ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

Sezione di Catania

INFN/AE-84/6
3 Luglio 1984

A. Italiano, M. Lattuada, G. D. Maccarrone, E. Recami, F. Riggi and D. Vinciguerra: HEAVY MESON MASS-SPECTRA BY GENERAL RELATIVISTIC METHODS

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

Istituto Nazionale di Fisica Nucleare
Sezione di Catania

INFN/AE-84/6
3 Luglio 1984

HEAVY MESON MASS-SPECTRA BY GENERAL RELATIVISTIC METHODS(*)

A. Italiano, M. Lattuada, G. D. Maccarrone, E. Recami, F. Riggi and D. Vinciguerra
INFN - Sezione di Catania, and Dipartimento di Fisica dell'Università di Catania

ABSTRACT

By applying the classical methods of General Relativity to elementary particles, one can get - in a natural way - the observed confinement of their constituents, avoiding any recourse to phenomenological models such as the bag model and allowing the deduction of the heavy meson [i. e. charmonium (J/ψ) and bottomonium (Υ)] mass-spectra.

1. - INTRODUCTION

It has been recently put forth a classical approach to the unification of the gravitational and strong interactions[1,2], according to the old idea by Riemann (and later Clifford) that the appearance of the elementary particles of matter is due to a strong, local space-curvature. It is also known that, avoiding recourse to phenomenological approaches such as the Bag model, this theory allows to explain some hadron phenomenology, particularly the so-called infrared slavery and the asymptotic freedom of the constituents.

(*) Work supported in part by M. P. I. and C. N. R.

The results of such an attempt are often similar to other Authors'[3], even if the starting points are very different.

From the empirical observation that the ratio r/R between the typical hadron radius $r \simeq 10^{-15}$ m and the Hubble radius $R \simeq 10^{26}$ m of our (gravitational) cosmos roughly equals the ratio s/S between the strengths of the gravitational field and the strengths of the nuclear field, one can - heuristically - infer that our gravitational macrocosmos and hadrons (conceived as strong micro-universes) could be systems physically similar.

To start with, let us regard cosmos and hadrons as being Newtonian balls; latter on we shall adopt Friedmann models, consistently with General Relativity.

As shown in a more detailed way in Refs. (1, 2), we assume cosmos and hadrons - both regarded as finite objects - to be systems governed by laws similar, and differing only for a global scale-transformation which carries R into r and the gravitational into the strong field. We shall take advantage of the fact that Einstein equations do not contain any inbuilt fundamental length, so that they can describe the space-time of cosmoses with any size.

Let us require that physical laws are covariant (besides under other transformations) also under the global space-time dilations

$$x'_\mu = \varrho x_\mu \qquad (\mu = 0, 1, 2, 3) \tag{1}$$

where ϱ may assume only a set of discrete values[4]. In particular, we choose

$$\varrho = \frac{r}{R} \approx \frac{s}{S} = \frac{Gm^2/\hbar c}{Ng^2/\hbar c} \approx 10^{-41} \tag{2}$$

where: (i) G and N are the gravitational and "strong" universal constants in vacuum, respectively; (ii) quantities m and g represent gravitational charge (= mass) and strong charge (or "strong mass"), respectively, of one and the same hadron (e. g. , nucleon or pion). We may choose units[1] such that m and g gets identified; and then $N = \varrho^{-1} G$.

Aim of this paper is showing some new results of the theory, mainly about the mass-spectra of heavy mesons.

2. - INSIDE A HADRON

Requiring the covariance of physical laws under dilations, eq. (1), we may assume:

a) Inside our (gravitational) "macro-cosmos", the Einstein equations with (attractive) cosmological term(5):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} = - \frac{8\pi G}{c^4} T_{\mu\nu} + g_{\mu\nu} \Lambda ; \tag{3}$$

b) Inside hadrons ("strong" micro-cosmoses), the scaled-down Einstein equations $(N = \varrho^{-1} G)$:

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\varrho}_{\varrho} = - \frac{8\pi N}{c^4} T_{\mu\nu} + \tilde{g}_{\mu\nu} H , \tag{4}$$

where H plays the role of a "scaled-down" cosmological constant.

Dimensional considerations, within the present "dilation-covariance" relativity, show that H is related to the ordinary cosmological constant Λ by:

$$H = \varrho^{-2} \Lambda .$$

Eqs. (4) admit, for a spherically symmetric strong field created by a strong charge g' (which may, e. g. , be identified with a quark), the known Schwarzschild solution. Thus for the solution of the "scaled-down" Einstein equations with cosmological term, one gets

$$ds^2 = \left(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3}\right)dt^2 - \left(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \tag{5}$$

wherefrom one can evaluate all non-zero Christoffel coefficients:

$$\Gamma^1_{oo} = \frac{1}{2}\left(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3}\right)\left(\frac{2Ng'}{c^2 r^2} + \frac{2Hr}{3}\right) ,$$

$$\Gamma^1_{11} = - \frac{1}{2}\left(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3}\right)^{-1}\left(\frac{2Ng'}{c^2 r^2} + \frac{2Hr}{3}\right) ,$$

$$\Gamma^1_{22} = - r\left(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3}\right) , \tag{6}$$

$$\Gamma^1_{33} = - r \sin^2\theta \left(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3}\right) ,$$

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{2}\left(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3}\right)\left(\frac{2Ng'}{c^2 r^2} + \frac{2Hr}{3}\right) ,$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r} \, ,$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta \, , \tag{6}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \, ,$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \mathrm{cotg}\,\theta \, .$$

If we confine ourselves to the radial motion of a <u>test</u>-constituent of strong charge g'' in the field described by the metric (5), we get the geodesic equation (in the vacuum) for the radial case :

$$\frac{d^2 r}{ds^2} = -\Gamma^1_{\mu\nu}\, u^\mu u^\nu = -\frac{1}{2}\Big[(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3})(\frac{2Ng'}{c^2 r^2} + \frac{2Hr}{3})\, u^o u^o -$$

$$- (1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3})^{-1}(\frac{2Ng'}{c^2 r^2} + \frac{2Hr}{3})\, u^1 u^1 - \tag{7}$$

$$- 2r(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3})\, u^2 u^2 + 2r\sin^2\theta\,(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3})\, u^3 u^3$$

which in the static limit ($v \ll c$) becomes

$$\frac{d^2 r}{dt^2} = -\frac{1}{2}c^2(1 - \frac{2Ng'}{c^2 r} + \frac{Hr^2}{3})(\frac{2Ng'}{c^2 r^2} + \frac{2Hr}{3}) \, . \tag{8}$$

3. - ABOUT QUARK CONFINEMENT

For large values of r, from eq.(8) one gets a confining radial force propor tional to $-r$:

$$F \approx -\frac{g'' c^2}{3} Hr \, . \tag{9}$$

Putting, in eq.(9), quantity $g'' = \frac{1}{3} m_p \simeq \frac{1.67}{3} \times 10^{-27}$ Kg; $H = 10^{30}\ m^{-2}$; $r = 1$ fm, one immediately gets

$$F \approx 16.7 \text{ ton}; \tag{10}$$

this value is consistent with previous heuristical analyses of the experimental data(6).

4. - THE QUARK-QUARK POTENTIAL $V_{q\bar{q}}$ AND THE DETERMINATION OF THE MESON MASS-SPECTRA

Let us pass now to the case when the test-constituent g" is actually a quark, as well as the source-constituent g'; of course, in such a case what precedes can be applied only approximately.

By integrating[7] eq. (8) after having multiplied it by the strong charge g", we get the quark-quark potential inside a hadron:

$$V_{q\bar{q}}(r) \simeq g''\frac{c^2}{2}\left[-\frac{2Ng'}{c^2 r} - \frac{2g'HN}{3c^2}r + \frac{2g'^2N^2}{c^4 r^2} + \frac{Hr^2}{3} + \frac{1}{2}\left(\frac{Hr^2}{3}\right)^2\right] , \tag{11}$$

To get our main results, let us borrow from the previous approach the expression of the potential, eq. (11), and insert it into the Schrödinger equation. We know that for the particular case of two particles interacting through a potential, depending on the distance r between the quarks, the time-independent Schrödinger equation takes the form

$$\left\{-\frac{\hbar^2}{2(g'+g'')}\nabla^2_{CM} - \frac{\hbar^2}{2g}\nabla^2 + \left[V_{q\bar{q}}(r) - E\right]\right\}\psi = 0 \tag{12}$$

where ∇^2_{CM} depends on the coordinates of the center of mass, and ∇^2 on the relative coordinates. By writing, as usual, the total wave function as the product

$$\psi = \psi_{CM}\,\psi_{rel} \tag{13}$$

eq. (12) yields, in particular:

$$\left(-\frac{h^2}{2g}\nabla^2 + V_{q\bar{q}}(r) - E_{rel}\right)\psi_{rel} = 0 \tag{14}$$

where $g = g' = g''$.

By writing eq. (14) in spherical polar coordinates and splitting, as costumary, the wave function into a product of functions of a single variable, we get the radial Schrödinger equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}R\right) + \frac{2g}{\hbar}\left[E + V^J_{q\bar{q}}(r)\right]R = 0 \tag{15}$$

where

$$V^J_{q\bar{q}}(r) = V_{q\bar{q}}(r) + \frac{J(J+1)\hbar^2}{2g^2r^2} \tag{16}$$

represents the complete form of the potential energy.

Since we derived an equation valid in the non-relativistic approximation, we are mostly concerned with the mass-spectra of heavy mesons, such as the charmonium (J/ψ) and the bottomonium (Υ).

In the case of the bottom quark, for example, with mass

$$g_b \simeq 5.2\ \mathrm{GeV}/c^2,$$

we should obtain the bottomonium spectrum.

Two plots of $V_{c\bar{c}}(r)$ and $V_{b\bar{b}}(r)$, that have been preliminary evaluated and used, are shown in Fig. 1 and Fig. 2, respectively.

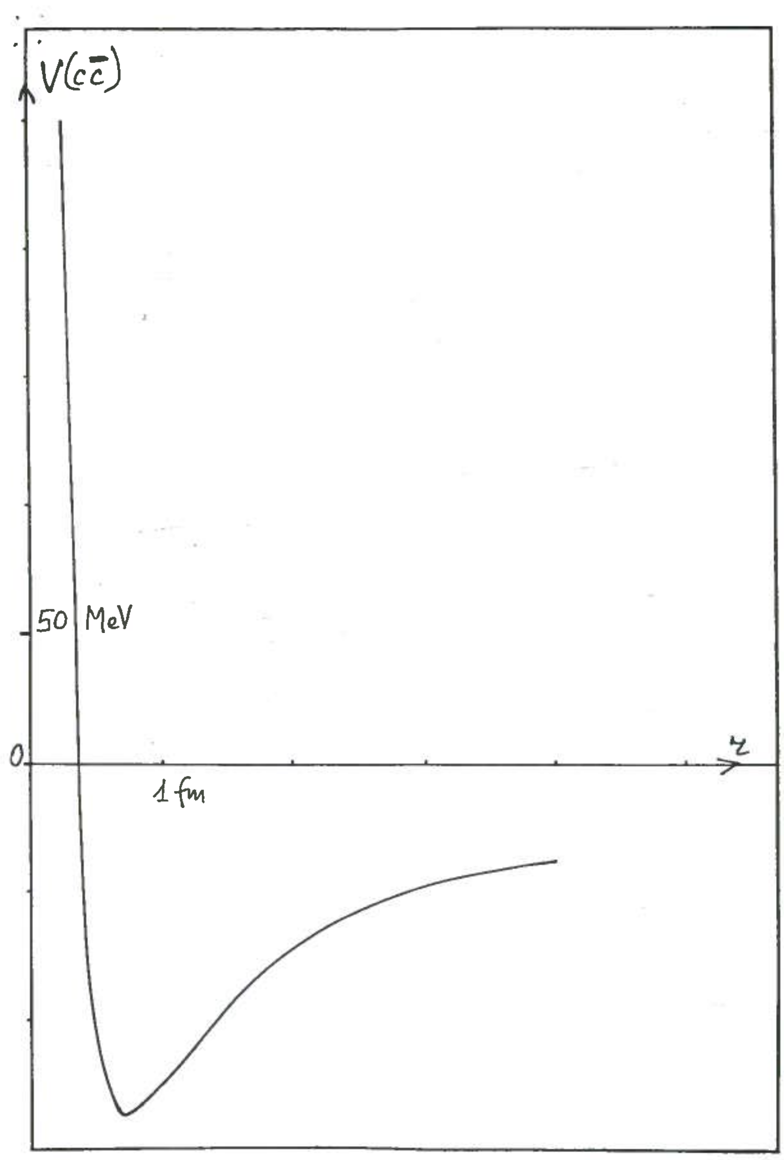

FIG. 1 - Potential energy $V_{c\bar{c}}(r)$ of the bound-system of a quark c and an antiquark $\bar{c}$ (charmonium), preliminarly derived from eq. (11).

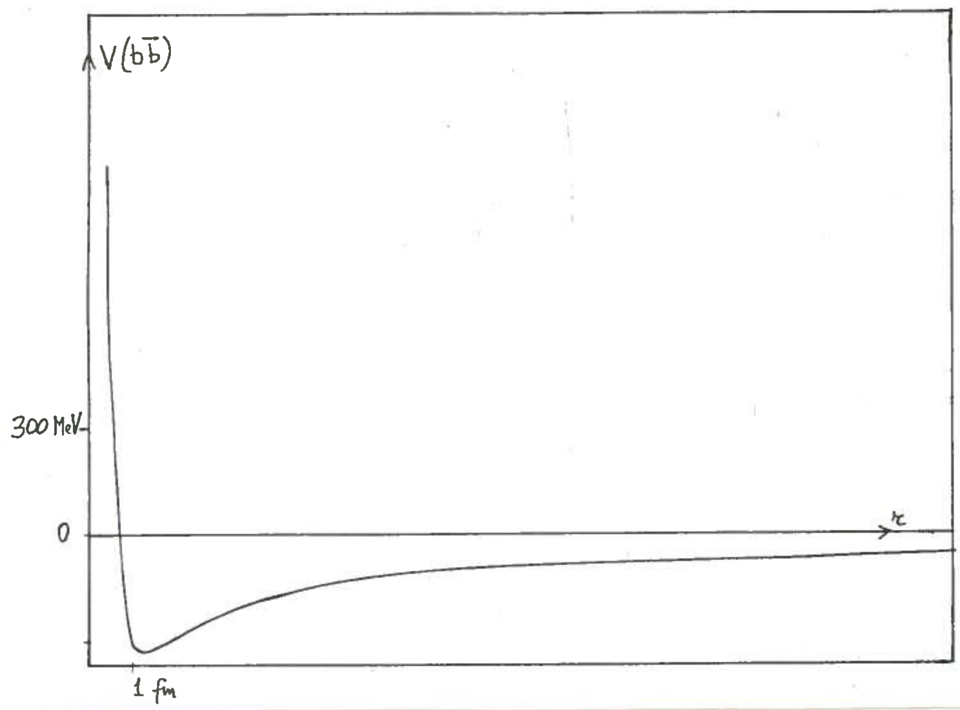

FIG. 2 - Potential energy $V_{b\bar{b}}(r)$ of the bound-system of a quark bottom b and its antiquark $\bar{b}$, preliminarly derived from eq. (11).

For instance, by solving the Schrödinger equation (16), with $V_{b\bar{b}}(r)$, one gets for the energy levels:

THEORETICAL (masses in MeV)	EXPERIMENTAL[8] (masses in MeV)
9992	10016
9862	9887
9474	9460

which are satisfactorily close to each other.

These preliminary results seem to be promising, and encourage us to go on with computer investigations. Further results will be reported elsewhere.

The authors acknowledge many helpful discussions with Professors M. Pavšič, P. K. Smrz and C. Spitaleri.

APPENDIX - The action for the "strong gravity" field

In analogy with General Relativity - and mainly for didactical purposes - let us derive the strong field equations (i. e., the "scaled-down" Einstein equations) from a properly chosen Action.

We can write the Action for the strong field as follows:

$$S_{sf} = \int (\tilde{G} - 2H) \sqrt{-\tilde{g}}\, d\Omega \quad , \tag{17}$$

which, by using the Gauss theorem, can be put[9] in such a form that its variation becomes:

$$\delta S_{sf} = - \frac{c^3}{16\pi N} \delta \int (\tilde{G} - 2H) d\Omega = - \frac{c^3}{16\pi N} \delta \int (\tilde{R} - 2H) \sqrt{-\tilde{g}}\, d\Omega \,. \tag{18}$$

We want to underline that our Action differs from the one usually adopted in General Relativity for the sign of the cosmological constant which, according to our point of view, has to describe the gravitational attraction among different portions of space due to the vacuum polarization properties (i. e. due to the vacuum emissions and absorptions of particle-antiparticle pairs)[10].

At this point, let us add to the Action - when in presence of strong field sources - also the term S_{sm} corresponding to the strong mass distribution[9]

$$S_{sm} = - \frac{1}{cN} \int A \sqrt{-\tilde{g}}\, d\Omega \,,$$

wherefrom:

$$\delta S_{sm} = \frac{1}{2cN} \int S_{\mu\nu} \delta \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}}\, d\Omega \;; \qquad (S_{\mu\nu} \equiv NT_{\mu\nu})$$

where $S_{\mu\nu}$ represents the "energy-momentum tensor" of the hadronic matter.

From the total Action, by following the standard procedures[9], one gets

$$- \frac{c^3}{16\pi N} \int (\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\sigma}_{\sigma} - H\tilde{g}_{\mu\nu} + \frac{8\pi}{c^4} S_{\mu\nu}) \delta \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}}\, d\Omega = 0;$$

wherefrom, since the $\delta\tilde{g}^{\mu\nu}$ are arbitrary, one finally derives the field equations ($S_{\mu\nu} \equiv NT_{\mu\nu} \equiv \varrho^{-1} GT_{\mu\nu}$):

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\sigma}_{\sigma} = - \frac{8\pi}{c^4} S_{\mu\nu} + H\tilde{g}_{\mu\nu} \tag{19}$$

which have to hold inside hadrons.

REFERENCES

(1) - E. Recami, in "Quarks and the Nucleus, vol. 8 of Progress in Particle and Nuclear Physics", ed. by D. Wilkinson (Pergamon, Oxford, 1982), p. 401; E. Recami, in "Old and New Questions in Physics, Cosmology, ... : Essay in Honour of W. Yourgrau", ed. by A. van der Merve (Plenum, New York, 1982), Chap. 24, p. 377; E. Recami, Found. of Physics 13, 341 (1983); P. Ammiraju, E. Recami and W. A. Rodrigues, Nuovo Cimento A78, 172 (1983).

(2) - E. Recami and P. Castorina, Lett. Nuovo Cimento 15, 347 (1976); P. Caldirola, M. Pavšič and E. Recami, Nuovo Cimento B48, 205 (1978); Phys. Letters A66, 9 (1978); P. Caldirola and E. Recami, Lett. Nuovo Cimento 24, 565 (1979); E. Recami, in "Relativity, Quanta and Cosmology in the Development of the Scientific Thought of A. Einstein", ed. by F. de Finis and M. Pantaleo (Johnson Rep. Co., New York, 1979), vol. 2, chap. 16, p. 581.

(3) - See e. g. A. Salam, in "Proceedings 19th Intern. Conf. on High-Energy Physics (Tokyo, 1978)", p. 937; A. Salam and J. Strathdee, Phys. Rev. D18, 4596 (1978); C. Sivaram and K. P. Sinha, Phys. Reports 51, 111 (1979).

(4) - Cf. also M. Pavšič, Lett. Nuovo Cimento 17, 44 (1976); P. Caldirola, Lett. Nuovo Cimento 18, 465 (1977); Nuovo Cimento A45, 548 (1978).

(5) - Cf. e. g. M. A. Markov, Zh. Eksp. Teor. Fiz. 51, 878 (1966); J. E. Charon, Compt. Rend. A. S. P. A266, 750 (1968); J. E. Charon, "Theorie de la relativité complexe" (A. Michel, Paris, 1977); D. D. Ivanenko, in "Astrofisica e Cosmologia, Gravitazione, Quanti e Relatività ... ", ed. by M. Pantaleo and F. de Finis (Giunti-Barbera, Firenze, 1979), p. 131; P. Roman and J. Haavisto, Int. Journ. Theor. Phys. 16, 915 (1978).

(6) - R. Arthur, Rev. Mod. Phys. 53, (1981); C. Edwards et al., Phys. Rev. Letters 48, (1982).

(7) - A. Italiano, Applicazione dei metodi della Relatività Generale alla fisica degli adroni (thesis work, under the supervision of E. Recami), Catania University, 1983, unpublished.

(8) - Cf. e. g. CERN Courier, June 1983, p. 185.

(9) - L. D. Landau and E. M. Lifshitz, Teoria dei campi (Editori Riuniti, MIR, Moscow, 1976).

(10) - See e. g. C. Sivaram and K. P. Sinha, Phys. Reports 51, 111 (1979), and references therein.