recall at last the whole Sect.4.3, as well as the two-dimensional theory (expounded in Sect.5) which, whenever possible, has been already expressed in a multi-dimensional language.

From the above Postulates (Sect.4), i.e. from the above requirements (i)-(iv), it follows that in four dimensions the GLTs are expected to be unimodular and special (see Sect.5.4 and eq.(31)): det G = +1, $\forall G \in \mathbb{G}$, and such that the LTs are orthogonal, $G^T G = +\mathbb{1}$, while the SLTs are antiorthogonal, $G^T G = -\mathbb{1}$ (cf. eqs.(30): notice that now, however, $\mathbb{1}$ is the fourdimensional unit matrix).

In analogy with Sect.5.5, the SLTs differ from the LTs only for the fact that they are anti-orthogonal, i.e. satisfy eq.(15) with the sign minus, still - however - with $g'_{\mu\nu} = g_{\mu\nu}$ (Sect.4.3). Alternatively, one might also say that a SLT is identical to its dual subluminal LT, provided that we require the primed observer S' to use the opposite metric-signature $g'_{\mu\nu} = -g_{\mu\nu}$ (without changing, however, the signs into the definition of time-like and space-like quantities! See also Smrz 1984).

In any case (cf. Sect.5.5), the fundamental theorem of fourdimensional ER writes ($u^2 < 1$; $U^2 > 1$):

$$\mathrm{SLT}(\underset{\sim}{U}) = \pm\, \mathscr{S}\cdot \mathrm{LT}(\underset{\sim}{u}) \, ; \qquad (\underset{\sim}{u} // \underset{\sim}{U}) \qquad (150a)$$

$$\mathscr{S} \equiv \mathscr{S}_4 \equiv i\,\mathbb{1} \, ; \quad \mathscr{S} \in \mathbb{G} \, ; \qquad (u = 1/U) \qquad (150b)$$

i.e., a generic SLT corresponding to a velocity $\underset{\sim}{U}$ will be formally expressed by the product of the dual (subluminal) LT, corresponding to the velocity $\underset{\sim}{u} // \underset{\sim}{U}$, with $u = c^2/U$, by the matrix $i\,\mathbb{1}$. Due to the imaginary unit i, the SLTs satisfy eq.(15) with the minus sign. The operation $\mathscr{S} \equiv \mathscr{S}_4 \equiv i\,\mathbb{1}$ plays the role of "transcendent SLT", since when $\underset{\sim}{u} \rightarrow 0$

we get:

$$\mathrm{SLT}(U = \infty) = \pm\, i\, \mathbb{1}\ . \tag{151}$$

Incidentally, the transcendent "transformation" $\mathscr{S}$ is simply given by eq.(150b), and does not affect the speed u (namely, does not operate any change $\beta \rightarrow \frac{1}{\beta}$, differently from what stated in some previous papers).

It is important to stress that the group-properties and space-isotropy can be preserved only by an operator $\mathscr{S}$ which is a 4x4 matrix symmetrical w.r.t. all the possible axis permutations (Maccarrone and Recami 1982a,1984). The expression $\mathscr{S}$ that appeared in Review I was suited only for the simple case of collinear boosts (and the GLTs as written in Review I formed a group only for collinear boosts). Misunderstanding this fact and overlooking some recent literature (e.g., Maccarrone and Recami 1982b), Marchildon et al. (1983) adopted the expression for $\mathscr{S}$ given in Review I also for the case of generic (non collinear) SLTs. They were led, of course, to incorrect conclusions about the SLTs.

The group $\mathbb{G}$ of the generalized Lorentz "transformations" (GLT), both sub- and Superluminal, will be

$$\underset{\sim}{\mathbb{G}} = \underset{\sim}{\mathscr{L}}{}_{+}^{\uparrow} \otimes \mathbb{Z} \qquad ; \qquad \underset{\sim}{\mathbb{Z}}(4) \equiv \{\sqrt[4]{-1}\} \equiv \{+1, -1, +i, -i\}\ ; \tag{152}$$

this is analogous to what seen in Sect.5.6, but now $\underset{\sim}{\mathscr{L}}{}_{+}^{\uparrow}$ is the fourdimensional proper orthochronous Lorentz group. Again we have that, if $G \in \underset{\sim}{\mathbb{G}}$, then ($\forall G \in \underset{\sim}{\mathbb{G}}$) also $- G \in \underset{\sim}{\mathbb{G}}$ and $\mathscr{S}G \in \underset{\sim}{\mathbb{G}}$; cf. eqs.(37). In particular, given a certain LT $\equiv L(\underset{\sim}{u})$ and the SLT = $+i\, L(\underset{\sim}{u})$, one has: $[iL(\underset{\sim}{u})][iL^{-1}(\underset{\sim}{u})] \equiv [iL(\underset{\sim}{u})][iL(-\underset{\sim}{u})] \equiv -\mathbb{1}$, while, on the contrary, it is: $[iL(\underset{\sim}{u})][-iL^{-1}(\underset{\sim}{u})] = [iL(\underset{\sim}{u})][-iL(-\underset{\sim}{u})] = +\mathbb{1}$; this shows that:

$$[iL(\underset{\sim}{u})]^{-1} = -\, iL^{-1}(\underset{\sim}{u}) = -\, iL(-\underset{\sim}{u})\ . \tag{153}$$

The group $\underset{\sim}{\mathbb{G}}$ is non-compact, non-connected and with discontinuities on the light-cone; its central elements, moreover, are $\underset{\sim}{C} = (+\mathbb{1}, -\mathbb{1}, +i\,\mathbb{1}, -i\,\mathbb{1})$. Let us recall from Sect.11.1 that: $-\mathbb{1} = \bar{P}\bar{T} = \mathrm{CPT} \in \underset{\sim}{\mathbb{G}}$, and that $\underset{\sim}{\mathbb{G}} = \mathscr{E}(\underset{\sim}{\mathscr{L}}{}_{+}^{\uparrow}, \mathrm{CPT}, \mathscr{S})$. See also Sect.11.3. Of course also $\det \mathscr{S} = +1$; $\mathscr{S}^{T}\mathscr{S} = -1$; and $\pm\mathscr{S} \in \underset{\sim}{\mathbb{G}}$ (cf. eq.(150b)).

In the particular case of a boost along x, our SLTs, eqs.(150), can be written (U = = 1/u), (see Maccarrone and Recami 1984, Maccarrone et al. 1983, and refs. therein):

$$\left.\begin{aligned} dt' &= \pm\, i\, \frac{dt - u dx}{\sqrt{1-u^2}} \equiv \mp\, i\, \frac{dx - U dt}{\sqrt{U^2-1}} \\ dx' &= \pm\, i\, \frac{dx - u dt}{\sqrt{1-u^2}} \equiv \mp\, i\, \frac{dt - U dx}{\sqrt{U^2-1}} \\ dy' &= \pm\, i\, dy \quad ; \qquad dz' = \pm\, i\, dz\ , \end{aligned}\right. \qquad \begin{bmatrix} \text{Superluminal case} \\ u^2 < 1\ ;\quad U^2 > 1 \\ u \equiv 1/U \end{bmatrix} \tag{154}$$

where we took advantage of the important identities (41): see Sect.5.6. Notice that under "transformations" (154) for the fourvelocity (Sect.7.2) it happens that $u'_\mu u'^\mu = -u_\mu u^\mu$; eqs. (154) are therefore associated with Superluminal motions, as we shall see better below. One should not confuse in the following the boost speeds u, U with the fourvelocity-components u_μ of the considered object.

Let us underline that our "formal" SLTs, eqs.(154), do form a group, $\mathbb{G}$, together with the ordinary (orthochronous and antichronous) LTs. It should be noticed that the generalized Lorentz "transformations" introduce only real or purely imaginary quantities, with exclusion of (generic) complex quantities. Let us moreover stress that the transcendent transformation $\mathcal{S}$ does not depend at this stage on any spatial direction, analogously to the transformation LT(u=0) = $\mathbb{1}$. This accords with the known fact (Sect.3.2) that the infinite speed plays for Ts a rôle similar to the one of the null speed for Bs; more generally, the dual correspondence (Sect.5.11)

$$u \rightleftarrows c^2/u \equiv U\ , \qquad\qquad \underline{u}//\underline{U}\ , \tag{155}$$

hold also in four dimensions.(See also beginning of Sect.14).

14.3.- Preliminary expression of GLTs in four dimensions

Sects.5.8 and 5.9 can be extended to four dimensions (see Maccarrone et al.1983). First of all

$$\mathbb{G} = \mathcal{D} \otimes \mathcal{L}_+^\uparrow \tag{156}$$

where $\mathcal{D}$ is the discrete group of the dilations D: $x'_\mu = \varrho x_\mu$ with $\varrho = \pm 1, \pm i$. Then, by using the formalism of Sect.5.8, we can end up with eqs.(45), <u>valid now also in four dimensions</u>.

In terms of the light-cone coordinates (46) and of the discrete scale-parameter ϱ, the GLTs in the case of generalized boosts along x can be written

$$\begin{aligned} &d\xi' = \varrho a d\xi \ ; \quad d\zeta' = \varrho a^{-1} d\zeta \ ; \quad dy' = \varrho dy \ ; \quad dz' = \varrho dz \ ; \\ &\varrho = \pm 1,\ \pm i \ ; \quad 0 < a < +\infty \ ; \quad -\infty < u \equiv u_x < +\infty \ , \end{aligned} \tag{157}$$

where a is any real, positive number. Eqs.(157) are such that $d\xi' d\zeta' - dy'^2 - dz'^2 = \varrho^2(d\xi\, d\zeta - dy^2 - dz^2)$. For many further details see the abovementioned Maccarrone et al. (1983).

It is more interesting to pass to the <u>scale invariant</u> light-cone coordinates (47). Eqs.(157) then become ($\alpha \equiv a\varrho$; $k' = \varrho^{-1}k$):

$$d\varphi' = \alpha d\varphi \ ; \quad d\psi' = \alpha^{-1} d\psi \ ; \quad d\eta'^{(2)} = d\eta^{(2)} ; \quad d\eta'^{(3)} = d\eta^{(3)} \ ; \tag{158}$$

$$\alpha \equiv a\varrho \; ; \quad \varrho^2 = \pm 1 \; ; \quad a \in (0,+\infty) \; ; \quad -\infty < u \equiv u_x < +\infty \; , \tag{158}$$

where, as usual, $\varrho = \pm 1$ yields the subluminal and $\varrho = \pm i$ the Superluminal x-boosts. Now, all the generalized boosts (158) preserve the quadratic form, its sign included:

$$\varphi'\psi' - (\eta'^{(2)})^2 - (\eta'^{(3)})^2 = \varphi\psi - (\eta^{(2)})^2 - (\eta^{(3)})^2 \; . \tag{159}$$

Actually, eqs.(158) automatically include in the Superluminal case the interpretation of the first couple of equations in (154), just as we obtained in Sect.5.6. In fact they yield ($U = 1/u$):

$$dt' = \pm \frac{dx - u\,dt}{\sqrt{1-u^2}} \equiv \mp \frac{dt - U\,dx}{\sqrt{U^2-1}} \; ;$$

$$dx' = \pm \frac{dt - u\,dx}{\sqrt{1-u^2}} \equiv \mp \frac{dx - U\,dt}{\sqrt{U^2-1}} \; ; \qquad \left[\begin{array}{l} \text{Superluminal case} \\ u^2 < 1 \; ; \quad U^2 > 1 \\ U \equiv 1/u \end{array}\right] \tag{154bis}$$

$$dy' = \pm i\,dy \; ; \qquad dz' = \pm i\,dz \; ,$$

where the imaginary units disappeared from the first two equations (cf. Sect.5.6). See Mignani and Recami (1973a), and Corben (1975,1976); see also Maccarrone et al.(1983), and Pavšič (1971). Moreover, from eqs.(158) one derives for the x-boost speed:

$$u = \frac{\alpha - \alpha^{-1}}{\alpha + \alpha^{-1}} \; ; \qquad \alpha \equiv \varrho a \; ; \qquad (0 < a < +\infty \; ; \quad -\infty < u \equiv u_x < +\infty) \tag{158'}$$

in particular in the Superluminal case ($\varrho = \pm i$), the boost speed follows to be faster than light:

$$u = \frac{a + a^{-1}}{a - a^{-1}} > 1 \; .$$

Actually, in the case of Superluminal boosts and in terms of the light-cone coordinates (46), eqs.(158) can be written:

$$d\xi' = \tilde{\alpha}\,d\xi \; ; \quad d\zeta' = \tilde{\alpha}^{-1} d\zeta \; ; \quad dy' = -i\,\frac{\tilde{\alpha}}{|\tilde{\alpha}|}\,dy \; ; \quad dz' = -i\,\frac{\tilde{\alpha}}{|\tilde{\alpha}|}\,dz \; ;$$

$$u^2 \equiv u_x^2 > 1 \; ; \qquad -\infty < \tilde{\alpha} < +\infty \; , \tag{158bis}$$

which are the transcription of eqs.(154bis) in terms of the coordinates (46); now $\tilde{\alpha}$ is just real. In particular

$$dt' = \frac{1}{2}(\tilde{\alpha} - \tilde{\alpha}^{-1})dt - \frac{1}{2}(\tilde{\alpha} + \tilde{\alpha}^{-1})dx \; ; \qquad dx' = \frac{1}{2}(\tilde{\alpha} - \tilde{\alpha}^{-1})dx - \frac{1}{2}(\tilde{\alpha} + \tilde{\alpha}^{-1})dt$$

so that for the relative boost-speed one has $u = \left.\frac{dx}{dt}\right|_{dx'=0} = (\tilde{\alpha} + \tilde{\alpha}^{-1})/(\tilde{\alpha} - \tilde{\alpha}^{-1})$; $u^2 > 1$.

Let us observe that our coordinates φ, ψ are so defined that u is subluminal (Superluminal) whenever in eqs.(158) the quantities α and α^{-1} have the same (opposite) sign.

The more difficult problem of the velocity composition law will be considered below. We shall consider below also the meaning of the aboveseen, automatic, "partial reinterpretation" of eqs.(154), - formal, but with good group theoretical properties -, into eqs. (154bis), - which lost, on the contrary, their group-properties: see Maccarrone and Recami (1984) -. Incidentally, let us explicitly remind that the reinterpretation we are (and we shall be) dealing with in this Sect.14 has nothing to do with the "switching procedure" (also known as "reinterpretation principle").

In analogy with Sect.5.7, the "partially" reinterpreted" eqs.(154bis) can be combined with the ordinary (ortho- and anti-chronous) LTs in a compact form and in terms of a continuous parameter $\vartheta \in (0,2\pi)$ as follows (Recami and Mignani 1973a):

$$dt' = \Omega\gamma_0(dt - dx\ \mathrm{tg}\,\vartheta)\ ; \quad dx' = \Omega\gamma_0(dx - dt\ \mathrm{tg}\,\vartheta)\ ;$$

$$dy' = -\Omega\delta dy\ ; \qquad dz' = -\Omega\delta dz\ ,$$

with

$$u \equiv \mathrm{tg}\,\vartheta; \quad \gamma_0' \equiv +(|1 - \mathrm{tg}^2\vartheta|)^{-1/2}\ ; \qquad \left[\begin{array}{c} 0 \leqslant \vartheta \leqslant 2\pi \\ u^2 \gtrless 1 \end{array}\right]$$

$$\delta \equiv +\sqrt{\frac{1-\mathrm{tg}^2\vartheta}{|1-\mathrm{tg}^2\vartheta|}}\ ; \quad \Omega \equiv \Omega(\vartheta) \equiv \frac{\cos\vartheta}{|\cos\vartheta|}\cdot\delta^2\ .$$

Eqs.(160) show, among the others, how the four various "signs" (real or imaginary, positive or negative) of dy' and dz' do succeed each other as functions of u, or rather of ϑ (notice that $-\infty < u < +\infty$). In brief, it is $dy' = -\Omega\gamma_0^{-1}\sqrt{1 - \mathrm{tg}^2\vartheta}\,dy$. Fig. 37 just shows it explicitly. (We should remember also Fig.12 in Sect.5.15).

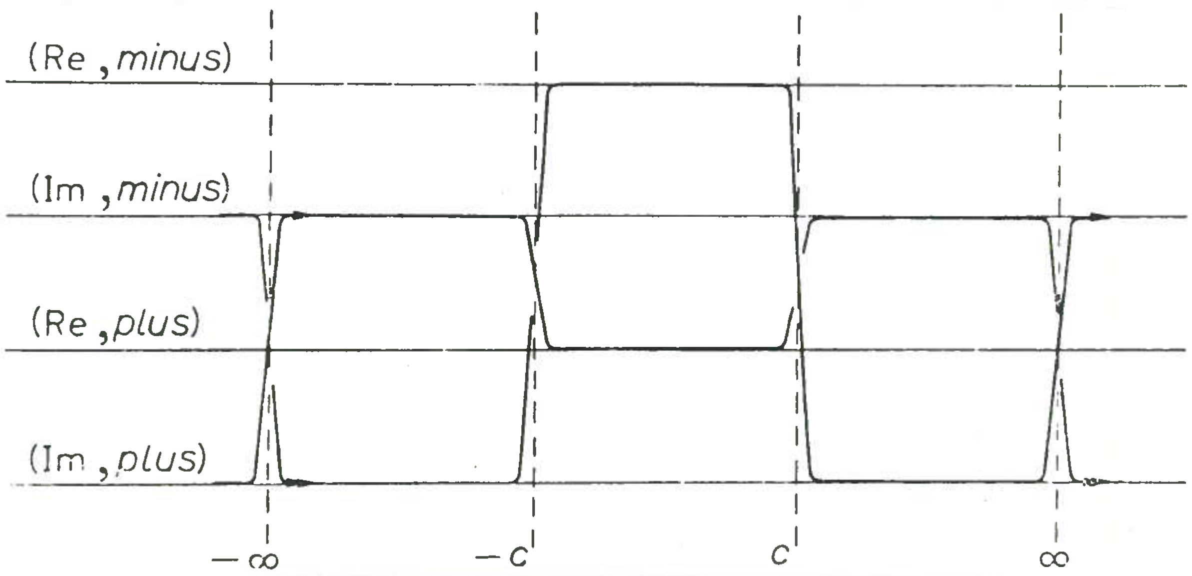


FIG. 37

As to eqs.(154bis), let us mention that recently Caldirola et al.(1980) discovered an early derivation due to Somigliana (1922). Somigliana looked for the most general linear transformations leaving invariant the electromagnetic wave propagation-equation, and found - besides the usual LTs - also eqs.(154bis), except for their double signs (actually necessary to the existence of the inverse transformations; for its procedure, see Caldirola et al.,1980).

14.4.- Three alternative theories

We preliminary saw from Fig.36 and from eqs.(154bis) that, if we look for SLTs satisfying eq.(15) of Sect.14.2 with the sign minus, we end up with "transformations" which carry in imaginary numbers for the transverse coordinates. As we mentioned many times, this problem disappears in (n,n) dimensions, and typically in (1,1) dimensions.

We deemed such problem (the problem of ER) has to be faced; and in the following we shall try to clarify its perpspectives (even if a lot of tachyon physics - as we saw - can be developed without trying to introduce Superluminal frames). We are mainly following,in other words, the approach by Mignani and Recami, and subsequent coworkers.

However other authros preferred to skip that problem, reducing it (even in four dimensions) to an essentially two-dimensional problem. Two alternative approaches have been proposed in such a direction.

14.4.1.- The fourdimensional approach by Antippa and Everett - A group of authors, initially inspired by a belief criticized in Sect.5.14, just assumed all tachyons to move exclusively along a priviliged direction, or rather along a unique "tachyon corridor". In this case the problem for tachyons becomes essentially two-dimensional, even in four dimensions. Such an approach does violate, however, not only space-isotropy, but also light-speed invariance. Those violations do not show up only for collinear boosts along the tachyon corridor. According to us, this approach avoids considering the real problem of SLTs in ER. It would then be better to investigate tachyons from the subluminal frames only (i.e., in the "weak approach" only). For details about this theory - which of course does not meet imaginaries;see Antippa and Everett (1971,1973), Antippa (1972,1975), Everett (1976), and Marchildon et al.(1979); see also Lemke (1976, 1977a,b), and Teli and Sukar (1978).

14.4.2.- The fourdimensional approach by Goldoni - The third theory is due to Goldoni (1972,1973), who developed an interesting approach in which a symmetry is postulated between the "slow" and "fast" worlds, and the tachyon rest-mass is real; he succeeded e.g. in producing the "tadpoles" dynamically (without supposing a non-zero vacuum expectation value of the fields).

Passing from the slow to the fast worlds, however, means interchange time with space. And in four dimensions, which space-axis has the time-axis to be interchanged with? The approach mainly followed by us is equivalent to answer: 'with all the three space-axes!', so to get transformations preserving the quadratic form, except for its sign (see eq.(15), Sect.14.2); afterwards, one has to tackle the appearance of imaginary transverse components. In order to avoid such difficulty, Goldoni introduced a different metric-signature for each observed tachyon, ending up with the four independent space-time metric-signatures: (---+); (+---); (-+--); (--+-). It follows that tachyons are not observable in Goldoni's approach, except for the fact that they can exchange with bradyons (only) internal quantum numbers. Some consequences for QFT may be appealing; but we deem that this approach too - at the relativistic level - avoids facing the real problem by a "trick".

Nevertheless, rather valuable seem the considerations developed by Goldoni (1975a,b, c) in general relativity.

14.5.- A simple application

Let us go back to Sect.14.4 and apply it to find out e.g. how a fourdimensional (space-time) "sphere" $t^2+x^2+y^2+z^2 = A^2$, that is to say

$$\frac{1}{2}\xi^2 + \frac{1}{2}\zeta^2 + y^2 + z^2 = A^2 , \qquad (161)$$

deforms under Lorentz transformations. In the ordinary subluminal case (eqs.(157) with $\varrho =$ $= +1$), eq.(161) in terms of the new (primed) coordinates rewrites ($0 < a < +\infty$):

$$\frac{1}{2} a^{-2} \xi'^2 + \frac{1}{2} a^2 \zeta'^2 + y'^2 + z'^2 = A^2 , \qquad \text{(subluminal case)} \qquad (162a)$$

which in the new frame is a fourdimensional ellipsoid.

In the case of a Superluminal boost (eqs.(158bis)), eq.(161) becomes, in terms of the new primed coordinates ($0 < a < +\infty$):

$$\frac{1}{2} a^{-2} \xi'^2 + \frac{1}{2} a^2 \zeta'^2 - y'^2 - z'^2 = A^2 \qquad \text{(Superluminal case)} \qquad (162b)$$

which in the new frame is now a fourdimensional hyperboloid.

Notice explicitly, however, that the present operation of transforming under GLTs a fourdimensional set of events has nothing to do with what one ordinarily performs (in fact, one usually considers a world-tube and then cuts it with different three-dimensional hyperplanes).

14.6.- Answer to the "Einstein problem" of Sect.3.2

We have still the task of interpreting physically the SLTs as given by eqs.(150), (154). Before going on, however, we wish to answer preliminary the "Einstein problem" mentioned in Sect.3.2 (cf. eq.(12)). We have seen in Sect.5.6, and later on in connection with eqs.(154bis) (Sect.14.3), that eq.(12) is not correct, coming from an uncritical extension of LTs to the Superluminal case. Let us consider an object with its centre at the space-origin O of its rest-frame; be it intrinsically spherical or, more generally, let it have intrinsic sizes $\Delta x_0 = 2x_0 \equiv 2r$, $\Delta y_0 = 2y_0$ and $\Delta z_0 = 2z_0$ along the three space axes, respectively. Instead of eq.(12), for the size along the boost motion-line x the theory of ER yields the real expression ($x_0 \equiv r$):

$$\Delta x' = \Delta x_0 \sqrt{U^2-1} \, . \qquad (U^2>1) \qquad (163a)$$

No problems arise, therefore, for the object size along the x-axis.

We meet problems, however, for the transverse sizes, which become imaginaries according to eqs.(154bis):

$$\Delta y' = i\Delta y_0 \, ; \qquad \Delta z' = i\,\Delta z_0 \, . \qquad (163b)$$

But let us go back to Sect.8.2 and Fig.19. If the considered object $P \equiv P_B$ is ellipsoidal in its rest-frame, then, when Superluminal, $P \equiv P_T$ will appear to be spread over the whole space confined between the double indefinite cone $\mathscr{C}$: $y^2/y_0^2+z^2/z_0^2=(Ut-x)^2/[x_0^2(U^2-1)]$, and the two-sheeted hyperboloid $\mathscr{H}$: $y^2/y_0^2+z^2/z_0^2=(Ut-x)^2/[x_0^2(U^2-1)]-1$ (cf. Recami and Maccarrone 1980). See Figs.17. The distance $2x_0'$ between the two vertices V_1 and V_2 of $\mathscr{H}$, which yields the linear size of P_T along x, is $2x_0'=2x_0\sqrt{U^2-1}$. For instance, for t=0, the position of the two vertices of $\mathscr{H}$ is given by $V_{1,2}=\pm x_0\sqrt{U^2-1}$. This incidentally, clarifies the meaning of eq.(163a).

Let us now turn our attention to the transverse sizes. The quantities y_0 and z_0 correspond to the intersections of the initial ellipsoid with the initial axes y and z, respectively (for t=0). We have then to look in the tachyonic case for the intersections of $\mathscr{H}$ with the transverse axes y and z. Since these intersections are not real, we shall formally get, still for t=0,

$$y_0' = iy_0 \, ; \qquad z_0' = iz_0$$

which do explain the meaning of eqs.(163b). In fact (see Figs. 38), the real quantities $y_0'/i=y_0$ and $z_0'/i=z_0$ have still the clear, simple meaning of semi-axes of $\mathscr{H}$. In other words, the quantities $|y_0'| = y_0'/i$ and $|z_0'| = z_0'/i$ just tell us the shape of the tachyon relevant surface (they express the transverse size of the "fundamental rectangles", i.e. allow to find out the fundamental asymptotes of P_T). See Recami and Maccarrone (1980);

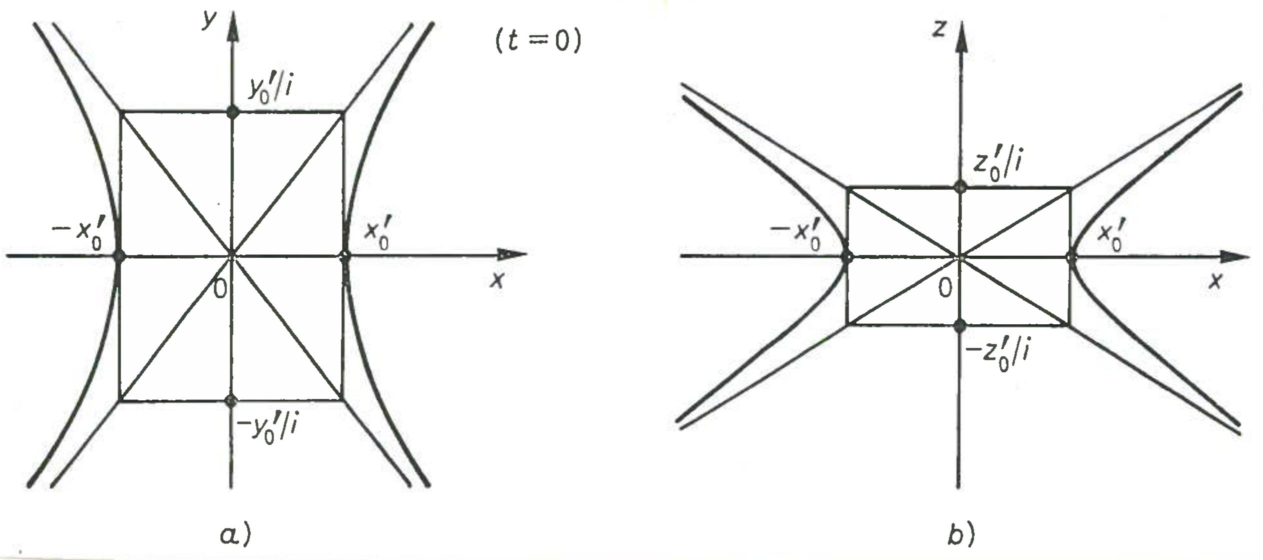


FIG. 38

see also Corben (1975), Gladkick (1978a,b), Terletsky (1978), Gott III (1974), and Fleury et al.(1973).

Even if <u>in a particular case</u> only, we have practically shown how to interpret also the last two equations in (154bis). We shall come back to this point; but let us add here the following. Eqs.(154bis) seem to transform each ellipsoidal (or spherical) surface $\mathcal{S}_1$ into a two-sheeted hyperboloid $\mathcal{S}_2$. Let us now consider the intersections of any surface $\mathcal{S}_1$ (see Fig. 39a) and of the corresponding $\mathcal{S}_2$ (Fig. 39b) with <u>all</u> the possible transver-

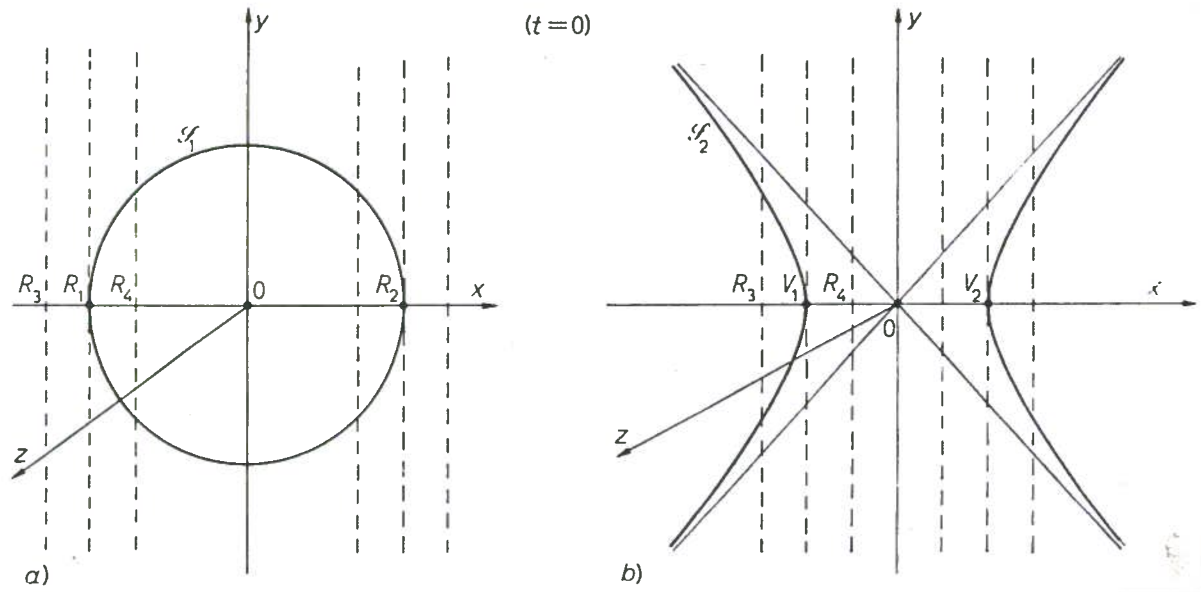


FIG. 39

sal planes $x=\bar{x}$. In Figs.39, for simplicity, the case of a Superluminal boost along x with speed $V=c\sqrt{2}$ and t=0 is considered; so that $\overline{OV_1}=\overline{OR_1}=x_0=r$ and all quantities $\overline{OR}$ have the same value both in Fig.(a) and in (b). It is immediate to realize that, when the intersections of $\mathcal{S}_1$ with the plane $x=\bar{x}$ are real, then the corresponding intersections of $\mathcal{S}_2$ are imaginary (with the same magnitude), and vice-versa. Namely, in the particular case considered, the intersections of $\mathcal{S}_1$ are real for $|\bar{x}|<r$ and imaginary for $|\bar{x}|>r$, while the intersections of $\mathcal{S}_2$ are on the contrary imaginary for $|\bar{x}|<r$ and real for $|\bar{x}|>r$. It is easy to understand that eqs.(154bis) operate in the planes (x,y) and (x,z) a mapping of ellipses $\mathcal{E}$ into hyperboles $\mathcal{h}$, in such a way that the real part of $\mathcal{E}$ goes into the imaginary part of $\mathcal{h}$, and vice versa (see Caldirola et al.1980). Cf. also Fig. 37.

14.7.- An auxiliary six-dimensional space-time M(3,3)

Equations (150), as well as (154), call imaginary quantities into play and therefore seem to require an 8-dimensional space C^4 (i.e. a 4-dimensional complex space-time) as the kinematical background. However, an essential teaching of SR appears to be that the four--position is given by one real and three imaginary coordinates - or viceversa - so that formally (with c=1): Time = ix Space. As noticed by Minkowski (1908) himself, one might formally write: 1s = ix(3×10^8) m. As a consequence, to interpret the SLTs it can be enough to assume (temporarily, at least) a 6-dimensional space-time M(3,3) as background; this was first suggested in Mignani and Recami (1976). Ever since, much work has been done on such spaces, with or without direct connection with the SLTs: see e.g. Dattoli and Mignani (1978), Vyšín (1978), Pappas (1978,1979,1982), Ziino (1979,1983), Strnad (1978,1979a,b, 1980), Pavšič (1981a,b), Johnson (1981), Froning (1981), Lewis (1981), Patty (1982), Conforto (1984) and particularly Cole (1978,1979,1980a,b,c,d,e); see also Tonti (1976), Jancewicz (1980), and Maccarrone and Recami (1982b). The idea of a possible multi-dimensional time, of course, was older (see e.g. Bunge 1959, Dorling 1970, Kalitzin 1975, Demers 1975).

Alternative approaches, that can be promising also w.r.t. tachyon theory, may be the ones which start from a complexification of space-time, via the introduction ab initio either of complex numbers (Gregory 1961,1962, Sudarshan 1963, Review I, Yaccarini 1974, Mignani and Recami 1974c, Cole 1977, Kálnay 1978, Moskalenko and Moskalenko 1978; see also Rosen 1962, Das 1966, Shin 1966, Kálnay and Toledo 1967, Baldo and Recami 1969, Recami 1970, Olkhovsky and Recami 1970, Hansen and Newman 1975, Hestenes 1975, Plebanski and Schild 1976, Charon 1977, Imaeda 1979, and Sachs 1982), or of octonions (see e.g. Casalbuoni 1978), or of twistors (see e.g. Penrose and McCallum 1973, Hansen and Newman 1975) and quaternions (see e.g. Edmonds 1972,1977,1978, Weingarten 1973, Mignani 1975,1978, Imaeda 1979). The most promising "alternative" approach is probably the last one: see the end of Sect.13.8 (and Souček 1981).

Let us mention, incidentally, that transformations in C^3-space are related to the group SU_3 of (unitary) intrinsic symmetries of elementary particles. It is not without meaning, possibly, that the M(3,3) formalism has been used to express the law of "trichro matism" (Demers 1975).

Let us confine ourselves to boosts along x. We are left with the problem of discussing the formal eqs.(154).

Let us consider (Maccarrone and Recami 1984a) the GLTs, eqs.(152), as defined in $M_6 = M(3,3) = (x,y,z,t_x,t_y,t_z)$; any observer s in M_6 is free to rotate the triad $\{\underset{\sim}{t}\} = (t_x,t_y,t_z)$ provided that $\{\underset{\sim}{t}\} \perp \{\underset{\sim}{x}\} = (x,y,z)$. In particular, the initial observer s_0 can always choose the axes t_x,t_y,t_z in such a way that, under a transcendent Lorentz transformation (without rotations: Møller 1962 pp.18-22, 45-46) $\mathscr{S} \equiv \mathscr{S}_6$, it is $x \rightarrow t_x$; $y \rightarrow t_y$; $z \rightarrow t_z$; $t_x \rightarrow x$; $t_y \rightarrow y$; $t_z \rightarrow z$; in agreement with the fact that the formal expression of $\mathscr{S} = i\mathbb{1}$ (where now $\mathbb{1}$ is the six-dimensional identity) is independent of any space direction.

Moreover, if observer s_0, when aiming to perform a Superluminal boost along x_j, rotates $\{\underset{\sim}{t}\}$ so that $t_j \equiv t$ (axis t being his ordinary time-axis: see Sect.4 and the following), then any transcendent boost can be formally described to operate as in Fig.40b.

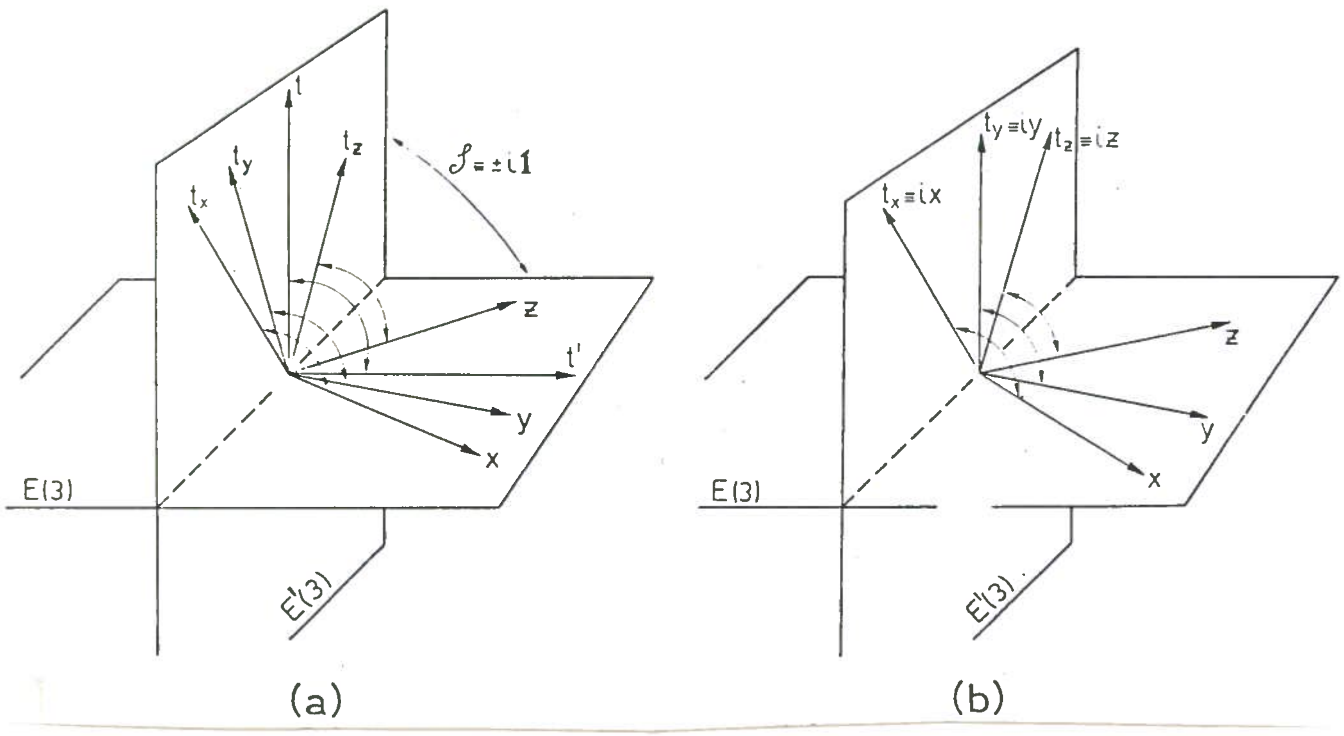


FIG. 40

What above means that the imaginary unit i can be reagrded as a 90°-rotation operator also in M_6; from the active point of view, e.g., it carries $\underset{\sim}{x} \equiv (x,y,z) \rightleftarrows (t_x,t_y,t_z) \equiv \underset{\sim}{t}$. Here the meaning of i, for one and the same observer, is analogous to its meaning in SR, where it is used to distinguish the time from the space coordinates, which are orthogonal to time. Therefore:

$$i \equiv e^{i\pi/2} \begin{cases} t \rightleftarrows x & \text{(two-dimensional case)} \\ \underset{\sim}{t} \rightleftarrows \underset{\sim}{x} & \text{(six-dimensional case).} \end{cases} \qquad (164)$$

Notice that in M_6 the GLTs are actually (linear) transformations, and not only mappings. What precedes (see e.g. eq.(164)) implies that

$$\text{for } LT_6, \qquad ds'^2_6 = + ds^2_6 \qquad (165a)$$

$$\text{for } SLT_6, \qquad ds'^2_6 = - ds^2_6 \qquad (165b)$$

with obvious meaning of the symbols. The GLTs, as always, can be considered either from the active or from the passive point of view (in the latter case, they will keep the 6-vector fixed and "rotate" on the contrary the six axes without changing - notice - their names during the "rotation").

The subluminal LTs in M_6, to be reducible in four dimensions to the ordinary ones in agreement with SR ($ds'^2_4 = + ds^2_4$), must be confined to those that call into play one time-axis, let it be $t \equiv t_1$, while t_2 and t_3 remain unchanged (or change in M_6 only in such a way that $dt'^2_2 + dt'^2_3 = dt^2_2 + dt^2_3$). As a consequence, because of eqs.(150), also the SLTs in M_6 must comply with some constraints (see Maccarrone and Recami 1984). For instance, when the boost speed U tends to infinity, the axis $t' \equiv t'_1$ tends to coincide with the boost axis x_i, and the axis x'_i with the axis $t_1 \equiv t$.

As to the signature in M_6 two alternative conventions are available. The first one is this: we can paint in blue (red) the axes called t_j (x_j) by the initial observer s_o, and state that the blue (red) coordinate squares must always be taken as positive (negative) for all observers, even when they are "rotated" so as to span the region initially spanned by the opposite-color axes. Under such a convention, a transcendent SLT acts as follows:

$$\begin{pmatrix} + \\ + \\ + \\ - \\ - \\ - \end{pmatrix} \quad \begin{array}{l} dt_z \rightarrow dt'_z = dz \\ dt_y \rightarrow dt'_y = dy \\ dt_x \rightarrow dt'_x = dx \\ dx \rightarrow dx' = dt_x \\ dy \rightarrow dy' = dt_y \\ dz \rightarrow dz' = dt_z \end{array} \quad \begin{pmatrix} + \\ + \\ + \\ - \\ - \\ - \end{pmatrix} \qquad (\text{under } \mathcal{S}) \qquad (166)$$

Notice that no imaginary units enter eqs.(166). The previous discussion on the action of $\mathcal{S}$ in M_6 was performed with such a metric-choice.

The second possible convention (still without changing the names - let us repeat - of the axes t_j, x_j during their "rotation") would consist in adopting the opposite six-dimensional metric in the r.h.s. of eqs.(166); it corresponds to changing the "axis signatu res" during their rotation:

$$\begin{pmatrix} + \\ + \\ + \\ - \\ - \\ - \end{pmatrix} \begin{matrix} dt_z \to idt'_z = idz \\ dt_y \to idt'_y = idy \\ dt_x \to idt'_x = idx \\ dx \to idx' = idt_x \\ dy \to idy' = idt_y \\ dz \to idz' = idt_z \end{matrix} \begin{pmatrix} - \\ - \\ - \\ + \\ + \\ + \end{pmatrix} \quad (\text{under } \mathscr{S}) \ . \qquad (167)$$

Such a second convention implies the appearance of imaginary units (merely due, however, to the change of metric w.r.t. eqs.(166)).

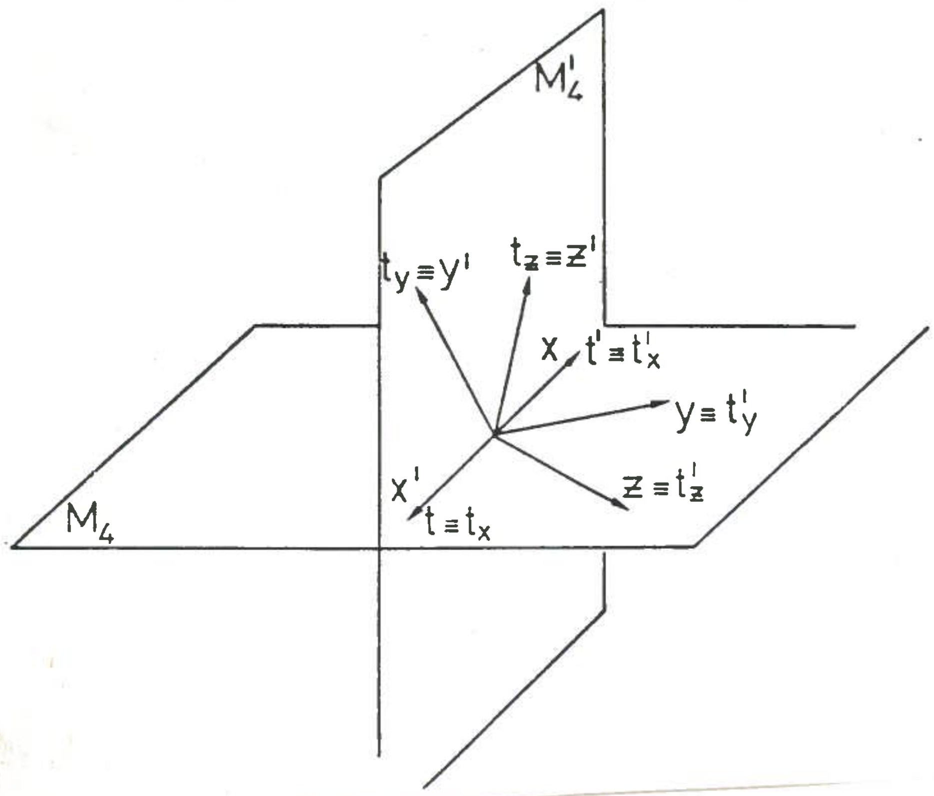
In any case, the axes called t_j by the subluminal observer s_o, and considered by s_o as subtending a three-temporal space $(t_x,t_y,t_z) \perp (x,y,z)$, are regarded by the Subluminal observer S'_∞, and by any other S', as spatial axes subtending a three-spatial space; and viceversa.

According to our Second Postulate (Sect.4) we have now to assume that s_o has access only to a 4-dimensional slice M_4 of M_6. When s_o describes bradyons B, we have to assume $M_4 \equiv (t_1 \equiv t;\ x,y,z)$, so that the coordinates t_2, t_3 of any B are not observable for s_o. With regard to SLTs we must e.g. specify, from the passive point of view, which is the "observability slice" M'_4 of M'_6 accessible to S' when he describes his own bradyons. By checking, e.g., eqs.(166) we realize that only two choices are possible: either (i) $M'_4 \equiv$ $\equiv (t'_x;x',y',z')$; or (ii) $M'_4 \equiv (t'_z,t'_y,t'_x,x')$. The first choice means assuming that each axis while rotating carries with itself the property of being observable or unobservable, so that the axes observable for S' are the transforms of the axes observable for s_o. The second choice, on the contrary, means assuming the observability (or unobservability) of each axis to be established by its position in M_6 (as judged by one and the same observer), so that two of the axes (i.e., t'_y, t'_z) observable for S' are the transforms of two axes (i.e., t_y, t_z) unobservable for s_o. In other words, the first choice is $M'_4 \perp M_4$, while the second choice is $M'_4 \equiv M_4$ (in M_6, when it is referred to one and the same observer). Notice that, roughly speaking, the above properties of the two choices get reversed when passing to the active point of view.

The first choice does not lead automatically, from eqs.(165) in six dimensions, to the ds^2_4 invariance (except for the sign) in four dimensions. It moreover calls all six co ordinates into play, even in the case of subluminal LTs obtained through suitable chains of SLTs and LTs. This choice, therefore, could be adopted only when whishing to build up a truly six-dimensional theory. The resulting theory would predict the existence in M'_4 of a "tachyon corridor" and would violate the light-speed invariance in M_4': in such a sense it would be similar to Antippa's (1975).

The <u>second choice</u>, once assumed in M_6 that $ds_6'^2 = -ds_6^2$ for SLTs, <u>does</u> lead automatically also to $ds_4'^2 = -ds_4^2$ in four dimensions (Maccarrone and Recami 1984). Moreover, it calls actually into play four coordinates only, in the sense that (cf. e.g. eqs.(166)) it is enough to know initially the coordinates $(t;x,y,z)$ in M_4 in order to know finally the coordinates $(t_z',t_y',t_x';x')$ in M_4'. We adopt the second choice since we want to try to go back from six to four dimensions, and since we like to have the light-speed invariance preserved in four dimensions even under SLTs. The "square brackets" appearing in eqs.(166),(167) just refer to such a choice.

To go on, let us start by adopting also the signature - first convention - associated with eqs.(166). If we consider in M_6 a (tangent) 6-vector dv lying on the slice $M_4(t_x=t;x,y,z)$, then a SLT - regarded from the <u>active</u> point of view - will "rotate" dv <u>in</u> to a vector dv' lying on the slice $M_4'(t_x,t_y,t_z;x)$: see Fig. 41. In other words, any SLT - as given by eqs.(150),(154) - leads from a bradyon B with observable coordinates in



<u>FIG. 41</u>

$M(1,3)\equiv(t;x,y,z)$ to a final tachyon T with "observable" coordinates in $M'(3,1)\equiv(t_1,t_2,t_3;w)$, where the w-axis belongs to $E(3)\equiv(x,y,z)$ and the t-axis belongs to $E'(3)\equiv(t_1,t_2,t_3)$: see Fig.40a. Formally: $(1,3) \xrightarrow{\text{SLT}} (3,1)$. From the <u>passive</u> point of view, the initial observer s_0 has access, e.g., only to the slice $(t_x\equiv t;x,y,z)$, while the final observer S' (e.g., S_∞') has access only to the slice $(t_z',t_y',t_x';x')$; so that the coordinates t_y, t_z (and y', z') are not observable (see also Poole et al. 1980, and Sobczyk 1981). Notice that x' comes from the "rotation" of the boost axis.

At this point two observations are in order: 1) Our Second Postulate (Sects.4 and 14.2) requires observer S' to consider his space-time ($t'_z, t'_y, t'_x; x'$) as related to three space axes and one time axis; actually renaming them, e.g., ξ'_1, ξ'_2, ξ'_3 and τ', respectively. This consideration is the core of our interpretation, i.e. the basis for understanding how S' sees the tachyons T in his M'_4; 2) The principle of relativity (Sect.4) requires that also s_o describe his tachyons (in M_4) just as S' describes his tachyons (in M'_4); and viceversa. If we understand how S' sees his tachyons in M'_4, we can immediately go back to the initial M(1,3) and forget about six dimensions.

In connection with M'_4, the effect of a Superluminal boost along x will be the following:

$$\begin{pmatrix} - \\ + \\ + \\ + \end{pmatrix} \qquad \begin{aligned} dx &\to dx' = \mp \frac{dx - Udt}{\sqrt{U^2-1}} \; ; \\ dt_x &\to dt'_x = \mp \frac{dt - Udx}{\sqrt{U^2-1}} \; ; \\ dt_y &\to dt'_y = \pm\, dy \; ; \\ dt_z &\to dt'_z = \pm\, dz \; . \end{aligned} \qquad \begin{bmatrix} U \equiv 1/u \\ u^2 < 1 \; ; \quad U^2 > 1 \end{bmatrix} \tag{168}$$

In eqs.(168) no imaginaries appear. But our signature-choice (166) implies that S' - from the metric point of view, since he uses the signature (+++-) - deals with t'_j as if they were actually time-components, and with x' as if it were actually a space-component.

We might say, as expected, that a tachyon T will appear in M'_4 to S' (and therefore also to s_o in M_4) as described by the same set of coordinates describing a bradyon B, provided that three out of those coordinates are regarded as time coordinates, and only one as a space coordinate. Since we do not understand the meaning of such a statement, we may seek recourse to some formal procedures so to deal eventually (at least formally, apparently) with one time and three space coordinates; we can hope to understand a posteriori such meaning, via the latter choice (see e.g. Mignani and Recami 1974c, and Maccarrone et al. 1983). One of the possible procedures is the following. Let us change the signature-choice, by passing from eqs.(166) to eqs.(167), in such a way that both (Sect.4.3.6) s_o and S' use the signature (+---), as if S' too dealt with one time and three space coordinates. With the choice (167), eqs.(168) transform into:

$$\begin{pmatrix} + \\ - \\ - \\ - \end{pmatrix} \quad \begin{aligned} dx &\to idx' = \mp i \frac{dx - Udt}{\sqrt{U^2-1}} \\ dt_x &\to idt'_x = \mp i \frac{dt - Udx}{\sqrt{U^2-1}} \\ dt_y &\to idt'_y = \pm idy \\ dt_z &\to idt'_z = \pm idz \end{aligned} \quad \begin{pmatrix} + \\ - \\ - \\ - \end{pmatrix} \qquad (168')$$

where now "imaginary units" do appear, which correspond to the metric change (166)$\rightleftarrows$(167). Eqs.(168') are of course equivalent to eqs.(168). Eqs.(168'), and therefore eqs.(168), coincide with our eqs.(154bis), provided that the second one of eqs.(164) is applied to the vector $(it'_z, it'_y, it'_x, ix')$. See the following.

14.8.- Formal expression of the Superluminal boosts: The First Step in their interpretation

We reached the point at which to attempt interpreting eqs.(154). At the end of the last Sect.14.7, we just saw how to transform eqs.(168') into eqs.(154bis). The result has been the same got in an "automatic" way in Sect.14.3.

This is a first step in the interpretation of SLTs. But we shall have to deal also with the imaginaries remained in the last two of eqs.(154), or of eqs.(168').

The first two equations in (168') - in fact - are true transformations carrying a couple of coordinates (t,x) belonging to the initial observability slice into a couple of coordinates (t',x') belonging to the final observability slice. In other words, t' and x' come from the "rotation" of x and t, such a rotation taking always place inside both the observability slices of s_o and S'. We can just eliminate the i's on both sides, getting the reinterpreted eqs.(39')-(39") of Sect.5.6.

On the contrary, the coordinates t'_y, t'_z - that S' must interpret as his transverse space-coordinates ξ'_2, ξ'_3 - are the transforms of the initial coordinates t_y, t_z (unobservable for s_o), and not of the initial coordintaes y, z. Precisely, the axes ξ'_2, ξ'_3 derive by applying to the axes t_y, t_z a 90°-"rotation" which takes place in M_6 outside the observability-slices of s_o and S'. As a consequence, in the first two equations in (168') we have to substitute dz' for idt'_z and dy' for idt'_y so that

$$\begin{pmatrix} - \\ - \end{pmatrix} \quad \begin{aligned} dz' &= \pm idz \\ dy' &= \pm idy \end{aligned} \qquad \text{(Superluminal x-boost)}$$

The i's remain here; in fact, the coordinates y', z' (regarded as spatial by S') are considered as temporal by s_0.

Notice that, from the active point of view, M_4 and M'_4 intersect each other in M_6 just (and only) along the plane $(x,t) = (t',x')$: see Figs.40 and 41.

Eqs.(168') have been thus transformed into eqs.(154bis).

While eqs.(150) or (154) for $U \rightarrow \infty$ (transcendent SLT) yield

$$\pm\mathcal{S}: \quad dt' = \pm\, idt\ ; \quad dx' = \pm\, idx\ ; \quad dy' = \pm\, idy\ ; \quad dz' = \pm\, idz, \qquad (169)$$

in agreement with the fact that the formal expression of $\mathcal{S} = i\,\mathbb{1}$ is direction-independent, after the (partial) reinterpretations of eqs.(154) into eqs.(154bis) we get that the transcendent SLT along x acts as follows:

$$dt' = \pm\, dx\ ; \quad dx' = \pm\, dt\ ; \quad dy' = \pm\, idy\ ; \quad dz' = \pm\, idz\ .$$

In this case, in fact, the reinterpretation follows by regarding i as a 90°-rotation operator in the complex plane $(x,t) = (t',x')$, and not in the planes (y,t) or (z,t). Consequently, even if all transcendent SLTs (without rotations) $\mathcal{S}$ are formally identical, they will differ from one another after the reinterpretation.

More details on this "interpretation First Step" can be found in Maccarrone and Recami (1984, Sect.7). We want to stress explicitly that the reinterpretation is a "local" phenomenon, in the sense that it clarifies how each observer S' renames the axes and therefore "physically interpret" his own observations. The interpretation procedure, thus, is frame-dependent in ER and breaks the generalized Lorentz-invariance. Eqs.(154), e.g., do form the group $\mathbb{G}$ together with the LTs; but the "partially interpreted" eqs.(154bis) do not. Moreover, the reinterpretation (when necessary) has to be applied only at the end of any possible chain of GLTs: to act differently would mean (besides the others) to use diverse signatures - in our sense - during the procedure, and this is illegal. (Notice once more that the reinterpretation we are discussing in Sect.14 has nothing to do with the Stückelberg-Feynman-Sudarshan "switching procedure", also known as "reinterpretation principle").

14.9.- The Second Step (i.e.: Preliminary considerations on the imaginary transverse components)

In Sects.14.3 and 14.7-14.8 we have seen how to interpret the first two equations in (154), so to pass to eqs.(154bis). We are left with the need for a second step in the interpretation of SLTs, to understand the geometrico-physical meaning of the last two equations in (154) or in (168').

How to perform this second step has been already discussed in Sect.14.6, when answering the Einstein problem. Namely, when applying a SLT in the chronotopical space, the pre sence of the "i's" in the transverse components causes the shape of a tachyon (e.g., intrinsically spherical) to appear essentially as in Figs.19d, 18 and 17 (see Sects.8.2 and 14.6). To be honest, we know how to interpret the last two equations in (154) only in some relevant cases (cf. Sect.14.6). This is a problem still open in part; we want at least to clarify and formalize that reinterpretation procedure at our best. This will be accomplished in the next Section 14.10, for a generic SLT.

Here let us make a comment. The "Lorentz" mappings (154) - after their interpretation - do not seem to carry one any more outside the initial Minkowski space-time M_4. Only for this reason we always used the convention of calling just "transformations" the SLTs (a use well justified in two, or six, dimensions), even if in four dimensions they seem to transform manifolds into manifolds, rather than points into points; on this respect, the critical comments in Sect.8.3 ought to be attentively reconsidered (see also Smrz 1984).

14.10.- The case of the generic SLTs

Let us extend the whole interpretation procedure (of the whole set of four equations constituting a SLT) to the case of a generic SLT "without rotations" (Møller 1962), i.e. of a Superluminal boost $L(\underset{\sim}{U})$ along a generic motion-line ℓ. In terms of the ordinary coordinates x^μ, according to eqs.(150) we shall have ($\underset{\sim}{u}//\underset{\sim}{U}$; $u\equiv 1/U$; $u^2<1$; $U^2>1$):

$$L(\underset{\sim}{U};\, x^\mu) = iL(\underset{\sim}{u};\, x^\mu) = \pm\, i \begin{pmatrix} \gamma & u\gamma n_s \\ -u\gamma n^r & \delta^r_s - (\gamma-1)n^r n_s \end{pmatrix} \qquad \gamma \equiv (1-u^2)^{-\frac{1}{2}} \tag{169}$$

where $L(\underset{\sim}{u})$ is the dual (subluminal) boost along the same ℓ. Quantity n is the unit vector individuating ℓ: $n_r n^r = -1 = -|\underset{\sim}{n}|^2$; it points in the (conventionally) positive direction along ℓ. Notice that u, U may be positive or negative. Eqs.(169) express $L(\underset{\sim}{U})$ in its formal, "original" form, still to be interpreted.

$L(\underset{\sim}{U};x^\mu)$ can be obtained from the corresponding Superluminal boost $L_o(x,U) \equiv B(x)$ along x through suitable rotations ($L_o(x,U) = iL_o(x,u)$; r,s = 1,2,3):

$$L(\underset{\sim}{U},x^\mu) = R^{-1}B(x)R\,; \qquad R \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & n_x & n_y & n_z \\ 0 & -n_y & 1-An_y^2 & -An_yn_z \\ 0 & -n_z & -An_yn_z & 1-An_z^2 \end{pmatrix} \qquad A \equiv (1+n_x)^{-1} \tag{170}$$

where B(x) is given by eqs.(154). Till now we dealt with the interpretation of eqs.(150) only in the case of Superluminal boosts along a Cartesian axis. To interpret, now, also the

$L(\underset{\sim}{U};x^{\mu})$ of eqs.(169), let us compare $L(\underset{\sim}{U})$ with $\bar{L}(\underset{\sim}{U})$, where

$$\bar{L}(\underset{\sim}{U};x^{\mu}) = R^{-1}\bar{B}(x)R \tag{171}$$

and $\bar{B}(x)$ is the (partially) reinterpreted version of eqs.(154), i.e. is given by eqs.(154 bis).

From eqs.(171) and (154bis) we get (Maccarrone et al.1983):

$$\bar{L}(\underset{\sim}{U};x^{\mu}) = \pm \begin{pmatrix} -\bar{\gamma} & -U\bar{\gamma}n_s \\ U\bar{\gamma}\,n^r & i\delta^r_s + (1+\bar{\gamma})n^r n_s \end{pmatrix}; \qquad \begin{bmatrix} U \equiv 1/u \\ r,s = 1,2,3 \end{bmatrix} \tag{172a}$$

where $\bar{\gamma} \equiv (U^2-1)^{-\frac{1}{2}}$ with $U=1/u$; $u^2<1$; $U^2>1$. Eq.(172a) can also be written

$$L(\underset{\sim}{U},x^{\mu}) = \pm \begin{pmatrix} -u\gamma & -\gamma n_s \\ \gamma n^r & i\delta^r_s + (i+u\gamma)n^r n_s \end{pmatrix}, \tag{172b}$$

where γ is defined in eqs.(169), with $|u|<1$. Notice explicitly that the four-dimensional SLTs in their original mathematical form are always <u>purely imaginary</u>; this holds in particular for a generic SLT "without rotations". <u>It will seem to contain complex quantities only in its</u> (partially) <u>reinterpreted form</u>. But this is a "local" fact, relative to the <u>final</u> frame, and due to a trivial effect of the relevant space-rotations: its interpretation is partly related to Fig. 42 (in the following).

Let us also recall that in the case of a chain of GLTs the interpretation procedure is to be applied only <u>at the end of the chain</u> (the reinterpretation, being frame-dependent, breaks the Lorentz invariance).

We have just to compare the matrix in eq.(172) with the matrix in eq.(169), including in it its imaginary coefficient, in order to get the interpretation of eqs.(169). Such a reinterpretation will proceed, as usual, <u>in two steps</u>; the first consisting now in the interpretation of the time coordinate and of the space-coordinate along ℓ; the second one consisting in the interpretation of the imaginary space-coordinates <u>transverse</u> to ℓ. For instance, let us compare eq.(169) with eq.(172b), apart from their double signs:

$$\begin{cases} dt' = i\gamma dt + i\gamma u n_s dx^s \,; \\ dx'^r = -iu\gamma n^r dt + i\delta^r_s dx^s - i(\gamma-1)n^r n_s dx^s \,; \end{cases} \tag{169}$$

$$\begin{cases} dt' = -u\gamma dt - \gamma n_s dx^s \,; \\ dx'^r = \gamma n^r dt + i\delta^r_s dx^s + (u\gamma+i)n^r n_s dx^s \,. \end{cases} \tag{172b}$$

FIRST STEP: recipe:

You can eliminate the imaginary unit in all the addenda containing γ as a multiplier, provided that you substitute t for $r_{\parallel}$ and $r_{\parallel}$ for t (notice that $r_{\parallel} = \underset{\sim}{r}\cdot\underset{\sim}{n} = -n_s x^s$);

SECOND STEP: In the second equations in (169) and (172b) if we put $\underset{\sim}{r} \equiv \underset{\sim}{x} \equiv (x,y,z)$ and $\underset{\sim}{r}' \equiv \underset{\sim}{x}' \equiv (x',y','z')$, we can write $\underset{\sim}{r} = \underset{\sim}{r}_{\parallel} + \underset{\sim}{r}_{\perp}$, where $\underset{\sim}{r}_{\parallel} \equiv (r_{\parallel})\underset{\sim}{n}$ and $\underset{\sim}{r}_{\perp} = \underset{\sim}{r} - r_{\parallel}\underset{\sim}{n} = \underset{\sim}{r}-(\underset{\sim}{r}\cdot\underset{\sim}{n})\underset{\sim}{n}$. Then eq.(172b) can be written in integral form as $\underset{\sim}{r}' = \underset{\sim}{r}'_{\parallel} + \underset{\sim}{r}'_{\perp} = \gamma(t-ur_{\parallel})\underset{\sim}{n} + i\underset{\sim}{r}_{\perp}$; and - after having applied the "first step" recipe - we are left only with $\underset{\sim}{r}'_{\perp} = i\underset{\sim}{r}_{\perp}$; i.e. only with the imaginary terms (not containing γ as a multiplier):

$$d(\underset{\sim}{r}'_{\perp})^r = id(\underset{\sim}{r}_{\perp})^r \equiv i(\delta^r_s + n^r n_s)dx^s \, , \qquad (173)$$

which enter only the expression dx'^r. (Of course $\underset{\sim}{r}_{\perp}$ is a space vector orthogonal to ℓ and therefore corresponds to two further coordinates only). Since eqs.(173) refer to the space-coordinates orthogonal to the boost direction, their imaginary "signs" have to be interpreted so as we did (Fig.19) in Sect.14.6 (and 14.9) for the transverse coordinates y' and z' in the case of Superluminal x-boosts: see Fig. 42.

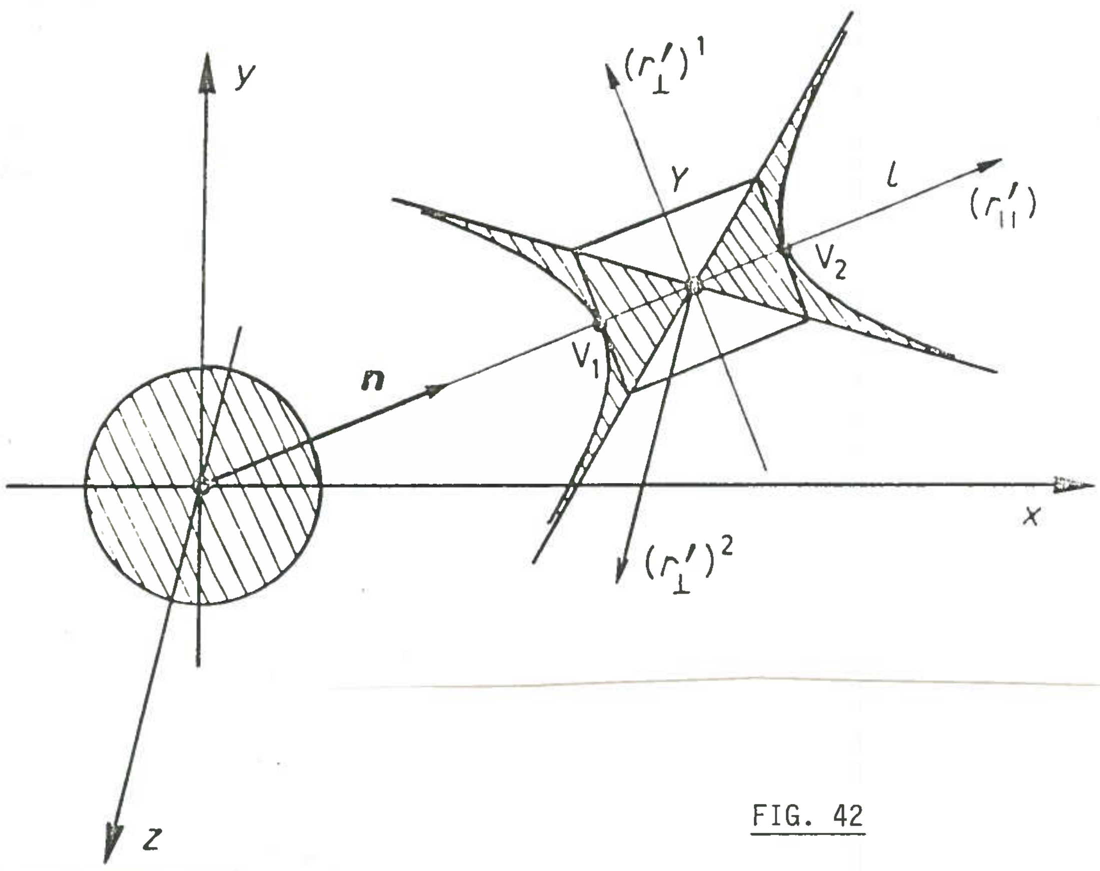


FIG. 42

This means that, if the considered SLT is applied to a body P_B initially at rest (e.g., spherical in its rest frame), we shall finally obtain a body P_T moving along the motion-line ℓ with Superluminal speed V=U, such a body P_T being no longer spherical or ellipsoidal in shape, but appearing on the contrary as confined between a two-sheeted hyperboloid and a double cone, both having as symmetry axis the boost motion-line ℓ. Fig. 42 refers to the case in which P_B is intrinsically spherical; and the double-cone semi-angle α is given by $tg\alpha = (V^2-1)^{-\frac{1}{2}}$. More in general, the axis of the tachyon shape will not coincide with ℓ (but will depend on the tachyon speed V=U).

More precisely, the vector $\underset{\sim}{R}'_\perp$, apart from its imaginary "sign", - i.e. the vector $\underset{\sim}{R}_\perp$ - can be described by the two coordinates $R_\perp^{(1)}=Y_0$; $R_\perp^{(2)}=Z_0$ such as in Sects. 14.6 and 14.9: see Figs.38 and 42.

We see once more that this reinterpretation "second step" works only in particular special cases. To clarify a bit more the present situation, Maccarrone et al.(1983) emphasized the following points: (i) one is not supposed to consider (and reinterpret) the GLTs when they are applied just to a "vacuum point"; actually, we know from SR that each observer has a right to consider the vacuum as at rest w.r.t. himself; (ii) one should then apply - and eventually reinterpret - the GLTs, in particular the SLTs, only to transform the space-time regions associated with physical objects; these are considered as extended objects (Kálnay 1978), the point-like situation being regarded only as a limiting case; (iii) the extended-type object is referred to a frame with space-origin in its center of symmetry.

Many problems remain still open, therefore, in connection with such a "second step" of the interpretation (cf. Sects.14.14-14.16).

14.11.- Preliminaries on the velocity composition problem

Let us apply a SLT in the form (172a) along the generic motion-line ℓ with Superluminal speed U=1/u ($U^2>1$; $u^2<1$) to a bradyon P_B having initial four-velocity u^μ and velocity $\underset{\sim}{v}$. Again, one should pay attention to not confuse the boost speeds u, U with the four-velocity components u^μ of P_B. For the purpose of generality, v and U should not be parallel. We get

$$u'^o = -\bar{\gamma}(u^o - Uu_\parallel) \quad ; \qquad u'^r = -\bar{\gamma}(u_\parallel - Uu^o)n^r + iu_\perp^r \qquad (174)$$

where $u_\parallel = -u^s n_s$; $u_\perp^r = u^r + u^s n_s u^r = u^r - u_\parallel n^r$, and $\underset{\sim}{n}$ is still the unit vector along ℓ, while $\gamma \equiv (U^2-1)^{-\frac{1}{2}}$ so as in eq.(172a). Notice that u'_o is real, while the second equation in (174) rewrites ($u'_\parallel = -u'^s n_s$):

$$u'_\parallel = -\bar{\gamma}(u_\parallel - Uu^o) = -(u_\parallel - Uu^o)/\sqrt{U^2-1} \quad ; \qquad u'_\perp = iu_\perp \, , \qquad (U^2>1) \qquad (175)$$

where $u'_\parallel$ is real too and only $u'_\perp$ is purely imaginary; $u_\parallel$, $u'_\parallel$ ($u_\perp$, $u'_\perp$) are the longitudinal (transverse) components w.r.t. the boost-direction.

If we define the 3-velocity $\underset{\sim}{V}'$ for tachyons in terms of the 4-velocity u^μ (j=1,2,3):

$$u'^j \equiv \frac{V'^j}{\sqrt{V'^2-1}} \quad ; \qquad u'^o \equiv \frac{1}{\sqrt{V'^2-1}} \quad , \tag{176}$$

eqs.(175) yield:

$$\begin{aligned} V'_{\parallel} &= \frac{U - v_{\parallel}}{Uv_{\parallel} - 1} \equiv \frac{1 - uv_{\parallel}}{v_{\parallel} - u} \quad ; \\ V'_{\perp} &= i\,\frac{v_{\perp}\sqrt{U^2-1}}{Uv_{\parallel} - 1} \equiv \frac{iv_{\perp}\sqrt{1-u^2}}{v_{\parallel} - u} \quad . \end{aligned} \qquad (U^2 > 1 \ ; \ \ u^2 < 1 \ ; \ U \equiv 1/u) \tag{177}$$

It may be noticed that: $V'_{\parallel} = 1/\tilde{v}_{\parallel}$; $V'_{\perp} = i\tilde{v}_{\perp}/\tilde{v}_{\parallel}$, where $\tilde{\underline{v}}$ is the transform of $\underline{v}$ under the dual (subluminal) Lorentz transformation $L(\underline{u})$ with $u=1/U$; $\underline{u}//\underline{U}$. Again $V'_{\parallel}$ is real and $V'_{\perp}$ pure imaginary. However, V'^2 is always positive so that $|\underline{V}'|$ is real and even more Superluminal; in fact:

$$V'^2 = V'^2_{\parallel} + V'^2_{\perp} = V'^2_{\parallel} - |V'_{\perp}|^2 > 1 \ . \tag{178}$$

More in general, eqs.(177) yield for the magnitudes

$$1 - V'^2 = \frac{(1-v^2)(1-U^2)}{(1-\underline{U}\cdot\underline{v})} \quad , \qquad (v^2 < 1 \ ; \ U^2, \ V'^2 > 1) \tag{179}$$

which, incidentally, is a G-covariant relation. Let us recall that eqs.(174), (175) and (177) have been derived from the (partially) reinterpreted form of SLTs; therefore they do not possess group-theoretical properties any longer. For instance, eqs.(177) cannot be applied when transforming (under a SLT) a speed initially Superluminal.

Eq.(179) shows that under a SLT a bradyonic speed v goes into a tachyonic speed V'. But we have still to discuss the fact that the tachyon 3-velocity components transverse to the SLT motion-line are imaginary (see the second equation in (177)).

We shall proceed in analogy with Sects.14.6 and 14.10. Let us initially consider, in its c.m. frame, a spherical object with center at O, whose external surface expands in time for $t \geqslant 0$ ("symmetrically exploding spherical bomb"):

$$0 \leqslant x^2 + y^2 + z^2 \leqslant (R + vt)^2 \qquad\qquad (t \geqslant 0) \tag{180}$$

where R and v are fixed quantities. In Lorentz-invariant form (for the subluminal observers), the equation of the "bomb" world-cone is (Maccarrone et al.1983):

$$\begin{aligned} &(x_{\mu} + b_{\mu})(x^{\mu} + b^{\mu}) \leqslant \frac{\left[(x^{\mu}+b^{\mu})u_{\mu}\right]^2}{u_{\mu}u^{\mu}} \leqslant (1-v^2)^{-1}(x_{\mu}+b_{\mu})(x^{\mu}+b^{\mu}) \ ; \\ &x^{\mu} \geqslant 0 \ , \end{aligned} \tag{180'}$$

where $x^{\mu} = (t,x,y,z)$ is the generic event inside the (truncated) world-cone, vector u^{μ} is the "bomb" center-of-mass fourvelocity, and $b^{\mu} \equiv u^{\mu}R/v$. One can pass to Superluminal observers S' just recalling that (Sect.8.2) the SLTs invert the quadratic-form sign (cf. however also Sect.8.3). If S' just moves along the x-axis with Superluminal speed -U, the first limiting equality in eq.(180') transforms, as usual, into the equation of a double cone symmetrical w.r.t. the x-axis and travelling with speed V=U along the axis $x \equiv x'$. The second inequality in eq.(180') transforms on the contrary into the equation

$$(1-v^2V^2)x'^2-(V^2-1)(y'^2+z'^2)-2t'V(1-v^2)x'-2RvV\sqrt{V^2-1}\;x' \leqslant$$
$$\leqslant (V^2-1)R^2-2t'Rv\sqrt{V^2-1}-(V^2-v^2)t'^2 \,. \qquad (x' \geqslant t'/V) \qquad (181)$$

When it is $vV < 1$, the equality sign in eq.(181) corresponds to a two-sheeted hyperboloid whose position relative to the double cone does change with time (Fig. 43). The distance between the two hyperboloid vertices, e.g., reads $V_2-V_1=2(1-v^2V^2)^{-1}\left[t'v(V^2-1)+R\sqrt{V^2-1}\right]$. When in eq.(181) it is $vV > 1$ the geometrical situation gets more complicated.

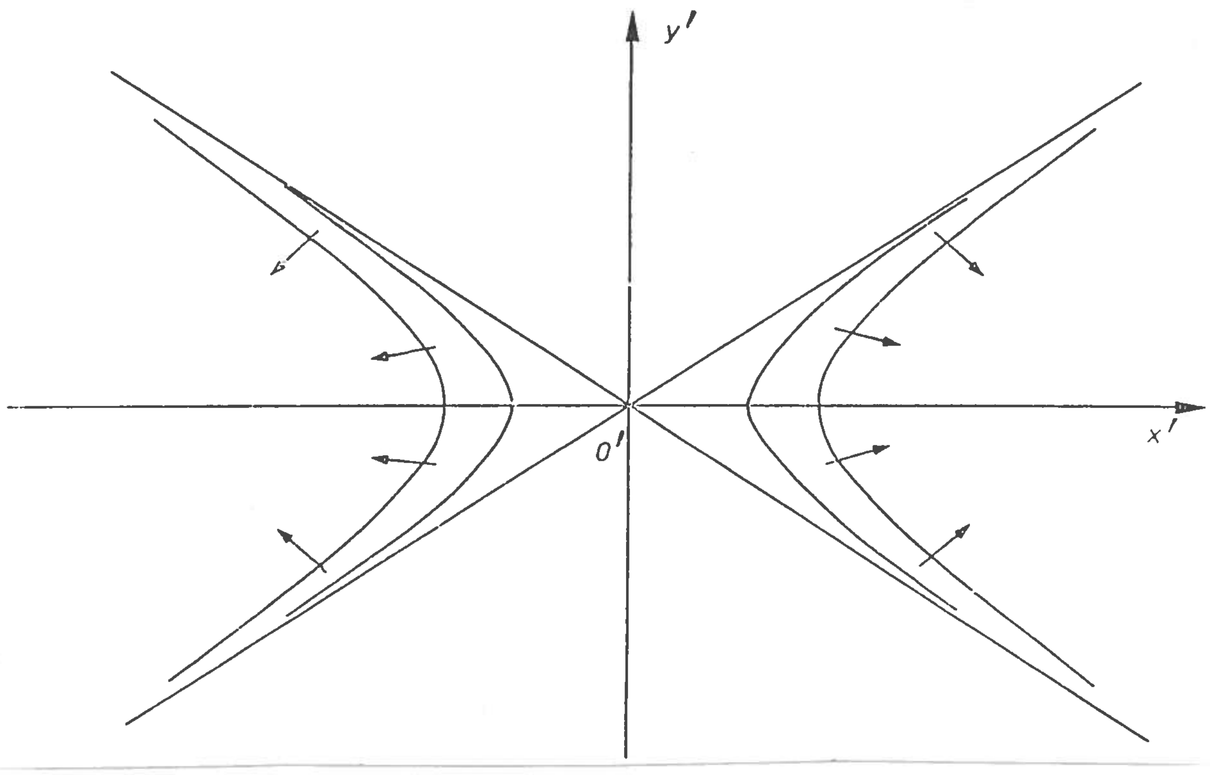


FIG. 43

But, in any case, the "bomb" is seen by the Superluminal observers to explode remaining always confined within the double cone.

This means that: (i) as seen by the subluminal observers s a (bradyonic) bomb explodes in all space directions, sending its "fragments" also - e.g. - along the y and z axes with speeds v_y and v_z respectively; (ii) as seen by the Superluminal observers S', however, the (tachyonic) bomb looks to explode in two "jets" which remain confined within the double cone, in such a way that no "fragments" move along the y' or z' axis. In other words, the speeds V'_y, V'_z of the tachyonic bomb fragments "moving" along the y', z' axes, respectively, would result to be imaginary (Maccarrone et al.1983; see also Corben 1974,1975).

14.12.- Tachyon four-velocity

Let us refer, for the particular case of Superluminal x-boosts in four dimensions, to eqs.(154) and (154bis). Let us recall that in this particular case the SLTs - after their partial interpretation - coincide with the ones proposed by Mignani and Recami (Review I).

We want to reconsider *ab initio* the problem of introducing the 3- and 4-velocity vectors for tachyons.

In agreement with eqs.(150), we have seen that, if a subluminal LT carries from the "rest-frame" s_o to a frame s endowed with velocity $\underline{u}$ relative to s_o, then the dual SLT must carry from s_o to the frame S' endowed with velocity $U_x=u_x/u^2$; $U_y=u_y/u^2$; $U_z=u_z/u^2$, such that $U^2=1/u^2$. By refering to the auxiliary space-time M_6 and to the names attributed to the axes by the initial observer s, the second observer S' is expected to define the 3-velocity of the observed object as follows (Sect.14.6):

$$V'_x = \frac{d\tilde{t}_x}{d\tilde{x}} \; ; \quad V'_y = \frac{d\tilde{t}_y}{d\tilde{x}} \; ; \quad V'_z = \frac{d\tilde{t}_z}{d\tilde{x}} \; , \qquad \text{(Superluminal boost)} \tag{182}$$

where the tilde indicates the transformation accomplished by the dual, subluminal LT (actually, $d\tilde{t}_y=dt_y$; and $d\tilde{t}_z = dt_z$); the tilde disappears when the considered SLT is a transcendent Lorentz boost: $V'_x \equiv dt_x/dx$; $V'_y \equiv dt_y/dx$; $V'_z \equiv dt_z/dx$. However, due to our Postulates, S' in his terminology will of course define the 3-velocity of the observed tachyon in the ordinary way

$$V'_x \equiv \frac{dx'}{dt} \; ; \quad V'_y \equiv \frac{dy'}{dt} \; ; \quad V'_z \equiv \frac{dz'}{dt} \tag{183}$$

where dx', dy', dz' are a priori given by eqs.(154).

Identifying eqs.(183) with (182), on the basis of eqs.(154bis) we get (see Fig.44):

$$V'_x \equiv \frac{dx'}{dt'} = \frac{d\tilde{t}}{d\tilde{x}} \; ; \quad V'_y \equiv \frac{dy'}{dt'} = \frac{d\tilde{y}}{d\tilde{x}} \; ; \quad V'_z \equiv \frac{dz'}{dt'} = \frac{d\tilde{z}}{d\tilde{x}} \; , \tag{184}$$

where, in the present case, $d\tilde{y}=dy$; $d\tilde{z}=dz$. Namely, apart from the sign, the SLTs yield the final relations ($dt_x \equiv dt$):

$$V'_x \equiv \frac{dx'}{dt'} = \frac{dt-udx}{dx-udt} \; ; \quad V'_y \equiv \frac{dy'}{dt'} = i\,\frac{dy\sqrt{1-u^2}}{dx-udt} \; ; \quad V'_z \equiv \frac{dz'}{dt'} = i\,\frac{dz\sqrt{1-u^2}}{dx-udt} \; , \tag{184'}$$

relating the observations made by s on P_B with the observations made by S' on P_T (trans-

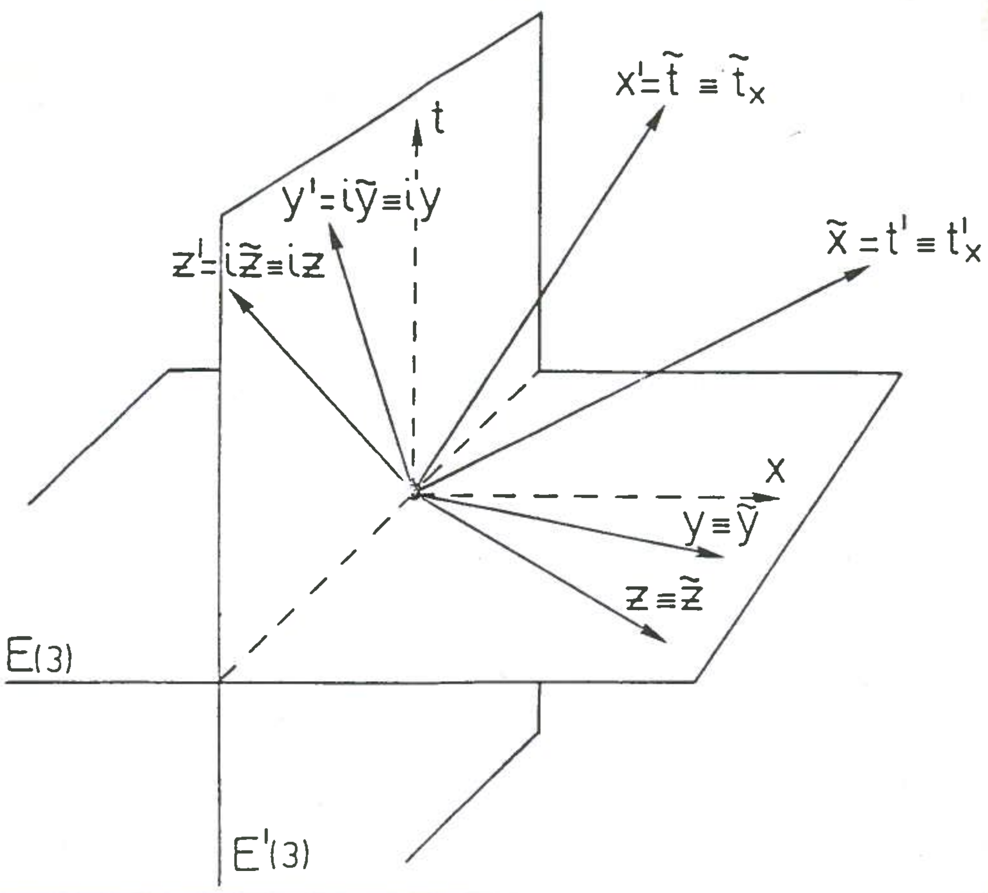


FIG. 44

form of P_B under the considered Superluminal boost). The imaginary units in the transverse-components mean a priori that the tachyon P_T moves, w.r.t. S', with velocity $\underset{\sim}{V}'$ in the M'_4 space-time (see the following). From eqs.(184') one immediately sees that

$$V'_x \tilde{v}_x = 1 \tag{185}$$

and in particular $V'_x v_x = 1$ when SLT= $\mathcal{S}$. Notice, therefore, that the dual correspondence $V' \rightleftarrows c^2/v$ holds only for the velocity components along the SLT direction; that correspondence does not hold for the transverse components (even if $V'_y \neq v_y$ and $V'_z \neq v_z$), nor for the magnitudes V' and $\tilde{v}$. In fact ($v \equiv |v|$; v^2, $u^2 < 1$):

$$V'_x = \frac{1-uv_x}{v_x-u} = \frac{1}{\tilde{v}_x}\ ; \qquad V'_y = \frac{iv_y\sqrt{1-u^2}}{v_x-u} = i\,\frac{\tilde{v}_y}{\tilde{v}_x}\ ,$$

$$V'_z = \frac{iv_z\sqrt{1-u^2}}{v_x-u} = i\,\frac{\tilde{v}_z}{\tilde{v}_x}\ ; \tag{186}$$

cf. also Sect.14.11. That is to say, the transverse components V'_y, V'_z are connected with the longitudinal component V'_x in the same way as in the ordinary subluminal SR (Maccarrone and Recami 1984). Eqs.(186), as well as eqs.(158'), confirm that eqs.(154) are actually associated with Superluminal motion, notwithstanding their appearance. Eqs.(186) can be written, in terms of the Superluminal-boost speed ($U^2 > 1$):

$$V'_x = \frac{U-v_x}{Uv_x-1} \; ; \qquad V'_y = \frac{iv_y\sqrt{U^2-1}}{Uv_x-1} \; ; \qquad V'_z = \frac{iv_z\sqrt{U^2-1}}{Uv_x-1} \tag{186'}$$

which express the velocity-composition law in the case of Superluminal boosts.

Let us stress, again (see eqs.(178) and Sect.14.11), that from eqs.(186)-(186') one can verify that always

$$\underset{\sim}{V}'^2 > 1 \tag{186"}$$

even if $V'_y \leqslant 0$ and $V'_z \leqslant 0$, so that $1 < V'^2 \leqslant V_x'^2$. This means that $V' \equiv |\underset{\sim}{V}'|$ is always real and Superluminal. See also eq.(179).

In terms of four-velocities, the composition of a subluminal generic fourvelocity v_μ with a Superluminal x-boost fourvelocity U_μ will yield

$$\begin{aligned} V'_o &= v_1 u_o + v_o u_1 = -(v_1 U_1 + v_o U_o) \\ V'_1 &= v_o u_o + v_1 u_1 = -(v_o U_1 + v_1 U_o) \\ V'_{2,3} &= i\, v_{2,3} \end{aligned} \qquad \left[\begin{array}{l}\text{Superluminal boost} \\ v_\mu v^\mu = +1; \; U_\mu U^\mu = -1\end{array}\right] \tag{187}$$

which do coincide with eqs.(186'). The x-boost Superluminal speed is $-U$ with $U \equiv 1/u$. Let us repeat that eqs.(186)-(187) should not be applied, when starting from a Superluminal speed $|\underset{\sim}{v}| > 1$, since applying the (partial) interpretation broke the group-properties.

We shall come back to the problem of the imaginaries in the transverse components of eqs.(186), (186') in Sect.14.15.

14.13.- Tachyon four-momentum

Let us apply the SLTs to the fourmomentum, defined in a G-covariant way as follows:

$$p_\mu \equiv m_o v_\mu \; ; \qquad v_\mu \equiv dx_\mu / d\tau_o \; . \qquad (v^2 \gtrless 1) \tag{188}$$

Then p_μ is a G-vector, and we can apply to it eqs.(154), or (154bis). The latter yield, for the tachyon fourmomentum (obtained by applying a Superluminal boost along x to a bradyon B with 3-velocity $\underset{\sim}{v}$; $|\underset{\sim}{v}| \equiv v < 1$):

$$\begin{aligned} p'_o &= \pm \frac{p_1 - u p_o}{\sqrt{1-u^2}} \equiv \mp \frac{p_o - U p_1}{\sqrt{U^2-1}} \; ; \qquad p'_1 = \pm \frac{p_o - u p_1}{\sqrt{1-u^2}} \equiv \mp \frac{p_1 - U p_o}{\sqrt{U^2-1}} \; ; \\ p'_2 &= \pm i p_2 \; ; \qquad p'_3 = \pm i p_3 \; , \end{aligned} \tag{189}$$

wherefrom, among the others, $p'_{2,3} = \pm im_o v_{2,3} = \pm im_o v_{y,z} / \sqrt{1-v^2} = \pm imv_{y,z}$. Do not confuse the fourvelocity components $v_{2,3}$ with the three-velocity components $v_{y,z}$; and so on. Attention must be paid, moreover, to the fact that $\underset{\sim}{v}$, v_μ refer to the initial bradyon (in the initial frame), while $\underset{\sim}{U}$ and its dual velocity u refer to the SLT.

Eqs.(189) can be rewritten (Maccarrone and Recami 1984):

$$p'_o = \pm m \frac{v_x - u}{\sqrt{1-u^2}} \equiv \mp m \frac{1-Uv_x}{\sqrt{U^2-1}} \; ; \qquad p'_1 = \pm m \frac{1-uv_x}{\sqrt{1-u^2}} \equiv \mp m \frac{v_x - U}{\sqrt{U^2-1}} \; ;$$

$$p'_2 \equiv m_o V'_2 = \pm imv_y = \pm im_o v_2 \; ; \qquad p'_3 \equiv m_o V'_3 = \pm imv_z = \pm im_o v_3 \; .$$

Notice that, even if these equations express the fourmomentum of the final tachyon $T \equiv P_T$, nevertheless m and v_x, v_y, v_z represent the relativistic mass and the 3-velocity components of the initial bradyon $B \equiv P_B$ (in the initial frame), respectively; in particular

$$m = \frac{m_o}{\sqrt{1-v^2}} \; . \qquad (v^2 \equiv \underset{\sim}{v}^2 < 1)$$

By comparing eqs.(189) with the velocity-composition law (186)-(186'), it follows even for tachyons that

$$p'_o = \frac{m_o}{\sqrt{V'^2-1}} \; ; \qquad p'_2 = \frac{m_o V'_x}{\sqrt{V'^2-1}} \; ;$$
$$p'_2 = \frac{m_o V'_y}{\sqrt{V'^2-1}} \; ; \qquad p'_3 = \frac{m_o V'_z}{\sqrt{V'^2-1}} \; . \qquad (V'^2 \equiv \underset{\sim}{V}'^2 > 1) \qquad (190)$$

Since V'_y and V'_z are imaginary, V'_2 and V'_3 are imaginary as well, in agreeement with the relations $V'_2 = \pm iv_2$; $V'_3 = \pm iv_3$.

Finally, comparing eqs.(190) with (188), one derives that even in the tachyon case the 4-velocity and the 3-velocity are connected as follows:

$$V'_o = \frac{1}{\sqrt{V'^2-1}} \; ; \qquad V'_1 = \frac{V'_x}{\sqrt{V'^2-1}} \; ;$$
$$V'_2 = \frac{V'_y}{\sqrt{V'^2-1}} \; ; \qquad V'_3 = \frac{V'_z}{\sqrt{V'^2-1}} \; , \qquad (191)$$

when $V'^2 \equiv \underset{\sim}{V}'^2$. In conclusion, the eqs.(188)-(191) that we derived in the tachyonic case from eqs.(154bis) are self-consistent and constitute a natural extension of the corresponding subluminal formulae. For instance, it holds in G-covariant form :

$$v'_o = (\sqrt{|1-\underset{\sim}{v}'^2|})^{-1} \; ; \quad v'_{1,2,3} = v'_{x,y,z}(\sqrt{|1-\underset{\sim}{v}'^2|})^{-1} \; . \qquad (\underset{\sim}{v}'^2 \gtrless 1)$$

Since v_μ, like x_μ and p_μ, is a G-vector, we may apply the SLTs directly to v_μ. By applying a Superluminal boost, one gets

$$v'_o = \mp \frac{v_o - Uv_1}{\sqrt{U^2-1}} \; ; \qquad v'_1 = \mp \frac{v_1 - Uv_o}{\sqrt{U^2-1}} \; ;$$

$$v'_{2,3} = \pm i v_{2,3} = \pm i \frac{v_{y,z}}{\sqrt{1-\underset{\sim}{v}^2}} \; . \qquad (U^2 > 1 \; ; \; \underset{\sim}{v}^2 < 1) \qquad (192)$$

14.14.- Is linearity strictly necessary?

We might have expected that transformations $\mathcal{T}$: $x_\mu \rightarrow x'_\mu$ mapping points of M_4 into points of M_4 (in such a way that $ds^2 \rightarrow - ds^2$) did not exist. Otherwise, real linear SLTs: $dx_\mu \rightarrow dx'_\mu$ of the tangent vector space associated with the original manifold map $\mathcal{T}$ should have existed (Rindler 1966, Smrz 1984). But we saw, already at the end of Sect.3.2, that real linear SLTs (meeting the requirements (ii)-(iv) of Sect.4.2) do not exist in four dimensions.

On the contrary, the results in Sect.8.2, as well as in Sects.14.6 and 14.11, seem to show that in the Superluminal case in M_4 we have to deal with mappings that transform manifolds into manifolds (e.g., points into cones). In Sect.8.3 we inferred the SLTs: $dx_\mu \rightarrow dx'_\mu$ to be linear but not real; just as we found in the present Section 14.

We may however - and herhaps more soundly - make recourse to non linear (but real) SLTs.

If we consider SLTs: $dx_\mu \rightarrow dx'_\mu$ real but not linear, then Superluminal maps $\mathcal{T}$: $M_4 \rightarrow M_4$ (carrying points into points) do not exist. We already realized this. The important point, in this case, is that the "Superluminal mappings" $\mathcal{T}$ (trsnforming then manifolds into manifolds) be compatible with the Postulates of SR; in particular (Sect.4.2): (i) transform inertial motion into inertial motion; (ii) preserve space isotropy (and homogeneity); (iii) preserve the light-speed invariance.

To meet the group-theoretical requirements, we have to stick to eqs.(154) and to their integral form. But their reinterpretation - accomplished in this Sect.14 and anticipated in Sect.8 - does comply with conditions (i)-(iii) above. For example, it leads from a point-like bradyon moving with constant velocity to a tachyon spatially extended, but still travelling with constant velocity. The problem is now to look for real, non-linear SLTs (i.e., mappings of the tangent vector space), and substitute them for the linear non-real eqs.(154bis); with the hope that the new (non-linear) SLTs can yield more rigorously the same results met before, thus solving the problems left open by the previous "second step" reinterpretation. For a discussion of such topics see also Smrz (1984).

14.15.- An attempt

A temptative approach to real, non linear SLTs can be suggested by investigating the difficulty mentioned at the end of Sect.14.12 (i.e., the still present difficulty of the imaginaries in the transverse components of eqs.(186), (186')).

The 3-velocity $\underset{\sim}{W}'$ of the tachyon "barycenter", i.e. of the vertex of the "enveloping cone" $\mathscr{C}$ (Figs.18 and 42), must be real, in any case. For example (see Sect.14.12), in the trivial case in which $v_y=v_z=0$, it is simply $W'\equiv W'_x=V'_x=V'$. More generally, when concerned with the overall velocity $\underset{\sim}{W}'$ of the considered tachyon T, the imaginaries in the transverse components essentially record the already mentioned fact that, by composing $\underset{\sim}{U}$ with $\underset{\sim}{v}$, one gets a velocity $\underset{\sim}{V}'$ whose magnitude V' is smaller than V'_x (Sect.14.12). In the particular case when $\underset{\sim}{U}$ and $\underset{\sim}{v}$ are directed along x and y, respectively, and $|\underset{\sim}{v}| \ll 1$, one may conclude that (Fig. 45):

$$|\underset{\sim}{W}'| \simeq \sqrt{U^2-v_y^2} \equiv \sqrt{\underset{\sim}{U}^2-\underset{\sim}{v}^2}\ ; \qquad \mathrm{tg}\,\alpha \equiv \frac{W'_y}{W'_x} \simeq v\,\frac{\sqrt{U^2-1}}{U}\ , \tag{193}$$

which yield also the direction α of $\underset{\sim}{W}'$ (Maccarrone and Recami 1984). Notice that $W'_x = |\underset{\sim}{W}'|\cos\alpha$ and $W'_y = |\underset{\sim}{W}'|\sin\alpha$, but $W'_x \neq V'_x$ and $W'_y \neq V'_y$.

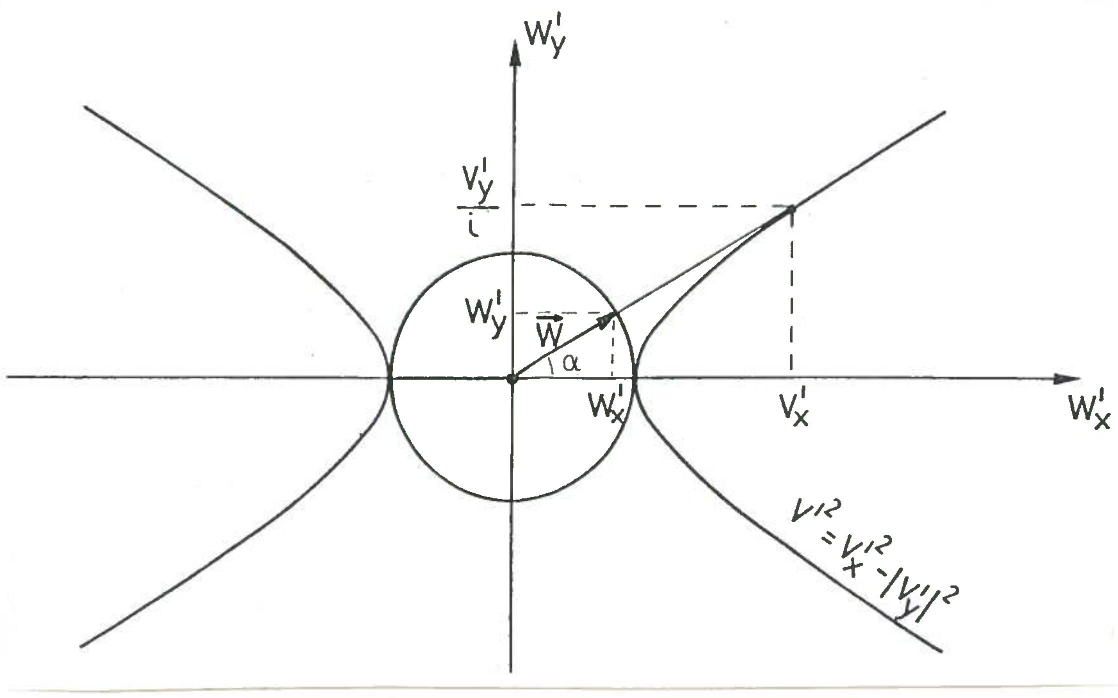


FIG. 45

The second equation in (193) can be obtained from the following intuitive analysis. Let us recall what seen in Sects.8.2 and 14.6 for an (intrinsically) spherical object P, initially at rest w.r.t. a certain frame s_o and with its center C at the space-origin O of s_o. When travelling along x with Superluminal speed $|\underset{\sim}{W}'| = W' = W'_x$ it will appear to s_o as in Fig.19d (where for simplicity only the plane (x,y) is shown). It is trivial to extend the previous picture by requiring that, when $C \neq O$, for instance $C \equiv (0, \bar{y})$, the shape of P will be obtained by shifting the shape in Fig.19d along y by the quantity $\Delta y = \bar{y}$ (if the laboratory containing P travels again with speed W', parallel to the x-axis).

If P is now supposed to move slowly along y in the lab, and the laboratory travels parallel to x with speed W' w.r.t. s_o, it is sensible to expect that P will appear to s_o with a shape still similar to Fig.19d, but travelling along a (real) line inclined w.r.t. the x-axis by an angle α. It is what we showed in Sect.14.10: see Fig. 42.

The "reinterpretation" of the cone-vertex velocity (i.e. of the overall tachyon velocity) suggested by the previous intuitive remarks is, then, the one shown in Fig. 45; where we consider for simplicity $W'_z = V'_z = 0$. Recall that the magnitude of the tachyon overall velocity is $W' \equiv |\underset{\sim}{W}'| = W'^2_x + W'^2_y = V'^2_x - |V'_y|^2$, since $V'_y = iv_y\sqrt{U^2-1}/(Uv_x-1)$ is imaginary. According to the interpretation here proposed for the velocity transverse components, the direction of W' is given by $tg\alpha \equiv W'_y/W'_x = (V'_y/i)/V'_x$ (see eq.(188)).

14.16.- Real, non-linear SLTs: A temptative proposal

The interpretation proposed in the previous Sect.14.15 has been shown by Maccarrone to correspond to the real, non-linear transformations ($|\underset{\sim}{W}'| = |\underset{\sim}{V}'|$):

$$W'_x = \frac{1}{\tilde{v}_x} A \; ; \qquad W'_y = \frac{\tilde{v}_y}{\tilde{v}_x} A \; ; \qquad W'_z = \frac{\tilde{v}_z}{\tilde{v}_x} A \; ;$$
$$A \equiv \left[(1-\tilde{v}^2_y-\tilde{v}^2_z)/(1+\tilde{v}^2_y+\tilde{v}^2_z)\right]^{1/2} \qquad (194)$$

where (Sect.14.12) $\underset{\sim}{\tilde{v}}$ is given by the dual, subluminal Lorentz transformation (v^2, $u^2 < 1$):

$$\tilde{v}_x = \frac{v_x - u}{1-uv_x} \; ; \qquad \tilde{v}_y = \frac{v_y\sqrt{1-u^2}}{1-uv_x} \; ; \qquad \tilde{v}_z = \frac{v_z\sqrt{1-u^2}}{1-uv_x} \; . \qquad (|\underset{\sim}{v}| < 1)$$

In terms of the 4-velocity, eqs.(194) write (cf. eqs.(191)):

$$W'_o = \frac{\tilde{v}_x}{\sqrt{1-\tilde{v}^2}} \; ; \qquad W'_1 = \frac{A}{\sqrt{1-\tilde{v}^2}} \equiv B \; ;$$
$$W'_2 = \tilde{v}_y B \; , \qquad W'_3 = \tilde{v}_z B \; . \qquad (W'_\mu W'^\mu = -1 \; ; \; \underset{\sim}{\tilde{v}}^2 \equiv \tilde{v}^2) \qquad (195)$$

Eqs.(195) should then hold for all tangent vectors. We are therefore led to the real SLTs: $dx_\mu \rightarrow dx'_\mu$:

$$dt' = \tilde{v}_x \sqrt{(1-v^2)/(1-\tilde{v}^2)}\,dt \;; \qquad dx' = B\sqrt{1-v^2}\,dt \equiv C dt \;;$$

$$dy' = \tilde{v}_y\, C dt \;; \qquad dz' = \tilde{v}_z\, C dt \;, \tag{196}$$

which are non-linear, but carry $ds^2 \rightarrow -ds^2$, transform inertial motion into inertial motion, and preserve space isotropy (and homogeneity) since they do not explicitly depend on the space-time position nor on any particular space direction. Notice, moreover, that $d\tau_o = dt'\sqrt{W'^2-1} = dt'\sqrt{(1-\tilde{v}^2)}/\tilde{v}_x$.

Since any kind of real, non-linear SLTs, so as eqs.(196), constitute a reinterpretation of eqs.(154), we do not expect them to possess group-theoretical properties (which still appear possessed only by SLTs in their mathematical, formal expression (154)).

14.17.- Further remarks

Let us recall here the following further points:

(i) At the beginning of Sect.14.7 we mentioned the possibility of introducing ab initio a complex space-time.

(ii) At the end of Sect.13.8 we stressed the possible rôle of quaternions in the description of tachyons (see also Souček 1981, Mignani 1978, Edmonds 1978).

(iii) Kálnay (1978,1980 , Kálnay and Toledo 1967) showed in particular how to describe the four-position of extended-type objects (cf. e.g. Santilli 1983) by complex numbers (see also Olkhovsky and Recami 1970). According to that author, genuine physical information goes lost when physics is eclusively constrained to real variables.

(iv) Further considerations on the issues of this Sect.14 can be found (besides in the quoted literature Maccarrone et al. 1983, Maccarrone and Recami 1984) in Smrz (1984).

15.- ON TACHYON ELECTROMAGNETISM

We preliminarily introduced the "generalized Maxwell equations" (in terms of the four-potential) already in Sect.10.5. The method followed there is noticeable since it does not depend on the explicit form of the SLTs.

If we now make recourse, however, to SLTs in their form (154bis), we can generalize Maxwell equations in a more convincing way for the case in which both sub- and Super-luminal charges are present. It is noteworthy that, even if imaginary quantities enter the last two equations in (154bis), nevertheless the generalized Maxwell equations can be expressed in purely real terms (see e.g. Recami and Mignani 1974a, Corben 1978); we already mentioned, actually, that this seems to happen for all the fundamental classical equations for tachyons (Review I). Therefore, it is not strictly necessary to pass to a multi-dimensional space-time for exploiting tachyon electromagnetism; but interesting work has been done for example in six dimensions (see e.g. Dattoli and Mignani 1978, Cole 1980a, Patty 1982).

Before going on, let us recall that the ordinary Maxwell equations read (μ,ν = 0,1, 2,3):

$$\partial_\nu F^{\mu\nu} = j^\mu \; ; \qquad \partial_\nu \tilde{F}^{\mu\nu} = 0 \quad , \tag{197}$$

where $j^\mu \equiv (\varrho_e, \underset{\sim}{j})$ and $\tilde{F}$ is the tensor dual to the electromagnetic tensor $F^{\mu\nu}$:

$$\tilde{F}_{\alpha\beta} \equiv \frac{1}{2}\, \varepsilon_{\alpha\beta\gamma\delta} F^{\gamma\delta} \quad . \qquad (\alpha,\ \beta,\ \gamma,\ \delta = 0,1,2,3) \tag{197'}$$

Notice that $\tilde{\tilde{F}}_{\mu\nu} = F_{\mu\nu}$. Typically, the present duality effects the exchanges

$$\underset{\sim}{E} \rightarrow i\underset{\sim}{H} \; ; \qquad \underset{\sim}{H} \rightarrow -i\underset{\sim}{E} \; . \tag{198}$$

In terms of the autodual electromagnetic tensor (Review I)

$$T_{\mu\nu} \equiv F_{\mu\nu} + \tilde{F}_{\mu\nu} \quad ; \qquad \tilde{T}_{\mu\nu} = T_{\mu\nu} \quad , \tag{199}$$

which is invariant under the "duality exchanges" (198), the eqs.(197) write

$$\partial_\nu T^{\mu\nu} = j^\mu \quad ; \qquad \tilde{T}_{\mu\nu} = T_{\mu\nu} \quad . \tag{200}$$

When in presence also of ordinary magnetic monopoles (Dirac 1931), i.e. also of a magnetic current $g^\mu = (\varrho_m, \underset{\sim}{g})$, eqs.(197) and (200) get "symmetrized":

$$\partial_\nu F^{\mu\nu} = j^\mu \quad ; \qquad \partial_\nu \tilde{F}^{\mu\nu} = i g^\mu \tag{201a}$$

$$\partial_\nu T^{\mu\nu} = j^\mu + i g^\mu \quad ; \qquad \tilde{T}_{\mu\nu} = T_{\mu\nu} \; . \tag{201b}$$

Eqs.(200), (201) are covariant, besides under the Lorentz group, also (among the others) under the duality transformations: that is to say, under eqs.(198) and under more general rotations in the space $\underset{\sim}{\mathcal{F}} = \underset{\sim}{E} + i\underset{\sim}{H}$ (see e.g. Amaldi 1968; Amaldi and Cabibbo 1972, Ferrari 1978).

At lsat, let us recall the under subluminal x-boosts the electric and magnetic field components transform as follows ($u^2 < 1$):

$$E_x = E'_x \;;\quad E_y = \frac{E'_y + uH'_z}{\sqrt{1-u^2}} \;;\quad E_z = \frac{E'_z - uH'_y}{\sqrt{1-u^2}} \;;$$
$$H_x = H'_x \;;\quad H_y = \frac{H'_y - uE'_z}{\sqrt{1-u^2}} \;;\quad H_z = \frac{H'_z + uE'_y}{\sqrt{1-u^2}} \,. \qquad (202)$$

15.1.- Electromagnetism with tachyonic currents: Two alternative approaches

Let us suppose the existence of slower and faster than light electric charges, corresponding to the two fourcurrents $j^{\mu}(s) \equi (\varrho(s), \underset{\sim}{j}(s))$ and $j^{\mu}(S) \equiv (\varrho(S), \underset{\sim}{j}(S))$.

In analogy with what we mentioned in Sect.10.5, the electromagnetic tensor $F^{\mu\nu}$ may not be any more a tensor under the SLTs; i.e., it cannot be expected a priori to be a G--tensor (Sect.7.2). According to the way one solves this problem, different theories follow (see Recami and Mignani 1974a).

It is then sound to pass and investigate how the $\underset{\sim}{E}$ and $\underset{\sim}{H}$ components are expected to transform under SLTs. Let us confine to Superluminal x-boosts.

(i) If one wishes ordinary Maxwell equations (197) to be G-covariant, one has to postulate (with a unique choice for the sign, for simplicity's sake) that ($U^2 > 1$):

$$E'_x = -E_x \;;\quad E'_y = i\gamma(E_y - UH_z) \;;\quad E'_z = i\gamma(E_z + UH_y) \;;$$
$$H'_x = -H_x \;;\quad H'_y = i\gamma(H_y + UE_z) \;;\quad H'_z = i\gamma(H_z - UE_y) \,, \qquad (203)$$

with $\gamma \equiv 1/\sqrt{U^2-1}$. Notice that eqs.(203) leave G-covariant also eqs.(201a), (201b); see Recami and Mignani (1974a).

This choice was adopted by Corben. In his approach, let us repeat, Maxwell equations hold in their ordinary form also when in presence of both sub- and Super-luminal currents (i.e., when $u_\mu u^\mu = \pm 1$ in eqs.(203):

$$\partial_\nu F^{\mu\nu} = j^\mu \;;\qquad \partial_\nu \tilde{F}^{\mu\nu} = 0 \;;\qquad j^\mu \equiv \varrho_e u^\mu \;;\qquad u^\mu u_\mu = \pm 1 \;; \qquad (203bis)$$

for details on such an interesting theory - which correspond to assume $F^{\mu\nu}$ to be a G-tensor - see Corben (1975,1976,1978a).

(ii) On the contrary, one can try to generalize the subluminal transformations (202) for the Superluminal case, and only a <u>posteriori</u> deduce if $F^{\mu\nu}$ is a G-tensor or not, and finally derive how Maxwell equations get generalized. In eqs.(202) each couple of components E_y, H_z and E_z, H_y transform just as the couple of coordinates x,t (cf. Fig.7a); and the components E_x, H_x both transform just as the coordinate y or z.

Substituting the plane (E_y,H_z), or the plane (E_z,H_y), for the plane (x,t), it is then natural (cf. Fig.7b) to extend the subluminal transformations by allowing the axes E'_y,H'_z (or E'_z,H'_y) to "rotate" beyond 45°, untill when E'_y coincides with H_z and H'_z with E_y for $U \rightarrow \infty$: see Figs. 46. This corresponds to extend the two-dimensional Lorentz transformations so as in Sect.5.6, eq.(39").

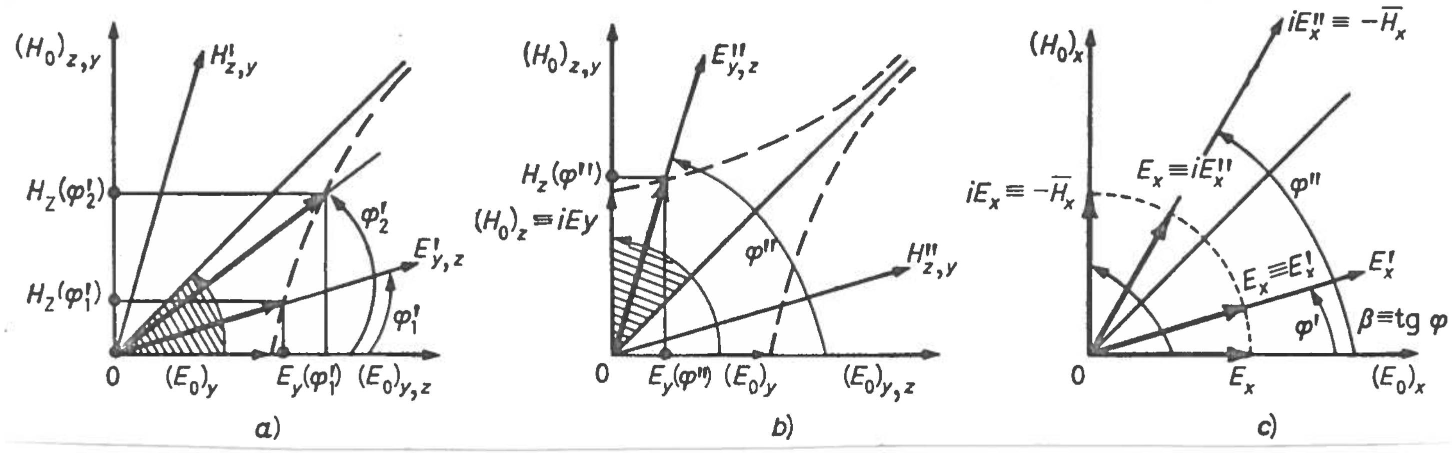


FIG. 46

Then, we may extend the transformations for E_x (and H_x) by analogy with the last two equations in (154bis) or in (160); that is to say: $E_x=iE'_x$; $H_x=iH'_x$; where for simplicity we confined ourselves to $-\frac{\pi}{2} < \vartheta < +\frac{\pi}{2}$. In such an approach, the quantities $T^{\mu\nu}$, $F^{\mu\nu}$, $A_{\mu,\nu}$ are not G-tensors, since under SLTs they transform as tensors except for an extra i (see e.g. Review I, and Recami and Mignani 1976,1977). Notice that, due to the invariance of $T^{\mu\nu}$ under the "duality" transformations, we may identify $iE'_x \equiv -H_x$; $iH'_x \equiv E_x$, in Heaviside-Lorentz units (i.e., in rationalized Gaussian units).

In Review I it has been shown that the assumption of the previous Superluminal transformations for the components of $\underset{\sim}{E}$ and $\underset{\sim}{H}$ leads to generalize eqs.(200) in the following (G-covariant) form

$$\partial_\nu T^{\mu\nu} = j^\mu(s) - i j^\mu(S) \; ; \quad \tilde{T}^{\mu\nu} = T^{\mu\nu} \; , \qquad (v^2 \gtrless 1) \qquad (204)$$

which constitute the "extended Maxwell equations" - valid in presence of both sub- and Super-luminal "electric" currents - according to Mignani and Recami (1975b,c,1974d), and Recami and Mignani (1976,1974a,b).

If we confine to subluminal observers, eqs.(204) easily write (Recami and Mignani 1974a):

$$\left.\begin{aligned} &\mathrm{div.}\, \underset{\sim}{D} = +\varrho(s) \; ; \qquad \mathrm{div.}\, \underset{\sim}{B} = -\varrho(S) \; ; \\ &\mathrm{rot}\, \underset{\sim}{E} = -\partial \underset{\sim}{B}/\partial t + \underset{\sim}{j}(S) \; ; \\ &\mathrm{rot}\, \underset{\sim}{H} = +\partial \underset{\sim}{D}/\partial t + \underset{\sim}{j}(s) \; . \end{aligned}\right. \qquad \begin{bmatrix} v^2 \gtrless 1 \\ s \leftrightarrow v^2 < 1 \\ S \leftrightarrow v^2 > 1 \end{bmatrix} \qquad (205)$$

Therefore, according to the present theory, if both sub- and Super-luminal "electric" charges exist, Maxwell equations get fully symmetrized, even if (ordinary) magnetic monopoles do not exist.

Actually, the generalization of eq.(202) depicetd in Figs.46, as well as the extended Maxwell eqs.(204)-(205) seem to comply with the very spirit of SR and to "complete" it.

15.2.- Tachyons and magnetic monopoles

The "subluminal" eqs.(201b) seem to suggest that a multiplication by i carries electric into magnetic current, and viceversa. Comparison of eqs.(201b) with the generalized equation (204) suggests that:

(i) the covariance of eqs.(201b) under the duality transformations, e.g. under eqs. (198), besides under LTs, corresponds to the covariance of eqs.(204) under the operation $\mathscr{S} \equiv \mathscr{S}_4$ (Sect.14.2), i.e. under SLTs. In other words, the covariance of eq.(201b) under the transtion charges $\rightleftarrows$ monopoles corresponds to the covariance of eqs.(204) under the transition bradyons $\rightleftarrows$ tachyons;

(ii) when transforming eqs.(201b) under SLTs (in particular under the Superluminal transformations previously defined for the electric and magnetic field components) electric and magnetic currents go one into the other. Eqs.(205) show, more precisely, that a Superluminal "electric" positive charge will contribute to the field equations in a way similar to the one expected to come from a magnetic south pole; and analogously for the currents. This does not mean, of course, that a Superluminal charge is expected to behave just as an ordinary monopole, due to the difference in the speeds (one sub-, the other Super-luminal!). Since eqs.(205) are symmetric even if ordinary monopoles would not exist, ER seems to suggest - at least in its most economical version - that only a unique type of charge exists (let us call it the electromagnetic charge), which, if you like, may be called "electric" when subluminal, and "magnetic" when Superluminal (Mignani and Recami 1975b, Recami and Mignani 1976,1977). The universality of electromagnetic interactions seems therefore recovered even at the classical level (i.e., in SR).

Let us exploit point (ii), by finding out the conditions under which the generalized equations (118)-(118') of Sect.10.5, written there in terms of four-potentials, are equivalent to the present extended Maxwell equations written in the form (204):

$$\left\{ \begin{array}{l} \Box \bar{A}^{\mu} = \bar{J}^{\mu} \\ \partial_{\mu} \bar{A}^{\mu} = 0 \end{array} \right\} \Longleftrightarrow \left\{ \begin{array}{l} \partial_{\nu} T^{\mu\nu} = \bar{J}^{\mu} \\ \tilde{T}^{\mu\nu} = T^{\mu\nu} \end{array} \right\} \; ; \qquad (v^2 \gtrless c^2) \qquad (206)$$

where $\bar{J}_{\mu} \equiv j_{\mu}(s) - i j_{\mu}(S)$. From the identity $\Box \bar{A}^{\mu} = -\partial_{\nu}^{2} \bar{A}^{\mu} + (\partial^{\mu} \partial_{\nu} \bar{A}^{\nu} + \varepsilon^{\mu\nu\varrho\sigma} \partial_{\nu} \partial_{\varrho} A_{\sigma})$ we can derive that eq.(206) holds provided that we set ($v^2 \gtrless c^2$):

$$T_{\mu\nu} \equiv \bar{A}_{\nu,\mu} - \bar{A}_{\mu,\nu} + \varepsilon_{\mu\nu\varrho\sigma}\bar{A}^{\sigma,\varrho} \quad . \tag{207}$$

It is remarkable that eq.(207) can be explicited into one of the two following conditions

$$F_{\mu\nu} \equiv A_{\nu,\mu} - A_{\mu,\nu} - i\,\varepsilon_{\mu\nu\varrho\sigma}B^{\sigma,\varrho} \quad , \tag{208a}$$

$$\overset{*}{F}_{\mu\nu} \equiv B_{\nu,\mu} - B_{\mu,\nu} + i\,\varepsilon_{\mu\nu\varrho\sigma}A^{\sigma,\varrho} \quad , \tag{208b}$$

where $\overset{*}{F}_{\mu\nu} \equiv \frac{i}{2}\varepsilon_{\mu\nu\varrho\sigma}F^{\varrho\sigma} \equiv i\tilde{F}_{\mu\nu}$ (so that $T_{\mu\nu} \equiv F_{\mu\nu} - i\overset{*}{F}_{\mu\nu}$, in agreement with eqs.(118')). Eq.(208b) is a consequence of the identity (Finzi and Pastori 1961): $B_{\nu,\mu} - B_{\mu,\nu} + i\,\varepsilon_{\mu\nu\varrho\sigma}A^{\sigma,\varrho} = \frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}(A_{\beta,\alpha} - A_{\alpha,\beta} - i\,\varepsilon_{\alpha\beta\gamma\delta}B^{\delta,\gamma}) \equiv \overset{*}{F}_{\mu\nu}$. Eq.(208a) is nothing but the Cabibbo and Ferrari (1962; see also Ferrari 1978) relation. In fact, those authors showed that the electomagnetism with ordinary charges and monopoles can be rephrased in terms of <u>two</u> fourpotentials A^μ and B^μ; and, in particular, gave the "Dirac term" the form of the last addendum in the r.h.s. of eq.(208a).

We gave a <u>new</u> physical interpretation of the Cabibbo-Ferrari relation. Moreover, while the ordinary approach with the two four-potentials A^μ, B^μ meets difficulties when confronting the gauge requirements, such difficulties disappear in our theory since B^μ is essentially the transform of A^μ under a suitable SLT.

15.3.- On the universality of electromagnetic interactions

Eqs.(205) say that, grosso modo, a "tachyon electron" (electric charge -e) will behave as a (Superluminal!) north magnetic charge (+g), and so on; in the sense that the tachyonic electron will bring into the field equations a contribution exactly <u>at the places</u> where contribution was on the contrary expected from a magnetic charge.

Since when passing, in the four-momentum space, on the other side of the light cone the topology does change (see e.g. Shah 1977), it is not easy to find out the relation between +g and -e. Mignani and Recami put forth the most naive proposal:

$$g = -e \; ; \tag{209}$$

in such a case (when quantizing) we expect to have

$$eg = \pm\,\alpha\hbar c \tag{209'}$$

where α is the fine-structure constant, instead of the Dirac-Schwinger relation $eg = \pm\hbar c$. But this point needs further investigation (on the basis, e.g., of Singe's work). In any case, in the present approach, SR itself is expected to yield a relation between g and e, so to provide a theory with a unique independent coupling constant. In ordinary classical electromagnetism with monopoles two coupling constants, on the contrary, do appear: and this violates at a classical level the universality of electromagnetic interactions, at variance with what one expects in SR (only at the quantum level the universality gets recovered, in the ordinary theory without tachyons).

As a work-hypothesis, let us assume eqs.(209)-(209') to be valid in our "tachyonic" theory; that is to say, in general, $ge = n\alpha\hbar c$.

We know that, quantizing the ordinary theory with subluminal monopoles, we end up on the contrary with the different relation $eg = \frac{1}{2} n\hbar c$ (Dirac 1931), or $eg = n\hbar c$ (Schwinger 1966). To avoid contradiction, we have at least to show that, when quantizing the present approach (with "tachyon monopoles"), we end up neither with Dirac's, nor with Schwinger's relation.

In fact (Recami and Mignani 1977), let us quantize this theory by using Mandelstam's method, i.e. following Cabibbo and Ferrari (1962). In that approach the field quantities describing the charges (in interaction with the electromagnetic field) are defined so that

$$\varphi(x,P') = \varphi(x,P)\, \exp(-\frac{ie}{2}\int_{\Sigma} F_{\mu\nu}\, d\sigma^{\mu\nu}) \tag{210}$$

where Σ is a surface delimited by the two considered space-like paths P and P', ending at point x. In other words, the field quantities φ are independent of the gauge chosen for the fourpotential A^{μ} but are path-dependent. When only subluminal electric charges are present, then $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ and eq.(210) does not depend on the selected surface Σ (it depends only on its boundary P-P'). If also subluminal magnetic monopoles are present, then $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} - i\,\varepsilon_{\mu\nu\varrho\sigma} B^{\sigma,\varrho}$, where B^{μ} is a second fourpotential, and the following condition must be explicitly imposed:

$$\exp(-\frac{ie}{2}\oint_{\Sigma-\Sigma'} F_{\mu\nu}\, d\sigma^{\mu\nu}) = 1\,, \tag{211}$$

wherefrom Dirac relation $eg = n\hbar c/2$ follows.

However, if "magnetic monopoles" cannot be put at rest, as in the case of "tachyon monopoles", then eq.(210) is again automatically satisfied, without any recourse to Dirac condition.

15.4- Further remarks

(i) It may be interesting to quote that the possible connection between tachyons and "monopoles" in the sense outlined above (Recami and Mignani 1974a) was first heuristically guessed by Arzelies (1958) - who predicted that $\underset{\sim}{E} \rightleftarrows \underset{\sim}{H}$ for $U > c$ - and later on by Parker

(1969), in its important and pioneering two-dimensional theory (see also Weingarten 1973).

(ii) As to the first considerations about the motion of a charged tachyon in an external field, see Bacry (1972) and Bacry et al.(1974). Notice, incidentally, that even a zero-energy charged tachyon may radiate (Rhee 1969) subtracting energy to the field.

(iii) The interactions of tachyon soliton charges have been studied, e.g., by van der Merwe (1978), by means of Bäcklund transformations.

(iv) If we consider the quanta inside the Cauchy-Fresnel "evanescent waves", since the momentum component normal to the reflecting plane is imaginary, the one parallel to that plane is larger than the energy. Such partial "tachyon properties" of those quanta have been studied particularly by Costa de Beauregard (1973; see also Costa de Beauregard et al.1971), whose reserach group even performed an experimental investigation (Huard and Imbert 1978). Further experimental work is presently being performed for example by Alzetta at Pisa.

15.5.- "Experimental" considerations

The very first experiments looking for tachyons, by Alväger et al.(1963,1965,1966), have been already mentioned in Sect.3.1. Let us add that a major reserach for charged tachyons was first carried on by Alväger and Kreisler (1968).

Most experiments (see Hänni and Hugentobler 1978; see also, e.g., Perepelitsa (1977a) looked for the Cherenkov radiation supposedly emitted by charged tachyons in vacuum. In Sect.10.3 we have however seen that we should not expect such a radiation to be emitted.

Searches for tachyons were performed in the cosmic radiation (see e.g. Ramana Murthy 1971), and in elementary particle reactions (see e.g. Baltay et al.1970, Danburg et al. 1971, Ramana Murthy 1973, and Perepelitsa 1976). Also tachyonic monopoles were looked for (see e.g. Bartlett and Lahana 1972, Perepelitsa 1977b, and Bartlett et al.1978).

We indirectly discussed many experimental topics in Sect.13, were the possible rôle was shown of tachyons in elementary particle physics and quantum mechanics; and we refer the reader to that Section.

Let us add here that - even if one does not stick to the conservative attitude of considering tachyons only as "internal lines" in interaction processes - any sound experimental project ought to take account (Corben 1975) of the drastic "deformation" caused by the huge velocity of the observed objects w.r.t. us: see e.g. the results on the tachyon shape presented in Sects.8.2 and 14.6. As noticed by Barut (1978), one may wonder if we have really correctly looked for Tachyons so far.

Within the classical theory of tachyons, it would be important to evaluate how charged tachyons would electromagnetically interact with ordinary matter: for instance, with an electron. The calculations can be made on the basis of the generalized Maxwell equations, either in Corben's form or in Mignani and Recami's (Sect.15.1). If we take seriously, however, Sect.8 on the shape of tachyons, we have to remember that a pointlike charge will appear - when Superluminal - to be spread over a double cone $\mathscr{C}$; it would be nice (see Sect.10.3) first to know the L -function of the space-time coordinates yielding the distribution of the tachyon charge-density over $\mathscr{C}$.

16.- CONCLUSIONS

Most tachyon classical physics can be obtained without resorting to Superluminal observers; and in such a classical physics extended to tachyons the ordinary causal problems can be solved.

The elegant results of ER in two dimensions, however, prompt us to look for its multi-dimensional extensions (i.e., to try understanding the meaning and the possible physical relevance of all the related problems: Sect.14).

Tachyons may have a rôle as objects exchanged between elementary particles, or between black-holes (if the latter exist). They can also be classically emitted by a black-hole, and have therefore a possible rôle in astrophysics.

For future research, it looks however even more interesting to exploit the possibility of reproducing quantum mechanics at the classical level by means of tachyons. On this respect even the appearance of imaginary quantities in the theories of tachyons can be a relevant fact, to be further studied.

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REFERENCES

Abers E., Grodsky I.T. and Norton R.E., 1967 Phys. Rev. 159, 1222.

Agodi A. 1972 Lezioni di Fisica Teorica (Catania University: unpublished).

Agudin J.L. 1971 Lett. Nuovo Cimento 2, 353.

Agudin J.L. and Platzeck A.M. 1982 Phys. Letters A90, 173.

Aharonov Y., Komar A. and Susskind L. 1969 Phys. Rev. 182, 1400.

Akiba T. 1976 Prog. Theor. Phys. 56, 1278.

Alagar Ramanujam G. and Namasivayam N. 1973 Lett. Nuovo Cimento 6, 245.

Alagar Ramanujam G., Savariraj G.A. and Shankara T.S. 1983 Pramana 21, 393.

Alvager T., Blomqvist J. and Ermann P. 1963 Annual Report of Nobel Research Institute, Stockholm (unpublished).

Alvager T., Ermann P. and Kerek A. 1965 Annual Report of Nobel Research Institute, Stockholm (unpublished).

Alvager T., Ermann P. and Kerek A. 1966 Preprint (Stockholm: Nobel Institute).

Alvager T. and Kreisler M.N. 1968 Phys. Rev. 171, 1357.

Amaldi E. 1963 Old and New Problems in Elementary Particles, ed. by G.Puppi (New York).

Amaldi E. and Cabibbo N. 1972 Aspects of Quantum Theory, ed. by A.Salam and E.P.Wigner (Cambridge).

Ammiraju P., Recami E. and Rodrigues W. 1983 Nuovo Cimento A78, 192.

Antippa A.F. 1972 Nuovo Cimento A10, 389.

Antippa A.F. 1975 Phys. Rev. D11, 724.

Antippa A.F. and Everett A.E. 1971 Phys. Rev. D4, 2198.

Antippa A.F. and Everett A.E. 1973 Phys. Rev. D8, 2352.

Arcidiacono G. 1974 Collectanea Mathematica (Barcelona) 25, 295.

Arons M.E. and Sudarshan E.C.G. 1968 Phys. Rev. 173, 1622.

Arzeliès H. 1955 La Cinématique Relativiste (Paris: Gauthier-Villars), p. 217.

Arzeliès H. 1957 Compt. Rend. A.S.P. 245, 2698.

Arzeliès H. 1958 Dynamique Relativiste (Paris: Gauthier-Villars), vol. 2, p. 101.

Arzeliès H. 1974 Compt. Rend. A.S.P. A279, 535.

Bacry H. 1972 Phys. Today 25(11), 15.

Bacry H., Combe Ph. and Sorba P. 1974 Rep. Math. Phys. 5, 145.

Baldo M. and Recami E. 1969 Lett. Nuovo Cimento 2, 643.

Baldo M., Fonte G. and Recami E. 1970 Lett. Nuovo Cimento 4, 241.

Banerjee A. 1973 Curr. Sci. (India) 42, 493.

Banerjee A. and Dutta Choudhury S.B. 1977, Austr. J. Phys. 30, 251.

Banerji S. and Mandal D.R. 1982 J. Phys. A: Math. Gen. 15, 3181.

Barashenkov V.S. 1975 Sov. Phys. Usp. 17, 774 (English translation of Usp. Fiz. Nauk 114, 133 (1974).

Barnard A.C.L. and Sallin E.A. 1969 Phys. Today 22(10), 9.

Barrett T.W. 1978 Nuovo Cimento B45, 297.

Bartlett D.F. and Lahana M. 1972 Phys. Rev. D6, 1817.

Bartlett D.F., Soo D. and White M.G. 1978 Phys. Rev. D18, 2253.

Barut A.O. 1978a Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 143.

Barut A.O. 1978b Phys. Letters A67, 257.

Barut A.O. 1978c Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 227.

Barut A.O. and Duru I.H. 1978 Proc. Roy. Soc. London A333, 217.

Barut A.O. and Nagel J. 1977 J. Phys. A: Math. Gen. 10, 1223.

Barut A.O., Maccarrone G.D. and Recami E. 1982 Nuovo Cimento A71, 509.

Barut A.O., Schneider C.K.E. and Wilson R. 1979 J. Math. Phys. 20, 2244.

Bell J.S. 1979 private communication.

Benford G.A., Book D.L. and Newcomb W.A. 1970 Phys. Rev. D2, 263.

Berley D. et al. 1975 Berkeley 1974 Proceedings PEP Summer Study (Berkeley), p. 450.

Bernardini C. 1982 Nuovo Cimento A67, 298.

Berzi V. and Gorini V. 1969 J. Math. Phys. 10, 1518.

Bhat P.N., Gopalakrishnan N.V., Gupta S.K. and Tonwar S.C. 1979 J. Phys. G: Nucl. Phys. 5, L13.

Bilaniuk O.M. and Sudarshan E.C.G. 1969a Phys. Today 22(5), 43.

Bilaniuk O.M. and Sudarshan E.C.G. 1969b Nature 223, 386.

Bilaniuk O.M., Deshpande V.K. and Sudarshan E.C.G. 1962 Am. J. Phys. 30, 718.

Bilaniuk O.M., Brown S.L., De Witt B., Newcomb V.A., Sachs M., Sudarshan E.C.G. and Yoshikawa S. 1969 Phys. Today 22(12), 47.

Bilaniuk O.M., Csonka P.L., Kerner E.H., Newton R.G., Sudarshan E.C.G. and Tsandoulas G.N. 1970 Phys. Today 23(5), 13; 23(11), 79.

Biretta J.A., Cohen M.H., Unwin S.C. and Pauliny-Toth I.I.K. 1983 Nature 306, 42.

Bjorkeen J.D.and Drell S.D. 1964 Relativistic Quantum Mechanics (New York: McGraw-Hill) vol. 1, p. 86.

Bjorkeen J.D., Kogut J.B. and Soper D.E. 1971 Phys. Rev. D3, 1382.

Blanfort R.D., McKee C.F. and Rees M.J. 1977 Nature 267, 211.

Bludman S.A. and Ruderman M.A. 1970 Phys. Rev. D1, 3243.

Bohm D. 1965 The Special Theory of Relativity (New York)

Bohm D. and Vigier J.P. 1954 Phys. Rev. 96, 208.

Bohm D. and Vigier J.P. 1958 Phys. Rev. 109, 882.

Bolotovsky B.M. and Ginzburg V.L. 1972 Usp. Fiz. Nauk 106, 577

Bondi H. 1964 Relativity and Common Sense (New York: Doubleday).

Boratav M. 1980 Ronda 1980 Proceedings - Fundamental Physics, p. 1.

Broido M.M. and Taylor J.O. 1968 Phys. Rev. 17, 1606.

Brown G.E. and Rho H. 1983 Phys. Today 36(2).

Browne I.W.A., Clark R.R., Moore P.K., Muxlow T.W.B., Wilkinson P.N., Cohen M.H. and Porcas R.W. 1982 Nature 299, 788.

Bugrij A.I., Jenkovsky L.L. and Kobylinsky N.A. 1972 Lett. Nuovo Cimento 5, 389.

Bulbeck A.R. and Hurst C.A. Answer to Agudin and Platzeck, Preprint (Adelaide Univ.).

Bunge M. 1959 Br. J. Philos. Soc. 9, 39.

Cabibbo N. and Ferrari E. 1962 Nuovo Cimento 23, 1147.

Caldirola P. and Recami E. 1978 Epistemologia (Genova) 1, 263.

Caldirola P. and Recami E. 1980 Italian Studies in the Philosophy of Science, ed. by M.L. Dalla Chiara (Boston: Reidel), p. 249.

Caldirola P., Maccarrone G.D. and Recami E. 1980 Lett. Nuovo Cimento 29, 241.

Caldirola P., Pavšič M. and Recami E. 1978 Nuovo Cimento B48, 205.

Camenzind M. 1970 Gen. Rel. Grav. 1, 41.

Camenzind M. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 89.

Cao Sh-L. 1984 Preprint (Peking: Beijing Normal University).

Carey A.L., Ey C.M. and Hurst C.A. 1979 Hadronic J. 2, 1021.

Carrol A. et al. 1975 Berkeley 1975 Proceedings PEP Summer Study (Berkeley), p. 176.

Casalbuoni R. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 247.

Castellino A. 1984 Un approccio teorico allo studio di alcune apparenti espansioni superluminali in astrofisica, MS-Thesis, supervisor E.Recami (Catania Univ.: Phys. Dept.).

Castorina P. and Recami E. 1978 Lett. Nuovo Cimento 22, 195.

Catara F., Consoli M. and Eberle E. 1982 Nuovo Cimento B70, 45.

Cavaliere A. Morison P. and Sartori L. 1971 Science 173, 525.

Cavalleri G. and Spinelli G. 1973 Lett. Nuovo Cimento 6, 5.

Cavalleri G. and Spinelli G. 1977 Phys. Rev. D15, 3065.

Cavalleri G. and Spinelli G. 1978 Lett. Nuovo Cimento 22, 113.

Cawley R.G. 1969 Ann. of Phys. 54, 132.

Cawley R.G. 1970 Int. J. Theor. Phys. 3, 483.

Cawley R.G. 1972 Lett. Nuovo Cimento 3, 523.

Charon J.E. 1977 Théorie de la Relativité Complexe (Paris: A. Michel).

Chew G.F. 1968 Science 161, 762.

Ciborowski J. 1982 Preprint (Warsaw: Inst. Exp. Phys.).

Clavelli L., Feuster S. and Uretsky J.L. 1973 Nuclear Phys. B65, 373.

Cohen M.H. and Unwin S.C. 1982 Proceedings I.A.U. Symposium no. 97, p. 345.

Cohen M.H., Cannon W., Purcell G.H., Shaffer D.E., Broderick J.J., Kellermann K.I. and Jauncey D.L. 1971 Astrophys. J. 170, 207.

Cohen M.H., Kellermann K.I., Shaffer D.B., Linfield R.P., Moffet A.T., Romney J.D., Seielstad G.A., Pauliny-Toth I.I.K., Preuss E., Witzel A., Schillizzi R.T. and Geldzahler B.J. 1977 Nature 268, 405.

Cole E.A. 1977 Nuovo Cimento A40, 171.

Cole E.A. 1978 Nuovo Cimento B44, 157.

Cole E.A. 1979 Phys. Letters A75, 29.

Cole E.A. 1980a J. Phys. A: Math. Gen. 13, 109.

Cole E.A. 1980b Nuovo Cimento B55, 269.

Cole E.A. 1980c Phys. Letters A76, 371.

Cole E.A. 1980d Lett. Nuovo Cimento 28, 171.

Cole E.A. 1980e Nuovo Cimento A60, 1.

Coleman S. 1970 Acausality in Subnuclear Phenomena, ed. by A.Zichichi (New York: Academic Press), Part A, p. 283.

Comer R.P. and Lathrop J.D. 1978 Am. J. Phys. 46, 801.

Conforto G. 1984 Preprint (Cosenza University: Dept. of Mathem.).

Corben H.C. 1974 Lett. Nuovo Cimento 11, 533.

Corben H.C. 1975 Nuovo Cimento A29, 415.

Corben H.C. 1976 Int. J. Theor. Phys. 15, 703.

Corben H.C. 1977a Lett. Nuovo Cimento 20, 645.

Corben H.C. 1977b three Preprints (West Hill, Ont.: Scarborough College, Aug., Sept. and Nov.).

Corben H.C. 1978a Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 31.

Corben H.C. 1978b Lett. Nuovo Cimento 22, 116.

Costa de Beauregard D. 1972 Found. Phys. 2, 111.

Costa de Beauregard D. 1973 Int. J. Theor. Phys. 7, 129.

Costa de Beauregard D. 1983 Old and New Questions in Physics, Cosmology, Philosophy: Essays in Honor of W.Yourgrau, ed. by A.van der Merwe (New York: Plenum), p. 87.

Costa de Beauregard D. 1984 The Wave-Particle Dualism, ed. by S.Diner et al. (Dordrecht Reidel), p. 485.

Costa de Beauregard D., Imbert Ch. and Ricard J. 1971 Int. J. Theor. Phys. 4, 125.

Csonka P.L. 1970 Nuclear Phys. B21, 436.

Cunningham C.T. 1975 Preprint DAP-395 (Pasadena: Caltech).

Dadhich N. 1979 Phys. Letters A70, 3.

Dar A. 1964 Phys. Rev. Letters 13, 91.

Das A. 1966 J. Math. Phys. 7, 45, 52, 61.

Dattoli G. and Mignani R. 1978 Lett. Nuovo Cimento 22, 65.

Davies P.C.W. 1975 Nuovo Cimento B25, 571.

Dell'Antonio G.F. 1961 J. Math. Phys. 2, 572.

Demers P. 1975 Can. J. Phys. 53, 1687.

Dent W.A. 1972 Science 175, 1105.

De Sabbata V.1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam, North-Holland), p. 99.

De Sabbata V., Pavšič M. and Recami E. 1977 Lett Nuovo Cimento 19, 441.

Des Coudres Th. 1900 Arch. Neerland Sci.(II) 5, 652.

d'Espagnat B. 1981 Found. Phys. 11, 205.

Dhar J. and Sudarshan E.C.G. 1963 Phys. Rev. 174, 1808.

Dhurandhar S.V. 1978 J. Math. Phys. 19, 561.

Dhurandhar S.V. and Narlikar J.V. 1978 Gen. Rel. Grav. 9, 1089.

Di Jorio M. 1974 Nuovo Cimento B22, 70.

Dirac P.A.M. 1931 Proc. Roy. Soc. London A133, 60.

Dorling J. 1970 Am. J. Phys. 38, 539.

Duffey G.H. 1975 Found. Phys. 5, 349.

Duffey G.H. 1980 Found. Phys. 10, 959.

Ecker G. 1970 Ann. of Phys. 58, 303.

Edmonds J.D. 1972 Lett. Nuovo Cimento 5, 572.

Edmonds J.D. 1974 Found. Phys. 4, 473.

Edmonds J.D. 1976 Found. Phys. 6, 33.

Edmonds J.D. 1977a Found. Phys. 7, 835.

Edmonds J.D. 1977b Lett. Nuovo Cimento 18, 498.

Edmonds J.D. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North Holland), p. 79.

Eeg J.O. 1973 Phys. Norvegica 7, 21.

Einstein A. 1911 Ann. der Phys. 35, 898.

Einstein A. and Bergmann P. 1938 Ann. Math. 39, 683.

Elder J.D. 1970 Phys. Today 23(10), 15, 79.

Epstein R.L. and Geller M.J. 1977 Nature 265, 219.

Ey C.M. and Hurst C.A. 1977 Nuovo Cimento B39, 76.

Enatsu H., Takenaka A. and Okazaki M. 1978 Nuovo Cimento A43, 575.

Eriksen E. and Voyenli K. 1976 Found Phys. 6, 115.

Everett A.E. 1976 Phys. Rev. D13, 785, 795.

Federighi I. 1983 Boll. Soc. Ital. Fis. 130, 92.

Feinberg G. 1967 Phys. Rev. 159, 1089.

Feinberg G. 1970 Scient. Am. 222(2), 68

Feinberg G. 1978 Phys. Rev. D17, 1651.

Feinberg G. 1979 Phys. Rev. D19, 5812.

Feldman L.M. 1974 Am. J. Phys. 42, 179.

Fermi E. 1951 Elementary Particles (New Haven: Yale Univ. Press).

Ferrari E 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 203.

Ferretti I. and Verde M. 1966 Atti Accad. Sci. Torino, Cl. Sci. Fis. Mat. Nat., p.318.

Feynman R.P. 1049 Phys. Rev. 76, 749, 769.

Finkelstein A.M., Kreinovich V.Ja. and Pandey S.N. 1983 Report (Pulkovo: Sp. Astrophys. Observatory).

Finzi B. and Pastori M. 1961 Calcolo Tensoriale e Applicazioni (Bologna: Zanichelli).

Flato M. and Guenin M. 1977 Helv. Phys. Acta 50, 117.

Fleury N., Leite-Lopes J. and Oberlechner G. 1973 Acta Phys. Austriaca 38, 113.

Foster J.C. and Ray J.R. 1972 J. Math. Phys. 13, 979.

Fox R., Kuper C.G. and Lipson S.G. 1969 Nature 223, 597.

Fox R., Kuper C.G. and Lipson S.G. 1970 Proc. Roy. Soc. London A316, 515.

Frank P. and Rothe H. 1911 Ann. der Phys. 34, 825.

Freed K. 1972 J. Chem. Phys. 56, 692.

Froning H.D. 1981 Spec. in Sci. and Techn. 4, 515.

Fronsdal C. 1968 Phys. Rev. 171, 1811.

Fronsdal C. 1969a Phys. Rev. 182, 1564.

Fronsdal C. 1969b Phys. Rev. 185, 1768.

Fukunda R. 1978 Phys. Letters B73, 33.

Fuller R.W. and Wheeler J.A. 1962 Phys. Rev. 128, 919.

Galilei G. 1632 Dialogo ... sopra i due massimi sistemi del mondo: Tolemaico e Copernicano (Firenze: G.B.Landini Pub.).

Galilei G. 1953 Dialogue on the Great World Systems: Salusbury translation, ed by. G. De Santillana (Chicago: Univ. of Chicago Press), p. 199.

Garuccio A. 1984 private communication.

Garuccio A., Maccarrone G.D., Recami E. and Vigier J.P. 1980 Lett. Nuovo Cimento 27, 60.

Gatlin L.L. 1980 Int. J. Theor. Phys. 19, 25.

Giacomelli G. 1970 Evolution of Particle Physics, ed. by M.Conversi (New York), p. 148.

Girard R. and Marchildon L. 1984 Found. Phys. 14, 535.

Gladkikh V.A. 1978a Fizika (Is. Tomsk Univ.) 6, 69, 130.

Gladkikh V.A. 1978b Fizika (Is. Tomsk Univ.) 12, 52.

Gleeson A.M. and Sudarshan E.C.G. 1970 Phys. Rev. D1, 474.

Gleeson A.M., Gundzik M.G., Sudarshan E.C.G. and Pagnamenta A. 1972a Phys. Rev. A6, 807.

Gleeson A.M., Gundzik M.G., Sudarshan E.C.G. and Pagnamenta A. 1972b Fields and Quanta 2, 175.

Glück M. 1969 Nuovo Cimento A62, 791.

Göbel R. 1976 Comm. Math. Phys. 46, 289.

Gödel K. 1973 A.Einstein: Philosopher-Scientists, ed. by P.A.Schilpp (La Salle, Ill.: Open Court), p. 558.

Goldhaber A.S. and Smith F. 1975 Rep. Prog. Phys. 38, 731 (see pp. 757-760).

Goldoni R. 1972 Lett. Nuovo Cimento 5, 495.

Goldoni R. 1973 Nuovo Cimento A14, 501, 527.

Goldoni R. 1975a Acta Phys. Austriaca 41, 75.

Goldoni R. 1975b Acta Phys. Austriaca 41, 133.

Goldoni R. 1975c Gen. Rel. Grav. 6, 103.

Goldoni R. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 125.

Gondrand J-C. 1971 Report CEA-B18-199 (Saclay: CEN), in French.

Gorini V. 1971 Comm. Math. Phys. 21, 150.

Gorini V. and Zecca A. 1970 J. Math. Phys. 11, 2226.

Gott III J.R. 1974a Nuovo Cimento B22, 49.

Gott III J.R. 1974b Astrophys. J. 187, 1.

Greenberg O.W. 1962 J. Math. Phys. 3, 859.

Gregory C. 1961 Nature 192, 1320.

Gregory C. 1962 Phys. Rev. 125, 2136.

Gregory C. 1965 Nature 206, 702.

Gregory C. 1972 Nature Phys. Sci. 239, 56.

Grodsky L.T. and Streater R.F. 1968 Phys. Rev. Letters 20, 695.

Grøn Ø. 1978 Lett. Nuovo Cimento 23, 97.

Grøn Ø. 1979 private communication.

Guasp M. 1983 Sobre la importancia del movimento en el concepto de la realidad fisica (Valencia: Ed. Valencia 2000).

Guenin M. 1976 Phys. Letters B62, 81.

Gurevich L.E. and Tarasevich S.V. 1978 Sov. Astron. Lett. 4, 183.

Gurin V.S. 1983, private communication.

Gurin V.S. 1984 Fizika 16, 87.

Hadjioannou F.T. 1966 Nuovo Cimento 44, 185.

Hahn E. 1913 Arch. Math. Phys. 21, 1.

Halpern M. and Malin S. 1969 Coordinate conditions in general relativity, Report (Hamilton: Colgate Univ.).

Hamamoto S. 1972 Prog. Theor. Phys. 48, 1037.

Hamamoto S. 1974 Prog. Theor. Phys. 51, 1977.

Hansen R.O. and Newman E.T. 1975 Gen. Rel. Grav. 6, 361.

Havas P. 1974 Causality and Physical Theories, ed. by W.B.Rolnick (New York).

Hawking S.W. and Ellis G.F.R. 1973 The Large-Scale Structure of Space-Time (Cambridge: Cambridge Univ. Press).

Heaviside O. 1892 Electrical Papers (London), Vol. 2, p. 497.

Hegerfeld G.C. 1974 Phys. Rev. D10, 3320.

Heisenberg W. 1972 Aspects of Quantum Theory, ed. by A.Salam and E.P.Wigner (Cambridge: Cambridge Univ. Press).

Hestenes D. 1975 J. Math. Phys. 16, 556.

Hettel R.O. and Helliwell T.M. 1973 Nuovo Cimento B13, 82.

Hilgevoord J. 1960 Dispersion Relations and Causal Description (Amsterdam: North--Holland), p. 4.

Honig E., Lake K. and Roeder R.C. 1974 Phys. Rev. D10, 3155.

Hoyle F. and Narlikar J.V. 1974 Action-at-a-distance (San Francisco: Freeman).

Huard S. and Imbert C. 1978 Opt. Comm. 24, 185.

Ignatowski W.V. 1910 Phys. Zeits. 21, 972.

Imaeda K. 1979 Nuovo Cimento B50, 271.

Ishikawa K.I. and Miyashita T. 1983 Gen. Rel. Grav. 15, 1009.

Israel W. 1967 Phys. Rev. 164, 1775.

Ivanenko D.D. 1979 Relativity, Quanta and Cosmology, ed. by F.De Finis and M.Pantaleo (New York: Johnson Rep. Co.), Vol. 1, p. 295.

Jackiw R. and Rebbi C. 1976 Phys. Rev. Letters 37, 172.

Jadczyk A.Z. 1970 Preprint no. 213 (Wroclaw Univ.: Inst. Theor. Phys.).

Jaffe J. and Shapiro I. 1974 Phys. Rev. D6, 405.

Jammer M. 1979 Problems in the Foundations of Physics: Proceedings of the 72nd Course of the Varenna Int. School of Physics, ed. by G.Toraldo di Francia.

Jancewicz B. 1980 Electromagnetism with use of bivectors, Preprint (Wroclaw Univ.: Theoret. Phys. Dept.).

Janis A.I., Newman E.T. and Winicour J. 1968 Phys. Rev. Letters 20, 878.

Jehle M. 1971 Phys. Rev. D3, 306.

Jehle M. 1972 Phys. Rev. D6; 441.

Johnson L.E. 1981 External Tachyons/ Internal Bradyons, unpubl. Report (New Concord, Ohio).

Johri V.B. and Srivastava S.K. 1978 Preprint (Gorakhpur Univ.: Phys. Dept.).

Jones L.W. 1977 Rev. Mod. Phys. 49, 717.

Jones R.T. 1963 Journ. Franklin Inst. 275, 1.

Jordan T.F. 1978 J. Math. Phys. 19, 247.

Jue C. 1973 Phys. Rev. D8, 1757.

Kalitzin N. 1975 Multitemporal Theory of Relativity (Sofia: Bulg. Ac. Sc.).

Kálnay A.J. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 53.

Kálnay A.J. 1980 Lett. Nuovo Cimento 27, 437.

Kálnay A.J. and Toledo B.P. 1967 Nuovo Cimento 43, 997.

Kamoi K. and Kamefuchi S. 1977 Lett. Nuovo Cimento 19, 413.

Kastrup H.A. 1962 Ann. der Phys. 7, 388.

Kellermann K.I. 1980 Ann. N.Y. Acad. Sci. 336, 1.

Keszthelhyi T. and Nagy K.L. 1974 Acta Phys. Ac. Sc. Hungaricae 37, 259.

Kirch D. 1977 Umshau Wiss. Tech. 77, 758.

Kirzhnits D.A. and Polyachenkov L. 1964 Sov. Phys. JETP 19, 514.

Kirzhnits D.A. and Sazonov V.N. 1974 Einsteinian Symposium - 1973 - Academy of Sciences USSR (Moscow: Nauka), in Russian.

Klein O. 1929 Zeit. fur Phys. 53, 157.

Knight C.A., Robertson D.S., Rogers A.E.E., Shapiro I.I., Whitney A.R., Clark T.A., Goldstein R.M., Marandino G.E. and Vanderberg N.R. 1971 Science 172, 52.

Korff D. and Fried Z. 1967 Nuovo Cimento A52, 173.

Kowalczyński J.K. 1973 Phys. Letters A65, 269.

Kowalczyński J.K. 1979 Phys. Letters A74, 157.

Kowalczyński J.K. 1984 Int. J. Theor. Phys. 23, 27.

Kreisler M.N. 1969 Phys. Teacher 7, 391.

Kreisler M.N. 1973 Am. Scientists 61, 201.

Krōlikowski W. 1969 Report "P" no. 1060/VII/PH (Warsaw: Inst. Nucl. Res.).

Krüger J. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 195.

Kyselka A. 1981 Int. J. Theor. Phys. 20, 13.

Lake K. and Roeder R.C. 1975 Lett Nuovo Cimento 12, 641.

Lalan V. 1937 Bull. Soc. Math. France 65, 83.

Landau L. and Lifshitz E. 1966a Mecanique (Moscow: MIR).

Landau L. and Lifshitz E. 1966b Thēorie du Champ (Moscow: MIR).

Laplace P.S. 1845 Mecanique Celeste, in Ouvres (Paris: Imprimerie Royal), Tome IV, Book X, Chapt. VII, p. 364.

Leibowitz E. and Rosen N. 1973 Gen. Rel. Grav. 4, 449.

Leiter D. 1971a Lett. Nuovo Cimento 1, 395.

Leiter D. 1971b Nuovo Cimento A2, 679.

Lemke H. 1976 Nuovo Cimento A32, 181.

Lemke H. 1977a Int. J. Theor. Phys. 60.

Lemke H. 1977b Phys. Letters A60.

Lewis B.L. 1981 Report (Washington: Naval Res. Lab.).

Liaofu L. and Chongming X. 1984 Intern. Conf. on Relativity and Gravitation (GR10), ed. by B.Bertotti, F.de Felice and A.Pascolini (Rome: CNR), p. 749.

Lightman A.P., Press W.H., Price R.H. and Teukolski S.A. 1975 Problem Book in Relativity and Gravitation (Princeton: Princeton Univ. Press), p. 405.

Ljubicic A., Pisk K. and Logan B.A. 1979 Phys. Rev. D20, 1016.

Lucretius Caro T. ca. 50 B.C. De Rerum Natura, ed. by M.T.Cicero (Rome), Book 4, lines 201-203.

Lugiato L. and Gorini V. 1972 J. Math. Phys. 13, 665.

Maccarrone G.D. and Recami E. 1980a Found. Phys. 10, 949.

Maccarrone G.D. and Recami E. 1980b Nuovo Cimento A57, 85.

Maccarrone G.D. and Recami E. 1982a Report INFN/AE-82/12 (Frascati: INFN), pp. 1-39.

Maccarrone G.D. and Recami E. 1982b Lett. Nuovo Cimento 34, 251.

Maccarrone G.D. and Recami E. 1984a Found. Phys. 14, 367.

Maccarrone G.D., Pavšič M. and Recami E. 1983 Nuovo Cimento B73, 91.

Mackley F. 1973 Am. J. Phys. 41, 45.

Majorana E. 1932 Nuovo Cimento 9, 335.

Maitsev V.K. 1981 Teor. Mat. Fiz. 47, 177.

Mann R.B. and Moffat J.W. 1982 Phys. Rev. D26, 1858.

Mannheim P.D. 1977 Preprint SLAC-PUB-1885 (Stanford Univ.: SLAC).

Marchildon L., Antippa A.F. and Everett A.E. 1983 Phys. Rev. D27, 1740.

Marchildon L., Everett A.E. and Antippa A.F. 1979 Nuovo Cimento B53, 253.

Marques G.C. and Swieca J.A. 1972 Nuclear Phys. B43, 205.

Marscher A.P. and Scott J.S. 1980 Pubbl. Astron. Soc. Pacific 92, 127.

Marx E. 1970 Int. J. Theor. Phys. 3, 299.

Mathews P.M. and Seetharaman M. 1973 Phys. Rev. D8, 1815.

McLaughin D. 1972 J. Math. Phys. 13, 784, 1099.

Mensky M.B. 1976 Comm. Math. Phys. 47, 97.

Miller J.G. 1979 Phys. Rev. D19, 442.

Mignani R. 1975 Lett. Nuovo Cimento 13, 134.

Mignani R. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 67.

Mignani R. and Recami E. 1973a Nuovo Cimento A14, 169 (Erratum: A16, 208).

Mignani R. and Recami E. 1973b Lett. Nuovo Cimento 7, 388.

Mignani R. and Recami E. 1974a Nuovo Cimento A24, 438.

Mignani R. and Recami E. 1974b Lett. Nuovo Cimento 11, 421.

Mignani R. and Recami E. 1974c Lett. Nuovo Cimento 9, 357.

Mignani R. and Recami E. 1974d Lett. Nuovo Cimento 9, 367.

Mignani R. and Recami E. 1975a Int. J. Theor. Phys. 12, 299.

Mignani R. and Recami E. 1975b Nuovo Cimento A30, 533.

Mignani R. and Recami E. 1975c Lett. Nuovo Cimento 13, 539.

Mignani R. and Recami E. 1976a Phys. Letters B65, 143.

Mignani R. and Recami E. 1976b Lett. Nuovo Cimento 16, 449.

Mignani R. and Recami E. 1977 Lett. Nuovo Cimento 18, 5

Mignani R., Recami E. and Lombardo U. 1972 Lett. Nuovo Cimento 4, 624.

Milewski B. 1973 private communication.

Minkowski H. 1908 Space and Time: address delivered at the 80th Assembly of German Scientists and Physicians (Cologne, Sept. 21).

Møller C. 1962 The Theory of Relativity (Oxford: Oxford Univ. Press), p. 234.

Moore R.L., Readhead A.C.S. and Baath L. 1983 Nature 306, 44.

Moskalenko V.A. and Moskalenko T.V. 1978 Is. Akad. Nauk Mold. SSR, Ser. Fiz. Tek. Mat., 1.

Mukunda N. 1969 Completeness of the solutions of the Majorana equations, Preprint (Bombay: Tata Inst.).

Murphy J.E. 1971 Tachyons fields and causality, Preprint (New Orleans: Louisiana State Univ.).

Mysak L. and Szekeres G. 1966 Can. J. Phys. 44, 617.

Nambu Y. 1950 Prog. Theor. Phys. 5, 82.

Naranan S. 1972 Lett. Nuovo Cimento 3, 623.

Narlikar J.V. and Dhurandhar S.V. 1976 Pramãna 6, 388.

Narlikar J.V. and Dhurandhar S.V. 1978 Lett. Nuovo Cimento 23, 513.

Narlikar J.V. and Sudarshan E.C.G. 1976 Mon. Not. R. Astron. Soc. 175, 105.

Neeman Y. 1974 High-Energy Astrophysics and its Relation to Elementary Particle Physics, ed. by K.Brecher and G.Setti (Cambridge: The MIT Press), p. 405.

Newton R.G. 1967 Phys. Rev. 162, 1274.

Newton R.G. 1970 Science (AAAS) 167, 1569.

Nielsen H.B. 1977 Fundamentals of the Quark Model, ed. by Y.Barbour and A.T.Davies (Scottish Univ. Summer School).

Nielsen H.B. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 169.

Nielsen H.B. 1979 private communications.

Nielsen H.B. and Minomiya M. 1978 Preprint NBI-HE-78-10 (Copenhagen: Niels Bohr Inst.).

Nielsen N.K. and Olesen P. 1978 Nuclear Phys. B144, 376.

Nishioka M. 1983 Hadronic J. 6, 794.

Olkhovsky V.S. and Recami E. 1970a Report IFT/70 (Kiev: Ukr. Acad. Sci.).

Olkhovsky V.S. and Recami E. 1970b Vysnik Kiyvskogo Universitetu, Ser. Fiziki (Kiev) 11, 58.

Olkhovsky V.S. and Recami E. 1970c Lett. Nuovo Cimento 4, 1165.

Olkhovsky V.S. and Recami E. 1971 Lett. Nuovo Cimento 1, 165

Oor L.J. and Browne I.W.A. 1982 Mon. Not. R. Astron. Soc. 200, 1067.

Pahor S. and Strnad J. 1976 Nuovo Cimento B33, 821.

Pappas P.T. 1978 Lett. Nuovo Cimento 22, 601.

Pappas P.T. 1979 Lett. Nuovo Cimento 29, 429.

Pappas P.T. 1982 Nuovo Cimento B68, 111.

Parisi G. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 233.

Parker L. 1969 Phys. Rev. 188, 2287.

Parmentola J.A. and Yee D.D.H. 1971 Phys. Rev. D4, 1912.

Patty C.E. 1982 Nuovo Cimento B70, 65.

Pauliny-Toth I.I.K., Preuss E., Witzel A., Graham D., Kellermann K.I. and Ronnaug R. 1981 Astron. J. 86, 371.

Pavšič M. 1971 The extended special theory of relativity, unpublished preprint (Ljubljana Univ.).

Pavšič M. 1972 Obz. za Matem. in Fiz. 19(1), 20.

Pavšič M. 1978 Tachyons, Monopoles, and Related Topics, ed . by E.Recami (Amsterdam: North-Holland), p. 105.

Pavšič M. 1981a Lett. Nuovo Cimento 30, 111.

Pavšič M. 1981b J. Phys. A: Math. Gen. 14, 3217.

Pavšič M. and Recami E. 1976 Nuovo Cimento A36, 171 (Erratum: A46, 298).

Pavšič M. and Recami E. 1977 Lett. Nuovo Cimento 19, 273.

Pavšič M. and Recami E. 1982 Lett. Nuovo Cimento 34, 357 (Erratum: 35, 354).

Pearson T.J., Unwin S.C., Cohen M.H., Linfield R.P., Readhead A.C.S., Seielstad G.A., Simon R.S. and Walker R.C. 1981 Nature 290, 365.

Perepelitsa V.F. 1976 Report ITEF-86 (Moscow: Inst. Theor. Exp. Phys.).

Perepelitsa V.F. 1977a Phys. Letters B67, 471.

Perepelitsa V.F. 1977b Report ITEF-81 (Moscow: Inst. Theor. Exp. Phys.).

Perepelitsa V.F. 1980a Report ITEF-100 (Moscow: Inst. Theor. Exp. Phys.).

Perepelitsa V.F. 1980b Report ITEF-165 (Moscow: Inst. Theor. Exp. Phys.).

Perepelitsa V.F. 1981 Report ITEF-104 (Moscow: Inst. Theor. Exp. Phys.).

Peres A. 1969 Lett. Nuovo Cimento 1, 837.

Peres A. 1970 Phys. Letters A31, 361.

Pirani F.A.E. 1970 Phys. Rev. D1, 3224.

Plebanski J.F. and Schild A. 1976 Nuovo Cimento B35, 35.

Poole C.P., Farach H.A. and Aharonov Y. 1980 Found. Phys. 10, 531.

Pooley G. 1981 Nature 290, 363.

Porcas R.W. 1981 Nature 294, 47.

Porcas R.W. 1983 Nature 302, 753.

Prasad G. and Sinha B.B. 1979 Nuovo Cimento B52, 105.

Preparata G. 1976 Current Induced Reactions, ed. by J.G.Korney, G.Kramer and D. Schildknecht D. (Berlin: Springer).

Rafanelli K. 1974 Phys. Rev. D9, 2746.

Rafanelli K. 1976 Nuovo Cimento B35, 17.

Rafanelli K. 1978 Phys. Rev. 17, 640.

Ramachandran G. Tagare S.G. and Kolaskar B. 1972 Lett. Nuovo Cimento 4, 141.

Ramana Murthy P.V. 1971 Lett. Nuovo Cimento 1, 908.

Ramana Murthy P.V. 1972 Phys. Rev. D7, 2252.

Ramanujam G.A. and Namasivayan N. 1973 Lett. Nuovo Cimento 6, 245.

Ray J.R. 1975 Lett. Nuovo Cimento 12, 249.

Ray J.R. 1980 Lett. Nuovo Cimento 27, 32.

Ray J.R. and Foster J.C. 1973 Gen. Rel. Grav. 4, 371.

Ray J.R. and Zimmerman J.C. 1976 Lett. Nuovo Cimento 15, 457.

Ray J.R. and Zimmerman J.C. 1977 Preprint (Clemson University).

Raychaudhuri A.K. 1974 J. Math. Phys. 15, 256.

Readhead A.C.S., Hough D.H., Ewing M.S. and Romney J.D. 1983 Astrophys. J. 265, 107.

Rees M.J. 1966 Nature 211, 46.

Reichenbach H. 1971 The Direction of Time, ed. by M.Reichenbach (Berkeley: Univ. of California Press), p. 264.

Recami E. 1968 Possible causality effects and comments on tachyons, virtual particles, resonances, Report IFUM-088/SM (Milan Univ., Italy: Phys. Inst., Aug. 1968).

Recami E. 1969 Giornale di Fisica (Bologna) 10, 195.

Recami E. 1970 Accad. Naz. Lincei, Rendic. Sc. (Roma) 49, 77.

Recami E. 1973 Enciclopedia EST Mondadori, Annuario 1973 (Milano: Mondadori), pp. 85-94, in Italian.

Recami E. 1974 unpublished work: many seminars: private communications (e.g. to D.Sciama); computer calculations, and unpublished pieces of work in coll. with H.B.Nielsen et al.

Recami E. 1975 Scientia 109, 721.

Recami E. 1977a Lett. Nuovo Cimento 18, 501.

Recami E. 1977b Topics in Theoretical and Experimental Gravitation Physics, ed. by V.De Sabbata and J.Weber (New York: Plenum), p. 305.

Recami E. 1978a Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 3.

Recami E. 1978b Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 1-285.

Recami E. 1978c Found. Phys. 8, 329.

Recami E. 1978d Report INFN/AE-78/2 (Frascati: INFN).

Recami E. 1979a Albert Einstein 1879-1979: Relativity, Quanta and Cosmology, ed. by F. de Finis and M.Pantaleo (New York: Johnson Rep. Co.), Vol. 2, Chapt. 16, p. 537. This ref. appeared also in Italian: Centenario di Einstein: Astrofisica e Cosmologia, Gravitazione, Quanti e Relatività, ed. by M.Pantaleo (Florence: Giunti-Barbera, 1979), Chapt. 18, p. 1021; and in Russian: Astrofizika, Kvanti i Teorya Otnositelnosti, ed. by F.I.Fedorov (Moscow: MIR, 1982), p. 53.

Recami E. 1982a Progress in Particle and Nuclear Physics, vol. 8: Quarks and the Nucleus, ed. by D.Wilkinson (Oxford: Pergamon), p. 401.

Recami E. 1982b Old and New Questions in Physics, Cosmology, Philosophy: Essays in Honor of W.Yourgrau, ed. by A.van der Merwe (New York: Plenum), p. 377

Recami E. and Maccarrone G.D. 1980 Lett. Nuovo Cimento 28, 151.

Recami E. and Maccarrone G.D. 1983 Lett. Nuovo Cimento 37, 345.

Recami E. and Mignani R. 1972 Lett. Nuovo Cimento 4, 144.

Recami E. and Mignani R. 1973a Lett. Nuovo Cimento 8, 110.

Recami E. and Mignani R. 1973b Lett. Nuovo Cimento 8, 780.

Recami E. and Mignani R. 1974a Riv. Nuovo Cimento 4, 209 (Erratum: 4, 398).

Recami E. and Mignani R. 1974b Lett. Nuovo Cimento 9, 479.

Recami E. and Mignani R. 1976 Phys. Letters B62, 41.

Recami E. and Mignani R. 1977 The Uncertainty Principle and Foundations of Quantum Mechanics, ed. by W.C.Price and S.S.Chissick (London: J.Wiley), Chapt. 4, p. 21.

Recami E. and Modica E. 1975 Lett. Nuovo Cimento 12, 263.

Recami E. and Rodrigues W.A. 1981 Found. Phys. 12, 709 (plus Erratum).

Recami E. and Shah K.T. 1979 Lett. Nuovo Cimento 24, 115.

Recami E. and Ziino G. 1976 Nuovo Cimento A33, 205.

Recami E., Maccarrone G.D., Nielsen H.B., Corben H.C., Rodonõ M. and Genovesi S. 1976 unpublished work.

Regge T. 1981 Cronache dell'Universo (Torino: Boringhieri), p. 21.

Rhee J.W. 1969 Techn. Report 70-025 (College Park).

Rindler W. 1966 Special Relativity (Edinburgh: Oliver and Boyd).

Rindler W. 1969 Essential Relativity (New York: Van Nostrand Reinhold), Sect. 38.

Robinett L. 1978 Phys. Rev. 18, 3610.

Rolnick W.B. 1969 Phys. Rev. 183, 1105.

Rolnick W.B. 1972 Phys. Rev. D6, 2300.

Rolnick W.B. 1974 Causality and Physical Theories, ed. by W.B.Rolnick (New York), p. 1.

Rolnick W.B. 1979 Phys. Rev. D19, 3811.

Root R.G. and Trefil J.S. 1970 Lett. Nuovo Cimento 3, 412.

Rosen N. 1970 Relativity, ed. by M.Carmeli, S.I.Fickel and L.Witten (New York).

Rosen N. 1962 Ann. of Phys. 19, 165.

Rosen N. and Szamosi G. 1980 Nuovo Cimento B56, 313.

Saavedra I. 1970 Lett. Nuovo Cimento 4, 873.

Sachs M. 1982 General Relativity and Matter (Dordrecht: Reidel).

Sachs R. and Wu W. 1980 General Relativity for Mathematicians (Berlin: Springer).

Sala K.L. 1979 Phys. Rev. A19, 2377.

Salam A. 1978 Proceedings of the XIX Intern. Conf. on High-Energy Physics, Tokyo 1978, p. 937.

Salam A. and Strathdee J. 1978 Phys. Rev. D13, 4596.

Saltzman F. and Saltzman G. 1969 Lett. Nuovo Cimento 1, 859.

Sanders R.H. 1974 Nature 248, 390.

Santilli R.M. 1983 Lett. Nuovo Cimento 37, 545.

Schener P.A.G. and Readhead A.C.S. 1979 Nature 277, 182.

Schillizzi R.T. and de Bruyn A.G. 1983 Nature 303, 26.

Schmidt H. 1958 Zeits. fur Phys. 151, 365, 408.

Schmutzer E. 1968 Relativistische Physik (Leipzig: B.G.Teubner).

Schulman L.S. 1971 Nuovo Cimento B2, 38.

Schwartz C. 1982 Phys. Rev. D25, 356.

Schwinger J. 1966 Phys. Rev. 144, 1084.

Science News 1981 119, 229 (unsigned).

Sen Gupta N.D. 1966 Nuovo Cimento 44, 512.

Severi F. 1955 Cinquant'anni di Relatività, ed. by M.Pantaleo (Florence: Giunti Editrice Universitaria).

Shaffer D.B., Cohen M.H., Jauncey D.L. and Kellermann K.I. 1972 Astrophys. J. Letters 173, L147.

Shah K.T. 1977 Lett. Nuovo Cimento 18, 156.

Shah K.T. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 49.

Shankara T.S. 1974 Found. Phys. 4, 97.

Shankara T.S. 1979 private communication.

Shanks 1980 Gen. Rel. Grav. 12, 1029.

Shapiro I.I., Hinteregger H.F., Knight C.A., Punsky J.J., Robertson D.S., Rogers A.E.E., Whitney A.R., Clark T.A., Marandino G.E. and Goldstein R.M. 1973 Astrophys. J. Letters 183, L47.

Shay D. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 185.

Shay D. and Miller K.L. 1977 Nuovo Cimento A38, 490.

Shay D. and Miller K.L. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 189.

Shenglin C. and Yongzhen L. 1983 Proceedings Third Grossmann Meeting on General Relativity, ed. by H.Ing (Amsterdam: Science Press), p. 1319.

Shenglin C., Xinghua X., Yongznen L. and Zugan D. 1984 Preprint (Peking: Beijing Normal University).

Shin E.E.H. 1966 J. Math. Phys. 7, 174.

Schroer B. 1971 Phys. Rev. 3, 1764.

Sivaram C. and Sinha K.P. 1979 Phys. Reports 51, 111.

Smith H.J. and Hoffeit D. 1963 Nature 198, 650.

Smrz P. 1983 private communications.

Smrz P. 1984 Perspectives of Superluminal Lorentz Transformations, Lett. Nuovo Cimento 41, 327.

Sobczyk G. 1981 Acta Phys. Polonica B12, 407.

Somigliana C. 1922 Rend. Accad. Naz. Lincei (Roma) 31, 53; also appeared in Memorie Scelte (Torino: 1936), p. 469.

Sommerfeld A. 1904 K. Med. Akad. Wet. Amsterdam Proc. 8, 346.

Sommerfeld A. 1905 Nachr. Ges. Wiss. Gottingen, Feb. 25, p. 201.

Souček J. 1979a Preprint (Prague: Czech. Acad. Sc., Math. Inst.).

Souček J. 1979b Czech. J. Phys. B29, 315.

Souček J. 1981 J. Phys. A: Math. Gen. 14, 1629.

Souček J., Janis V. and Souček V. 1981 Reports KMA/1 and KMA/2 (Prague: Mathem. Phys. Faculty).

Srivastava S.K. 1977 J. Math. Phys. 18, 2092.

Srivastava S.K. 1982 J. Math. Phys. 23, 1981.

Srivastava S.K. 1984 J. Math. Phys. 25, 693.

Srivastava S.K. and Patnak M.P. 1977 J. Math. Phys. 18, 483.

Stapp H.P. 1977 Nuovo Cimento B40, 191.

Stephas P. 1983 Nuovo Cimento A75, 1.

Stoyanov D.Tz. and Todorov I.T. J. Math. Phys. 9, 2146.

Streit L. and Klauder J.R. 1971 Tachyon quantization, Preprint (Syracuse University).

Strnad J. 1970 Fortsch. Phys. 18, 237.

Strnad J. 1971 Fizika 10, 217.

Strnad J. 1979a Lett. Nuovo Cimento 25, 73.

Strnad J. 1979b Lett. Nuovo Cimento 26, 535.

Strnad J. 1980 J. Phys. A: Math. Gen. 13, L389.

Strnad J. and Kodre A. 1975a Lett. Nuovo Cimento 12, 261.

Strnad J. and Kodre A. 1975b Phys. Letters A51, 139.

Stuckelberg E.C.G. 1941 Helv. Phys. Acta 14, 321, 588.

Sudarshan E.C.G. 1963 J. Math. Phys. 4, 1029.

Sudarshan E.C.G. 1968 Report NYO-3399-191/SU-1206-191 (Syracuse Univ.: Phys. Dept.).

Sudarshan E.C.G. 1969a Arkiv f. Phys. 39, 585.

Sudarshan E.C.G. 1969b Proc. Ind. Acad. Sci. 69, 133.

Sudarshan E.C.G. 1970a Symposia on Theoretical Physics and Mathematics (New York), Vol. 10, p. 129.

Sudarshan E.C.G. 1970b Phys. Rev. D1, 2473.

Sudarshan E.C.G. 1970c Proceedings of the VIII Nobel Symposium, ed. by N.Swartholm (New York), p. 335.

Sudarshan E.C.G. 1970d Physics of complex mass particles, Report ORO-3992-5 (Austin: Texas University).

Sudarshan E.C.G. 1970e Report CPT-81/AEC-30 (Austin: Texas University).

Sudarshan E.C.G. 1972 Report CPT-166 (Austin: Texas University).

Sudarshan E.C.G. and Mukunda N. 1970 Phys. Rev. D1, 571.

Sum C.P. 1974 Lett. Nuovo Cimento 11, 459.

Szamosi G. and Trevisan D. 1978 Preprint (Windsor Univ., Ont.: Phys. Dept.).

Talukdar B., Sen M and Sen O. 1981 J. Math. Phys. 22, 377.

Tanaka S. 1960 Prog. Theor. Phys. (Kyoto) 24, 171.

Tanaka S. 1979 private communication.

Tangherlini F.R. 1959 Thesis (Stanford Univ.: Phys. Dept.).

Taylor E.F. and Wheeler J.A. 1966 Space-Time Physics (San Francisco: Freeman).

Taylor J.C. 1976 Gauge Theories of Weak Interactions: Cambridge monographs on mathematical physics (Cambridge: Cambridge Univ. Press.), Vol. 3.

Teli M.T. and Sutar V.K. 1978 Lett. Nuovo Cimento 21, 127.

Teli M.T. 1978 Lett. Nuovo Cimento 22, 489.

Teli M.T. and Palaskar D. 1984 Lett. Nuovo Cimento 40, 121.

Terletsky Ya.P. 1960 Doklady Akad. Nauk SSSR 133, 329 (English translation: Sov. Phys. Dokl. 5, 782 (1961).

Terletsky Ya.P. Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 47.

Thankappan V.K. 1977 On waves, particles and superluminal velocities, preprint (Calicut Univ.: Phys. Dept.).

t'Hooft G. 1976 Phys. Rev. Letters 37, 8.

Thomson J.J. 1889 Phil. Mag. 28, 13.

Thoules D.J. 1969 Nature 224, 506.

Tolman R.C. 1917 The Theory of Relativity of Motion (Berkeley, Cal.), p. 54

Tonti E. 1976 Appl. Math. Modelling 1, 37.

Toyoda T. 1973 Prog. Theor. Phys. 49, 707.

Trefil J.S. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 27.

Trigg G.L. 1970 Phys. Today 23(10), 79.

Ugarov V.A. 1979 Special Theory of Relativity (Moscow: Nauka), p. 297-303.

Unwin S.C. et al. 1983 Astrophys. J. 271, 536.

Van Dam H. and Wigner E.P. 1965 Phys. Rev. B138, 1576.

Van Dam H. and Wigner E.P. 1966 Phys. Rev. B142, 838.

Van der Merwe P.T. 1978 Lett. Nuovo Cimento 22, 11.

Van der Spuy E. 1971 Nuovo Cimento A3, 822.

Van der Spuy E. 1973 Phys. Rev. D7, 1106.

Van der Spuy E. 1978 Tachyons, Monopoles, and Related Topics, ed. by E.Recami (Amsterdam: North-Holland), p. 175.

Vaidya P.C. 1971 Curr. Sci. (India) 40, 651.

Velarde G. 1971 Report JEN-252 (Madrid: Junta En. Nucl.), in Spanish.

Vigier J.P. 1979 Lett. Nuovo Cimento 24, 258, 265.

Vigier J.P. 1980 Lett. Nuovo Cimento 29, 476.

Vilela-Mendes R. 1976 Phys. Rev. D14, 600.

Volkov A.B. 1971 Canad. J. Phys. 49, 1697.

Volkov M.K. and Pervushin V.N. 1977 Sov. Phys. Usp. 20, 89.

Voulgaris G.V. 1976 unpublished report (Athens Univ.: Astron. Dept.), in Greek.

Vyšín V. 1977a Nuovo Cimento A40, 113.

Vyšín V. 1977b Nuovo Cimento A40, 125

Vyšín V. 1978 Lett. Nuovo Cimento 22, 76.

Weinberg S. 1972 Gravitation and Cosmology (New York: J.Wiley), p. 415.

Weingarten D. 1973 Ann. of Phys. 76, 510.

Wheeler J.A. 1968 Topics in Nonlinear Physics, ed. by N.J.Zapusky (Berlin).

Wheeler J.A. and Feynman R.P. 1945 Rev. Mod. Phys. 17, 157.

Wheeler J.A. and Feynman R.P. 1949 Rev. Mod. Phys. 21, 425.

White T.H. 1939 The Once and Future King (Berkeley: P.Putnam's Sons Pub.).

Whitney A.R., Shapiro I.I., Rogers A.E.E., Robertson D.S., Knight C.A., Clark T.A., Goldstein R.M., Maramdino G.E. and Vandenberg N.R. 1971 Science 173, 225.

Wigner E.P. 1939 Ann. of Math. 40, 149.

Wigner E.P. 1976 private communication.

Wimmel H.K. 1971a Lett. Nuovo Cimento 2, 363 (Erratum: 2, 674).

Wimmel H.K. 1971b Tachyon Mechanics and Classical Tunnel Effect, Report IIP-6/95 (Garching bei Munchen: Max-Planck Inst. fur Plasma-physik).

Wolf K.B. 1969 Nuclear Phys. B11, 159.

Yaccarini A. 1973 Can. J. Phys. 51, 1304.

Yaccarini A. 1974 Can. J. Phys. 52, 40.

Yaccarini A. 1975 Unified space-time formalism applied to tachyons, Report (Quebec: Univ. Laval).

Yamamoto H. 1969 Prog. Theor. Phys. 42, 707.

Yamamoto H. 1970a Prog. Theor. Phys. 43, 520.

Yamamoto H. 1970b Prog. Theor. Phys. 44, 272.

Yamamoto H. 1976 Prog. Theor. Phys. 55, 1998.

Yamamoto H. and Kudo K. 1975 Prog. Theor. Phys. 53, 275.

Yokoyama K-i. 1972 Prog. Theor. Phys. 47, 352.

Zeldovich Ya.B. 1972 Magic without Magic: J.A.Wheeler, ed. by J.R.Klauder (San Francisco), p. 279.

Zeldovich Ya.B. 1974a Phys. Letters B52, 341.

Zeldovich Ya.B. 1974b Zurn. Eksp. Teor. Fiz. Pisma Red. 20, 338 (English translation in JETP Letters).

Zeldovich Ya.B. and Novikov I.D. 1971 Stars and Relativity (Chicago), p. 93.

Ziino G. 1979 Phys. Letters A70, 87.

Ziino G. 1983 Lett. Nuovo Cimento 37, 188.